# Efficiency Measurements in Multi-activity Data Envelopment Analysis with Shared Inputs: An Application to Farmers' Organizations in Taiwan

by

Po-Chi Chen
*Dept. of International Business, Chung Hua University, Hsinchu, Taiwan, R.O.C.*

Shih-Hsun Hsu
*Department of Agricultural Economics, National Taiwan University, Taipei, Taiwan, R.O.C.*

Ching-Cheng Chang
*Institute of Economics, Academia Sinica, and Department of Agricultural Economics, National Taiwan University, Taipei, Taiwan, R.O.C.*

Ming-Miin Yu
*Dept. of Transportation and Navigation Science, National Taiwan Ocean University, Keelung, Taiwan, R.O.C.*

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## Abstract

A multi-activity DEA model with variable returns to scale is proposed to provide information on the efficiency performance of organizations with inputs shared among several closely-related activities. The model is applied to study the case of 279 farmers' cooperatives in Taiwan. The results provide overall and individual performance measures for each activity as well as suggestions for future improvement. Inter-firm networking should be strengthened and thus policies that promote the consolidation of TFCs will not be sufficient to meet the public goal of institutional reform.

***Keywords:*** multi-activity DEA, shared inputs, efficiency measure, directional distance function

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## I. INTRODUCTION

As in many developing countries around the world, Taiwan's farmers' cooperatives (TFCs) have played an important role in assisting the government to promote certain policy goals for agricultural development. Each TFC has consisted of four departments to provide credit, extension, insurance, and marketing services to their members, who are mostly farmers or residents located in rural areas. Profits from the credit departments have been used for improving cooperative marketing, insurance and extension services, whereas the activities of the extension, insurance, and marketing services have attracted savings to the TFCs which have been able to serve as loanable funds that can be made available to eligible members. As the favorable conditions for agricultural production have declined over time, the TFCs have also begun to take on a broader role in promoting village construction and enhancing farmers' welfare, thereby helping to bring about wider development. After Taiwan became a member of the WTO in 2002, the TFCs were given a new role to minimize the impact of WTO entry through the promotion of local products in global markets.

The close linkages among the services and the close ties between the cooperatives and the government have made TFCs the most important but also most controversial financial institutions in Taiwan's rural communities. Previous studies (e.g., Wang et al., 2008) have examined their performance and focused on how the subsidized credits impaired their incentives to minimize costs and gave rise to a detrimental effect on the TFC's competitiveness. However, the multi-purpose nature of the TFCs and the complementary effect of inter-firm networking to serve rural development purposes are often overlooked. In this study, we propose the adoption of a multi-activity data envelopment analysis (MDEA) method by Beasley (1995) and Tsai and Mar Molinero (1998, 2002) to examine the role of teamwork in the efficiency performance of the TFCs. The efficiency measure derived from the traditional data envelopment analysis (DEA) model implicitly assumes that each TFC is equally efficient in all activities, and that the TFC is free to apply any of its inputs to any of its outputs in the most desirable way. In comparison, the MDEA identifies the particular strengths and weaknesses of the TFCs by distinguishing which department operates in the most efficient manner as well as under the most productive scale. It allows us to determine how much of the internally shared inputs are associated with each department. The primal and dual relationships of the MDEA model are also used to estimate the status of returns to scale for the whole team and the four departments individually.

The remainder of this paper is organized as follows. The next section describes the methodology of MDEA followed by a description of the empirical model. Section Three discusses the data and Section Four presents the empirical results. Section Five concludes.

## 2. METHODOLOGY

The MDEA model was first introduced by Beasley (1995) in a ratio form in which the multi-activity production technology could be constructed as a piecewise linear combination of all the observed inputs and outputs. Mar Molinero (1996) subsequently revised the model into a linear form using Shephard's distance function. Cook et al. (2000) also used a model similar to that of Beasley (1995) to evaluate multi-component efficiencies of a sample of Canadian banks and discussed how the assumptions regarding Beasley's nonlinear model could be relaxed into a linear one. Tsai and Mar Molinero (1998, 2002) extended Mar Molinero's MDEA model to encompass variable returns to scale (RTS) and applied it to the National Health Service in the UK.

Our model provides an alternative measure by adopting Luenberger (1992)'s directional distance functions. The directional distance function generalizes Shephard's input and output distance functions by simultaneously scaling inputs and outputs,[1] but not necessarily along the rays from the input and output origin (Fukuyama, 2003). Therefore, the directional-distance-function-based DEA model is not as restrictive as those based on Shephard's distance function in that the latter require equi-proportional adjustments of the inputs and outputs. We will show how the efficiency performance can be further generalized to the case where there are multi-activity entities like the TFCs.

Another reason for using the directional distance function comes from an extension of incorporating an undesirable (or "bad") output as a byproduct of desirable ("good") production activities. In the case of TFCs, excessive bad loans or default risk exceeding a certain limit are viewed as a "bad" byproduct as the credit departments are trying to maximize their loan provisions to their members. If we seek a reduction in the bad output and simultaneous increases in the good output, then the directional distance function will be a preferred method because it allows non-proportional adjustments of the good and bad outputs. In this section, we describe the method for generating efficient surfaces of a production possibility set in which activity-specific inputs and shared inputs are used to jointly produce the desirable and undesirable outputs.

### 2.1 Traditional DEA with Undesirable Outputs

Let $x = (x_1, x_2, \ldots, x_N) \in R_+^N$ denote an input vector and $u = (u_1, u_2, \ldots, u_G) \in R_+^G$ an output vector, where $u$ is composed of desirable outputs ($y$) and undesirable outputs ($b$), i.e., $u = (y, b) = (y_1, y_2, \ldots, y_M;\ b_1, b_2, \ldots b_R) \in R_+^{M+R}$. The directional distance function seeking to

[1] Details of the relationship between directional distance functions and Shephard distance functions can be found in Chung et al. (1997) and Färe and Grosskopf (2000).

increase the desirable outputs and decrease the undesirable outputs and inputs directionally can be defined by the following formulation:

$$\vec{D}(x, y, b; g) = \sup\{\beta : (x - \beta g_x, y + \beta g_y, b - \beta g_b) \in T\}, \tag{1}$$

where the nonzero vector $g = (g_x, g_y, g_b)$ determines the "directions" in which inputs, desirable outputs and undesirable outputs are scaled, and the technology reference set $T = \{(x, u) : x \text{ can produce } u\}$ satisfies the assumptions of variable returns to scale, strong disposability of desirable outputs and inputs, and weak disposability of undesirable outputs.

Suppose there are $k = 1, \cdots, K$ DMUs in the data set. Each DMU uses inputs $x^k = (x_1^k, x_2^k, \cdots, x_N^k) \in R_+^N$ to jointly produce desirable outputs $y^k = (y_1^k, y_2^k, \cdots, y_M^k) \in R_+^M$ and undesirable outputs $b^k = (b_1^k, b_2^k, \cdots, b_R^k) \in R_+^R$. The piecewise reference technology allowing for variable returns to scale can be constructed as follows:

$$\begin{aligned}
T = \{(x, y, b) :\ & \sum_{k=1}^{K} z^k y_m^k \geq y_m, && m = 1, \ldots\ldots, M. \\
& \sum_{k=1}^{K} z^k b_r^k = b_r, && r = 1, \ldots\ldots, R. \\
& \sum_{k=1}^{K} z^k x_n^k \leq x_n, && n = 1, \ldots\ldots, N. \\
& z^k \geq 0, && k = 1, \ldots\ldots, K. \\
& \sum_{k=1}^{K} z^k = 1\},
\end{aligned} \tag{2}$$

where the $z_k$ are the intensity variables to shrink or expand the individual observed activities of DMU *k* for the purpose of constructing convex combinations of the observed inputs and outputs.

Relative to the reference technology $T$ constructed in (2), traditionally, for each DMU

$k' = 1, \cdots, K$, the directional distance function can be obtained by solving the following linear programming problem with $g = (g_x, g_y, g_b) = (x^{k'}, y^{k'}, b^{k'})$, i.e., when the direction chosen is based on observed inputs and outputs:

$$\vec{D}(x^{k'}, y^{k'}, b^{k'}; -x^{k'}, y^{k'}, -b^{k'}) = \max\ \beta^{k'}$$

$$\begin{aligned}
\text{s.t.}\quad & \sum_{k=1}^{K} z^k y_m^k \geq (1+\beta^{k'}) y_m^{k'} && m = 1, \cdots, M; \\
& \sum_{k=1}^{K} z^k b_r^k = (1-\beta^{k'}) b_r^{k'} && r = 1, \ldots\ldots, R; \\
& \sum_{k=1}^{K} z^k x_n^k \leq (1-\beta^{k'}) x_n^{k'} && n = 1, \cdots, N; \\
& z^k \geq 0, && k = 1, \cdots, K; \\
& \sum_{k=1}^{K} z^k = 1;
\end{aligned} \tag{3}$$

where $\beta^{k'}$ measures the maximum inflation of all desirable outputs and deflation of all inputs and undesirable outputs that remain technically feasible and can serve as a measure of technical inefficiency. If $\beta^{k'} = 0$, then DMU $k'$ operates on the frontier of *T* with technical efficiency. If $\beta^{k'} > 0$, then DMU $k'$ operates inside the frontier of *T*.

The efficiency measurement constructed in (3) expands all desirable outputs and contracts all inputs and undesirable outputs at the same rate $\beta$. It can be further generalized to accommodate different expansion and contraction ratios as follows:

$$\vec{D}(x^{k'}, y^{k'}, b^{k'}; -x^{k'}, y^{k'}, -b^{k'}) = \max\ \omega_1 \beta_1^{k'} + \omega_2 \beta_2^{k'} + \omega_3 \beta_3^{k'} = \beta^{k'}$$

$$\text{s.t.}\quad \sum_{k=1}^{K} z^k y_m^k \geq (1+\beta_1^{k'}) y_m^{k'} \qquad m = 1, \cdots, M;$$

$$\sum_{k=1}^{K} z^k b_r^k = (1-\beta_2^{k'}) b_r^{k'} \qquad r = 1,\dots\dots, R;$$

$$\sum_{k=1}^{K} z^k x_n^k \le (1-\beta_3^{k'}) x_n^{k'} \qquad n = 1,\cdots, N; \tag{4}$$

$$z^k \ge 0, \qquad k = 1,\cdots, K;$$

$$\sum_{k=1}^{K} z^k = 1.$$

The measure $\vec{D}(x^{k'}, y^{k'}, b^{k'}; -x^{k'}, y^{k'}, -b^{k'})$ given in (4) is maximized hyperbolically $\beta^{k'} = \omega_1\beta_1^{k'} + \omega_2\beta_2^{k'} + \omega_3\beta_3^{k'}$ by comparing the observed $(y^{k'}, b^{k'}, x^{k'})$ with the frontier ( $(1+\beta_1^{k'})y_m^{k'}$, $(1-\beta_2^{k'})b_r^{k'}$, $(1-\beta_3^{k'})x_n^{k'}$ ), where $(1+\beta_1^{k'})y_m^{k'}$, $(1-\beta_2^{k'})b_r^{k'}$, and $(1+\beta_3^{k'})x_n^{k'}$ maximize the value of $\omega_1\beta_1^{k'} + \omega_2\beta_2^{k'} + \omega_3\beta_3^{k'}$. The coefficients $\omega_1$, $\omega_2$ and $\omega_3$ are associated with the priorities given to the inputs and outputs and their sum is normalized to unity. The improvement expressed in terms of the percentage of desirable outputs, undesirable outputs, and inputs can be measured by $\beta_1^{k'}$, $\beta_2^{k'}$, and $\beta_3^{k'}$, respectively, and then used to calculate the weighted inefficiency score $\beta^{k'}$ (Yu and Fan, 2006). Note that if we set $\beta_1^{k'} = \beta_2^{k'} = \beta_3^{k'}$, then model (4) degenerates to model (3).

The dual of (4) is shown to be:

$$\min \quad -\sum_{m=1}^{M} u_m^{k'} y_m^{k'} + \sum_{n=1}^{N} v_n^{k'} x_n^{k'} + \sum_{r=1}^{R} \rho_r^{k'} b_r^{k'} + \delta^{k'}$$

$$\text{s.t} \quad -\sum_{m=1}^{M} u_m^{k'} y_m^{k} + \sum_{n=1}^{N} v_n^{k'} x_n^{k} + \sum_{r=1}^{R} \rho_r^{k'} b_r^{k} + \delta^{k'} \ge 0 \qquad k = 1,\dots\dots, K.$$

$$\sum_{m=1}^{M} u_m^{k'} y_m^{k'} \ge \omega_1 \tag{5}$$

$$\sum_{r=1}^{R} \rho_r^{k'} b_r^{k'} \geq \omega_2$$

$$\sum_{n=1}^{N} v_n^{k'} x_n^{k'} \geq \omega_3$$

$$u_m^{k'}, v_n^{k'} \geq 0, \ \ \rho_r^{k'}, \delta^{k'} \ \ free,$$

where $u_m^{k'}, v_n^{k'}$, and $\rho_r^{k'}$ are multipliers for desirable outputs, inputs, and undesirable outputs, respectively. Model (5) shows that a measure of technical inefficiency may be defined as follows:

$$TIE^{k'} = \frac{-\sum_{m=1}^{M} u_m^{k'} y_m^{k'} + \sum_{n=1}^{N} v_n^{k'} x_n^{k'} + \sum_{r=1}^{R} \rho_r^{k'} b_r^{k'} + \delta^{k'}}{\sum_{m=1}^{M} u_m^{k'} y_m^{k'} + \sum_{n=1}^{N} v_n^{k'} x_n^{k'} + \sum_{r=1}^{R} \rho_r^{k'} b_r^{k'}}.$$

The first constraint in (5) is used to ensure that the cross efficiencies do not exceed unity.

As for the RTS classifications, Charnes et al. (1978) used the sum of the optimal intensity variable values as a measure for RTS. Banker et al. (1984) proposed that the shadow price $\delta^{k'}$ on the convexity constraints can be used to characterize the scale properties. Fukuyama (2003) indicated that the criteria to determine the status of RTS associated with the directional distance function based on $\delta^{k'}$ are as follows: (i) if $\delta^{k'} > 0$, then DRS prevail; (ii) if $\delta^{k'} = 0$, then CRS prevail; and (iii) if $\delta^{k'} < 0$, then IRS prevail. The scale efficiency index proposed by Färe et al. (1985) can also be used to test the nature of the RTS. This method states that the scale inefficiency of a DMU is due to DRS if the DMU scores the same value under NIRS technology, otherwise it is due to IRS.

### 2.2 An Extension to the MDEA with Shared Inputs

Following Tsai and Mar Molinero (1998, 2002)'s approach, the traditional DEA model is extended to a multi-activity fashion by allowing each activity to grade its performance and RTS property with its own technology frontier. This multi-activity efficiency measure provides a performance measure with activity-based information as part of the aggregated score.

Consider again that there are $k = 1, \cdots, K$ DMUs and that each engages in $I$ activities. Let $X_k^1, X_k^2, \cdots, X_k^I$ and $X_k^s = (x_{k,1}^s, x_{k,2}^s, \cdots, x_{k,L}^s)$ denote the dedicated input vector and shared inputs of DMU *k*, respectively, where $X_k^i$ is the input vector associated solely with the *i*th activity while $x_{k,l}^s$ is the *l*th input shared by the *I* activities. Because $x_{k,l}^s$ is a shared input, it is assumed that some portion $\mu_{k,l}^i$ ($0 < \mu_{k,l}^i < 1$, $\sum_{i=1}^{I} \mu_{k,l}^i = 1$) of this shared input is allocated to the *i*th activity. In the MDEA model, $\mu_{k,l}^i$ is a decision variable to be determined by the DMU. Thus, the *i*th activity employs $X_k^i$ and $\mu_k^i X_k^s$ to jointly produce desirable output $Y_k^i$ and undesirable output $B_k^i$ in which $\mu_k^i X_k^s = (\mu_{k,1}^i x_{k,1}^s, \mu_{k,2}^i x_{k,2}^s, \cdots, \mu_{k,L}^i x_{k,L}^s)$, $Y_k^i = (y_{k,1}^i, y_{k,2}^i, \cdots, y_{k,M_i}^i)$ and $B_k^i = (b_{k,1}^i, b_{k,2}^i, \cdots, b_{k,R_i}^i)$.

The production technology with variable returns to scale and shared inputs for the *i*th activity can be defined as follows:

$$
\begin{aligned}
T^i = \{ (x^i, y^i, b^i) : \; & \sum_{k=1}^{K} z_k^i y_{k,m_i}^i \geq y_{m_i}^i, && m_i = 1, \ldots\ldots, M_i. \\
& \sum_{k=1}^{K} z_k^i b_{k,r_i}^i = b_{r_i}^i, && r_i = 1, \ldots\ldots, R_i. \\
& \sum_{k=1}^{K} z_k^i x_{k,n_i}^i \leq x_{n_i}^i, && n_i = 1, \ldots\ldots, N_i. \\
& \sum_{k=1}^{K} z_k^i \mu_{k,l}^i x_{k,l}^s \leq \mu_{k,l}^i x_l^s, && l = 1, \ldots\ldots, L. \qquad (6) \\
& 0 < \mu_{k',l}^i < 1 && l = 1, \ldots\ldots, L.
\end{aligned}
$$

$$z_k^i \geq 0, \qquad k = 1,......,K.$$

$$\sum_{k=1}^{K} z_k^i = 1 \}$$

Then the directional distance function can be used as the basis for estimating the weighted-average inefficiency of each DMU ( $\beta^{k'}$ ) by solving the following MDEA model:

$$Max \qquad \beta^{k'} = \sum_{i=1}^{I} w^i \beta_{k'}^i \tag{7}$$

$$\text{s. t.} \quad \sum_{k=1}^{K} z_k^i y_{k,m_i}^i \geq (1+\beta_{k'}^i) y_{k',m_i}^i, \qquad m_i = 1,......,M_i. \quad i = 1,......,I \tag{8}$$

$$\sum_{k=1}^{K} z_k^i b_{k,r_i}^i = (1-\beta_{k'}^i) b_{k',r_i}^i, \qquad r_i = 1,......,R_i. \quad i = 1,......,I \tag{9}$$

$$\sum_{k=1}^{K} z_k^i x_{k,n_i}^i \leq (1-\beta_{k'}^i) x_{k',n_i}^i, \qquad n_i = 1,......,N_i. \quad i = 1,......,I \tag{10}$$

$$\sum_{i=1}^{I} \sum_{k=1}^{K} z_k^i \mu_{k',l}^i x_{k,l}^s \leq \sum_{i}^{I} (1-\beta_{k'}^i) \mu_{k',l}^i x_{k',l}^s, \qquad l = 1,......,L. \tag{11}$$

$$\sum_{i=1}^{I} \mu_{k',l}^i = 1 \qquad l = 1,......,L. \tag{12}$$

$$\sum_{k=1}^{K} z_k^i = 1 \qquad i = 1,......,I \tag{13}$$

$$z_k^i \geq 0, \qquad k = 1,......,K. \quad i = 1,......,I \tag{14}$$

$$0 < \mu_{k',l}^i < 1, \quad \beta_{k'}^i \geq 0, \tag{15}$$

where $w^i$ is a positive number which represents the relative importance given to the various activities and their sum is standardized to be equal to 1. This MDEA model is essentially designed to minimize the inputs and undesirable outputs and at the same time maximize the desirable outputs for each activity.

Here, we would also like to examine the returns to scale properties of each DMU. Therefore, the dual form of the above model is described as follows:

$$Min \quad -\sum_{i=1}^{I}\sum_{m_i=1}^{M_i} u_{m_i}^i y_{k',m_i}^i + \sum_{i=1}^{I}\sum_{n_i=1}^{N_i} v_{n_i}^i x_{k',n_i}^i + \sum_{i=1}^{I}\sum_{r_i=1}^{R_i} \rho_{r_i}^i b_{k',r_i}^i + \sum_{l=1}^{L} v_l^s x_{k',l}^s + \sum_{i=1}^{I} \delta_{k'}^i \tag{16}$$

$$\text{s.t} \quad -\sum_{m_i=1}^{M_i} u_{m_i}^i y_{k,m_i}^i + \sum_{n_i=1}^{N_i} v_{n_i}^i x_{k,n_i}^i + \sum_{r_i=1}^{R_i} \rho_{r_i}^i b_{k,r_i}^i + \sum_{l=1}^{L} v_l^s \mu_{k',l}^i x_{k,l}^s + \delta_{k'}^i \geq 0, \tag{17}$$

$$\sum_{m_i=1}^{M_i} u_{m_i}^i y_{k',m_i}^i + \sum_{n_i=1}^{N_i} v_{n_i}^i x_{k',n_i}^i + \sum_{r_i=1}^{R_i} \rho_{r_i}^i b_{k',r_i}^i + \sum_{l=1}^{L} v_l^s \mu_{k',l}^i x_{k',l}^s \geq w^i, \tag{18}$$

$$u_{m_i}^i, v_{n_i}^i, v_l^s \geq 0, \ \rho_{r_i}^i, \delta^i \ free, \tag{19}$$

where $u_{m_i}^i, v_{n_i}^i, \rho_{r_i}^i, v_l^s$ are multipliers for desirable outputs, inputs, undesirable outputs, and shared inputs, respectively. When the equality holds in equation (18), an aggregate measure of technical inefficiency may be defined as follows:

$$TIE_{k'} = \frac{-\sum_{i=1}^{I}\sum_{m_i=1}^{M_i} u_{m_i}^i y_{k',m_i}^i + \sum_{i=1}^{I}\sum_{n_i=1}^{N_i} v_{n_i}^i x_{k',n_i}^i + \sum_{i=1}^{I}\sum_{r_i=1}^{R_i} \rho_{r_i}^i b_{k',r_i}^i + \sum_{l=1}^{L} v_l^s x_{k',l}^s + \sum_{i=1}^{I} \delta_{k'}^i}{\sum_{i=1}^{I}\sum_{m_i=1}^{M_i} u_{m_i}^i y_{k',m_i}^i + \sum_{i=1}^{I}\sum_{n_i=1}^{N_i} v_{n_i}^i x_{k',n_i}^i + \sum_{i=1}^{I}\sum_{r_i=1}^{R_i} \rho_{r_i}^i b_{k',r_i}^i + \sum_{l=1}^{L} v_l^s x_{k',l}^s}.$$

This measure is the weighted result of *I* activities' individual inefficiency (see Appendix A for the proof). Moreover, the constraint (17) ensures that the efficiencies do not exceed unity (see Appendix B).

Following the similar criteria stated above, the shadow price $\delta^i$ can be used to determine the RTS status for each activity. As Tsai and Mar Molinero (1998, 2002) indicated, there are two interesting consequences regarding the RTS properties in the MDEA model. First, different activities are allowed to operate under different RTS since each activity may have its own production technology. Second, the overall status of the RTS of each DMU depends on the individual RTS of all activities' $\delta^i$ (i.e., $\sum_{i=1}^{I}\delta^i$ ). Thus a DMU may appear to be operating under CRS and to be scale efficient when it is actually operating under IRS in some activities and under DRS in the others and is scale inefficient. Thus, the scale efficiency in the context of a multi-activity DEA is much more complex than the traditional DEA model would suggest.

## 3. DATA AND VARIABLE SPECIFICATION

The empirical application is implemented using the data from the *Farmers' Association Yearbook* of 2003 published by the Taiwan Provincial Farmers' Association. The total number of TFCs is 279, 78 of which are deleted either because their credit departments were taken over by commercial banks or because of the problem of missing data.

Regarding the specification of the variables, for the marketing activity the specific input of operating expenditures ($x_1^1$) is used to produce two outputs, namely, the income from marketing (operating income, $y_1^1$) and other income ($y_2^1$). Similarly, the insurance department employs the specific input of operating expenditures ($x_1^2$) to produce total insurance income ($y_1^2$). The extension department uses operating expenditures ($x_1^3$) to carry out extension services ($y_1^3$), farmers' education ($y_2^3$), and rural welfare programs ($y_3^3$). The credit departments employ two inputs, namely, loanable funds ($x_1^4$) and capital expense ($x_2^4$) to produce two desirable outputs, i.e., total loans ($y_1^4$) and non-loan receipts ($y_2^4$), and one undesirable output, namely, non-performing loans ($b_1^4$).

Among the four departments, there are two shared inputs: labor ($x_1^s$), which is defined as the number of employees and managers, and fixed assets ($x_2^s$), which include the net present values of land, buildings, machines, equipment and other fixed capital.

Finally, it is known that the uncontrollable variables (i.e., environmental variables) such as location characteristics, labor union power, and government regulations, etc. (Fried et al., 1999) are not traditional inputs, but could influence the efficiency of a DMU. Therefore, if the DMUs operate in different environments, the following constraint can be added to the MDEA model to incorporate such effects:

$$\sum_{k=1}^{K} z_k^i e_{k,h_i}^i \le e_{h_i}^i, \qquad h_i = 1,......,H_i. \tag{20}$$

where $e_{k,h_i}^i$, $h_i = 1,......,H_i$, are the environmental variables with a positive effect on the efficiency faced by the *i*th activity of DMU *k*. Here, the ratio of associate members to total members is used as a proxy for the location effects because a TFC that is located in an urban area tends to have more non-regular (or associate) members than regular members. Table 1 provides the sample means and standard deviations for all variables and the relationship for them is given in Figure 1.

## 4. EMPIRICAL RESULTS

Three modifications are made prior to our applications. First, the impacts of the environmental variables are normally undetermined. Since the TFCs with higher ratios of associate members are more likely to be located in the urban areas with tougher competition from the commercial banks, their credit departments are expected to perform better than those with lower ratios. Therefore, the sign of the environmental variable is expected to be positive for the credit department. However, for the other three departments, the impacts of the environmental variables are undetermined. Therefore, the inequality signs in constraint (20) for the marketing, extension and insurance activities are changed into equalities.

Second, the weights in the objective function of the MDEA model (i.e., $w^i$ in equation (7)) are viewed as pre-specified parameters. Tsai and Mar Molinero (1998, 2002) believed that activities may not be considered to be equally important, so they adopted the proportions of individual activities' current operating expenditures in relation to the total expenditures as the initial weights. Diez-Ticio and Mancebon (2002) and Yu and Fan (2006), on the other hand, chose to weight various activities equally, with the aim being not to introduce into the analysis any subjective element that is difficult to justify. Here we adopt both specifications and compare their differences. In Tsai and Mar Molinero's specification, the $w^i$ s are given by the survey results from the Council of Agriculture, the supervisory institution for the TFCs. They are 0.28, 0.11, 0.27, and 0.34 for the marketing, insurance, extension and credit departments, respectively.

Third, for the unknown allocation of shared inputs, i.e., $\mu_{k,l}^i$, proper bounds should be specified to obtain feasible solutions for these fractions (Cook et al., 2000). For the labor share, the number of employees associated with each activity is available in the published yearbook of the TFC. Therefore, the ratios can be computed for the entire sample period for each TFC, from which the largest and smallest ones are chosen as the upper and lower bounds for the shares of the labor input. These bounds are also used as the bounds for the other shared input, i.e., the fixed assets.

Table 2 reports the summary statistics of inefficiencies where unequal weights are specified. Note that the inefficiency score $\beta$ should be larger than or equal to zero and that a higher score indicates a more inefficient status. The results diverge from 0.000 to 0.398 with a sample mean of 0.222. This suggests that on average there is room for TFCs to expand their outputs by 22.2% and decrease their inputs and undesirable outputs by the same proportion to become a fully efficient unit. The second column also shows that, out of the 201 TFCs, only 13 (6.47%) can be considered to be globally efficient.

As for individual activities, the performances of marketing and credit departments are in general much better than those of insurance and extension departments. The mean values of the insurance and extension departments' $\beta$ are 0.412 and 0.559, respectively, with high standard deviations, while the means of the other two departments are 0.042 and 0.041 with much smaller standard deviations. The priority given by the managers of TFCs to the marketing and credit departments, as a consequence of earning more profit, could be the major reason which explains this phenomenon. Nevertheless, the lower average and the more divergent performance of the extension and insurance departments suggest that the challenge to improve the overall efficiency lies in these two departments.

We also compute the efficiency scores using equal weights following Diez-Ticio and Mancebon (2002). The results in Table 3 show that the mean value of overall inefficiency is 0.263 with 0.043, 0.041, 0.420, and 0.550 for the marketing, credit, insurance, and extension departments, respectively. When compared with the results presented in Table 2, it can be found that the overall efficiency deteriorates significantly because the weights assigned to the activities with high efficiency scores are lower than the weights assigned to the activities with low efficiency scores. However, the mean values for the four activities do not change significantly. In addition, Table 3 presents the Kendall rank correlation coefficients between the two measurements and the results strongly reject the null hypothesis of independence in ranking. This implies that changing the priority regarding individual activities will neither influence the mean values nor their relative rankings.

For comparison purposes, the traditional DEA efficiency scores are computed and listed in the last column of Table 2. It can be found that the mean value of the traditional DEA is very close to zero with 87.56% of TFCs being located on the technology frontier. The high efficiency scores may be explained in terms of two aspects. First, as Diez-Ticio and Mancebon (2002) indicated, the achievement of maximum efficiency in the MDEA model requires that good productive behavior be demonstrated on the part of every activity, while in the traditional DEA model it is possible for them to compensate each other. Thus, a DMU will reach the production frontier in the traditional DEA model if it in only one of the activities that it carries out outperforms the other DMUs. Second, it is known that, for any fixed sample size, the greater the number of input and output variables in a DEA, the higher the dimensionality of the programming solution space, and thus the higher the scores for the DMUs (Jenkins and Anderson, 2003; Huhhes and Yaisawarng, 2004). In other words, the traditional DEA model which incorporates all activities' input and output variables into an integrated model has less discriminating power than the MDEA model. Although the MDEA model is much more technically demanding, it is more discriminating than the traditional DEA model.

Next, the nature of the RTS of TFCs is explored in Table 4 where the numbers and percentages of TFCs operating under decreasing, constant and increasing RTS by activity are summarized. It can be found that the status of RTS differs considerably among the four activities. Table 3 also indicates that more than 50 percent of TFCs operate under insufficient scales in their credit, insurance, and extension departments, suggesting that their efficient performance in three out of four departments can be improved through expansion. However, for the marketing department, DRS prevail suggesting that this department is either over-capitalized or over-staffed, and should be contracted in most TFCs. Besides the implications on the need for intra-TFC realignment, this result suggests that the marketing service of agricultural products at the local level has reached a limit. It is thus necessary for the marketing services to operate over broader geographical areas through strategic alliances or consolidations into a regional or even national operation.

Finally, the overall status of RTS can be obtained by aggregating the RTS results for all four activities. Table 4 also demonstrates that only 1.5 percent of the TFCs operate under the optimal scale. The number of TFCs considered to be too large (i.e., DRS) is almost identical with the number of those considered to be too small (i.e., IRS). Therefore, although recent legislation has increased the pressures on TFCs to consolidate, it is very important to take into account the discrepancies in RTS to ensure that the TFCs are operating under the most productive scale.

In terms of the policy aspect, the results above suggest that the TFCs should pay more attention to improving the efficiency of their insurance and extension departments despite the fact that these two departments are by nature non-profit-oriented operations. As for the improvements in the scale efficiency, it is found that in most TFCs the four departments operate under an improper returns-to-scale status. Thus, how to enhance the inter-firm networking is as important as the intra-firm consolidation. Policies that promote the consolidation of TFCs will not be sufficient to meet the public goal of institutional reform.

## 5. CONCLUSIONS

This study proposes a modified MDEA model that decomposes the efficiency measures into components that reflect the multi-purpose characteristics or multi-activity nature of a production entity. The directional distance functions are used to construct a non-radial measure of performance in which the optimal input/output adjustment and the optimal allocation of shared inputs among different activities are taken into consideration simultaneously. In this article, an empirical study on Taiwan's farmers' cooperatives is used to offer policy suggestions as to how TFCs can effectively reallocate their fixed resources among different departments in a team production environment. For a team production system, such a measure can be used for rewarding the individual groups of a team based on their relative contributions to the team's overall performance.

The empirical results suggest that there exist significant divergences in terms of the performance among the four departments of the TFCs. The MDEA overcomes the inflexibility of alternative approaches by allowing the allocation of shared inputs to be optimally determined. It ensures that multi-activity efficiencies are fully realized by first generating efficiency scores based on a comparison of individual activities among peers and then embedding them into a maximization of the overall achievement with constraints on shared inputs. In so doing, an individual department benefits from an additional efficiency gain which can be difficult to achieve without reallocating the shared inputs among its team members. Furthermore, the wide divergences in the RTS statuses among the TFCs and their four departments warrant continuing deregulation of the TFCs by easing restrictions on their ability to acquire or consolidate with other TFCs and to operate over broader geographical areas. To our knowledge, the MDEA technique has been applied to the performance evaluation of the education and healthcare sectors, but this is the first time it has been applied to agricultural cooperatives. Due to the particular characteristics of agricultural production, not only do the farmers' cooperatives engage in several parallel missions, but the farmers themselves are often involved in several business activities at the same time for various reasons. Thus the proposed method can also be applied to a wide range of agribusiness entities in the future.

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Table1. Summary Statistics of All Variables

| Category | Variable name | unit | Mean | Std. Dev. |
|---|---|---|---|---|
| **1. Marketing department** | | | | |
| Specific inputs | Operating expenditure ( $x_1^1$ ) | NT$ millions | 83.27 | 113.06 |
| Outputs | Operating income ( $y_1^1$ ) | NT$ millions | 85.11 | 114.94 |
| | Other income ( $y_2^1$ ) | NT$ millions | 4.76 | 8.99 |
| **2. Insurance department** | | | | |
| Specific inputs | Operating expenditure ( $x_1^2$ ) | NT$ millions | 1.36 | 3.88 |
| Outputs | Operating income ( $y_1^2$ ) | NT$ millions | 2.26 | 3.75 |
| **3. Extension department** | | | | |
| Specific inputs | Operating expenditure ( $x_1^3$ ) | NT$ millions | 17.33 | 30.67 |
| Outputs | No.of extension duties ( $y_1^3$ ) | Thousands | 0.33 | 0.37 |
| | Farmers' education ( $y_2^3$ ) | NT$ millions | 2.11 | 3.22 |
| | Welfare activity( $y_3^3$ ) | Thousands of persons | 5.13 | 10.69 |
| **4. Credit department** | | | | |
| Specific inputs | Loanable funds ( $x_1^4$ ) | NT$ millions | 4,931.87 | 4,551.49 |
| | Capital expense ( $x_2^4$ ) | NT$ millions | 23.72 | 18.13 |
| Desirable outputs | Total loans ( $y_1^4$ ) | NT$ millions | 1,857.38 | 1,973.20 |
| | Non-loan receipts ( $y_2^4$ ) | NT$ millions | 2,885.12 | 2,798.16 |
| Undesirable outputs | Non-performing loans( $b_1^4$ ) | NT$ millions | 365.82 | 442.08 |
| **5. Shared input** | | | | |
| | Labor ( $x_1^s$ ) | No. of persons | 67.91 | 37.20 |
| | Fixed assets ( $x_2^s$ ) | NT$ millions | 236.59 | 258.79 |
| **6. Environmental variable** | | | | |
| | Membership ratio( $e_1$ ) | % | 36.50 | 23.96 |

Table 2. Summary Statistics of Inefficiency Measures of TFCs

| | Multi-activity DEA | | | | | Traditional DEA |
|---|---|---|---|---|---|---|
| | Overall | Marketing | Insurance | Extension | Credit | |
| Mean | 0.222 | 0.041 | 0.412 | 0.559 | 0.042 | 0.003 |
| SD | 0.112 | 0.036 | 0.272 | 0.331 | 0.051 | 0.008 |
| Max | 0.398 | 0.207 | 0.981 | 0.987 | 0.254 | 0.043 |
| Min | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| No. of fully efficient units | 13 | 52 | 29 | 31 | 84 | 176 |
| % of fully efficient units | 6.47 | 25.87 | 14.43 | 15.42 | 41.79 | 87.56 |

Table 3. Comparison for Different Specifications on Efficiency Weights

| | Overall | Marketing | Insurance | Extension | Credit |
|---|---|---|---|---|---|
| Using COA weights | 0.222 | 0.041 | 0.412 | 0.559 | 0.042 |
| Using equal weights | 0.263 | 0.043 | 0.420 | 0.550 | 0.041 |
| t statistics[a] | 3.459* | 0.455 | 0.291 | -0.274 | -0.224 |
| Kendall's rank test | 0.796* | 0.930* | 0.971* | 0.978* | 0.967* |

a. the difference in means of these two groups of efficiencies scores are compared.
* Significant at the 1%

Table 4. Numbers and Percentages in Total of TFCs Experiencing DRS, CRS or IRS

| | Overall | Marketing | Insurance | Extension | Credit |
|---|---|---|---|---|---|
| IRS | 92(45.8%) | 65(32.3%) | 105(52.2%) | 106(52.7%) | 122(60.7%) |
| CRS | 3 (1.5%) | 5 (2.5%) | 33(16.4%) | 5 (2.5%) | 6 (3.0%) |
| DRS | 106(52.7%) | 131(65.2%) | 63(31.3%) | 90(44.8%) | 73(36.3%) |

a Percentages may not add to 1 because of rounding.

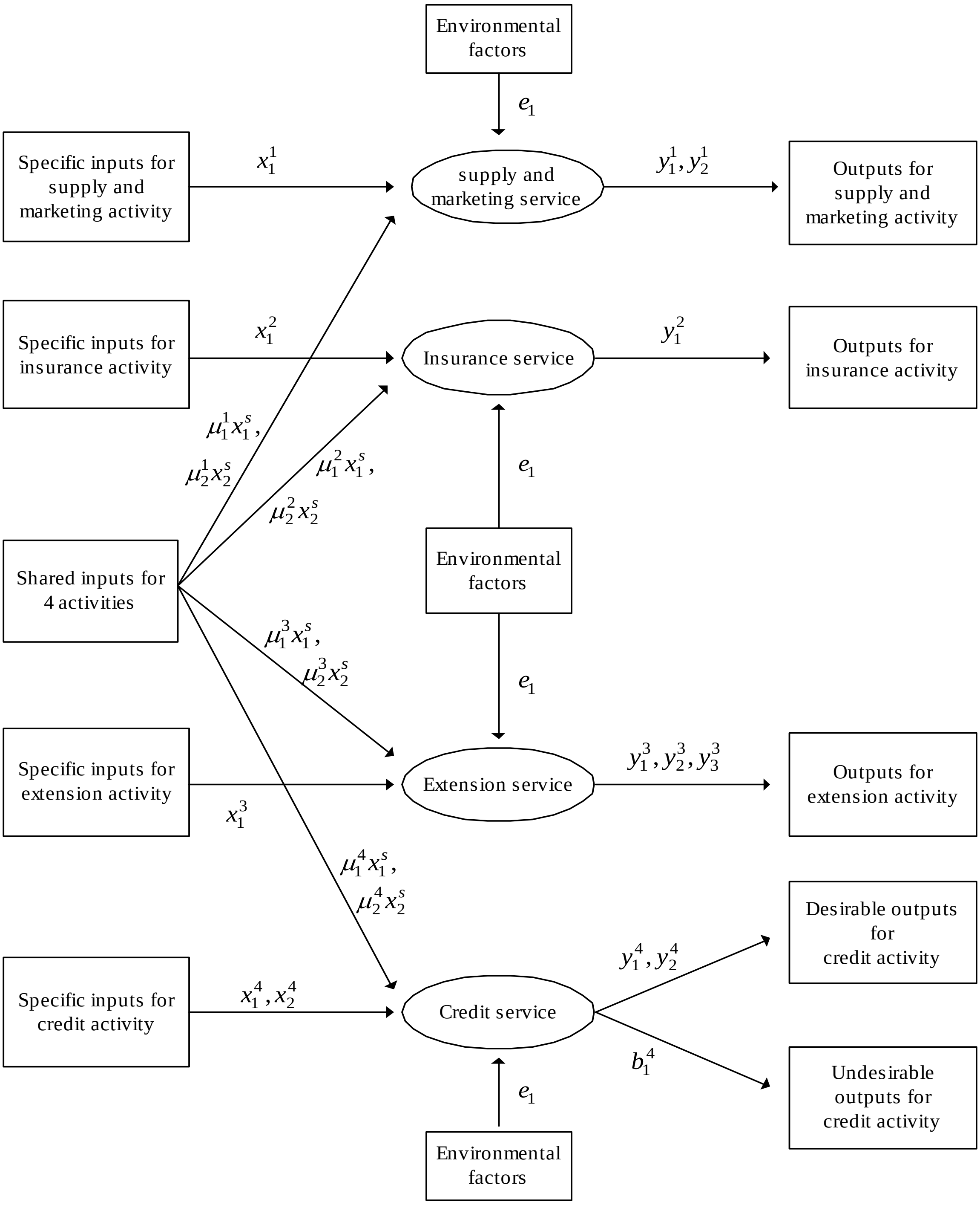


Figure 1. The Team Production Process of a TFC

## Appendix A.

For notational ease, the proof is shown in the matrix form. In addition to the notation defined above, we also denote $u^i = (u_1^i, u_2^i, \cdots, u_{M_i}^i)$, $v^i = (v_1^i, v_2^i, \cdots, v_{N_i}^i)$, $\rho^i = (\rho_1^i, \rho_2^i, \cdots, \rho_{R_i}^i)$, and $v^s = (v_1^s, v_2^s, \cdots, v_L^s)$. The technical inefficiency measure is defines as follows:

$$TIE_{k'} = \frac{-\sum_{i=1}^{I} u^i y_{k'}^i + \sum_{i=1}^{I} v^i x_{k'}^i + \sum_{i=1}^{I} \rho^i b_{k'}^i + v^s x_{k'}^s + \sum_{i=1}^{I} \delta_{k'}^i}{\sum_{i=1}^{I} u^i y_{k'}^i + \sum_{i=1}^{I} v^i x_{k'}^i + \sum_{i=1}^{I} \rho^i b_{k'}^i + v^s x_{k'}^s}$$

$$= \frac{(-u^1 y_{k'}^1 + v^1 x_{k'}^1 + \rho^1 b_{k'}^1 + v^s \mu_{k'}^1 x_{k'}^s + \delta_{k'}^1) + \cdots + (-u^I y_{k'}^I + v^I x_{k'}^I + \rho^I b_{k'}^I + v^s \mu_{k'}^I x_{k'}^s + \delta_{k'}^I)}{\sum_{i=1}^{I} u^i y_{k'}^i + \sum_{i=1}^{I} v^i x_{k'}^i + \sum_{i=1}^{I} \rho^i b_{k'}^i + v^s x_{k'}^s}$$

$$= \frac{(-u^1 y_{k'}^1 + v^1 x_{k'}^1 + \rho^1 b_{k'}^1 + v^s \mu_{k'}^1 x_{k'}^s + \delta_{k'}^1)}{\sum_{i=1}^{I} u^i y_{k'}^i + \sum_{i=1}^{I} v^i x_{k'}^i + \sum_{i=1}^{I} \rho^i b_{k'}^i + v^s x_{k'}^s} \times \frac{u^1 y_{k'}^1 + v^1 x_{k'}^1 + \rho^1 b_{k'}^1 + v^s \mu_{k'}^1 x_{k'}^s}{u^1 y_{k'}^1 + v^1 x_{k'}^1 + \rho^1 b_{k'}^1 + v^s \mu_{k'}^1 x_{k'}^s}$$

$$+ \cdots + \frac{(-u^I y_{k'}^I + v^I x_{k'}^I + \rho^I b_{k'}^I + v^s \mu_{k'}^I x_{k'}^s + \delta_{k'}^I)}{\sum_{i=1}^{I} u^i y_{k'}^i + \sum_{i=1}^{I} v^i x_{k'}^i + \sum_{i=1}^{I} \rho^i b_{k'}^i + v^s x_{k'}^s} \times \frac{u^I y_{k'}^I + v^I x_{k'}^I + \rho^I b_{k'}^I + v^s \mu_{k'}^I x_{k'}^s}{u^I y_{k'}^I + v^I x_{k'}^I + \rho^I b_{k'}^I + v^s \mu_{k'}^I x_{k'}^s}$$

$$= \frac{(-u^1 y_{k'}^1 + v^1 x_{k'}^1 + \rho^1 b_{k'}^1 + v^s \mu_{k'}^1 x_{k'}^s + \delta_{k'}^1)}{u^1 y_{k'}^1 + v^1 x_{k'}^1 + \rho^1 b_{k'}^1 + v^s \mu_{k'}^1 x_{k'}^s} \times \frac{u^1 y_{k'}^1 + v^1 x_{k'}^1 + \rho^1 b_{k'}^1 + v^s \mu_{k'}^1 x_{k'}^s}{\sum_{i=1}^{I} u^i y_{k'}^i + \sum_{i=1}^{I} v^i x_{k'}^i + \sum_{i=1}^{I} \rho^i b_{k'}^i + v^s x_{k'}^s}$$

$$+ \cdots + \frac{(-u^I y_{k'}^I + v^I x_{k'}^I + \rho^I b_{k'}^I + v^s \mu_{k'}^I x_{k'}^s + \delta_{k'}^I)}{u^I y_{k'}^I + v^I x_{k'}^I + \rho^I b_{k'}^I + v^s \mu_{k'}^I x_{k'}^s} \times \frac{u^I y_{k'}^I + v^I x_{k'}^I + \rho^I b_{k'}^I + v^s \mu_{k'}^I x_{k'}^s}{\sum_{i=1}^{I} u^i y_{k'}^i + \sum_{i=1}^{I} v^i x_{k'}^i + \sum_{i=1}^{I} \rho^i b_{k'}^i + v^s x_{k'}^s}$$

$$= TIE_{k'}^1 \times w^1 + \cdots + TIE_{k'}^I \times w^I$$

$$= \sum_{i}^{I} w^i TIE_{k'}^i$$

## Appendix B.

Here, we use the activity 1 of DMU $k$ as an example to present this proof. The technical inefficiency of activity 1 is defined as follows:

$$TIE_k^1 = \frac{-u^1 y_k^1 + v^1 x_k^1 + \rho^1 b_k^1 + v^s \mu_k^1 x_k^s + \delta_k^1}{u^1 y_k^1 + v^1 x_k^1 + \rho^1 b_k^1 + v^s \mu_k^1 x_k^s}.$$

So the technical efficiency can be calculated by the following formulation and should not exceed 1.

$$TE_k^1 = 1 - \frac{-u^1 y_k^1 + v^1 x_k^1 + \rho^1 b_k^1 + v^s \mu_k^1 x_k^s + \delta_k^1}{u^1 y_k^1 + v^1 x_k^1 + \rho^1 b_k^1 + v^s \mu_k^1 x_k^s} \leq 1.$$

Then, we have

$$\frac{(u^1 y_k^1 + v^1 x_k^1 + \rho^1 b_k^1 + v^s \mu_k^1 x_k^s) - (-u^1 y_k^1 + v^1 x_k^1 + \rho^1 b_k^1 + v^s \mu_k^1 x_k^s + \delta_k^1)}{u^1 y_k^1 + v^1 x_k^1 + \rho^1 b_k^1 + v^s \mu_k^1 x_k^s} \leq 1$$

$$\Rightarrow 2u^1 y_k^1 - \delta_k^1 \leq u^1 y_k^1 + v^1 x_k^1 + \rho^1 b_k^1 + v^s \mu_k^1 x_k^s$$

$$\Rightarrow -u^1 y_k^1 + v^1 x_k^1 + \rho^1 b_k^1 + v^s \mu_k^1 x_k^s + \delta_k^1 \geq 0$$

Thus, we obtain the constraint (17) as $i$ =1. Note that we can use the similar method to show that the combination of the constraints in equation (17) ensures that the aggregate efficiency for DMU $k$ should not exceed 1.