Optimal crop protection against climate risk in a dynamic cost-loss decision-making model

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ABSTRACT

Extreme meteorological events have increased over the last decade and it is widely accepted that it is due to climate change. Some of these extremes like drought or frost episodes largely affect agricultural outputs that could face a serious decline. Therefore, additional efforts on technology and adaptation become crucial to reduce the effect of climate change on agricultural production. Protection against adverse weather extremes is a significant component of risk management and large protection decisions need dynamic treatment, such as the vineyards or citrus protection against frost (Murphy et al., 1985), the water management for cereals irrigation in drought shortage contexts, the "options and futures" exchange markets decisions for major crops in USA, and many other risk management problems. The goal of this paper is to obtain the analytical expressions for the optimal actions and minimal expected expenses associated with a dynamic cost-loss model with finite horizon. The optimal policy expressions for this type of situations can help in the management of climate risk in agriculture in a more general approach than numerical solutions.

Key words: information value, cost-loss ratio, dynamic decision models
Classification JEL : C6 and Q1.

1 Introduction

Agriculture sector is the most vulnerable to the extreme meteorological events. The possible increase in water shortage and extreme weather events may cause lower harvestable yields and higher yield variability. Agriculture in European Union accounts for only 2% of the total GDP, but accounts for 5.6% of total employment. Even if the vulnerability of the overall economy to changes that affect agriculture is therefore low, locally effects may be large (Olesen and Bindi, 2002).

Vulnerability has increased over the last decade as one of the climate change consequences. (IPCC, 2007). There is now concern that global warming has the potential for affecting the climatic regimes of entire regions and increases the extreme meteorological events (IPCC, 2007). Many studies document the repercussions for agriculture, and show that although significant variations exits between different regions and different scales (IPCC, 2007; Olesen and Bindi, 2002; Parry et al., 2004) the implications are substantial. As well, demographic changes are altering vulnerability to water shortages and agricultural production can face serious consequences at local and regional levels.

However, the resulting effects depend on the direction of change and the availability of resources and infrastructure to cope with change. (Olesen and Bindi, 2002). Some of the climate risks, --e.g. drought or frost episodes that largely affect agricultural outputs (Iglesias and Quiroga, 2007)-- can be managed by farmers across protection systems or some kind of insurance instrument. Therefore, it is important to enhance climate management tools in agriculture.

Major agricultural decisions involve more than a period, the decision making process being dynamic in nature. Management of weather-sensitive enterprises frequently requires that decisions be made in the face of uncertainty regarding future weather events and for main situations the opportunities for and constraints on future decisions depend on previous decisions and weather events. (Wilks, 1991).

Therefore, dynamic treatment is essential to risk management problems. When farmers decide about protecting harvest from a meteorological risk, do not obtain immediate results, since the decision taken over every period influences total results. To take no protective action in a given period is enough to loss the overall harvest if the weather is “adverse” (that is, some meteorological event able to destroy the harvest if it has not been protected). For example, frost protection is just one phase of an orchard management process, but the decisions made relative to it have a direct and significant economic effect on orchard production since, if the crop is lost to frost, the decisions pertinent to orchard operation for the rest of the season may have limited or no effect on current year production. (Baquet et al. 1976).

Classic agricultural decision-making problems include the vineyards or citrus protection against froze (Katz and Murphy, 1997; Murphy et al., 1985; Katz et al., 1982; Baquet et al.,1976), the water management for cereals irrigation in drought shortages contexts, the “options and futures” exchange markets decisions for major crops in USA, and much more risk management decisions.

Most of these problems are dynamic in nature (e.g., Katz 1993, Brown et al, 1986; Mjelde et al. 1988), because the action taken by a decision maker on the current occasion does have an effect on any actions to be taken on future occasions. Therefore, it is necessary to deal with all the periods as a whole and consider the decision as dynamic, so this creates a decision-making problem that is inherently more complex. As widely analyzed, in general, the optimal policy in a dynamic context cannot be reached across the sequence or the static optimal decisions in each period involved (Cerdá, 2001).

In this paper, the focus is on the analytical expressions for the optimal policy and minimal expected expenses associated for a dynamic cost-loss model with finite horizon in order to asses management of climate risk in agriculture. A dynamic cost-loss ratio decision-making problem version is formalized in section 2. Analytical results concerning the structure of the optimal policy and the corresponding minimal expected expenses for this model are presented in section 3, including technical derivations. Section 4 presents two case studies and some numerical examples. Finally, some conclusions are discussed.

2 The model

The cost-loss ratio situation is a decision-making problem widely analyzed in the literature in assessing the economic value of weather forecast (e.g., Murphy 1977; Murphy et al, 1985, Murphy and Ehrendorfer, 1987; Katz, 1993; Palmer, 2002; Katz and Ehrendorfer, 2006). The model involves two possible actions, protect $(\alpha = 1)$, and not protect $(\alpha = 0)$, and two possible events, adverse weather ($\theta = 1$) and no adverse weather ($\theta = 0$). The decision maker is assumed to incur a cost C>0 if protective action is taken, a loss L>0 if protective action is not taken and adverse weather occurs, and no cost or loss otherwise.

The model is a representation of a sequence of related decisions that must be made in the face of uncertainty regarding future weather events, which will affect the consequence of the decisions. We consider the finite horizon dynamic version of this problem (Murphy et al, 1985) where the risk of losing the harvest remains in a set of N periods (the length of the decision sequence is arbitrary but finite), and the farmer has to manage this risk making a decision every period between protecting or not protecting the crop.

When a protective action is taken, the cost incurred is γL_i, where $0 < \gamma < 1$ y L_i is the value of the harvest in the period i (that can be $L_i = L$ if the harvest has suffered no damage until the period i, or $L_i = 0$ if has been destroyed before the period i).

The state of weather for every period can be “adverse weather”, with $\theta_i = 1$, or “non adverse weather”, with $\theta_i = 0$. In every period, the farmer incurs into the following additional cost: If there is “adverse weather”, the cost reaches the total value of the harvest in period i (L_i) if he did not protect the crop for this period, or equals zero if the crop has been protected. This can be written as a cost $(1-\alpha_i)L_i$, where $\alpha_i = 1$ if some protective action is taking in i, and $\alpha_i = 0$ in other case. Therefore, the accumulated expense to the farmer will be the cumulative sum of the costs, which can be calculated for the entire N periods as: $\sum_{i=1}^{N} \gamma \alpha_i L_i + \theta_i [L_i - \alpha_i L_i]$. The structure is “dynamic” in the sense that decisions and events at a particular point in the sequence will influence future decisions and their consequences (Wilks, 1991).

Once the harvest is lost, there is no possibility to recover it, so if the weather is adverse in a period ($\theta_i = 1$), the part of the harvest remaining for the following period i+1, (that is, the quantity which is

possible to loss in the case of not protection), will be $\alpha_i L_i$. That is because on one hand, if the farmer protects the harvest over the period i ($\alpha_i = 1$), no loss is incurred and the value in i+1 is the same L_i, and on the other hand, if he does not protect the harvest over the period i ($\alpha_i = 0$), the harvest is totally damaged and the value remaining for the period i+1 is zero.
However, if the weather is "non adverse" in the period i, $\theta_i = 0$, the harvest value for the period i+1 is the same that in the period i, that is: L_i.

So, the state of the harvest value in the i+1 period (and the maximum loss for the farmer in this period) can be expressed as a function of the state of nature in the last period (i), being:
$L_{i+1} = L_i - \theta_i(1-\alpha_i)L_i \qquad i = 1,...,N$. The goal of the analysis is to identify sequences of decisions that are optimal, in the sense that a measure of overall performance (such as total expected monetary expense or gain) is minimized or maximized over the full sequence of decisions considered.

So, a farmer who minimizes the expected expense (that implies neutral risk aversion), should solve:

$$\begin{aligned} &\underset{\{\alpha_i\}_1^N}{Min}\, E\left\{\sum_{i=1}^{N} \gamma\alpha_i L_i + \theta_i[L_i - \alpha_i L_i]\right\} \\ &s.a. \quad L_{i+1} = L_i - \theta_i(1-\alpha_i)L_i \qquad i = 1,...,N \\ &\qquad\; L_1 = L \end{aligned} \tag{1}$$

where θ_i is a random variable that in the i-period takes the following values $\begin{cases} 1, & \text{with prob } P_\theta \\ 0, & \text{with prob } 1\text{-}P_\theta. \end{cases}$

We assume that the random variables $\theta_1,...,\theta_N$ are independent.

$\alpha_i \in \{0,1\} \qquad i = 1,...,N$

Figure 1 shows the time structure of the dynamic decision problem. The farmer begins a period i with a harvest value L_i, so that is the maximum loose value if the weather is "adverse" and he takes no protective action for this period.

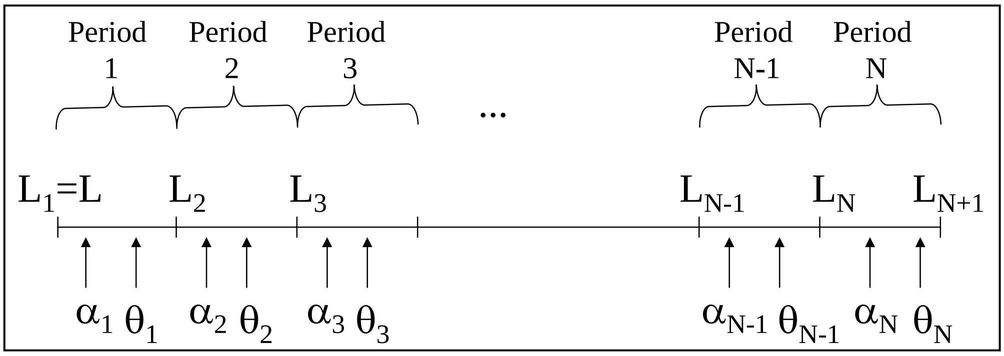


Figure 1 Decision problem temporal structure

So farmers must decide between protect or not protect (α_i), and of course the decision has to be taken before the value taken by the weather random variable (θ_i) is known. However, climatological (or prior) information is available, consisting of probability of adverse weather, $\mathrm{P}_\theta = \Pr\{\theta = 1\}$ based on statistical or historical information.

3 Results

3.1 Analytical results

Proposition 1: ***Optimal policy for the dynamic cost-loss problem with climatological information and finite horizon (N periods).*** The following optimal solution is obtained to problem (1), for K=1,2,...,N:

A) If $N > K^* = N - \frac{P_\theta - \gamma}{\gamma P_\theta}$ (that occurs if and only if $\gamma < P_\theta$): The optimal policy would be not protect until period K^* and protect from this period to the end. The optimal control and the optimal value function are:

i) If $K > K^* = N - \frac{P_\theta - \gamma}{\gamma P_\theta}$: the optimal control would be: $\alpha_K = 1$ (to protect in K)

In addition, the optimal value function: $J_K^* \{L_K\} = [N - K + 1]\gamma L_K = \sum_{i=K}^{N} \gamma L_K$

ii) If $K < K^* = N - \frac{P_\theta - \gamma}{\gamma P_\theta}$: the optimal control would be: $\alpha_K = 0$ (not protect in the K period) and the optimal value function:

$$J_K^* \{L_K\} = E\left[\theta_K L_K + \left[\sum_{i=K+1}^{[K^*]} \theta_i \prod_{j=K}^{i-1} (1-\theta_j)\right] L_K + \sum_{i=[K^*]+1}^{N} \gamma L_K \prod_{j=K}^{[K^*]} (1-\theta_j)\right] =$$

$$= P_\theta L_K + (1-P_\theta) L_K \left[1-(1-P_\theta)^{[K^*]-K}\right] + \left[N - [K^*]\right]\gamma (1-P_\theta)^{[K^*]-K+1} L_K$$

iii) If $K = K^* = N - \frac{P_\theta - \gamma}{\gamma P_\theta}$, the agent is indifferent between protecting or not.

B) If $N \le K^* = N - \frac{P_\theta - \gamma}{\gamma P_\theta}$ (if and only if $\gamma \ge P_\theta$): Optimal control would be: $\alpha_K = 0 \quad \forall K = 1,...,N$, that is to protect in none of the periods, and the optimal value function would be:

$$J_K^* \{L_K\} = E\left[\theta_K L_K + \left[\sum_{i=K+1}^{N} \theta_i \prod_{j=K}^{i-1} (1-\theta_j)\right] L_K\right] = P_\theta L_K + (1-P_\theta) L_K \left[1-(1-P_\theta)^{N-K}\right]$$

Proof: Using the Induction over K method, we start the analysis on the final period.

Final period: Being L_{N+1} determined, with no contribution to the objective function: $J^*_{N+1}\left\{L_{N+1}\right\}=0$

N period (with L_N determined): Bellman equation corresponding this period is:

$J^*_N\left\{L_N\right\}=\underset{\alpha_N\in\{0,1\}}{Min}\,E\left[\gamma\alpha_N L_N+\theta_N[L_N-\alpha_N L_N]+J^*_{N+1}\left\{L_{N+1}\right\}\right]$, which calculating the expectation becomes: $J^*_N\left\{L_N\right\}=\underset{\alpha_N\in\{0,1\}}{Min}\,\gamma\alpha_N L_N+P_\theta[L_N-\alpha_N L_N]$.

The solution to this problem is:

If $\gamma<P_\theta$, (in which case we have: $N>K^*=N-\dfrac{P_\theta-\gamma}{\gamma P_\theta}$), the optimal decision for the N period is to protect, so: $\alpha_N=1$. In this case, the accumulated expenses would be: $J^*_N\left\{L_N\right\}=\gamma L_N$.

If $\gamma>P_\theta$, (in which case we have: $N<K^*=N-\dfrac{P_\theta-\gamma}{\gamma P_\theta}$), the better decision for the N period is not to protect, then: $\alpha_N=0$, and the accumulated expenses are: $J^*_N\left\{L_N\right\}=E\left[\theta_N L_N\right]=P_\theta L_N$.

If $\gamma=P_\theta$, (so we have: $N=K^*=N-\dfrac{P_\theta-\gamma}{\gamma P_\theta}$), the agent is indifferent between the both possibilities.

So for the N period the Proposition 1 is satisfied.

Induction hypothesis: We consider the $K+1$ period where L_{K+1} has already been determined. We assume that Proposition 1 is certain for $K+1$ (following the induction hypothesis) and prove that in that case the proposition is also true for the K period.

A) If we consider $N>K^*=N-\dfrac{P_\theta-\gamma}{\gamma P_\theta}$ (which it is true if and only if $\gamma<P_\theta$):

i) Assuming that $K+1>K^*=N-\dfrac{P_\theta-\gamma}{\gamma P_\theta}$, then, in $K+1$, the proposition 1 implies that the agent chooses to protect the harvest ($\alpha_{K+1}=1$), being the optimal value function:

$$J^*_{K+1}\left\{L_{K+1}\right\}=\sum_{i=K+1}^{N}\gamma L_{K+1}=\sum_{i=K+1}^{N}\gamma\left[1-\theta_K\left(1-\alpha_K\right)\right]L_K=\left[N-K\right]\gamma\left[1-\theta_K\left(1-\alpha_K\right)\right]L_K\quad.$$

Given this, in the K period (with L_K determined), Bellman equation corresponding the K period would be: $J^*_K\left\{L_K\right\}=\underset{\alpha_K\in\{0,1\}}{Min}\,E\left[\gamma\alpha_K L_K+\theta_K\left(1-\alpha_K\right)L_K+J^*_{K+1}\left\{L_{K+1}\right\}\right]=$

$= \underset{\alpha_K \in \{0,1\}}{Min} E\left[\gamma\alpha_K L_K + \theta_K\left(1-\alpha_K\right)L_K + \left[N-K\right]\gamma L_K\left[1-\theta_K\left(1-\alpha_K\right)\right]\right]$. Calculating the expectation becomes:

$J_K^*\left\{L_K\right\} = \underset{\alpha_K \in \{0,1\}}{Min} \left[\gamma\alpha_K L_K + P_\theta\left(1-\alpha_K\right)L_K + \left[N-K\right]\gamma L_K\left[1-P_\theta\left(1-\alpha_K\right)\right]\right]$. So:

If $\alpha_K = 0 \Rightarrow$ expected expense from *K* to *N* period would be: $P_\theta L_K + \left[N-K\right]\gamma L_K\left(1-P_\theta\right)$.

If $\alpha_K = 1 \Rightarrow$ expected expense from *K* to *N* period would be: $\gamma L_K + \left[N-K\right]\gamma L_K$.

Therefore, in this case, the optimal solution is:

$$\alpha_K = 1 \Leftrightarrow \gamma \cancel{L_K} + \left[N-K\right]\gamma \cancel{L_K} < P_\theta \cancel{L_K} + \left[N-K\right]\gamma \cancel{L_K}\left(1-P_\theta\right) \Leftrightarrow$$

$$\gamma < P_\theta - P_\theta\left[N-K\right]\gamma \Leftrightarrow \gamma + P_\theta\left[N-K\right]\gamma < P_\theta \Leftrightarrow \left[1+P_\theta\left[N-K\right]\right]\gamma < P_\theta \Leftrightarrow \gamma < \frac{P_\theta}{1+P_\theta\left[N-K\right]},$$

And this is always satisfied since $K > N - \frac{P_\theta - \gamma}{P_\theta \gamma} = K^*$. In addition, the optimal value function would be:

$J_K^*\left\{L_K\right\} = \gamma L_K + \left[N-K\right]\gamma L_K = \left[N-K+1\right]\gamma L_K = \sum_{i=K}^{N}\gamma L_K$ as we wanted to prove.

However, another possibility is that $K \le K^* < K+1$. In that case, we have: $\gamma > \frac{P_\theta}{1+P_\theta\left[N-K\right]}$, moreover, the optimal solution is $\alpha_K = 0$, and the optimal value function equals:

$J_K^*\left\{L_K\right\} = P_\theta L_K + \left[N-K\right]\gamma L_K\left(1-P_\theta\right)$.

While $K+1 > K^*$, it implies $\left[K^*\right] = K$, so Proposition 1 is satisfied.

ii) If we assume $K+1 \le K^* = N - \frac{P_\theta - \gamma}{\gamma P_\theta}$, then, in the $K+1$ period, as Proposition 1 is satisfied (in accordance with the induction hypothesis), the agent would chose not to protect the harvest ($\alpha_{K+1} = 0$) (or would be indifferent if $K+1 = K^*$). Therefore, the optimal value function can be calculated as:

$$J_{K+1}^*\left\{L_{K+1}\right\} = E\left[\theta_{K+1}L_{K+1} + \left[\sum_{i=K+2}^{\left[K^*\right]}\theta_i\prod_{j=K+1}^{i-1}\left(1-\theta_j\right)\right]L_{K+1} + \sum_{i=\left[K^*\right]+1}^{N}\gamma L_{K+1}\prod_{j=K+1}^{\left[K^*\right]}\left(1-\theta_j\right)\right].$$

We have to prove that Proposition 1 is sustained for the K period (with L_K determined). For this period, Bellman equation is: $J_K^*\left\{L_K\right\} = \underset{\alpha_K \in \{0,1\}}{Min} E\left[\gamma\alpha_K L_K + \theta_K\left(1-\alpha_K\right)L_K + J_{K+1}^*\left\{L_{K+1}\right\}\right] =$

$$= \underset{\alpha_K \in \{0,1\}}{Min} E\Big[\gamma\alpha_K L_K + \theta_K\left(1-\alpha_K\right)L_K + \theta_{K+1}L_{K+1} + \left[\sum_{i=K+2}^{\left[K^*\right]}\theta_i \prod_{j=K+1}^{i-1}\left(1-\theta_j\right)\right]L_{K+1} +$$

$$+\left[N-\left[K^*\right]\right]\gamma L_{K+1}\prod_{i=K+1}^{\left[K^*\right]}\left(1-\theta_i\right)\Big].$$

This can be written as a function of L_K if we take into account the state equation:

$$J_K^*\left\{L_K\right\} = \underset{\alpha_K \in \{0,1\}}{Min} E\Big[\gamma\alpha_K L_K + \theta_K\left(1-\alpha_K\right)L_K + \theta_{K+1}\left(1-\theta_K\left(1-\alpha_K\right)\right)L_K +$$

$$+\left[\sum_{i=K+2}^{\left[K^*\right]}\theta_i \prod_{j=K+1}^{i-1}\left(1-\theta_j\right)\right]\left(1-\theta_K\left(1-\alpha_K\right)\right)L_K + \left[N-\left[K^*\right]\right]\gamma\left(1-\theta_K\left(1-\alpha_K\right)\right)L_K\prod_{i=K+1}^{\left[K^*\right]}\left(1-\theta_i\right)\Big].$$

We calculate the expectation with respect $\left\{\theta_i\right\}$ distribution function assuming that $\left\{\theta_i\right\}$ are random variables independently distributed:

$$E\Big[\gamma\alpha_K L_K + \theta_K\left(1-\alpha_K\right)L_K + \theta_{K+1}\left(1-\theta_K\left(1-\alpha_K\right)\right)L_K + \left[\sum_{i=K+2}^{\left[K^*\right]}\theta_i \prod_{j=K+1}^{i-1}\left(1-\theta_j\right)\right]\left(1-\theta_K\left(1-\alpha_K\right)\right)L_K +$$

$$+\left[N-\left[K^*\right]\right]\gamma\left(1-\theta_K\left(1-\alpha_K\right)\right)L_K\prod_{i=K+1}^{\left[K^*\right]}\left(1-\theta_i\right)\Big] = \gamma\alpha_K L_K + P_\theta\left(1-\alpha_K\right)L_K + P_\theta\left(1-P_\theta\left(1-\alpha_K\right)\right)L_K +$$

$$+E\left[\left[\sum_{i=K+2}^{\left[K^*\right]}\theta_i \prod_{j=K+1}^{i-1}\left(1-\theta_j\right)\right]\left(1-\theta_K\left(1-\alpha_K\right)\right)L_K\right] + E\left[\left[N-\left[K^*\right]\right]\gamma\left(1-\theta_K\left(1-\alpha_K\right)\right)L_K\prod_{i=K+1}^{\left[K^*\right]}\left(1-\theta_i\right)\right],$$

where:

$$E\left[\left[\sum_{i=K+2}^{\left[K^*\right]}\theta_i \prod_{j=K+1}^{i-1}\left(1-\theta_j\right)\right]\left(1-\theta_K\left(1-\alpha_K\right)\right)L_K\right] = E\Big[\big[\theta_{K+2}\left(1-\theta_{K+1}\right) + \theta_{K+3}\left(1-\theta_{K+2}\right)\left(1-\theta_{K+1}\right) + \ldots +$$

$$+\theta_{\left[K^*\right]}\left(1-\theta_{K+1}\right)\left(1-\theta_{K+2}\right)\ldots\left(1-\theta_{\left[K^*\right]-2}\right)\left(1-\theta_{\left[K^*\right]-1}\right)\big]\left(1-\theta_K\left(1-\alpha_K\right)\right)L_K\Big],$$

as we are considering identically and independently distributed random variables $\left\{\theta_i\right\}$ with $E\left[\theta_i\right] = P_\theta \quad \forall i \in \left\{1,\ldots,N\right\}$, we have:

$$E\left[\left[\sum_{i=K+2}^{\left[K^*\right]}\theta_i \prod_{j=K+1}^{i-1}\left(1-\theta_j\right)\right]\left(1-\theta_K\left(1-\alpha_K\right)\right)L_K\right] = P_\theta\left[\left(1-P_\theta\right) + \left(1-P_\theta\right)^2 + \ldots + \left(1-P_\theta\right)^{\left[K^*\right]-K-1}\right]\left(1-P_\theta\left(1-\alpha_K\right)\right)L_K =$$

$$= \cancel{P_\theta}\frac{\left(1-P_\theta\right) - \left(1-P_\theta\right)^{\left[K^*\right]-K}}{\cancel{P_\theta}}\left(1-P_\theta\left(1-\alpha_K\right)\right)L_K$$, and:

$$E\left[\left[N-\left[K^*\right]\right]\gamma\left(1-\theta_K\left(1-\alpha_K\right)\right)L_K\prod_{i=K+1}^{\left[K^*\right]}\left(1-\theta_i\right)\right]=\left[N-\left[K^*\right]\right]\gamma\left(1-P_\theta\left(1-\alpha_K\right)\right)L_K\left(1-P_\theta\right)^{\left[K^*\right]-K}.$$

So, replacing both mathematical expectation expressions, Bellman equation for the K period can be written as:

$$J_K^*\left\{L_K\right\}=\underset{\alpha_K\in\{0,1\}}{Min}\quad\gamma\alpha_K L_K+P_\theta\left(1-\alpha_K\right)L_K+\left[1-P_\theta\left(1-\alpha_K\right)\right]L_K\left[1-\left(1-P_\theta\right)^{\left[K^*\right]-K}\right]+$$
$$+\left[N-\left[K^*\right]\right]\gamma\left(1-P_\theta\right)^{\left[K^*\right]-K}\left(1-P_\theta\left(1-\alpha_K\right)\right)L_K.$$

Hence:

If $\alpha_K=0\Rightarrow$ expected cumulative expense from *K* to *N* would be:

$$P_\theta L_K+\left(1-P_\theta\right)L_K\left[1-\left(1-P_\theta\right)^{\left[K^*\right]-K}\right]+\left[N-\left[K^*\right]\right]\gamma\left(1-P_\theta\right)^{\left[K^*\right]-K}\left(1-P_\theta\right)L_K.$$

If $\alpha_K=1\Rightarrow$ expected expense from K to N equals:

$$\gamma L_K+L_K\left[1-\left(1-P_\theta\right)^{\left[K^*\right]-K}\right]+\left[N-\left[K^*\right]\right]\gamma\left(1-P_\theta\right)^{\left[K^*\right]-K}L_K.$$

We want to prove that $\alpha_K=0$, which is optimal just in the case:

$$P_\theta L_K+\left(1-P_\theta\right)L_K\left[1-\left(1-P_\theta\right)^{\left[K^*\right]-K}\right]+\left[N-\left[K^*\right]\right]\gamma\left(1-P_\theta\right)^{\left[K^*\right]-K}\left(1-P_\theta\right)L_K<$$

$$<\gamma L_K+L_K\left[1-\left(1-P_\theta\right)^{\left[K^*\right]-K}\right]+\left[N-\left[K^*\right]\right]\gamma\left(1-P_\theta\right)^{\left[K^*\right]-K}L_K\Leftrightarrow$$

$$\Leftrightarrow P_\theta+\left(1-P_\theta\right)\left[1-\left(1-P_\theta\right)^{\left[K^*\right]-K}\right]-P_\theta\left[N-\left[K^*\right]\right]\gamma\left(1-P_\theta\right)^{\left[K^*\right]-K}<\gamma+\left[1-\left(1-P_\theta\right)^{\left[K^*\right]-K}\right]\Leftrightarrow$$

$$\Leftrightarrow\left(1-P_\theta\right)^{\left[K^*\right]-K}P_\theta-\left[N-\left[K^*\right]\right]\gamma\left(1-P_\theta\right)^{\left[K^*\right]-K}P_\theta<\gamma\Leftrightarrow$$

$$\Leftrightarrow\gamma>\frac{P_\theta\left(1-P_\theta\right)^{\left[K^*\right]-K}}{1+P_\theta\left[N-\left[K^*\right]\right]\left(1-P_\theta\right)^{\left[K^*\right]-K}}.$$

However, as we have that $K+1\le K^*\Rightarrow K<K^*=N-\frac{P_\theta-\gamma}{\gamma P_\theta}$, this imply: $\gamma>\frac{P_\theta}{1+P_\theta\left[N-K\right]}$,

Therefore, a sufficient condition to satisfy $\gamma>\frac{P_\theta\left(1-P_\theta\right)^{\left[K^*\right]-K}}{1+P_\theta\left[N-\left[K^*\right]\right]\left(1-P_\theta\right)^{\left[K^*\right]-K}}$, would be: $\frac{P_\theta}{1+P_\theta\left[N-K\right]}\ge\frac{P_\theta\left(1-P_\theta\right)^{\left[K^*\right]-K}}{1+P_\theta\left[N-\left[K^*\right]\right]\left(1-P_\theta\right)^{\left[K^*\right]-K}}$, what it is satisfied $\forall K<K^*$ as we prove behind:

We denote n the number of periods that separate K from $\left[K^*\right]$, therefore: $n=\left[K^*\right]-K$, and, $K=\left[K^*\right]-n,$ donde $1\leq n\leq\left[K^*\right]-1$. Taking into account this notation over the expression:

$$\frac{P_\theta}{1+P_\theta\left[N-K\right]}\geq\frac{P_\theta\left(1-P_\theta\right)^{\left[K^*\right]-K}}{1+P_\theta\left[N-\left[K^*\right]\right]\left(1-P_\theta\right)^{\left[K^*\right]-K}}\quad\Leftrightarrow \text{ we have:}$$

$$\frac{\cancel{P_\theta}}{1+P_\theta\left[N-\left(\left[K^*\right]-n\right)\right]}\geq\frac{\cancel{P_\theta}\left(1-P_\theta\right)^{\left[K^*\right]-\left(\left[K^*\right]-n\right)}}{1+P_\theta\left[N-\left[K^*\right]\right]\left(1-P_\theta\right)^{\left[K^*\right]-\left(\left[K^*\right]-n\right)}}\quad\Leftrightarrow$$

$$1+P_\theta\left[N-\left[K^*\right]\right]\left(1-P_\theta\right)^n\geq\left[1+P_\theta\left[N-\left[K^*\right]+n\right]\right]\left(1-P_\theta\right)^n\quad\Leftrightarrow 1\geq\left(1-P_\theta\right)^n\left(1+P_\theta n\right).$$

Denoting $X(u)=\left(1-P_\theta\right)^u\left(1+P_\theta u\right)$, it is verified that for every value of $u\in\left[1,\left[K^*\right]-1\right]$, $X(u)\leq 1$, because: $X(1)=1-P_\theta^2\leq 1$ and we have that,

$$\frac{dX(u)}{du}=\left(1-P_\theta\right)^u\left[\ln\left(1-P_\theta\right)\right]\left(1+P_\theta u\right)+\left(1-P_\theta\right)^u P_\theta\leq 0\Leftrightarrow\left[\ln\left(1-P_\theta\right)\right]\left(1+P_\theta u\right)+P_\theta\leq 0\Leftrightarrow$$

$$\Leftrightarrow\ln\left[\left(1-P_\theta\right)^{\left(1+P_\theta u\right)}\right]+P_\theta\leq 0\Leftrightarrow\left(1-P_\theta\right)^{\left(1+P_\theta u\right)}\leq\exp\left\{-P_\theta\right\}\Leftrightarrow\left(1-P_\theta\right)^{\left(1+P_\theta u\right)}\leq\frac{1}{\exp\left\{P_\theta\right\}},$$

What is always satisfied, since the left hand side of the expression decreases with u and $\left(1-P_\theta\right)^{\left(1+P_\theta\right)}<\dfrac{1}{\exp\left\{P_\theta\right\}}\ \forall P_\theta\in\left[0,1\right]$, what can be easily observed if we represent both functions graphically. In addition, $X(u)$ is a decreasing function in u and it is always less or equal to 1.

Particularly, $1\geq\left(1-P_\theta\right)^n\left(1+P_\theta n\right)$, with $n=\left[K^*\right]-K$ and therefore, the following condition is satisfied: $\dfrac{P_\theta}{1+P_\theta\left[N-K\right]}\geq\dfrac{P_\theta\left(1-P_\theta\right)^{\left[K^*\right]-K}}{1+P_\theta\left[N-\left[K^*\right]\right]\left(1-P_\theta\right)^{\left[K^*\right]-K}}\quad\forall K<K^*$ as we wanted to prove.

This is a sufficient condition to: $\gamma>\dfrac{P_\theta\left(1-P_\theta\right)^{\left[K^*\right]-K}}{1+P_\theta\left[N-\left[K^*\right]\right]\left(1-P_\theta\right)^{\left[K^*\right]-K}}\Rightarrow\alpha_K=0\quad\forall K<K^*$.

And for this case,

$J_K^*\left\{L_K\right\}=P_\theta L_K+\left(1-P_\theta\right)L_K\left[1-\left(1-P_\theta\right)^{\left[K^*\right]-K}\right]+\left[N-\left[K^*\right]\right]\gamma\left(1-P_\theta\right)^{\left[K^*\right]-K+1}L_K$, as we wanted to prove.

B) If $N \leq K^* = N - \frac{P_\theta - \gamma}{\gamma P_\theta}$ (what it possible if and only if $\gamma \geq P_\theta$). Then, in the $K+1$ period, since we assume that Proposition 1 is satisfied, the agent will choose not to protect the harvest value ($\alpha_{K+1} = 0$), and the optimal value function is: $J^*_{K+1}\left\{L_{K+1}\right\} = E\left[\theta_{K+1}L_{K+1} + \left[\sum_{i=K+2}^{N}\theta_i \prod_{j=K+1}^{i-1}\left(1-\theta_j\right)\right]L_{K+1}\right]$.

We have to prove that Proposition 1 is also satisfied in the K period, with the following. Bellman equation (given L_K): $J^*_K\left\{L_K\right\} = \underset{\alpha_K \in \{0,1\}}{Min}\, E\left[\gamma\alpha_K L_K + \theta_K\left(1-\alpha_K\right)L_K + J^*_{K+1}\left\{L_{K+1}\right\}\right] =$

$$= \underset{\alpha_K \in \{0,1\}}{Min}\, E\left[\gamma\alpha_K L_K + \theta_K\left(1-\alpha_K\right)L_K + \theta_{K+1}L_{K+1} + \left[\sum_{i=K+2}^{N}\theta_i \prod_{j=K+1}^{i-1}\left(1-\theta_j\right)\right]L_{K+1}\right].$$

Calculating the mathematical expectation and including the state equation into the Bellman equation, we have: $J^*_K\left\{L_K\right\} = \underset{\alpha_K \in \{0,1\}}{Min}\quad \gamma\alpha_K L_K + P_\theta\left(1-\alpha_K\right)L_K + \left[1-P_\theta\left(1-\alpha_K\right)\right]L_K\left[1-\left(1-P_\theta\right)^{N-K}\right]$.

Consequently,

If $\alpha_K = 0 \Rightarrow$ expected cumulative expense from K period to N final period would be: $P_\theta L_K + \left(1-P_\theta\right)L_K\left[1-\left(1-P_\theta\right)^{N-K}\right]$.

If $\alpha_K = 1 \Rightarrow$ expected cumulative expense from K period to N would be: $\gamma L_K + L_K\left[1-\left(1-P_\theta\right)^{N-K}\right]$.

We want to prove that $\alpha_K = 0$, what is satisfied if and only if:

$$P_\theta L_K + \left(1-P_\theta\right)L_K\left[1-\left(1-P_\theta\right)^{N-K}\right] \leq \gamma L_K + L_K\left[1-\left(1-P_\theta\right)^{N-K}\right] \Leftrightarrow$$

$P_\theta - P_\theta\left[1-\left(1-P_\theta\right)^{N-K}\right] \leq \gamma \Leftrightarrow P_\theta\left[\left(1-P_\theta\right)^{N-K}\right] \leq \gamma$, what is always maintained since $\gamma \geq P_\theta$.

Therefore, $\alpha_K = 0$ and the optimal value function is: $J^*_k\{L_k\} = P_\theta L_k + (1-P_\theta)L_k\left[1-(1-P_\theta)^{N-K}\right]$, as we wanted to prove in this case.

3.2 Optimal policy

From Proposition 1 we have that the optimal policy structure (see Figure 2) can be articulated as follow: the absence of protective actions is an optimal decision until a given period $K^* = N - \frac{P_\theta - \gamma}{\gamma P_\theta}$, where K^* depends on the total number of periods, the probability of suffering the loss and the protection cost. From this K^* period to the end, the optimal decision is to protect, in order to make sure that protection cost makes sense avoiding the loss.

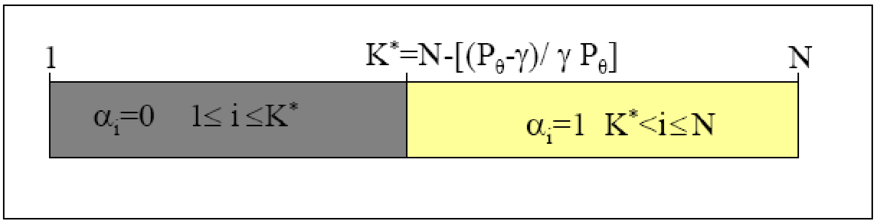


Figure 2 Optimal policy structure (N periods)

Of course, if $K^* > N$, (what is possible if $P_\theta < \gamma$), the optimal threshold period K* to start the protection is above the final period, what indicates that the optimal decision in this case would be to take none protection. In the opposite situation, if $K^* < 1$, (being $\gamma < \dfrac{P_\theta}{1+(N-1)P_\theta}$), the best would be to protect the harvest in every period. In Figure 3, we present a numerical example for the expected expense in the case of N=3, γ= 0.3, and L= 1. It is not surprising that the expected expense appears as an increasing function of P_θ that is the probability of suffering the loss.

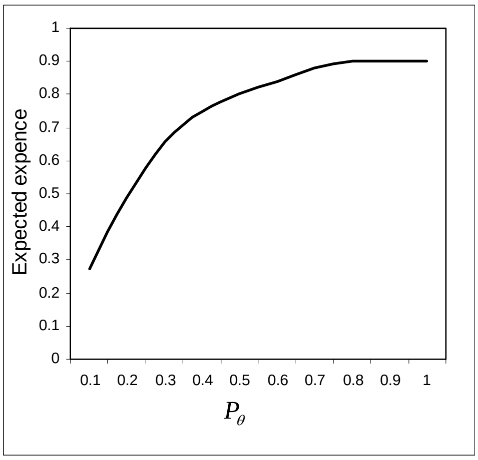


Figure 3 Expected expense when N=3, γ= 0.3, and *L*= 1

The optimal policy structure respond to the fact that in the case that a farmer could only afford to protect over part of the period, it is better to postpone protection as long as possible within the N-occasion time period, and so avoid unnecessary protection in the case of a loss L being incurred (Murphy et al., 1985).

4 Case studies

4.1 Frost protection in representative Spanish crops

The cost-loss dynamic model was motivated by a real state decision-making model for the fruit-frost problem (Katz et al., 1982). To minimize the damage caused by freezing temperatures, farmers may employ protective devices. Here we apply the analytical expressions developed in this study to two selected case studies in Spain. Crop production in Spain is characterized by the Mediterranean climate and we have selected the wine production in Burgos and the citrus production in Valencia being representative Mediterranean crops.

On one hand, Burgos is on the Northern Plateau, characterized by large seasonal differences in temperature and late spring frost. “Ribera del Duero” (RD) is a grapevine area with high quality wine production that it is mainly located in Burgos. (More than 75% of RD vineyards are planted over this district). Vines are susceptible to frost after budbreak and the critical frost danger period is over April. On Table 1 the number of days with temperature below 0ºC in April is presented for the 1980-2000 period in Burgos, and the climatologic probability of frost damage has been calculated.

On the other hand, Valencia is on the east coast of Spain, characterized by warm winters. However, occasional winter frost causes significant losses to orchadists. Crop production in Valencia is characterized for intensive irrigated horticulture and citrus fruits which production is among the most internationally recognized varieties. The major risk to this crop is the occurrence of winter freezes being January the cooler month on the region. On Table 1 the number of days with temperature below 0ºC in January is presented for the 1980-2000 period in Valencia, and the climatologic probability of frost damage has been calculated.

Some of the more common methods to avoid the losses of freezing temperatures are orchard heaters, wind machines, and overhead sprinkler irrigation. Considering the proposed case studies with the climatologic probability of freeze on Table 1, we can obtain the optimal protection policy from Proposition 1.

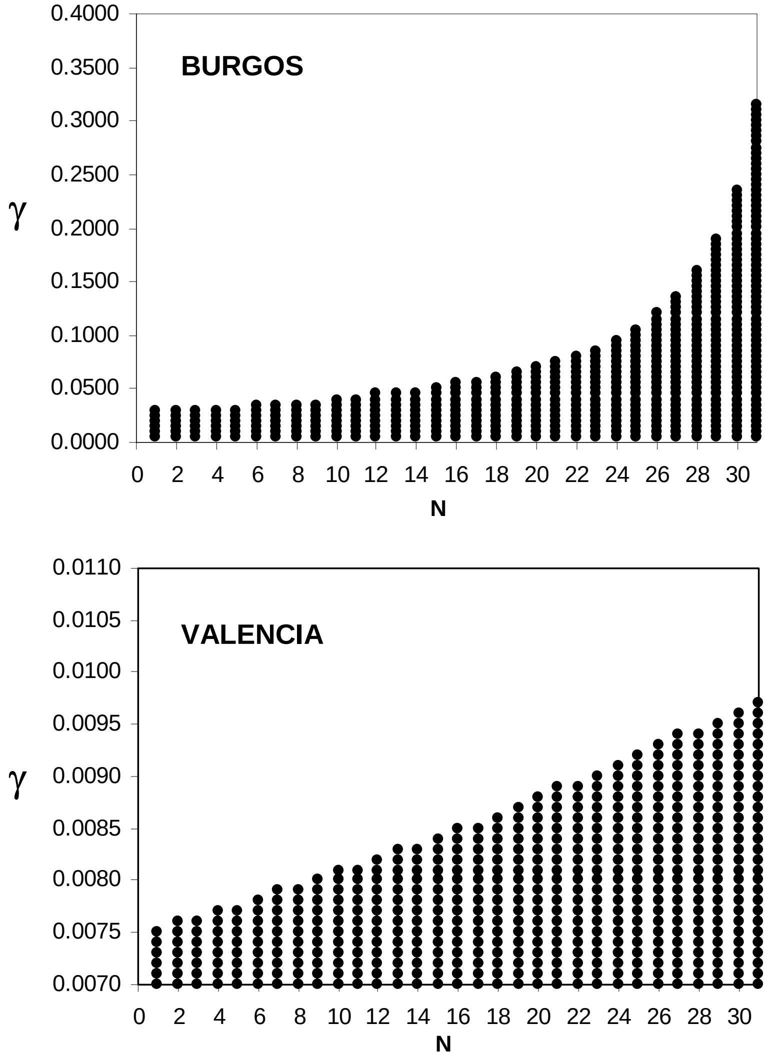
Table 1 Climatologic probability of freeze in the case studies (April in Burgos and January in Valencia)

Number of days with temperature below 0ºC in a month		
Year	Valencia (January)	Burgos (April)
1981	0.00	9.00
1982	0.00	4.00
1983	0.00	8.00
1984	0.00	1.00
1985	6.00	3.00
1986	0.00	14.00
1987	0.00	1.00
1988	0.00	6.00
1989	0.00	11.00
1990	0.00	5.00
1991	0.00	13.00
1992	0.00	10.00
1993	0.00	5.00
1994	0.00	14.00
1995	0.00	8.00
1996	0.00	8.00
1997	0.00	4.00
1998	0.00	8.00
1999	0.00	9.00
2000	0.00	3.00
Climatological probability (P_θ)	0.0097	0.2400

Source of data: INM (Spanish Institute of National Weather)

Figure 4 shows the optimal policy: a black dot denotes “protect” and the absence of a symbol indicates that optimal policy is “do not protect”.

Figure 4 Optimal policy for the case studies (wine production in Burgos and citrus production in Valencia)

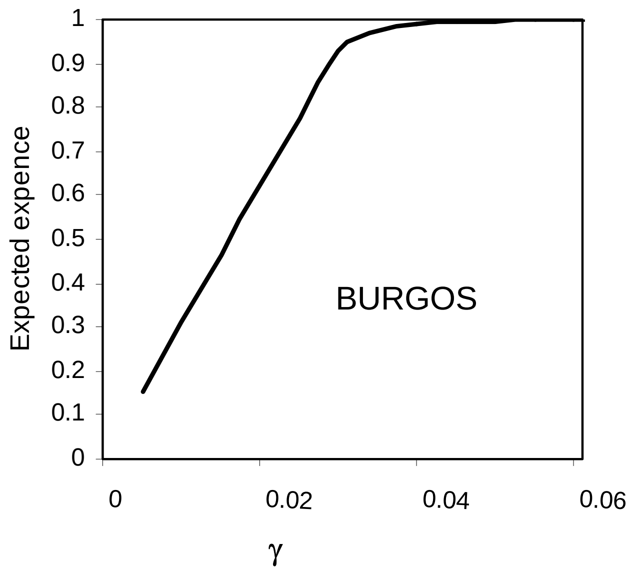


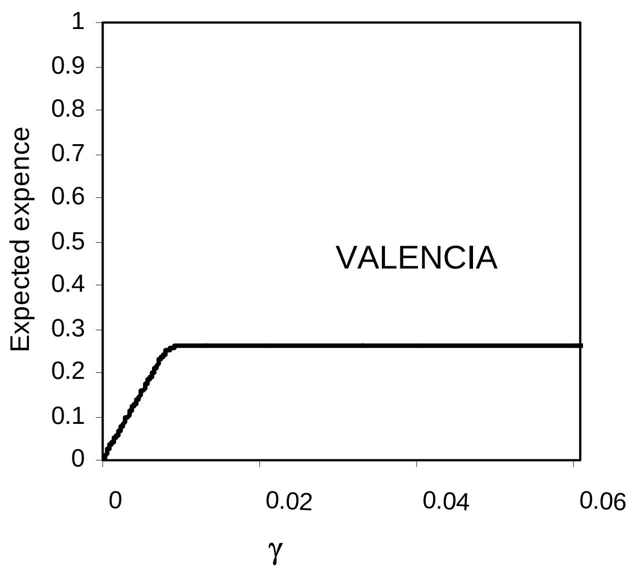
We can observe in the upper box in Figure 4 the optimal policy for the spring vineyard frost protection in Burgos. In April, the climatologic probability of freeze damage for the wine production in Burgos is about 0.24. Taking into account this information, we have that when cost-loss value is less than 0.035, the optimal policy would be protect from the beginning to the end of the period. For a cost-loss ratio between 0.035 and 0.315, the farmer should protect the harvest from a threshold period –that rises with the cost-- to the end of the month. Finally, for γ values over 0.315, no protection should be taken.

We can observe in the lower box in Figure 4 the optimal policy for the winter citrus frost protection in Valencia. In January, the climatologic probability of freeze in Valencia is about 0.0097. Consequently, with this smaller probability, it can be observed that protection should be optimal just in the case of low cost-loss ratios. For γ<0.0075, the farmer should protect everyday in the month. If 0.0076<γ<0.0097 a threshold exists from which he or she should begin the protection action, and finally, over 0.0098 no protection should be taken. However, it is important to notice that citrus fruit in Valencia is an intensive irrigated production with high productivity and harvest values, so potential losses can be significant and that imply minor cost-loss ratios.

Figure 5 shows the accumulated expected expense for each case study with a loss L normalized to 1.

Figure 5 Expected expense under the optimal policy in the case studies (wine production in Burgos and citrus production in Valencia)





4.2 Numerical results: sensitivity analysis over climatological probability

Some numerical examples are presented where numerical values of the cost-loss ratio γ , and the time horizon are specified. The climatologic probability, P_θ, has been ranged for a sensitivity analysis. Figure 6 illustrate the strategy variation as a function of the climatologic probability. A black dot denotes "protect" and the absence of a symbol indicates that the optimal policy is "not protect". Figure 7 shows the accumulated expected expense for each example.

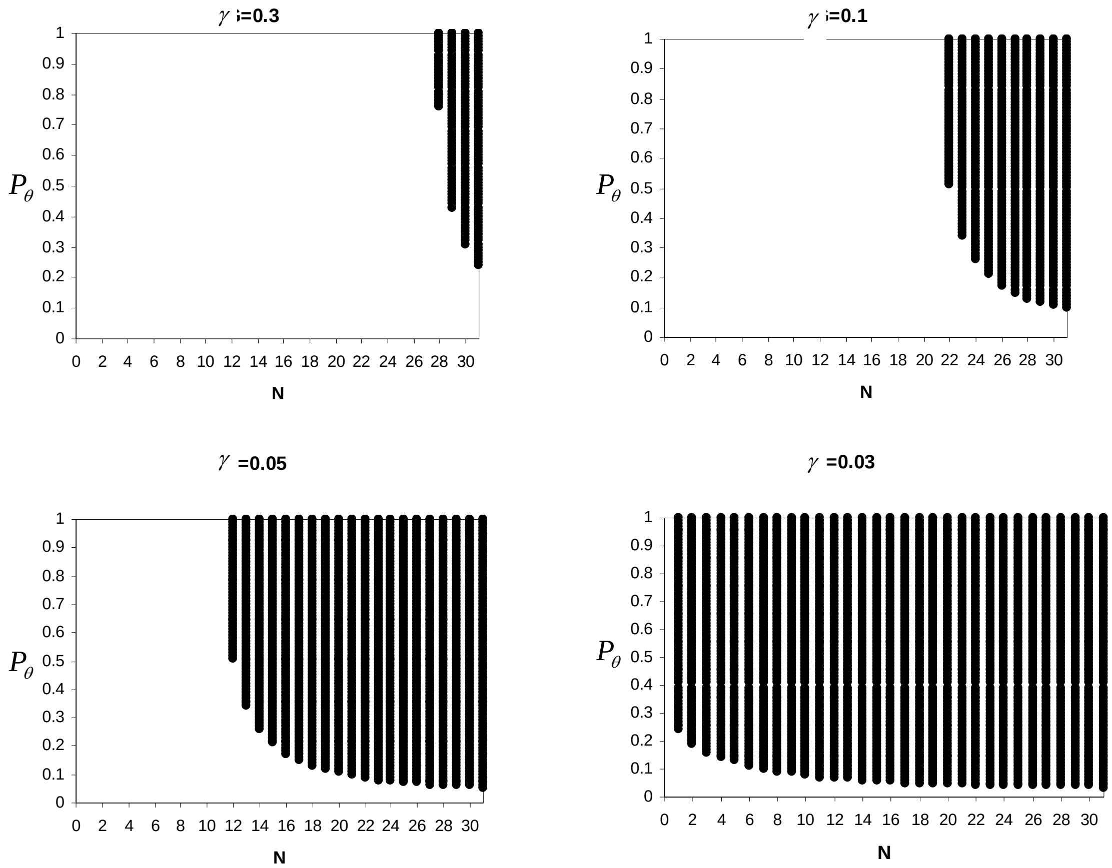


Figure 6 Optimal policy for N=31 time horizon, L=1 and for different values of protection cost-loss ratio: γ= 0.3, γ= 0.1, γ= 0.05 and γ= 0.03

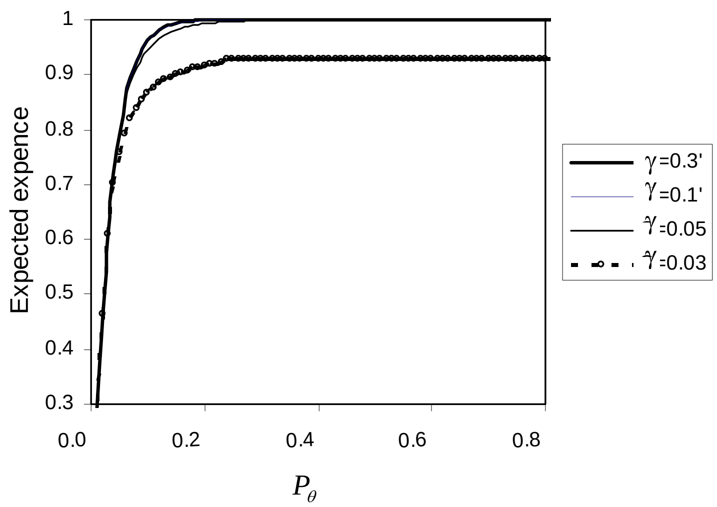


Figure 7 Accumulated expected expense for N=31 time horizon, L=1 and γ= 0.3, γ= 0.1, γ= 0.05 and γ= 0.03

Of course, if we compare the optimal policies on Figure 6, it is shown that protection is taken later in the cases with higher cost-loss ratio and also the accumulated expected expense in Figure 7 increases with this cost. However, this effect is far from linear, since the expense increases less than proportionally with the cost.

5 Conclusions

In this paper, we present an analytical solution to the Cost-Loss dynamic model in the case of climatological information. As we discussed in the paper, typical agricultural problems involved in a similar structure could include the vineyards or citrus protection against froze, the water management for cereals irrigation in drought shortages contexts, the “options and futures” exchange markets decisions for major crops in USA, and much more risk management.

We develop analytical expressions for the optimal policy, which have the structure proposed on the literature. It is never an optimal decision to protect in a period and not to protect in the following. There exists a threshold, from which it is optimal to protect to the end of the period in order to avoid the loss. This threshold depends on the climatologic probability, the cost-loss ratio and the time horizon.

The analytical solution can help to decision making in numerous cases suitable for this prototype model in a more accurate way than the numerical approaches in the literature. Two case studies have been developed as examples of application: the spring vineyard frost protection in Burgos and winter citrus frost protection in Valencia. The climatologic probability and the optimal policy have been calculated.

Then, some sensitivity analysis has been proposed ranging the climatologic probability between 0 and 1. It is shown that protection is taken later in the cases with higher cost-loss ratio but the accumulated expected expense increase less than proportionally with the cost. The model framework can also be used to expand the analytical solution to the case of probabilistic forecasts.

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