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On the Systemic Nature of Weather Risk

Wei Xu, Guenther Filler, Martin Odening

Department of Agricultural Economics,
Humboldt-Universität zu Berlin, D-10099 Berlin, Germany

Ostap Okhrin

Institute for Statistics and Econometrics,
CASE - Center for Applied Statistics and Economics,
Humboldt-Universität zu Berlin, D-10099 Berlin, Germany

Corresponding author: Wei Xu
Philippstraße 13, D-10115 Berlin, Germany

Email: xu.wei@agrar.hu-berlin.de
Phone: +49(0)30 2093 6431
Fax: +49(0)30 2093 6465

Contributed paper prepared for presentation at the International Association of Agricultural Economists Conference, Beijing, PR China, August 16-22, 2009

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On the Systemic Nature of Weather Risk

Abstract

Systemic weather risk is a major obstacle for the formation of private (nonsubsidized) crop insurance. This paper explores the possibility of spatial diversification of insurance by estimating the joint occurrence of unfavorable weather conditions in different locations. For that purpose copula methods are employed that allow an adequate description of stochastic dependencies between multivariate random variables. The estimation procedure is applied to weather data in Germany. Our results indicate that indemnity payments based on temperature as well as on cumulative rainfall show strong stochastic dependence even at a national scale. Thus the possibility to reduce risk exposure by increasing the trading area of the insurance is limited. Irrespective of their economic implications our results pinpoint the necessity of a proper statistical modeling of the dependence structure of multivariate random variables. The usual approach of measuring stochastic dependence with linear correlation coefficients turned out to be questionable in the context of weather insurance as it may overestimate diversification effects considerably.

Keywords: weather risk, crop insurance, copula
JEL Classification: C14, Q19

1 Introduction

Insurance of weather related production risks is a challenge for agricultural insurers. A well known precondition of insurability is that individual risks are independent or if the covariance among risks is small. This requirement rules out covariate or systemic risk. While such an assumption holds for some types of weather hazards, as for example hail damages, it does not for other types, at least at a regional level. Drought risk is a striking example for a peril that affects most if not all farmers in a whole region. Miranda and Glauber (1997) argue that the existence of systemic weather risk constitutes the main reason for the failure of private crop insurance markets unless efficient and affordable instruments for transferring this risk are available. This conjecture is based on the observation that existing crop insurance programs including drought risk are either subsidized (e.g. United States or Canada) or have negligible participation rates (as in Germany). However, there several possible instruments that allow handling of systemic risks, among them reinsurance or weather derivatives (Xu, Odening and Musshoff 2008). Alternatively, an insurance company may try to spatially diversify systemic weather risk by increasing its trading area. In order to identify appropriate measures for coping with systemic weather risk, it is necessary to quantify these risks. Clearly, the systemic nature of weather risk depends on the scale that one considers. At locations within a country or a state weather events like drought are highly correlated whereas these dependencies vanish at a national or global level. Thus the question arises how the dependence structure of weather events changes as a function of space and distance. The relationship between weather events at different locations is not only relevant when calculating joint losses from the viewpoint

of the insurer. It is also crucial for the hedging effectiveness of weather derivatives that insurance companies may wish to sell to farmers. It has been frequently stressed in the literature that the hedging effectiveness of weather derivatives is eroded by geographical basis risk (e.g. Woodard and Garcia 2008).

In view of the relevance of spatial dependence of weather events for insurance issues it is not surprising that many attempts for a quantification have been made. The usual approach is based on simple correlation coefficients between weather variables or indices which are measured at different locations (weather stations). With these correlation coefficients at hand de-correlation functions can be easily estimated, depicting correlation of weather variables as a function of the distance between weather stations. Examples of this kind of approach can be found in Woodard and Garcia (2008) or in Odening, Musshoff and Xu (2007). Goodwin (2001) applies the same technique to US yield data. Wang and Zhang (2003) also address the spatial characteristics of crop insurance. Using the concept of finite range positive dependence of spatial variables they calculate the effectiveness of risk pooling for cropping areas in the US. Their results are likewise based on correlations depending on lag distances.

The use of linear correlation of risks is computationally appealing but has some well-known pitfalls (cf. McNeil, Frey and Embrechts 2005). First of all, linear correlation cannot capture nonlinear dependence. Moreover, linear correlation in general does not contain all relevant information on the dependency structure of risks. That means, joint distributions with the same correlation coefficient may show a different behavior, particularly in their tails. This in turn may lead to an underestimation or overestimation of the likelihood of extreme insurance losses. An exception is the multivariate normal distribution where knowledge of the marginal distributions and the correlation matrix uniquely determines the joint distribution. However, there is much empirical evidence that weather indices as well as yield distribution are not normally distributed (e.g., Odening, Musshoff and Xu 2007, Goodwin and Ker 2002). The direct modeling and estimation of joint distributions of weather variables is in theory a response to the aforementioned problems, but it is practically affected by the shortness of available data series. Empirical data have to provide information on both the marginal distribution of weather indices and the dependence structure between them. A compromise between the restrictive application of linear correlations and the estimation of multivariate distributions is the use of copulas (Joe 1997, Nelsen 2006). Copulas avoid the direct estimation of multivariate distributions but allow for much greater flexibility in modeling the dependence structure compared to simple correlation coefficients. The basic idea of a copula function is to link marginal distributions together to a joint distribution (Sklar 1959). An advantage of copulas is that they can be determined independently from the marginal distributions of the risk variables using either parametric or nonparametric estimation procedures. Copulas became increasingly popular in the last years and have been applied to various problems in finance (c.f. Embrechts, McNeil and Straumann 1999, Cherubini, Luciano and Vecchiato 2004). Applications in agricultural economics, however, are rare. Vedenov (2008) analyzes the relationship between individual farm yields and area yields and Zhu, Ghosh and Goodwin (2008) investigate the dependence of prices and yields in the context of revenue insurance. To the knowledge of the authors copulas have not been used for the estimation of spatial dependence of weather events so far.

The objective of this paper is to model and to estimate the losses of a weather related insurance at different regional levels and different aggregation levels. We assume that indemnity payments directly or indirectly depend on weather indices measured at several

locations. The underlying question is to what extent weather risks exposures at different places can be diversified by increasing the selling area of the contracts. We are particularly interested in the tail behavior of the joint loss distribution as the probability of large losses is crucial for the required buffer fund of the insurer and the premium loading above the expected payoff and thus for the viability of an index-based crop insurance. For that purpose the probability distribution of the joint losses is estimated using copulas. Once the copula function and the marginal distributions of the weather indices have been determined the value-at-risk (VaR) of the insurers total losses can be calculated by means of stochastic simulation. By comparing results of different copula types with those from simple correlations we contribute to the discussion of an appropriate modeling of statistical dependencies in the context of weather insurance.

The remainder of the paper is organized as follows. Section 2 briefly reviews some basic properties of copula functions and explains their specification and estimation. Next, we describe the use of copulas in the particular context of simulating weather-dependent insurance losses. In section 3 this procedure is then applied to weather data in Germany. The results are presented in section 4. The paper ends with a discussion of the viability of crop insurance in Germany and some conclusions on the usefulness of the copula-based measurement of dependent risks.

2 Measuring spatial weather dependence with copulae

2.1 Identification and estimation of copulae

The rationale of using copulas if one is interested in the outcome of joint risks is given by Sklar's Theorem (Sklar 1959), which states that if F is a multivariate distribution function with margins $F_1, \ldots, F_n$ respectively then there exists a copula C such that

$$F(x_1, \ldots, x_n) = C\{F(x_1), \ldots, F(x_n)\}, \quad \forall\, x_1, \ldots, x_n \in \overline{R}. \tag{1}$$

Thus a copula $C(u_1, \ldots, u_n)$ can be understood as a multivariate distribution function with all margins being uniformly distributed on $[0, 1]$. $c(\cdot)$ denotes the density function of copula $C(\cdot)$ and the mathematical relation between $c(\cdot)$ and $C(\cdot)$ can be described as

$$c(u_1, \ldots, u_n) = \frac{\partial^n C(u_1, \ldots, u_n)}{\partial u_1 \cdots \partial u_n}, \quad \forall\, u_1, \ldots, u_n \in [0; 1] \tag{2}$$

and the multivariate density function is then given by

$$f(x_1, \ldots, x_n) = C\{F_1(x_1), \ldots, F_n(x_n)\} \cdot f_1(x_1) \cdots f_n(x_n), \tag{3}$$

where $f_1, \ldots, f_n$ are marginal densities.

(1) reveals that the information contained in the joint distribution $F(x_1, \ldots, x_n)$ can be partitioned into the information contained the margins $F(x_i)$ and the information on the dependence structure which is captured by the copula $C(\cdot)$. Note that the copula approach is very flexible, since the individual risks X_i can be modeled with any marginal distribution. One can show that margins together with the copula uniquely determine

the joint distribution (unlike margins and linear correlations). Vice versa, if the margins are continuous a unique copula corresponds to any joint distribution. However, a priori there are an infinite number of copula functions that could be used in (1) and thus the question arises how to choose the copula function appropriately in a sense of matching the multivariate data?

As with the estimation of any distribution function one can apply either parametric or non-parametric (e.g. kernel) approaches (Chen and Huang 2007). Vedenov (2008) argues that a nonparametric copula is a natural choice since there is no constructive way to determine the optimal copula function and thus the danger of misspecifying the copula is high. On the other hand if valuable prior information is available, parametric methods can improve the estimation results (Charpentier, Fermanian and Scaillet 2007, Genest, Ghoudi and Rivest 1995). In this paper we pursue a parametric approach. Important parametric copula types, which are frequently used in the existing literature, comprise, among others, the Gaussian copula and the family of Archimedean copulas (Haerdle, Okhrin and Okhrin 2008). The latter class includes the Clayton copula and the Gumbel copula.

The Gaussian copula which belongs to the class of elliptical copulas is derived from the multivariate Gaussian distribution and Sklar's theorem and has the form

$$C_G(u_1, \ldots, u_n, \mathbf{\Sigma}) = \mathbf{\Phi}_{\mathbf{\Sigma}}\{\Phi^{-1}(u_1), \ldots, \Phi^{-1}(u_n)\}, \tag{4}$$

where $\Phi(\cdot)$ is the cumulative density function of the standard normal distribution, and $\mathbf{\Sigma}$ is the Pearson Correlation Matrix. The probability density structure can be characterized by an elliptic shape ruling out tail dependence of the random variables. To proceed further we need a definition of the Archimedean copulae which are functions:

$$C(u_1, \ldots, u_n) = \phi\{\phi^{[-1]}(u_1) + \ldots + \phi^{[-1]}(u_n)\}, \tag{5}$$

where ϕ is called the generator function and $\phi(0) = 1$, $\phi(\infty) = 0$, $\phi^{[-1]}$ is its pseudoinverse.

Tail dependence can be captured by the Gumbel copula which reads

$$\begin{aligned} C_{Gu}(u_1, \ldots, u_n, \theta) &= \phi\left\{\phi^{-1}(u_1) + \cdots + \phi^{-1}(u_n)\right\} \\ &= \exp\left[-\left\{(-\ln u_1)^{\theta} + \cdots + (-\ln u_n)^{\theta}\right\}^{\frac{1}{\theta}}\right], 1 \leq \theta \leq \infty, \\ \phi(x, \theta) &= \exp\left(-x^{\frac{1}{\theta}}\right) \end{aligned} \tag{6}$$

where θ denotes a copula parameter that has to be estimated. The Gumbel copula shows a stronger linkage between positive values, more variability and more mass in the negative tail than the Gaussian copula (Okhrin 2007).

The Clayton copula is likewise an asymmetric Archimedean copula:

$$C_C(u_1, \ldots, u_n) = \left\{u_1^{-\theta} + \cdots + u_n^{-\theta} - (n-1)\right\}^{-\frac{1}{\theta}}, \; -1 \leq \theta \leq \infty, \theta \neq 0. \tag{7}$$

In contrast to the Gumbel copula the Clayton one assigns a higher probability to joint extreme negative events than to joint extreme positive events. It displays lower tail dependence and is characterized by zero upper tail dependence. Because of this feature the Clayton copula has been widely used in financial applications (e.g., Junker and May 2005, Blum, Dias and Embrechts 2002).

In general, three approaches are available to estimate the parameters of a copula. First, it is possible to estimate the copula parameters jointly with the parameters of the marginal distributions by means of exact maximum likelihood method (Cherubini, Luciano and Vecchiato 2004)

$$\begin{aligned}\tilde{\theta} &= (\hat{\theta}, \hat{\alpha}_1, \ldots, \hat{\alpha}_n) \\ &= \arg\max_{\theta} \sum_{j=1}^{k} \ln[c\{F(x_1 j; \alpha_1), \ldots, F(x_n j; \alpha_n);\ \theta\}] + \sum_{i=1}^{n}\sum_{j=1}^{k} \ln[f_j(x_{ij}, \alpha_i)]. \quad (8)\end{aligned}$$

Alternatively to this one-step estimation, one can apply a two-step procedure, where the parameters of the margins α are estimated first. Afterwards the copula parameters are determined, e.g. by maximum likelihood, treating the parameters of the margins as given. This procedure is called the inference for margin methods (IFM) (Joe 1997). The IFM is less efficient than the one-step maximum likelihood but computationally more attractive. The maximum of the log-likelihood of the copula parameter θ, $l(\theta)$, conditional on given α is

$$\hat{\theta} = \arg\max_{\theta} \sum_{j=1}^{k} \ln[c\left\{F\left(x_{1j}; \hat{\alpha}_1\right), \ldots, F\left(x_{nj}; \hat{\alpha}_n\right)\right\}; \theta]. \quad (9)$$

k denotes the number of samples. An alternative semi-parametric estimation procedure is the Canonical Maximum-Likelihood (CML) method (Haerdle, Okhrin and Okhrin 2008). The log likelihood function is now

$$\hat{\theta} = \arg\max_{\theta} \sum_{j=1}^{k} \ln[c\{\hat{F}\left(x_{1j}\right), \ldots, \hat{F}\left(x_{nj}\right); \theta\}]. \quad (10)$$

The resulting estimator is also called maximum-pseudolikelihood-estimator or rank-based maximum-likelihood estimator. The difference between (9) and (10) is that the parametric marginal distribution $F(x_{ij})$ is substituted by the empirical marginal distribution $\hat{F}(x_{ij})$. This is an advantage if the precise estimation of parametric margins is hampered by a limited number of observations. A detailed description of this estimation procedure can be found in Okhrin (2007). The empirical results in section 3 are based on the CML method.

Since different copula models imply very different dependence structures it is important to infer the correct one from the available data. The underlying test problem is equivalent to the goodness-of-fit tests for multivariate distributions. However, since the margins are estimated one cannot apply the standard test procedures directly. Chen and Fan (2005) propose a likelihood-ratio-test. Unfortunately, the test statistic does not follow a standard distribution so that either bootstrap or other computationally intensive methods have to be used. In this paper the choice of the copula type is therefore simply based on a comparison of the values of the maximized likelihood functions.

2.2 Copula-based simulation of insurance losses

With the estimated margins and the copula function at hand it is a straightforward task to assess the economic consequences of multiple risks. In the context of weather insurance particular interest lies in quantifying the likelihood of large payoffs due to the

joint occurrence of unfavorable weather conditions at different locations. Following Wang and Zhang (2003) we calculate the necessary size of the buffer fund that the insurer holds as a reserve to cover indemnity payments in extreme cases and to avoid a ruin. Formally, the buffer fund (BF) is defined as the value at risk (VaR) of the net losses of the insurer, i.e. the total indemnity payments minus the insurance premium

$$BF := P\left[\sum_{i=1}^{n} w_i \cdot \{L(X_i) - \pi_i\} \geq BF\right] = 1 - \alpha, \tag{11}$$

where $L(X_i)$ denotes the weather dependent indemnity payment for trading area i and π_i is the corresponding insurance premium. Here π_i is defined as $E[(L(X_i))]$. w_i denotes the weight of the ith insurance contract and $1-\alpha$ is the ruin probability. Dividing the buffer fund by the number of contracts gives the buffer load. The buffer load is the surcharge to the fair price of the insurance that ensures liquidity of the insurer. Other loading factors capturing administrative costs are ignored.

For the copula-based calculation of VaR we proceed as follows (cf. Giacomini and Haerdle 2005). Based on marginal distributions $F_n(x_n)$ which are specified using standard goodness of fit tests, and the estimated copula $C(u_1, \ldots, u_n; \theta)$ samples from the joint distribution $X \sim C(u_1, \ldots, u_n; \hat{\theta})$ of the weather variables can be generated using Monte Carlo simulation. There is an conditional inverse algorithm for simulating the full distribution of $x_1, \ldots, x_n$ by recursively simulating the conditional distribution of x_i given x_{i-1}. For each realization of the n-dimensional random vector of weather indices a (one dimensional) loss is calculated through aggregation of the indemnity payments resulting from the n insurance contracts. The indemnities $L(X_i)$ depend on the specification of the insurance contract. The contract design as well as the specification of the weather indices X_i is described in the next section.

3 Application

In what follows we apply the procedure explained in the previous section to weather data in Germany. The application is motivated by the fact that agricultural insurance companies in Germany are currently developing insurance products which protect farmers against multiple perils including drought risk. Unlike in many other countries agricultural insurance and reinsurance contracts in Germany are not subject to governmental subsidies. Thus a careful investigation of the stochastic properties of the insured risk is in the vital interest of potential suppliers of these contracts. Though we are aware that the trading area of insurance companies may not be confined to Germany we focus on this country simply for practical reasons of data availability.

The data sets consists of daily observations of precipitation and (average) temperature covering the period from January 1, 1973 until December 31, 2006, i.e. 34 years. These time series are available for 25 weather stations which are equally spread over Germany (see Figure 1). The choice of these particular weather stations was made with regard to maximizing the length of the time series.

Based on these daily observations three weather indices are derived that are used as underlyings for weather insurance. The first index is the cumulative rainfall index (CRI).

It measures the rainfall within the main vegetation period of most crops, which lasts from April 1 until June 30:

$$CRI_{i,t} = \sum_{j=\tau_b}^{\tau_e} P_{j,t,i},\ t = 1, \ldots, 34, \tag{12}$$

where $P_{j,i}$ symbolizes the daily precipitation at day j in year t and region i, $i = 1, \ldots, n$. $\tau_{b,t}$ and $\tau_{e,t}$ denote the begin (April 1) and the end (June 30) of the vegetation period, respectively. This index addresses drought risk (Martin, Barnett and Coble 2001). Indemnities are paid if CRI falls below a predetermined trigger level K_{CRI_i}

$$L_{CRI_{i,t}} = \max\left\{0, K_{CRI_{i,t}} - CRI_{i,t}\right\} \cdot V. \tag{13}$$

Herein V denotes the tick size which converts physical units into monetary terms. As we do not strive for an optimal contract design in the sense of maximizing the hedging effectiveness we set $V = 1$. Moreover, it is assumed that no policy limits apply.

The second index is a potential flood indicator PFI (Frich et al. 2002). It is also related to precipitation, however, it measures excessive rainfall rather than drought, which is also a serious source of yield shortfalls.

$$PFI_{i,t} = \max_{\tau}\left(\sum_{j=1+\tau}^{s+\tau} P_{j,t,i}\right),\ \tau = 0, \ldots, 365 - s. \tag{14}$$

The PFI equals the rainfall sum of the wettest s-day-period within a year. Here we choose $s = 5$. The insurance payoff for the PFI has the structure of a call option, i.e.

$$L_{PFI_{i,t}} = \max\left\{0, PFI_{i,t} - K_{PFI_{i,t}}\right\} \cdot V \tag{15}$$

The third index that we suggest is the "Growing Degree Days" (GDDs). The GDD index is intended to measure impact of temperature on the growth and the development of plants during a growing season (World Bank 2005)

$$GDD_{i,t} = \sum_{j=\tau_b}^{\tau_e} \max\left(0, T_{j,t,i} - \hat{T}\right), \tag{16}$$

where $T_{j,t,i}$ denotes daily temperature in degrees Celsius. τ_b and τ_e stand for the begin (March 1) and the end (October 31) of the growing season, respectively. The base temperature $\hat{T}$ is the minimum temperature that has to be exceeded before plant growth is stimulated. Though this threshold is plant specific we assume a constant value of $5°C$. Indemnities are calculated according to

$$L_{GDD_{i,t}} = \max\left\{0, K_{GDD_{i,t}} - GDD_{i,t}\right\} \cdot V \tag{17}$$

For all three indices we assume two alternative trigger levels, namely the 50% quantile and the 15% quantile of the respective index distribution.

The analysis of the spatial dependence of the aforementioned weather indices is carried out within three scenarios. The first scenario refers to only one state, Brandenburg (including Berlin). Brandenburg is located in North East Germany and thus affected by a dry continental climate. Four weather stations of our sample are located in this state (Berlin-Tempelhof, Neuruppin, Potsdam, and Lindenberg) and we expect significant dependence of the respective indices at this regional level. In the second and third scenario we consider

the entire German state. In order to limit the dimension of the estimation problem the country is divided into four regions (A-D) of comparable size. These regions represent the Eastern (A), the North-Western (B), the Western (C) and the Southern (D) part of Germany. The number of weather stations varies between 5 (regions C and D) and 9 (region A). Figure 1 depicts the number and the location of the available weather stations. Scenario 2 and 3 differ in the aggregation level of the weather indices. While in scenario 2 each region is represented by only one weather station (Potsdam, Hamburg, Duesseldorf, and Stuttgart, respectively) an average of all weather stations situated in the respective region underlies scenario 3.

Figure 1 about here

4 Results and Discussion

Marginal distributions and copulas have been estimated for the CRI, the PFI and the GDD index using the statistical procedures described in section 2. First of all, 36 marginal distributions (3 indices, 4 locations, 3 regional levels) have been selected in accordance with standard goodness of fit tests, i.e., Kolmogorov-Smirnoff test, χ^2 and Anderson-Darling test. The Lognormal, the Gamma and Beta distribution show the best fit for the rainfall-based indices (CRI and PFI), whereas the Weibull distribution fits the observations of the temperature-based index (GDD). Next, two different parametric copulas have been estimated, namely the Clayton copula and the Gumbel copula. The results are presented in table 1. The values of the maximized likelihood functions support the choice of the Clayton copula. In what follows we only discuss the results for this copula type.

Table 1 and table 2 about here

The main results of the simulations are summarized in table 2 and table 3. Table 2 displays the expected insurance payoffs for the regions under consideration. Obviously considerable differences exist between locations. This finding should be kept in mind when interpreting the effects of aggregating trading areas for weather insurance which are presented in table 3. As explained above the buffer load is derived from the buffer fund which is defined as the 99% quantile of loss distribution for the insurer minus the fair premium. The loss distribution is the outcome of 10.000 random draws from the estimated margins and the estimated copula. Dividing the buffer fund by the number of trading areas (n = 1, 2, 3, or 4, respectively) yields the buffer load. The buffer load is depicted for trading areas of growing size (A, A+B, A+B+C, and A+B+C+D) and for insurance contracts with two different trigger levels (15% and 50% quantile). Moreover, table 3 allow a comparison of the copula based approach that we propagate here and the traditional method of using Pearsons linear correlation coefficients[1]. Both methods use the same marginal distributions and differ only in the estimation of the dependence structure of the weather indices at different locations.

Table 3 about here

First of all, the size of the buffer load may appear surprisingly high in relation to the expected insurance payoff. Loosely speaking this finding indicates that the loss distribution

[1]The generation of multivariate random variables with arbitrary marginals following a given linear correlation matrix is carried out with the method of Iman and Conover (1982).

is wide and shows fat tails. Of course the buffer load depends on the trigger level. The values increase by a factor between 1,3 and 2,3 if the insurance contracts refer to the 50% quantile instead of the 15% quantile of the index distributions.

From a methodological viewpoint it is interesting to realize that the two methods of considering stochastic dependence between trading areas show considerable differences. In case of the drought insurance (CRI) the assumption of linear correlations tends to underestimate the risk of large joint indemnity payments for all three scenarios compared with the copula method. For the largest trading area (A+B+C+D) this underestimation varies between 11% and 21% in case of the 50% trigger value and lies between 18% and 31% in case of the 15% trigger. For the other two insurance contracts the differences of the buffer load are less pronounced. There are also cases in which the Clayton copula results in a lower buffer load than the linear correlation.

Table 3 also depicts spatial diversification effects of weather insurance. It can be seen that the decline of the buffer load is rather small if the trading area becomes larger. In some cases the buffer load even increases. This finding, which contradicts the intuitive expectation of diversification effects, can be explained by the heterogeneity of the weather indices in the trading areas that are pooled. As mentioned before, the index distributions are neither independent nor identically distributed. In order to assess the diversification effect more accurately we calculate the relative difference of the buffer load of a joint insurance of all four regions A-D compared to the sum of buffer loads associated with separate insurance for each region (last column in table 3). Recall that the buffer load in the in the $i.i.d.$ case declines with $1/\sqrt{n}$, which means that we could expect a reduction by 50% for $n = 4$ in that case. Apparently, the possibility of spatial diversification heavily depends on the insured weather event. For example, the buffer load for the contract based on the potential flood indicator (PFI) decreases considerably, particularly for scenario 2, irrespective of the trigger level and how the joint distribution has been computed. Opposed to that the decline of the buffer load is negligible for the insurance against low temperature. This result reflects the high stochastic dependence of the GDD at different locations. This holds for the state level as well as for the national level. Diversification is also modest for the drought insurance based on the CRI index. It is worth mentioning that the spatial diversification potential is not much higher in the whole country (Germany) than in a single state (Brandenburg). This is particularly true if the weather indices are calculated as an average value out of several weather stations (scenario S3). Finally, we emphasize again the difference between copulas and linear correlations: On the one hand the use of linear correlations underestimates the effect of spatial diversification for the insurance against excessive rain, on the other hand this effect is overestimated in case of the drought insurance and the GDD based insurance.

5 Conclusions

In this paper we have explored the risk that insurer will face when selling contracts with weather-based payoffs in Germany. Our results indicate that indemnity payments based on temperature as well as on cumulative rainfall show strong stochastic dependence at different regions in Germany. Thus the possibility to reduce risk exposure by increasing the trading area of the insurance is limited. Though the results are specific to Germany we conjecture that the situation in other EU countries of similar size and climate conditions

does not differ in principle. At the first glance our results contradict those of Wang and Zhang (2003) who found that the distance for the positive dependence of crop yields in the US is at most 570 miles and hence the required buffer load for national crop insurance is rather small. However, our results are not directly comparable. We analyze weather data whereas Wang and Zhang (2003) investigate regional crop yields. Regional differences in soil quality, for example, may lead to differences in yields even if weather conditions are similar. Moreover, our results rely on several assumptions that have an influence on the buffer load. Firstly, we did not take into account product diversification of the insurer. Secondly, only a single period has been considered and equity reserves that are built in years with premium surpluses have been ignored. Thirdly, the buffer load can be controlled by choice of the trigger value for the indemnity payments. It can be expected that the dependence of insurance payoffs at different locations becomes smaller if the trigger level is reduced. Defining absolute trigger values instead of relative ones (i.e. quantiles of the regional distributions) will effect the dependence structure of the payments, too. Fourthly, in our application the definition of trading areas took place on an ad-hoc basis. This procedure leaves room for a thorough identification of smaller and more homogenous climatic zones showing less dependence. Considering all these points our results must be interpreted with care. We do not state that private (unsubsidized) crop insurance is impossible in view of systemic weather risk at all. Anyhow, we believe that our finding may explain the reluctance of insurance companies to enter this market segment in Germany. Global reinsurance or transferring weather risk to the capital markets by means of weather bonds could of course alleviate this problem.

Irrespective of their economic implications our results pinpoint the necessity of a proper statistical modeling of the dependence structure of multivariate random variables. The usual approach of measuring stochastic dependence with linear correlation coefficients turned out to be questionable in the context of weather insurance. Considerable differences with regard to the weather risk assessment occurred in comparison with the more general copula method. Unfortunately, our empirical results are weakened by a rather small data base. The estimation of 4-dimensional distributions with only 34 observations is inevitably accompanied by large estimation errors. A solution to this problem could be the use of daily weather data. We suggest this as a direction for further research.

Acknowledgements

This work was supported by German Research Foundation (DFG) through the SFB 649 "Economic Risk".

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Table 1: **Maximum-Likelihood Estimation of Copulas**

Index/ Scenario	Gumbel		Clayton	
	θ	ML	θ	ML
I1: Cumulative Rainfall Index (CRI)				
S1	2.16	40.24	1.95	50.94
S2	1.47	12.38	0.65	12.27
S3	1.48	13.60	1.10	22.96
I2: Potential Flood Indicator (PFI)				
S1	1.43	12.46	0.85	18.13
S2	1.01	0.01	0.10	0.24
S3	1.14	1.29	0.35	3.69
I3: Temperature Index (GDD)				
S1	7.09	160.27	10.55	163.45
S2	2.87	64.88	2.45	64.47
S3	3.11	71.02	3.25	78.97

Table 2: **Expected Values of Insurance Payoffs for Different Indices and Regions**

Contract/ Scenario	A	B	C	D	A	B	C	D
	Indemnity trigger							
	50% Quantile				15% Quantile			
Contract based on CRI								
S1	15.52	12.81	13.62	17.62	1.54	0.77	2.42	2.39
S2	13.62	14.69	21.11	19.16	2.42	2.45	3.35	1.19
S3	13.47	11.78	16.11	13.66	1.79	0.82	3.53	1.55
Contract based on PFI								
S1	7.91	9.18	8.72	11.59	3.04	2.15	2.84	3.69
S2	8.72	6.44	5.22	8.60	2.84	0.90	1.08	1.47
S3	7.15	4.31	5.89	4.71	1.22	0.95	0.72	0.92
Contract based on GDD								
S1	73.89	59.81	78.89	72.76	8.06	4.11	3.24	4.56
S2	78.89	70.20	61.78	63.46	3.24	3.52	7.58	14.68
S3	65.08	70.75	73.44	72.06	4.40	2.46	11.21	13.17

Table 3: **Buffer Load for Weather Insurance in Different Trading Areas**

Contract/ Scenario	Trading Area					Effect of Diversification (%)
	Method	A	A+B	A+B+C	A+B+C+D	
a) Trigger = 50% Quantile						
Contract based on CRI						
S1	Linear correlation	56.01	53.01	55.14	60.63	88.87
	Clayton copula	56.01	59.20	61.06	68.65	98.58
S2	Linear correlation	64.79	65.83	60.84	54.46	69.85
	Clayton copula	64.79	69.88	69.44	69.43	88.31
S3	Linear correlation	69.66	53.56	60.25	55.90	79.37
	Clayton copula	69.66	58.52	70.49	65.88	93.41
Contract based on PFI						
S1	Linear correlation	57.32	39.50	38.26	40.98	73.62
	Clayton copula	57.32	39.08	32.69	31.26	53.41
S2	Linear correlation	54.30	37.84	26.16	21.42	52.25
	Clayton copula	54.30	32.88	23.10	18.87	44.39
S3	Linear correlation	35.33	24.40	20.11	18.05	71.67
	Clayton copula	35.33	21.35	15.82	13.40	51.08
Contract based on GDD						
S1	Linear correlation	500.87	352.71	402.48	425.30	99.21
	Clayton copula	500.87	351.14	400.14	423.17	99.58
S2	Linear correlation	503.37	375.26	380.90	351.43	95.05
	Clayton copula	503.37	367.09	376.19	349.90	97.85
S3	Linear correlation	446.08	442.63	451.33	447.77	96.25
	Clayton copula	446.08	458.71	469.42	470.76	98.94
b) Trigger = 15% Quantile						
Contract based on CRI						
S1	Linear correlation	31.66	27.80	27.97	31.12	81.45
	Clayton copula	31.66	32.96	32.86	37.89	97.45
S2	Linear correlation	33.34	30.68	27.97	24.52	56.07
	Clayton copula	33.34	34.65	34.37	35.57	80.10
S3	Linear correlation	44.16	27.22	33.42	29.93	69.72
	Clayton copula	44.16	31.64	41.97	39.34	91.14
Contract based on PFI						
S1	Linear correlation	39.58	26.39	24.96	27.08	68.25
	Clayton copula	39.58	26.05	21.04	19.40	46.27
S2	Linear correlation	37.91	24.87	17.34	13.83	48.59
	Clayton copula	37.91	21.93	15.20	12.45	41.66
S3	Linear correlation	25.22	15.76	11.70	9.70	61.94
	Clayton copula	25.22	15.10	10.23	7.75	46.54
Contract based on GDD						
S1	Linear correlation	346.20	220.30	264.27	283.54	98.90
	Clayton copula	346.20	221.23	265.40	285.55	99.37
S2	Linear correlation	358.43	261.64	261.82	233.46	92.91
	Clayton copula	358.43	250.37	255.50	229.85	96.76
S3	Linear correlation	317.26	314.88	315.60	309.99	94.87
	Clayton copula	317.26	330.46	333.54	334.64	98.52

	Region A	Region B	Region C	Region D
Scenario	Is represented by weather station(s) no.			
S1: Brandenburg (4 stations)	1	4	2	3
S2: Germany (4 stations, underlined)	2	14	20	22
S3: Germany (station mix, colored)	1,2,3,4,5, 6,7,8,11	9,10,12, 13,14,15	19,20,21, 24,25	16,17,18, 22,23

Figure 1: Distribution of Weather Stations and definition of Scenarios