Optimal Groundwater Extraction under the Linear Response and Plateau Technology

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Groundwater is one of the most cited examples of a common property resource. Conventional theory suggests that commonality or nonexcludability leaves resource owners or users little incentive to care about the future, resulting in overexploitation of the resource. Empirical evidence abounds, however, that the welfare gain from socially managing groundwater resources, as opposed to competitive exploitation with rent dissipation, is practically negligible (e.g., Allen and Gisser 1984; Gisser and Sanchez 1980; Lee et al. 1981; Nieswiadomy 1985). Koundouri (2004) in a thorough review of this literature concludes that the magnitude of such welfare improvements depends crucially on the elasticity of the derived water demand; a highly inelastic water demand entails negligible welfare gain. This important finding points us to a way to reconcile theory and empirical evidence; namely, by introducing into groundwater extraction models meaningful structure to arrive at an inelastic water demand, closing the gap between the social and private rates of extraction.

In this paper we incorporate into a simple groundwater mining model some plausible agronomic and hydrologic assumptions, which can lead to inelastic water demand. Inelastic water demand has been persistently found in the agricultural economics literature (e.g., Berbela and Gómez-Limón 2000; Bontemps and Couture 2002; Clark et al. 1986; Gardner and Young 1984; Moore et al. 1994; Shumway 1973). More interestingly, most of the studies cited above have found that water demand is particularly inelastic when water price is low. Our point of departure therefore is to look for a production function that entails inelastic water demand at low price. Here we assume that the crop production technology is approximated by a one-factor linear response and plateau (LRP) model in which yield responses linearly to applied water until some other factors become limiting and yield taps into a plateau. The LRP model derives its origin from the agronomic law of the minimum, also known as Liebig's law, which states yield responses to each factor until some other factor is limiting. Under such technology, water demand is perfectly inelastic up to the point where water price is too high to sustain profitable irrigation.

Another assumption we introduce into the model is that water availability is constrained by well-yield capacity, i.e., the maximal amount of water that can be pumped from the well in a given period. Well-yield capacity depends on some hydrological conditions of the aquifer. This is an important condition that unfortunately has largely been ignored in the economics literature of groundwater management. In many areas of U.S. where groundwater use is intense water demand frequently fails to be satisfied because of limited well-yield capacity; for example, drilling multiple wells is a common practice in the High Plains region where the Ogallala aquifer is the mainstay of agriculture.

The rest of the paper studies dynamic water use patterns in a simple groundwater mining model with the above two assumptions incorporated. Specifically, in a two-farm model both cooperative and non-cooperative solutions are derived and then compared so that insight is gained into welfare implications of commonality. We conclude with a discussion of groundwater management policy.

The two-farm model

Consider two adjacent farms $i = 1, 2$ overlying the same aquifer. By the Rule of Capture, both farms have unlimited access to the groundwater in the aquifer. The crop production technology adopted by the farms are approximated by a LRP function. Following Dillon (1977), the LRP function is written

$$F_i(w_i) = \begin{cases} a_i + b_i w_i & \text{for } w_i \in [0, W_i) \\ a_i + b_i W_i & \text{for } w_i \in [W_i, +\infty) \end{cases} \quad (1.1)$$

where w_i is the amount of water applied to the crop, a_i crop yield corresponding to non-irrigated or dryland farming, b_i the marginal product of applied water, and $W_i > 0$ the minimal amount of applied water needed for yield to reach the plateau.

Let $\varepsilon_i \in (0,1)$ denote irrigation efficiency. One unit of water pumped from the aquifer, that is, amounts to only ε_i units of water applied to the crop. Production function (1.1) thus can be expressed in terms of the amount of water pumped from the aquifer

$$G_i(x_i) = \begin{cases} a_i + b_i \varepsilon_i x_i & \text{for } 0 \le x_i \le W_i / \varepsilon_i \\ a_i + b_i W_i & \text{for } x_i > W_i / \varepsilon_i . \end{cases} \quad (1.2)$$

Let $\overline{s}_i$ and s represent the elevations of the farmland surface and water table in the aquifer, respectively. Note that the two adjacent farms may have different elevations, but the water table is the same for both by the “bathtub” analogy. The cost of pumping per unit of water per unit of lift, denoted by γ, is the same for both farms, too. This is an innocuous assumption because unitary pumping cost is determined by the engineering properties of the pump and energy price. Total pumping costs are assumed to be linear in pumping lift $\overline{s}_i - s$, i.e.,

$$P_i(s, x_i) = \gamma(\overline{s}_i - s) x_i . \quad (1.3)$$

Other operating costs are assumed away as they are irrelevant to the dynamic decision of groundwater extraction.

The dynamics of water table is governed by the differential equation

$$\dot{s}(t) = -\kappa \sum_{i=1}^{2} x_i(t), \qquad s(0) = s^0 \text{ given}, \quad (1.4)$$

where κ is a function of the specific yield of the aquifer, which represents the volumetric fraction of the aquifer that is occupied with water. Every unit of water withdrawn from the aquifer lowers water table by κ inches. Note that implicit in (1.4) is a zero recharge rate assumed on the aquifer. In addition, because it is practically impossible to drain an aquifer, we assume the aquifer to be bottomless, i.e., $s(t)$ is unbounded from below.

Assume that crop price is taken as given by the farms and, for simplicity purposes, equals unity. The farms' problems are to determine the optimal path of groundwater extraction that maximizes over an infinite horizon the present value of a stream of profits

$$\int_0^\infty e^{-rt}(G_i(x_i(t)) - P_i(s(t), x_i(t)))dt \tag{1.5}$$

subject to equations (1.2-1.4) and the hydrological constraint

$$0 \le x_i(t) \le X_i, \tag{1.6}$$

where r is the interest rate and X_i well-yield capacity in farm i.

An inspection of the farms' problems reveals that extracting more than W_i / ε_i units of water at any point of time generates no extra revenue but raises pumping costs in the future. An optimal path of extraction, therefore, is necessarily bounded above by W_i / ε_i. With this in mind, the two farms's problems can be written in a familiar form

$$\begin{aligned}
&\max_{x_i(\bullet)} \int_0^\infty e^{-rt}[a_i + [b_i\varepsilon_i - \gamma(\overline{s}_i - s(t))]x_i(t)]dt \\
&\text{s.t.} \quad \dot{s}(t) = -\kappa \sum_{i=1}^{2} x_i(t); \\
&\qquad\quad s(0) = s^0; \\
&\qquad\quad 0 \le x_i(t) \le Z_i = \min\{X_i, W_i / \varepsilon_i\}; \\
&\qquad\quad i = 1, 2.
\end{aligned} \tag{P.1}$$

The current-value Hamiltonians of (P.1) are

$$H_i = a_i + [b_i\varepsilon_i - \gamma(\overline{s}_i - s) - \kappa\lambda_i]x_i - \kappa\lambda_i x_j \text{ , } i, j = 1,2 \text{ and } i \neq j \text{ ;} \tag{1.7}$$

where λ_i are the current-value shadow price of the groundwater resource to farm i .

Maximization of (1.7) with respect to the constraint set $0 \leq x_i \leq Z_i$ yields the necessary condition

$$x_i(t) = \begin{cases} Z_i & \text{if } c_i < b_i\varepsilon_i \\ 0 & \text{if } c_i \geq b_i\varepsilon_i \end{cases} \text{ , } i = 1,2 \tag{1.8}$$

where $c_i = \gamma(\overline{s}_i - s) + \kappa\lambda_i$ is marginal economic cost composed of marginal pumping cost $\gamma(\overline{s}_i - s)$ and marginal user cost $\kappa\lambda_i$.

The canonical equations are (1.4) and

$$\dot{\lambda}_i(t) = r\lambda_i(t) - \gamma_i x_i(t) \text{ .} \tag{1.9}$$

The necessary transversality condition is

$$\lim_{t \to +\infty} e^{-rt} H_i(t) = 0 \text{ .} \tag{1.10}$$

The sufficient trasversality condition is that given $\tilde{x}_j(t)$ for any *admissible* pair $(\tilde{s}(t), \tilde{x}_i(t))$,

$$\lim_{t \to +\infty} e^{-rt} \lambda_i(t)(s(t) - \tilde{s}(t)) \leq 0 \text{ .} \tag{1.11}$$

The key to solving (P.1) is to understand the properties of the paths of λ_i . Indeed, λ_i are nonnegative throughout as shown in the following lemma.

Lemma 1. *The current-value shadow prices of groundwater stock in problem (P.1) are nonnegative, i.e.,* $\lambda_i(t) \geq 0, \ \forall t \geq 0$.

This makes economic sense in that the price of a freely disposable good should always be nonnegative. The proof of it is provided in Appendix 1.

It follows immediately that marginal economic cost is nondecreasing in time since $\dot{c}_i = \kappa r \lambda_i + \gamma \kappa x_j \geq 0$. Ruling out the case in which marginal economic cost c_i will never exceed marginal revenue of applied water $b_i \varepsilon_i$, the maximum of the Hamiltonian can be written

$$x_i(t) = \begin{cases} Z_i & \text{for} \quad t < T_i \\ 0 & \text{for} \quad t \geq T_i \end{cases}; \quad i = 1,2; \tag{1.12}$$

where $T_i \in [0, +\infty)$ is determined by the following complementary slackness condition

$$\begin{aligned} & T_i \geq 0; \\ & \gamma(\bar{s}_i - s(T_i)) + \kappa \lambda_i(T_i) - b_i \varepsilon_i \geq 0; \\ & T_i[\gamma(\bar{s}_i - s(T_i)) + \kappa \lambda_i(T_i) - b_i \varepsilon_i] = 0. \end{aligned} \tag{1.13}$$

By the sufficient trasversality condition (1.11) one can establish that $\lambda_i(t) = 0, \ \forall t \geq T_i$. Thus, (1.13) reduces to

$$\begin{aligned} & T_i \geq 0; \\ & \gamma(\bar{s}_i - s(T_i)) - b_i \varepsilon_i \geq 0; \\ & T_i[\gamma(\bar{s}_i - s(T_i)) - b_i \varepsilon_i] = 0. \end{aligned} \tag{1.14}$$

When $T_i > 0$, farm i practices irrigated farming until marginal pumping cost $\gamma_i(\bar{s}_i - s(T_i))$ equals marginal revenue $b_i \varepsilon_i$. When marginal pumping cost exceeds marginal revenue at the initial state, i.e., $\bar{s}_i - s^0 > b_i \varepsilon_i$; irrigation is not profitable at all and dryland farming is practiced throughout, i.e., $T_i = 0$.

Assume, without loss of generality, that $\bar{s}_1 - b_1 \varepsilon_1 / \gamma \geq \bar{s}_2 - b_2 \varepsilon_2 / \gamma$. Since $\bar{s}_i - b_i \varepsilon_i / \gamma$ solves the equation $\gamma(\bar{s}_i - s) = b_i \varepsilon_i$, it is the critical water table at which marginal pumping cost equals the marginal revenue. By definition, a relatively lower critical water table results from more favorable technical and environmental conditions for irrigation, in terms of land surface elevation $\bar{s}_i$, marginal product of applied water b_i, irrigation efficiency ε_i, and unitary pumping costs γ.

It follows immediately from this assumption and (1.14) that $s(T_1) \geq s(T_2)$, which in turn implies $T_1 \leq T_2$ since water table $s(\bullet)$ is nonincreasing in time. As expected, farm 1, who is assumed to have less favorable irrigation conditions and therefore a higher critical water table, will stop irrigation at least as early as farm 2 will. Integrating (1.4) and substituting from (1.12), the path of the water table then can be written

$$s(t) = \begin{cases} s^0 - \kappa t(Z_1 + Z_2) & \text{for } t \in [0, T_1) \\ s^0 - \kappa T_1 Z_1 - \kappa t Z_2 & \text{for } t \in [T_1, T_2) \\ s^0 - \kappa T_1 Z_1 - \kappa T_2 Z_2 & \text{for } t \in [T_2, +\infty). \end{cases} \tag{1.15}$$

And, given the terminal condition $\lambda_i(T_i) = 0$ the current-value shadow price can be solved in terms of the switching times

$$\lambda_i(t) = \begin{cases} \frac{\gamma}{r} Z_i \ (1\text{-}e^{-r(T_i - t)}) & \text{for } t \in [0, T_i) \\ 0 & \text{for } t \in [T_i, +\infty); \end{cases} \qquad i = 1, 2. \tag{1.16}$$

Note that the commonly owned stock of groundwater resource may be of different value to the two farms because of the difference in upper bound Z_i and switching time T_i.

It remains to solve for the Nash Equilibrium switching times for the two farms. Substituting for $s(T_i)$ from (1.15) into (1.14) to obtain

$$T_i \ = \max\{0, \tau_i\}, \quad i = 1, 2 \tag{1.17}$$

where

$$\tau_1 = \frac{s^0 - (\bar{s}_1 - b_1 \varepsilon_1 / \gamma)}{\kappa(Z_1 + Z_2)}; \tag{1.18}$$

$$\tau_2 = \frac{s^0 - \kappa \tau_1 Z_1 - (\bar{s}_2 - b_2 \varepsilon_2 / \gamma)}{\kappa Z_2}. \tag{1.19}$$

Farm 1, who is assumed to have less favorable irrigation conditions and a higher critical water table, abandons irrigated farming before farm 2 does so, simply because a higher critical water table will be reached earlier. As shown in (1.18), farm 1's switching time is equal to the amount of time needed for the water table to decline from the initial level s^0 to the critical level $\bar{s}_1 - b_1\varepsilon_1/\gamma$ at a constant rate $\kappa(Z_1+Z_2)$. In (1.19), farm 2's switching time equals the amount of time needed for the water table to decline from the initial level s^0 less the drawdown caused by farm 1 $\kappa\tau_1 Z_1$ to the critical level $\bar{s}_2 - b_2\varepsilon_2/\gamma$ at a constant rate κZ_2.

It is worth pointing out that the Markov-Perfect-Nash equilibrium solution derived above coincides with the solution in a rent-dissipation model in which the farms maximize their own instantaneous profits. The intuition behind this result is that marginal user cost is ineffective in determination of the extraction rate because the constraints are binding whenever marginal user cost is positive.

Next we consider that the two farms jointly determine the rates of extraction to maximize the present value of their total profits over an infinite horizon. Formally, the problem is written

$$\begin{aligned}
&\max_{\{x_i(\bullet)\}_{i=1,2}} \int_0^\infty e^{-rt}\sum_{i=1}^{2}[a_i + [b_i\varepsilon_i - \gamma(\bar{s} - s(t))]x_i(t)]dt \\
&\text{s.t.} \quad \dot{s}(t) = -\kappa\sum_{i=1}^{2} x_i(t); \\
&\qquad\ s(0) = s^0; \\
&\qquad\ 0 \le x_i \le Z_i = \min\{X_i, W_i / \varepsilon_i\}; \\
&\qquad\ i = 1, 2.
\end{aligned} \tag{P.2}$$

The joint current-value Hamiltonian is

$$H = \sum_{i=1}^{2} a_i + [b_i\varepsilon_i - \gamma(\bar{s}_i - s) - \kappa\lambda]x_i \tag{1.20}$$

where λ represents the current-value *social* shadow price of the groundwater stock.

Maximization of the Hamiltonian subject to the constraint set $0 \le x_i \le Z_i$ yields

$$x_i(t) = \begin{cases} Z_i & \text{if } c_i < b_i\varepsilon_i \\ 0 & \text{if } c_i \ge b_i\varepsilon_i \end{cases}, \; i = 1, 2; \tag{1.21}$$

where $c_i = \gamma(\overline{s}_i - s) + \kappa\lambda$ is marginal economic cost composed of marginal pumping cost $\gamma(\overline{s}_i - s)$ and marginal user cost $\kappa\lambda$. Note that (1.21) is different from (1.12) in that marginal user cost $\kappa\lambda$ in the cooperative model is different from – and presumably larger than – $\kappa\lambda_i$ in the non-cooperative model. As is well known, it is this difference that drives a wedge between the social and private rates of extraction.

The other necessary conditions includes equation (1.4) and

$$\dot{\lambda}(t) = r\lambda(t) - \gamma\sum_{i=1}^{2} x_i(t); \tag{1.22}$$

and the transversality condition,

$$\lim_{t\to+\infty} e^{-rt} H(t) = 0. \tag{1.23}$$

The sufficient transversality condition is that for any *admissible* triplet ($\tilde{s}(t), \tilde{x}_1(t), \tilde{x}_2(t)$),

$$\lim_{t\to+\infty} e^{-rt} \lambda(t)(s(t) - \tilde{s}(t)) \le 0, \tag{1.24}$$

where $\tilde{s}(t)$ is the stock path associated with the control paths $\tilde{x}_1(t)$ and $\tilde{x}_2(t)$ by way of (1.4).

We use a star notation to distinguish the solution paths of the state, costate, and control variables in the current model from those in the non-cooperative production model above. As in the non-cooperative model, the nonnegativity of the current-value shadow price $\lambda(t)$ implies that marginal economic costs in both farms are nonincreasing in time since $\dot{c}_i = \kappa r\lambda \ge 0$. Ruling out the case in which c_i will never exceed the marginal revenue $b_i\varepsilon_i$, the maximum of the Hamiltonian is written

$$x_i^*(t)=\begin{cases} Z_i & \text{for} \quad t<T_i^* \\ 0 & \text{for} \quad t\geq T_i^* \end{cases};\ i=1,2 \tag{1.25}$$

where T_i^* is determined by the following complementary slackness condition

$$\begin{aligned} &T_i^*\geq 0; \\ &\gamma(\bar{s}_i-s(T_i^*))+\kappa\lambda(T_i^*)-b_i\varepsilon_i\geq 0; \\ &T_i^*[\gamma(\bar{s}_i-s(T_i^*))+\kappa\lambda(T_i^*)-b_i\varepsilon_i]=0. \end{aligned} \tag{1.26}$$

Maintain the assumption that $\bar{s}_1-b_1\varepsilon_1/\gamma_1\geq\bar{s}_2-b_2\varepsilon_2/\gamma_2$. That is to say, farm 1 has relatively less favorable technical and environmental conditions for irrigation and, therefore, a higher critical water table as defined earlier. By (1.26) it must hold that $s(T_1)\geq s(T_2)$, which, on account of the nonincreasing water table, implies that $T_1\leq T_2$. Not surprisingly, the farm with less favorable irrigation conditions will abandon irrigation earlier. By the sufficient transversality condition (1.24) one can arrive at $\lambda(t)=0,\ \forall t\geq T_2$. With the terminal condition, $\lambda(T_2)=0$, the costate equation (1.22) can be solved in terms of the switching times. Integrating (1.22) and substituting from (1.25), the path of the current-value shadow price is written

$$\lambda^*(t)=\begin{cases} \dfrac{\gamma}{r}\displaystyle\sum_{i=1}^{2}Z_i\ (1\text{-}e^{-r(T_i^*-t)}) & \text{for } t\in[0,T_1^*) \\ \dfrac{\gamma}{r}Z_2\ (1\text{-}e^{-r(T_2^*-t)}) & \text{for } t\in[T_1^*,T_2^*) \\ 0 & \text{for } t\in[T_2^*,+\infty) \end{cases}. \tag{1.27}$$

Similarly, the path of water table $s(t)$ can be solved in terms of the switching times T_i^* by substituting from (1.25) and using the initial condition $s(0)=s^0$ as

$$s^*(t)=\begin{cases} s^0-\kappa t(Z_1+Z_2) & \text{for } t\in[0,T_1^*) \\ s^0-\kappa T_1^*Z_1-\kappa tZ_2 & \text{for } t\in[T_1^*,T_2^*) \\ s^0-\kappa T_1^*Z_1-\kappa T_2^*Z_2 & \text{for } t\in[T_2^*,+\infty) \end{cases}. \tag{1.28}$$

Finally, substitute for $\lambda^*(T_i^*)$ and $s^*(T_i^*)$ respectively from (1.27) and (1.28) into (1.26) to obtain the optimal switching times

$$T_i^* = \max\{0, \tau_i^*\}, \; i = 1, 2; \tag{1.29}$$

where τ_i^* solve the following two equations

$$\tau_1^* = \frac{s_0 - (\bar{s}_1 - \frac{b_1\varepsilon_1}{\gamma}) - \frac{\kappa}{r} Z_2 \; (1\text{-}e^{-r(\tau_2^* - \tau_1^*)})}{\kappa(Z_1 + Z_2)}; \tag{1.30}$$

$$\tau_2^* = \frac{s_0 - \kappa\tau_1^* Z_1 - (\bar{s}_2 - b_2\varepsilon_2 / \gamma)}{\kappa Z_2}. \tag{1.31}$$

Note that the solution paths of the control and state variables (1.25) and (1.28) in the cooperative model look almost identical to their respective counterparts (1.12) and (1.15) in the non-cooperative model. The only difference lies in the different switching times as can be seen by contrasting (1.18) to (1.30) and (1.19) to (1.31). Specifically, the difference for farm 1 is

$$\tau_1 - \tau_1^* = \frac{\lambda(T_1^*)}{\kappa(Z_1 + Z_2)} = \frac{Z_2(1\text{-}e^{-r(\tau_2^* - \tau_1^*)})}{r(Z_1 + Z_2)} \geq 0. \tag{1.32}$$

where the equality holds when irrigation is simultaneously abandoned by the two farms, i.e., $\tau_1^* = \tau_2^*$. Subtracting (1.31) from (1.30) yields

$$\tau_2^* - \tau_1^* = \frac{(\bar{s}_1 - \frac{b_1\varepsilon_1}{\gamma}) - (\bar{s}_2 - \frac{b_2\varepsilon_2}{\gamma})}{\kappa Z_2} + \frac{1\text{-}e^{-r(\tau_2^* - \tau_1^*)}}{r}. \tag{1.33}$$

One can verify that $\tau_1^* = \tau_2^*$ if and only if $\bar{s}_1 - b_1\varepsilon_1 / \gamma = \bar{s}_2 - b_2\varepsilon_2 / \gamma$, i.e., if and only if marginal pumping cost equals marginal revenue at the same critical water table for the two farms. Worth noting is that identical farms are a sufficient but not necessary condition for this to hold true.

As one should expect, the difference in switching time shown in (1.33) emanates from the discrepancy between the social and private shadow value of the groundwater resource. In cooperative production when farm 1 quits irrigation the social shadow price of the groundwater remaining in the aquifer is not necessarily zero because it may well have value to another farm; in non-cooperative production, however, a farm quits irrigation only if the resource's shadow value to itself is zero, regardless of the value to another farm.

The difference in switching time for farm 2 is easily obtained by subtracting (1.31) from (1.19)

$$\tau_2 - \tau_2^* = -(\tau_1 - \tau_1^*)\frac{Z_1}{Z_2} \leq 0 . \tag{1.34}$$

where, again, the equality holds whenever $\tau_1^* = \tau_2^*$ or $\bar{s}_1 - b_1\varepsilon_1 / \gamma = \bar{s}_2 - b_2\varepsilon_2 / \gamma$.

Unless the two farms face equally favorable conditions for irrigation so that $\bar{s}_1 - b_1\varepsilon_1 / \gamma = \bar{s}_2 - b_2\varepsilon_2 / \gamma$, non-cooperative production always leads the farm with less favorable irrigation conditions to irrigate longer than is socially desirable, leaving less water for the farm with more favorable irrigation conditions. If instead $\bar{s}_1 - b_1\varepsilon_1 / \gamma = \bar{s}_2 - b_2\varepsilon_2 / \gamma$, it must hold that $\tau_1 = \tau_2 = \tau_1^* = \tau_2^*$. That is to say, when that equality holds both farms pump the groundwater as fast as their respective hydrological and agronomic constraints permit and then switch to dryland farming simultaneously. But, this is exactly what a benevolent social planner would like them to do – commonality is completely innocuous.

Discussion

The backbone of the model is the linear technology with agronomic and hydrologic constraints. Conventional resource exploitation models have developed upon smooth technology, assuming away that constraints could bind. In our model, rather, smoothness is striped away so as to expose the behavioral implications of binding constraints. We have found that when a constraint binds the social rate of extraction is no different than the private, although the time when the constraint stops binding may be different under certain circumstances. It should be emphasized that by no means are the binding constraints being studied pathological. The agronomic constraint derives from Liebig's law of the minimum and is set by some other limiting factors in the field. That well-yield capacity could in reality restrain groundwater availability is best explained by the observation that multiple wells are frequently drilled in a single farm in many irrigation-intensive regions in the United States, such as the Southern High Plains.

Not only do the binding constraints introduced in the model render themselves relevant, but so does the water use behavior to which they lead. We have shown that when a constraint binds water demand is perfectly inelastic except at the switching point. But highly inelastic demand has been persistently found in the agricultural economics literature. Hence, one may infer that the inelastic demand is likely due to some binding constraints which stop demand from increasing even if it remains profitable to do so at the margin. Binding constraints offer a way to rationalize sticky water demand.

The linearity assumption in the model is responsible for the bang-bang behavior. If it is relaxed and we take a nonlinear plateau technology, say, the Liebig-Paris specification (Paris, 1992), after the constraint stops binding the rate of extraction, rather than jumping to zero, will decline along the nonlinear portion of the concave response curve, much as described in conventional groundwater extraction models. The assumption appears strong at first glance. But its practical significance should not be understated. Evidence of a LRP model approximating well the yield response curve abounds – and at least as much as of nonlinear plateau models – in the agronomic and agricultural economics literature [e.g., Bronson et al. (2006) in a recent study find a linear plateau yield-water relation; Dillon (1977) provides a list of empirical studies before the 1970's that support the LRP model]. Even if the actual crop production technology is a nonlinear Liebig model, estimating its curvature can be prohibitive to the producer. But a LRP model always serves as a safe approximation of any nonlinear plateau model. Again, the empirical evidence of inelastic water demand offers support for such reasoning.

When the situations described in our model indeed prevail in reality, conventional corrective measures prescribed for common property resources are seriously challenged. A system of quotas is completely ineffective when the quota is set higher than the binding constraint, and lowers the rate of extraction to a suboptimal level if the quota instead is set lower than the binding constraint. A tax on each unit of water extracted that is set equal to the marginal user costs can in theory internalize the externalities and close the gap between the social and private rates of extraction. Yet, in the situations described in our model, only the switching time needs to be altered. Also, a water tax is effective only when the producer needs to stop irrigation, and anytime before that period it is merely a financial burden on the producer. A tax system varying over time and among tax payers apparently is difficult to implement.

A more constructive approach seems to be something similar to the Conservation Reserve Program (CRP). For regions with heterogeneous hydrological and productive conditions, a payment can be made to producers who are willing to abandon irrigation earlier than they would otherwise. The regime, in light of our findings, has to allow farmland with less favorable irrigation conditions convert to dryland farming earlier, leaving more water to those with more favorable conditions. For regions with relatively homogeneous hydrological and productive conditions, however, there may not exist a need for any interventional effort to slow down water uss—at least not so for the reason of commonality.

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Appendix: Proof of Lemma 1

We assume $\lambda_i(t) < 0$ for some $t = \tau \geq 0$ and then derive a contradiction from the assumption. Because from (1.8) the rate of extraction x_i is nonnegative, an inspection of (1.9) suggests that $\lambda_i(\tau) < 0$ implies $\lambda_i(t) < 0$ for all $t \geq \tau$. Differentiating with respect to time both sides of equation $c_i = \gamma(\bar{s}_i - s) + \kappa\lambda_i$ yields $\dot{c}_i = \kappa r \lambda_i$ and $\ddot{c}_i = \kappa r \dot{\lambda}_i = \kappa r(r\lambda_i - \gamma x_i)$. It follows immediately that $\dot{c}_i(t) < 0$ and $\ddot{c}_i(t) < 0$ for $t \geq \tau$. This implies that $\lim_{t \to +\infty} c_i(t) < b_i \varepsilon_i$, which, on account of (1.8), leads to $\lim_{t \to +\infty} x_i(t) = \min\{X_i, W_i / \varepsilon_i\} > 0$. Because as time goes to infinity water table either decreases linearly in time or remains constant, $\lim_{t \to +\infty} e^{-rt} s(t) = 0$. Hence, the necessary transversality condition (1.10) reduces to $\lim_{t \to +\infty} e^{-rt} \lambda_i(t) = 0$. But by $\lambda_i(t) < 0$ for all $t \geq \tau$, $\lim_{t \to +\infty} e^{-rt} \lambda_i(t) < 0$, a contradiction.