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# **Altering Milk Components Produced by the Dairy Cow: Estimation by a Multiple Output Distance Function**

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# **Altering Milk Components Produced by the Dairy Cow: Estimation by a Multiple Output Distance Function\***

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**Abstract:** The impact of inputs on the four decomposed milk outputs of fluid milk, butterfat, protein, and other solids were estimated using an output distance function, with New York dairy farm level data. Differential impacts of inputs on component production indicate that milk component composition can be modified given component prices.

**Keywords:** milk component production, output distance function

## **Introduction**

Dairy farmers in New York now receive milk payments under the Federal Milk Marketing Order multiple-component pricing system. Payments are based on the quantities of the three main milk components: butterfat, protein, and other solids. Because the price of each component is determined by the value of that milk component in processing dairy products, and ultimately the prices of final dairy products, component prices vary over time. This provides an opportunity for dairy farmers to increase profits by altering individual milk components in response to component prices.

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Buccola and Iizuka (1997) estimated hedonic cost models to determine how farmers might respond to component price changes and found little opportunity to adjust components. Bailey et al. (2005) and Smith and Snyder (1978), investigated the economics of milk components by dairy breed, the factor thought most responsible for component composition. Although the role of breed and feed rations are believed to be the major determining factors affecting milk component composition, other inputs may also impact on milk composition. Thus, the objective of this paper is to examine the effects of inputs on each of the four decomposed milk outputs: fluid milk, butterfat, protein, and other solids at the farm level. This is accomplished by estimating an output distance function using New York dairy farm level data. Output distance functions can be used to estimate the production possibility set of the technology. However, inputs cannot be easily re-directed to various outputs since milk components are determined by the dairy cow and not the farmer. Thus we argue that it may not be plausible in this application to move along a production possibilities curve with a fixed vector of inputs. In contrast, however, the production possibilities set might be expanded or contracted in a non-parallel shift by altering the fixed input vector and in the process force a possible non-radial movement to a new area on the production possibility curve.

### **Output distance functions**

Production frontier models can be utilized in production technology with multiple outputs and inputs by incorporating a distance function (Shephard 1970; Brummer et al. 2002). The output distance function is an appropriate specification since most dairy farmers are interested in expanding output given their inputs. In contrast, the input distance function allows for the

contraction of inputs given fixed outputs. We observe very few farmers reducing the number of cows, land holdings, and other inputs, so believe the output specification is preferred.

We use a Cobb-Douglas and then the Translog functional forms to estimate the output distance function. The log-linear form of the Cobb-Douglas output distance function for farm  $i$  can be expressed as

$$(1) \quad \ln D_{oi} = \alpha_o + \sum_m \alpha_m \ln y_{mi} + \sum_k \beta_k \ln x_{ki} + \sum_j \delta_j d_{ji}$$

Because an output distance function is homogeneous of degree one in outputs, the imposition of homogeneity is accomplished by normalizing the outputs by one of the outputs. Hence, butterfat, protein, and other solids are normalized by fluid milk production ( $y_{mi}^* = y_{mi} / y_{li}$ ), so that milk production becomes the dependent variable and the independent output variables are represented as percentages of each component in milk, resulting in:

$$(2) \quad \ln D_{oi} - \ln y_{li} = \alpha_o + \sum_m \alpha_m \ln y_{mi}^* + \sum_k \beta_k \ln x_{ki} + \sum_j \delta_j d_{ji}$$

Although output distance functions are typically estimated allowing for technical inefficiency, we assume that these farms are technically efficient such that  $D_{oi}$  is equal to one so that  $\ln D_{oi}$  is equal to zero, and thus drops out of the equation.<sup>1</sup> To take into account unobserved random variations, a random error term ( $\varepsilon_i$ ) is added to the output distance function, assumed to be independently and identically distributed as  $N(0, \sigma^2)$ . Finally, the dependent variable in this equation is transformed to a positive  $\ln y_{li}$  so that the signs of the

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<sup>1</sup> An application specifying technical efficiency produced an average technical efficiency of 96% on these farms (minimum value of 90%)

estimated coefficients will be reversed, corresponding to those in a general production function, and resulting in the final estimated form as:

$$(3) \quad \ln y_{li} = \alpha_0 + \sum_m \alpha_m \ln y_{mi}^* + \sum_k \beta_k \ln x_{ki} + \sum_j \delta_j d_{ji} + \varepsilon_i$$

Several economic measures can be obtained from the estimation results. The coefficient estimates  $\beta_k = \partial \ln y_1 / \partial \ln x_k$  from the output distance function measures the increase in the primary output  $y_1$ , holding the output ratios of  $y_m^*$  constant. Thus, this estimated elasticity  $\beta_k$  includes the impact on the other outputs which keep their output ratios constant.

Single output, single input elasticities can be compute using the implicit function rule of differentiation. Rearranging the output distance function as:

$$(4) \quad G : 0 = \alpha_0 + \left(1 - \sum_m \alpha_m\right) \ln y_{li} + \sum_m \alpha_m \ln y_{mi} + \sum_k \beta_k \ln x_{ki} + \sum_j \delta_j d_{ji} + \varepsilon_i$$

permits using the implicit differentiation rule to obtain:

$$(5) \quad \frac{\partial \ln y_1}{\partial \ln x_k} = - \left( \frac{\partial G}{\partial \ln x_k} / \frac{\partial G}{\partial \ln y_1} \right) = - \frac{\beta_k}{1 - \sum_m \alpha_m}$$

$$(6) \quad \frac{\partial \ln y_m}{\partial \ln x_k} = - \left( \frac{\partial G}{\partial \ln x_k} / \frac{\partial G}{\partial \ln y_m} \right) = - \frac{\beta_k}{\alpha_m}$$

These computed values represent partial production elasticities. Equation (5) measures the production elasticity of the  $k^{th}$  input  $x_k$  for aggregate milk, and Equation (6) measures the production elasticity of the  $k^{th}$  input  $x_k$  for each individual component.

The conventional way to examine the relationships between outputs is to simply look at the production possibilities curve in  $y_m / y_n$  ( $m \neq n$ ) space. The product-product elasticities from the Cobb-Douglas production function are:

$$(7) \quad \frac{\partial \ln y_1}{\partial \ln y_m} = - \left( \frac{\partial G}{\partial \ln y_m} / \frac{\partial G}{\partial \ln y_1} \right) = - \frac{\alpha_m}{1 - \sum_m \alpha_m}$$

$$(8) \quad \frac{\partial \ln y_m}{\partial \ln y_n} = - \left( \frac{\partial G}{\partial \ln y_n} / \frac{\partial G}{\partial \ln y_m} \right) = - \frac{\alpha_n}{\alpha_m}$$

The Translog Output Distance Function is specified as:

(9)

$$\begin{aligned} -\ln y_{li} = & \alpha_0 + \sum_m \alpha_m \ln y_{mi}^* + 0.5 \sum_m \sum_n \beta_{mn} \ln y_{mi}^* \ln y_{ni}^* + \sum_k \alpha_k \ln x_{ki} + 0.5 \sum_k \sum_l \beta_{kl} \ln x_{ki} \ln x_{li} \\ & + \sum_k \sum_m \beta_{km} \ln x_{ki} \ln y_{mi}^* + \sum_j \delta_j d_{ji} + \varepsilon_i \end{aligned}$$

where again to enforce homogeneity of degree one in output quantities,

$$y_{mi}^* = y_{mi} / y_{li}.$$

Then, rearranging and implementing the implicit function differentiation rule results

in:

(10)

$$\begin{aligned} H : 0 = & \alpha_0 + \left( 1 - \sum_m \alpha_m \right) \ln y_{li} + \sum_m \alpha_m \ln y_{mi} + 0.5 \sum_m \sum_n \beta_{mn} (\ln y_{mi} - \ln y_{li})(\ln y_{ni} - \ln y_{li}) + \sum_k \alpha_k \ln x_{ki} \\ & + 0.5 \sum_k \sum_l \beta_{kl} \ln x_{ki} \ln x_{li} + \sum_k \sum_m \beta_{km} \ln x_{ki} (\ln y_{mi} - \ln y_{li}) + \sum_j \delta_j d_{ji} + \varepsilon_i \end{aligned}$$

$$(11) \quad \frac{\partial H}{\partial \ln x_k} = \alpha_k + 0.5 \sum_k \sum_l \beta_{kl} \ln x_l + \sum_k \sum_m \beta_{km} (\ln y_m - \ln y_l)$$



$$(12) \quad \frac{\partial H}{\partial \ln y_1} = \left(1 - \sum_m \alpha_m\right) + 0.5 \sum_m \sum_n \beta_{mn} (2 \ln y_1 - \ln y_m - \ln y_n) - \sum_k \sum_m \beta_{km} \ln x_k$$

$$(13) \quad \frac{\partial H}{\partial \ln y_m} = \alpha_m + 0.5 \sum_m \sum_n \beta_{mn} (\ln y_n - \ln y_1) + \sum_k \sum_m \beta_{km} \ln x_k$$

Where the elasticities can be computed as:

$$(14) \quad \frac{\partial \ln y_1}{\partial \ln x_k} = - \left( \frac{\partial H}{\partial \ln x_k} / \frac{\partial H}{\partial \ln y_1} \right)$$

$$(15) \quad \frac{\partial \ln y_m}{\partial \ln x_k} = - \left( \frac{\partial H}{\partial \ln x_k} / \frac{\partial H}{\partial \ln y_m} \right)$$

$$(16) \quad \frac{\partial \ln y_m}{\partial \ln y_n} = - \left( \frac{\partial H}{\partial \ln y_n} / \frac{\partial H}{\partial \ln y_m} \right)$$

## Data

Data were obtained from the New York Dairy Farm Business Summary (DFBS) with 105 participating farms in 2003, and 107 participating farms in 2004. Although 94 of the farms submitted data for both years, two years of data per farm precludes any consistent measure of management or firm affects and a fixed panel treatment would substantially reduce degrees of freedom. Thus, it was decided to tradeoff minor estimate bias with gains in efficiency, maximizing mean-squared error.

These data were submitted on a voluntary basis, and the average farm size at 448 cows is larger than the year 2004 average 95 cow New York dairy farm. Cow productivity of the DFBS farms is also higher at 21,059 pounds of milk per cow compared to the 2004 New York state average of 17,786. All outputs and inputs are measured on an accrual basis, reflecting what was actually produced and used during the year, rather than what was sold and produced.

Fluid milk and the three individual components – butterfat, protein, and other solids – are included in the model as output variables and are expressed as pounds per cow during a year. Outliers in the data distributions of butterfat, protein, and other solids may indicate data error, or data recorded under unusual circumstances such as diseases. Therefore, outliers were treated as missing data to prevent the possible distortion of regression results. Detailed criteria used to define the outliers are butterfat percentage less than 2 percent or greater than 5 percent, protein percentage less than 1 percent or greater than 5 percent, or other solids percentage less than 4 percent or greater than 8 percent. As a result of data correction, six missing data were deleted in each year. Fluid milk was calculated as total aggregate milk minus the weight of the other 3 outputs- butterfat, protein, and other solids.<sup>2</sup> This is in keeping with the distance function concept that the four outputs should be separate outputs.

Input variables were labor, purchased feed, crop expenses for grown feed, livestock inputs, and real estate inputs. Labor is the sum of family and hired labor hours per year which each farm put into milk production. Farms give cows both purchased feed and grown feed, and the variables, purchased feed and grown feed represent the expenditure of the purchase feed and the crop expenses for grown feed respectively. Expenditures on capital items, livestock, and real estate, include depreciation and holding costs. The holding costs of these items were calculated by applying interest rates to the average value during the year. Depreciation was added to these expenditures, which were then converted into quantities using capital price indices. The DFBS collects two types of expenditures on livestock: replacement and expansion. The replacement component reflects the livestock purchases necessary to maintain

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<sup>2</sup> Results using total aggregate milk as the dependent variable rather than fluid milk (by deleting butterfat, protein, and solids) were very similar.

a steady state amount of livestock. Expansion livestock in contrast, is expenditure on livestock to expand the business, and provides service in following years as well as the current year. A year dummy variable is also included to allow for unobserved technical changes and environmental aspects such as temperature and sunlight variation between the years.

## **Results**

### ***Cobb-Douglas Function Results***

The results of estimating the Cobb-Douglas output distance function (Table 2) indicate that all of the five production input factors have statistically significant effects on fluid milk production at the 0.05 level. Since the specification of the output distance function (3) is different from single-output production functions, the interpretations of the coefficient estimates  $\beta_k$  in Table 2 are different than a single output production function. In the case of a single-output production function, the coefficient estimates represent the (partial) production elasticity of the  $k^{\text{th}}$  input on the particular output  $y_m$ , but the coefficient estimates  $\beta_k$  from the output distance function represent the production elasticity of the  $k^{\text{th}}$  input on the dependent variable output, in this case fluid milk, holding the percentages of each milk component  $y_m^*$  constant as well as holding the other inputs constant. Thus, this estimated elasticity  $\beta_k$  includes the impact on the other outputs since their output ratios must stay constant, and the denominator of those output ratios is the dependent variable output. The coefficients on all inputs are positive, implying that increases in each input leads to increases in fluid milk and the other outputs except for butterfat, since the coefficient on butterfat is negative. This implies that the portion of butterfat goes down when fluid milk production increases on the

farm. This may reflect the fact that higher producing cows tend to have lower buttermilk percentages, although results are for the farm and not the individual cow.

The (partial) production elasticity  $\beta_{m,k}$  of each input on each of the four outputs computed by Equations (5) and (6) is reported in Table 3. The interpretations of these  $\beta_{m,k}$  elasticities are similar to the estimates from a single output production function.<sup>3</sup> According to Table 3, purchased feed has the largest impact on the production of each milk component, leading to increases in fluid milk, protein, and solid, but with a reduction in butterfat. The reduction in butterfat to increases in feed (and the other inputs) is troubling. It is reasonable to expect the butterfat percentage to fall with increased milk output per cow, but these values imply that the actual pounds of butterfat also falls. Yet, these regressions are across farm observations and are not based upon individual cows, but on the average cow on these farms. This implies that differences across farms, rather than changes within a representative farm, or changes to an individual cow, leads to decreases in butterfat production while other output components increase. Individual cows might still show increases in butterfat output from input increases. Unlike a firm where the manager can alter the inputs used among various outputs, the dairy cow cannot be asked to produce different milk components from a fixed input bundle, although it possible for the farmer to re-direct inputs to various cows.

Purchased feed has the most significant impact on all outputs, especially on other solids which increase 16.40 percent for only a one percent increase in expenditure on purchased feed. Protein, likewise, has an elastic response to purchased feed, increasing 2.73

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<sup>3</sup> The percentage change in the output  $y_m$  from a one percent change in an input, holding the other inputs and outputs constant.

percent for a one percent increase in purchased feed. The next input that has the largest impact is livestock expenditures, which increases other solids 10.64 percent for a one percent increase in livestock expenditures, and increases protein by 1.77 percent for a one percent increase in livestock expenditures. The other inputs have lower impacts on these components, with grown feed and then real estate lower. Labor shows the smallest impact on components

Table 4 shows the calculation of Equations (7) and (8) which is the substitution along the production possibilities set expressed in elasticities. As stated earlier, these elasticities may not be particularly relevant in this application since the dairy cow cannot alter milk composition without altering inputs. None-the-less, these are all positive elasticities except for any elasticity involving butterfat. Thus an increase in butterfat results in a decrease in other outputs, and vice versa.

### ***Translog Function Results***

The estimated translog did not improve the statistical fit of the model. The adjusted  $R^2$  did not increase with the additional log quadratic variables of the translog, with only 4 of those 36 quadratic terms being statistically significant at the  $p=0.05$  level. In addition, all the linear input coefficient terms became statistically insignificant in the translog estimation, when they had been statistically significant in the Cobb-Douglas estimation. An F test of whether the quadratic terms summed to a value greater than zero produced a value of only  $F(1, 273) = 0.46$ , convincing evidence that the Translog functional form does not add information to the production process. The computed elasticities computed at the various data points were also not stable, ranging from high positive values to low negative values. We conclude that the

translog does not add utility to the analysis and we elect not to report or utilize the translog estimates.<sup>4</sup>

## **Conclusions**

This study measured the responses of aggregate milk and individual milk component production to changes made in the dairy business. An output distance function was estimated using New York Dairy Farm Business Summary (DFBS) data from 105 farms in 2003 and 107 farms in 2004. The empirical results demonstrate the possibility of altering individual component productions. However, since the differences between the effects of each input on each output are relatively small, the farmer's ability to alter individual component productions may be limited. Yet, this is still important because, given the small profit margins that often occur in the dairy industry, this small ability provides the opportunity for farmers to increase profits by altering individual component production levels in response to each component price. Our results show that an increase in purchased feed would most significantly increase the production of milk components. The results also suggest that the butterfat has a negative relation with aggregate milk and other components production.

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<sup>4</sup> The translog estimates are available from the authors.

## References

- Bailey, K.W., Jones, C.M., and Heinrichs, A.J., 2005. Economic Returns to Holstein and Jersey Herds Under Multiple Component Pricing. *Journal of Dairy Science*, **88** (6), pp.2269-2280.
- Brummer, B., Glauben, T., and Thijssen, G., 2002. Decomposition of Productivity Growth Using Distance Functions: The Case of Dairy Farms in Three European Countries. *American Journal of Agricultural Economics*, **84** (3), pp.628-644.
- Buccola, S., and Iizuka, Y., 1997. Hedonic Cost Models and the Pricing of Milk Components. *American Journal of Agricultural Economics*, **79** (2), pp.452-462.
- Shephard, R.W., 1970. *Theory of Cost and Production Functions*. Princeton: Princeton University Press.
- Smith, B.J., and Snyder, S.D., 1978. Effects of Protein and Fat Pricing on Farm Milk Prices for the Five Major U.S. Dairy Breeds. *American Journal of Agricultural Economics*, **60** (1), pp.126-131.
- Sumner, D.A., and Leiby, J.D., 1987. An Econometric Analysis of the Effects of Human Capital on Size and Growth among Dairy Farms. *American Journal of Agricultural Economics*, **69** (2), pp.465-470.

*Table 1. Description of Variable Names and Sample Statistics*

<b>Variable names</b>	<b>Description</b>	<b>Mean</b>	<b>Std. Dev.</b>
MILK_COW	Milk production per cow per year (pounds)	21,307	3,367
BUTTERFAT_COW	Butterfat production per cow per year (pounds)	775	106
PROTEIN_COW	Protein production per cow per year (pounds)	639	94
OTHERSOLIDS_COW	Other solid production per cow per year (pounds)	1,208	195
LABOR	Labor per year (months)	88	5
PURCHASED FEED	Expenses for purchased feed per year (dollars)	283,268	19,860
GROWN FEED	Crop expenses of grown feed per year (dollars)	167,550	8,916
LIVESTOCK	Purchased livestock and livestock supplies (dollars)	179,585	8,916
REAL ESTATE	Depreciation and repairs of real estate (dollars)epreciation and repairs of Real Estate	119,761	7,264
YEAR	Dummy for year (2003=0 and 2004=1)	105 farms in 2003 107 farms in 2004	

*Table 2. Estimate of Cobb-Douglas Component Output Distance Function*

Variable Name	Coefficient	t-Value
BUTTERFAT_COW	-0.2555	-2.90
PROTEIN_COW	0.1626	1.58
OTHERSOLIDS_COW	0.0270	0.61
LABOR	0.0559	2.10
PURCHASED FEED	0.4436	18.85
GROWN FEED	0.1929	8.13
LIVESTOCK	0.2878	8.38
REAL ESTATE	0.1099	4.32
YEAR	-0.0628	-4.04
Constant	2.5690	13.99
Number of Observations = 319, Adjusted R-squared = 0.9857		

Coefficient signs are reversed because positive rather than negative MILK\_COW values were used as the dependent variable.



*Table 3. Elasticity of Output Changes to Input Changes (Cobb-Douglas Output Distance Function)*

	MILK COW	BUTTERFAT COW	PROTEIN COW	OTHERSOLIDS COW
LABOR	0.05	-0.22	0.34	2.07
PURCHASED FEED	0.42	-1.74	2.73	16.40
GROWN FEED	0.18	-0.76	1.19	7.13
LIVESTOCK	0.27	-1.13	1.77	10.64
REAL ESTATE	0.10	-0.43	0.68	4.06

*Table 4. Elasticity among Outputs (Cobb-Douglas Output Distance Function)*

	MILK COW	BUTTERFAT COW	PROTEIN COW	OTHERSOLIDS COW
MILK COW	1.00	-4.17	6.55	39.40
BUTTERFAT COW	-0.24	1.00	-1.57	-9.44
PROTEIN COW	0.15	-0.64	1.00	6.01
OTHERSOLIDS COW	0.03	-0.11	0.17	1.00

Note: Output Elasticities are Reciprocals

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