Implications of the New Growth Theory to Agricultural Trade Research and Trade Policy

Proceedings of a Conference of the International Agricultural Trade Research Consortium

Edited by Terry L. Roe

April 1997

The International Agricultural Trade Research Consortium
The New Growth Theory: Its Logic and Trade Policy Implications

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1. Introduction

Trade economists have long asserted that growth effects are more important than static effects, but have shied away from growth effects since they are purportedly hard to understand and difficult to measure. The development of endogenous growth and trade theory has removed first objection. Measuring growth effects of trade liberalization remains a challenge, however, some progress has been made on the econometric and computer simulation fronts.\(^2\)

The goal of this paper is to illustrate the basic logic of endogenous growth and trade models using a very simple approach that is based on Tobin's famous q-theory of investment. This approach shows that so-called new growth and trade models are not much difficult than standard trade models with monopolistic competition and scale economies. Indeed, we show that one of the most popular new growth and trade models - the product innovation model of Grossman and Helpman (1991) - has a static economy representation that resembles the specific-factors model. That is to say, the steady-state growth balance is characterized by a time-invariant allocation of primary resources across sectors. Moreover all the dynamic aspects of the model depend upon the amount of real resources devoted to the innovation sector. What this means is that analysing growth effects of trade policy boils down to little more than studying the impact of trade policy on the employment in one sector in a static model.

The plan of the rest of the paper is as follows. The next section (section 2) presents the basic logic of trade and endogenous growth model, focusing on the well-known production innovation model of growth. Section 3 uses this logic to study several openness-and-growth

links. Section 4 draws a number of conclusions for policy and the final sector contains our concluding remarks.

2. Trade and Endogenous Growth Models

This section illustrates the logic of endogenous growth models in an open economy context. Rather than covering all the models at a superficial level, we choose to go into a simplified version of one popular model - Romer's product innovation model - in some depth.

2.1 An Open-Economy Product Innovation Growth Model

In the Romer (1990) model of product innovation, long-run growth is driven by the ceaseless accumulation of product innovations. Creating an innovation involves a one time cost, yet it provides a flow of services, so innovations can be thought of as knowledge capital. The open-economy version of the Romer (1990), namely Rivera-Batiz and Romer (1991a, b), shares the same basic structure. All these models include three other factors besides knowledge capital. In order to present our ideas as simply as possible, we work with a simplified Romer model; The particular set of simplifying assumptions of this section are those of Baldwin and Forslid (1996). We turn now to the precise assumptions.

2.1.1 Assumptions

We start with two identical countries each of which as two sectors (X and I) and two factors of production (capital K and labour L). Units of capital are produced by the perfectly competitive I sector and sold to unrelated firms in the X sector. The X sector consists of a differentiated consumer good produced by monopolistically competitive firms. Each unique X variety is produced with labour (the variable input) and a unit of capital (the fixed cost). X sector output is sold directly to domestic and foreign representative consumers. This trade in X, however, is subject to home and foreign frictional (i.e. 'iceberg') import barriers measured by \( \tau \).
Factors are not traded. Since each new variety requires a unit of K and we ignore depreciation, the number of varieties rises in tandem with the capital stock. K accumulation thus increases X-sector competition, thereby driving down the reward to each unit of variety-specific capital. Consequently, the incentive to develop new varieties tends to vanish as K rises. This would result in a zero long-run growth rate were it not for Romer's assumption of a very specific learning curve in the I sector. Namely, he assumes that the I-sector's unit labour requirement falls as the sector's cumulative production rise (cumulative I-sector output equals the capital stock since depreciation is assumed away). Romer (1990) justifies this sector-specific, labour-saving externality as stemming from knowledge spillovers (i.e., the more K we have made, the easier it is to make more).

The X-sector cost function for a typical variety is $\pi + w_a X_i$ where $\pi$ is K's rental rate, $w$ is the wage and $X_i$ is per-firm output; The matching production function is $x_i = L_{xi}/a$, where $L_{xi}$ is the i-th firm's labour employment (presuming the firm owns a unit of K). By choice of units $a=1$.

Turning to the I-sector, the production and marginal cost functions for K are:

$$Q_K = L_I / b , \quad F = wb$$

$$b \equiv \left( \beta (K + \lambda K^*)^\eta \right)^{-1}, \quad 0 < \eta, \quad 0 \leq \lambda \leq 1$$

(2-1)

where $Q_K$ is the flow of output (i.e. K since K depreciation is assumed away), $L_I$ is I-sector employment, "b" is the I-sector unit labour input coefficient, and F is the marginal cost of K. Due to I-sector production externalities - "knowledge spillovers" - b is assumed to be diminishing in K and K* (the domestic and foreign capital stocks). The basic idea here is that b falls with K and K* due to learning effects; Note that K and K* are cumulative I-sector production. Also, $\lambda$ is a parameter that governs the extent to which externalities, or learning effects, occur internationally, $\beta$ is a parameter that reflects the fundamental productivity of I-

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3Numbered endnotes refer to the attached 'Supplemental Guide to Calculations'.
sector labour, \( \eta \) is the spillover elasticity (a measure of the importance of the production externalities, and \( F \) is \( K \)'s average and marginal cost. Notice that I-sector labour productivity rises (i.e. the unit labour requirement falls) as \( K \) and/or \( K^* \) grow. From the I-sector production function and symmetry of countries (\( K=K^* \)), we have:

\[
g = \frac{\dot{K}}{K} = \frac{Q_K}{K} = \frac{L_i}{P} (1 + \lambda) \eta K^{-1}
\]

where \( g \) is the growth rate of \( K \). Note that when \( \eta = 1 \) - as in Romer (1990) - a constant \( L_i \) yields constant capital stock growth. The dual of this is that \( F \) falls at the rate of capital stock growth.

The I-sector externalities in the Romer model have a knife-edge property. That is, if long-run growth is to be constant, the spillover elasticity \( \eta \) must be exactly equal to unity. For instance, if \( \eta < 1 \), the \( K \) stock has a steady-state level and the model has a zero long-run growth rate. However if \( \eta > 1 \), the economy tends to have an accelerating growth rate. As far as our analysis is concerned, the important point is that if \( \eta < 1 \), the 'Romer model' is not an endogenous growth model. It is an exogenous growth model. 1

Turning to the demand side, representative consumer preferences are:

\[
\int_{t=0}^{\infty} e^{-\rho t} \ln(C)dt ; \quad C = \left( \sum_{i=1}^{K+K^*} c_i^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \quad \sigma > 1 , \quad 0 < \rho < 1
\]

where \( \rho \) is the pure time-preference parameter, \( C \) is the consumption aggregate and \( c_i \) is consumption of variety \( i \). Full employment (with one-unit of \( K \) per variety) implies that \( K+K^* \) is the global number of varieties.

2.1.2 A Broader Interpretation

Romer (1990), and Grossman and Helpman (1991) cast their models in terms of knowledge capital. The underlying economic logic of the model, however, admits a broader

\[F is an unusual notation for marginal cost, but with perfect competition the I sector's marginal cost is the X-sector's fixed cost.\]
interpretation. For instance, instead of assuming that the appropriable knowledge is embedded in a blueprint or design (as in Romer (1990), and Grossman and Helpman (1991)), we could suppose it is 'embedded' in labour (this would make it human capital) or in a machine (as suggested in Romer's earlier work). Even more generally, the mechanics of the model does not require us to be specific about the nature of the capital. For instance, instead of assuming that the X sector fixed costs involves a unit of knowledge capital (e.g., a product innovation), we could assume that it requires one unit of variety-specific physical capital (e.g., a variety-specific machine), or alternatively one unit of variety-specific human capital (e.g., a variety-specific skill). The I-sectors corresponding to the three interpretations are: (1) The innovation (i.e. R&D) sector, (2) the investment good (i.e. machine) sector, and (3) the instruction (i.e. education/training) sector.\textsuperscript{5} The stories used to rationalize the I-sector externalities may be quite different for physical, human and knowledge capital, yet these differences have no impact on the model's mechanics. Here we adopt an agnostic interpretation by referring to the required unit as a generic unit of K.

2.1.3 Intermediate Results

On the demand side there are three important intermediate results. First, a typical nation's nominal income, denoted as $Y$, equals factor income $wL+\pi K$. Second, utility optimization yields standard CES demand functions for all X varieties:

$$c_j = \frac{p_j^{-\sigma}}{\sum_{i=1}^{X+K} p_i^{1-\sigma}} E$$

(2-4)

where E is nominal expenditure. Lastly, intertemporal optimization yields the standard Euler equation $E/E = r - \rho$, where $r$ is the rate of return on savings.\textsuperscript{3}

Turning to the X-sector intermediate results, we note that due to the assumed Dixit-

\textsuperscript{5}This coincidence of initial letters explains our terminology 'I-sector'.

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Stiglitz monopolistic competition, each X-firm perceives demand functions for its variety in both markets that are characterized by a constant demand elasticity equal to 1/\( \sigma \). Consequently, its optimal price is \( w/(1-1/\sigma) \) in its domestic market and \( w\tau/(1-1/\sigma) \) in its export market (as always, constant markups implies that frictional trade barriers are passed 100% on to foreign consumers). Since each variety is sold in both countries, the model is marked by intra-industry trade in differentiated products.

Because \( K \) is variety-specific, \( K \)'s reward is the Ricardian surplus of a typical variety. With constant markup pricing, this equals \( 1/\sigma \) times the value of a firm's sales, so \( \pi \) equals \( (sE+sE^*)/\sigma \), where \( s \) and \( s^* \) are the firm's home and export market shares, and \( E \) and \( E^* \) are home and foreign consumer expenditure on \( X \).

The model analytics are greatly simplified by balanced trade in \( X \); Sufficient conditions for this are that the countries are identical, or that \( X \) is the only traded good (both assumptions apply here). Trade balance requires that home expenditure on foreign varieties equals foreign expenditure on home varieties, so:

\[
\pi = \frac{sE}{\sigma} + \frac{s^*E^*}{\sigma} = \frac{E}{\sigma(s+s^*)} = \frac{E}{\alpha K}
\]

The last expression results from the fact that with symmetry of firms and countries, \( K(s+s^*)=1 \). As we shall see, this simple expression for \( K \)'s rental rate - in particular the simple dependency of \( \pi \) on the aggregate \( K \) stock - is extremely handy in solving the model.

By perfect competition in the I sector, the supply price, i.e. replacement cost, of capital (denoted as \( P_K \)) equals marginal cost \( F \).

2.1.4 Dynamic Analysis with Tobin's q

Tobin (1969) introduced a powerful and intuitive way of characterizing optimal investment in a general equilibrium context. As it turns out, this so-called q-theory approach also provides a
powerful way of analysing new and old growth models. More precisely, Tobin's q is the ratio of the stock market value of a unit of capital to the replacement cost of that unit. We shall see, Tobin's q is a simple function of K's steady-state growth rate in endogenous growth models (and a simple function of K's steady-state level in exogenous growth models). Furthermore, the q-approach helps us to see that the 'static-economy representation' of the Romer model is closely related to the well-known specific factors model.

Mainstream endogenous growth models, including the simple one presented above, all make assumptions such that constant real investment yields constant capital-stock growth and therefore constant output growth. It seems natural, therefore, to take real investment as the main state variable, even though the key dynamic equation - the Euler equation - involves expenditure. A simple change of variables permits substitution of real investment - i.e. $L_1$ in this model- for $E$. With I as nominal investment, nominal income $Y$ equals $E+I$, where $I$ equals $wL_t$. We have, therefore, that $E=wL+K\pi-wL_1$, and using $\pi=E/K\sigma$:

\[ E = \frac{w(L-L_1)}{1 - 1/\sigma} \quad (2-6) \]

Taking $L$ as numeraire ($w=1$) and real investment (namely $L_1$) as a state variable, we see $E=0$ in steady state because $L_1=0$ by definition of steady state. From the Euler equation, $E=0$ implies $r=p$ in steady state. Moreover since the only state variable $L_1$ is a "jumper", the system is always in steady state, so $r$ always equals $p$. This result greatly facilitates calculation of the present value of introducing a new variety.

To calculate Tobin's q, we start with the numerator. The stock market value of a unit of capital, $J$, is the present value of the $\pi$ stream, namely:

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\[ J(t) = \int_{s=t}^{\infty} e^{-r(s-t)} \pi(s) \, ds = \frac{\pi(t)}{\rho + g} \quad (2-7) \]

A unit of K can be bought for \( P_K \), so the denominator of \( q \) is \( P_K \). Due to I-sector competition, \( P_K = F \). Thus using (2-1), (2-2), (2-5) and (2-8):

\[ q[L] = \frac{\pi/ (\rho + g)}{F} = \frac{(L - L_I)\beta(1 + \lambda)}{(\sigma - 1)(\rho + L_I \beta(1 + \lambda))} \quad (2-8) \]

Tobin's \( q \) is a simple, monotonically decreasing function of the state variable \( L_I \).

Using Tobin's famous \( q = 1 \) steady-state condition we can solve for the steady-state allocation of \( L \) as:

\[ \bar{L}_I = \frac{L}{\sigma} - \frac{\rho(\sigma - 1)}{\sigma \beta(1 + \lambda)}, \quad \bar{L}_X = L - L_I \quad (2-9) \]

where "bars" denote steady-state values. Using the I-sector production function:

\[ \bar{g} = \frac{\beta L(1 + \lambda) - (\sigma - 1)\rho}{\sigma} \quad (2-10) \]

The remaining dynamic features of the model all follow from the steady state \( g \). In particular, real income grows at \( g/(\sigma - 1) \) since nominal income is time-invariant - namely \( Y = (\sigma L - L_I)/(\sigma - 1) \) - yet the consumer price index (the standard CES price index) falls at \( g/(\sigma - 1) \). The steady-state investment rate is straightforward to calculate. Nominal spending on investment is \( L_I \), so the investment rate is \( L_I/Y \). Using the expression for \( Y \):

\[ \text{investment rate} = \frac{I}{Y} = \frac{(\sigma - 1)\bar{L}_I}{\sigma L - \bar{L}_I} \quad (2-11) \]

This is clearly increasing in \( \bar{L}_I \). With trade balance, it is also the savings rate.
The ability to first solve for the static allocation of resources, then to characterize the
dynamics is an example of the 'block recursive' nature of growth models in steady state. In fact
although analytic concepts are clearest when expressing \( q \) in terms of \( L_{I} \), we have the possibility
of skipping straight to the dynamic part. The reason being that with \( g = \beta L_{I}(1+\lambda) \), \( q \) can also be
expressed as a simple monotonically decreasing function of \( g \), namely:

\[
q[g] = \frac{L\beta(1+\lambda)-g}{(\sigma-1)(\rho+g)}
\]  \hspace{1cm} (2-12)

and \( q=1 \) defines \( g \) directly.

2.1.5 Specific-Factors Representation of the Romer Model

In steady state the sectoral division of primary factors is time invariant, so the steady state
resembles a static economy.

To address this issue, note that \( q \) is the value of the marginal product of labour in the I-
sector. By definition the value of the marginal product of labour is the value of the output of a
unit of labour. Since it takes \( F \) units of labour to produce a unit of \( K \) that is worth \( J \), \( J/F \) is the
value of the marginal product of I-sector labour. This fact brings us to the static economy
representation of the Romer model.

In Figure 1, the horizontal axis's
length is the typical country's \( L \) endowment. I-
sector employment is measured from the left and X-sector employment measured from the
right, so every point defines a sectoral division of \( L \). The value of the marginal product of \( L \) in
the I sector - denoted as VMPL\(_{I} \) in the figure - is a decreasing function of \( L_{I} \). Since \( X \) is
imperfectly competitive, the proper concept is
the value of the marginal revenue product of L in the X rather than the value of the marginal product of \(L_x\). This, which we denoted as VMRPL\(_x\), equals unity for all divisions of L. To understand this, note that the marginal physical product of L in X is 1 (recall \(a=1\) by choice of units), so the marginal revenue product is \(p_x(1-1/\sigma)\). Since the optimal \(p_x\) equals \(1/(1-1/\sigma)\), VMRPL\(_x\)=1 for all \(L_x\). As usual the intersection of VMPL\(_t\) and VMRPL\(_x\) identifies the equilibrium division of \(L, L_t\). This is the static representation of the Romer model. The steady-state growth rate \(\ddot{g}\) is read off the I-sector production function in growth rate form (i.e. \(Q/K\)), which we have graphed in the lower quadrant of the figure.

2.1.6 Policy and Welfare Analysis

The recursive nature of steady-state growth models opens the door to a very simple approach to the analysis of the growth effects of policy and parameter changes. The specific-factors diagram illustrates the approach. In steady-state (this model is always in steady state)\(^8\) \(q=1\), but policy or parameter changes can lead to incipient changes in \(q\). For instance, consider a decrease in the rate of pure time preference \(\rho\). From (2-7), this increases \(J\) without altering \(F\) and so shifts the \(q\) schedule upwards (as shown by the dashed line in the figure). If \(L_t\) were unchanged, pure profits would appear in the I sector. These incipient profits direct I-firms to engage more labour, so in fact \(L_t\) increases instead of \(J\) (the steady state \(J\), of course, remains equal to \(F\)). The resulting change in the underlying static economy - viz. the rise in \(L_t\) - has obvious dynamic implications. In particular, it raises \(\ddot{g}\) and thereby the long-run growth of real GDP. We use the adjective 'incipient' since \(q\) always equals unity in steady state, but holding \(L_t\) constant, \(q\) would exceed 1. It is obvious from the diagram that any incipient increase in \(q\) will have a pro-growth effect and any incipient decrease in \(q\) has an anti-growth effect.

We conclude the analysis of the Romer model with its welfare implications. Private agents devote resources to capital formation up to the point where the present value of capital's income equals its marginal cost, i.e. \(J=F\). However, creating a new unit of capital also lowers the cost of all future capital formation due to the I-sector externality. Private investors do not
perceive this, so the laissez-faire economy under invests and grows too slowly. An ad valorem I-sector subsidy would restore optimality. (see Baldwin and Forslid (1996) for a formal derivation of the optimal subsidy).

2.2 Process Innovation Models

Given space constraints, we do not detail other main endogenous growth models as extensively as we have done with the product innovation model. In this subsection we briefly present models of endogenous growth that focus on process innovation. The next subsection looks at models that focus on human capital.

A separate line of thought has been pursued by the process-innovation models. Unlike the product-innovation models, these innovations may drive some existing firms out of business. The reason is simple. Instead of creating a new product that can be sold side-by-side with existing products, process innovations allow the innovator to make existing products more cheaply. This means the owner of the innovation may find it worthwhile to charge a price that drives her competitors out of business. The profits earned by the innovator from exploiting this edge compensate her for the research and development costs. However even as the most recent innovator is luxuriating in profits, other firms are developing processes that have even lower manufacturing costs. When one of them succeeds, the previous king-of-the-hill is deposed. This ceaseless search for cost-lowering innovations leads to a ceaseless accumulation of knowledge and drives manufacturing productivity ever higher. "Creative destruction" is the term that Schumpeter used to describe this process.

As with product-innovation models, the private-versus-social wedge in these models appears in the innovation sector. However, the technology spillover takes a different form. The "production function" for innovations does not change in the sense that the amount of resources necessary to obtain a new innovation does not change. What does change is the nature of the

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resulting innovation. Each new innovation is clearly better than the last. There is a spillover in that each new innovator gets to start working directly on the frontier of knowledge.

One model in the process-innovation vein, which is especially important for understanding real-world process innovation, is the "quality ladders" of Helpman and Grossman (1991) and related work by Aghion and Howitts (1992). This is a model where each subsequent process innovation allows the innovator to raise the quality of her product without raising the manufacturing cost. This is important to understanding modern technical progress since for many innovative industries, such as personal computers and consumer electronics, it is difficult to classify advances as product innovations or process innovations. Take the example of portable radios. Electronic firms make more or less the same products using more or less the same process, but each year the radios are clearly superior -- lighter, more durable, more powerful, with better sound, etc.

It is important to note that in spite of the different way technological progress is modelled, the process innovation and the product innovation class of models have similar predictions about the determinants of growth rates.

2.3 Human Capital and Growth

Lucas (1988) opened a new line of thinking in the endogenous growth literature by focusing on the contribution of human capital to GDP growth. Human capital is the skill of workers. It is considered capital since skill embodied in workers can produce a flow of service for an extended period, just as a capital good can. Models that focus on human capital accumulation do not differ in their fundamental economic structure from the models already discussed. The basic logic of ceaseless accumulation must be respected. In particular, the economy-wide rate of return on investing in human capital cannot be forever falling or rising. Moreover, to assure the well-behaved nature of the self-interested investment decisions of private citizens and firms, the models must have some way of introducing a wedge between the private and social (i.e. economy-wide) rates of return.
We start with a trivial re-labelling. First, it is obvious that by calling it "human capital" instead of "capital" we could interpret the endogenous growth models presented above as showing that the human capital can be a source of growth. What is interesting about models that focus on human capital is that they provide a particularly convincing story behind the existence of the private-social wedge. People invest in raising their skill level because they anticipate that there will be a lot of capital that will allow them to turn high skill levels into high salaries. Likewise, firms invest in new plant and equipment because they expect there to be a lot of skilled workers around to make the factories turn a profit. The wedge exists since undertaking the investments in human capital do not take account of the output-boosting effect of the capital formation that their actions will induce. One interesting application of this approach is to understand the spectacular growth experienced by Japan and Germany after WW-II. In both cases, the economies suffered a large loss of physical and human capital, however the loss in physical capital was by far the larger of the two. Given the implications of diminishing returns to K and H taken separately, this means that the postwar economies of Japan and Germany were extremely good places to invest physical capital. They had lots of human capital compared with physical capital, so the return on physical capital was extraordinarily high. Following the usual logic that a high rate of return attracts a high rate of investment, the theory predicts that Japan and Germany should have experience above normal growth during this period.

3. Trade and Growth Links: The Theory

The theory of trade and growth developed in two distinct phases. In the 1970s, trade economists explored the old (neoclassical) growth theory in the context of the old (Heckscher-Ohlin) trade model. In the late 1980s and early 1990s, trade economists explored new (endogenous) growth theory in the context of new (imperfect competition) trade models. Since the old-growth-old-trade models assumed perfect competition and constant returns, only a

The second-phase literature - i.e. new-trade-new-growth theory - is summarized and synthesized in Grossman and Helpman (1991), and Rivera-Batiz and Romer (1991a, b). By allowing for imperfect competition and increasing returns, the models in this literature extends the range of trade-and-growth links that can be considered. Nevertheless, since this literature worked with extreme - albeit common - forms of market structure and input-output relationships, it focused on a very limited range of trade and growth links. For example, Rivera-Batiz and Romer (1991a, b) list three trade and growth effects:

1. The redundancy effect:
   By eliminating duplication of innovation activities in different countries, trade permits a more efficient utilization of I-sector resources worldwide. The resulting increase in I-sector labour productivity tends to boost growth.

2. The integration effect:
   If the I-sector is subject to economy-wide scale economies, trade can boost I-sector labour productivity by increasing the extent of the market. In the simplified Romer model, this takes the form of a production externality that spillovers internationally.

3. The reallocation effect:
   As usual, opening to trade alters the equilibrium allocation of resources across sectors. Growth is increased if the trading equilibrium involves more resources in the I-sector; Growth is diminished if trade lowers the amount of resources employed in the I-sector.
Grossman and Helpman (1991, Chapter 9) suggest an alternative classification of effects by listing four ways in which trade can affect growth: Market size, redundancy, international knowledge spillovers and the allocation effect. The four Grossman-Helpman channels and three Rivera-Batiz-Romer channels are simply two ways of categorizing the same set of effects.

The redundancy and integration effects refer to very specific economic mechanisms. These are essentially shifts in the efficiency of investment spending. The third effect, however, is a grab bag of all sorts of effects. The 'reallocation effect' includes anything that changes the rate of investment. Since investment is the key to growth, it would seem important to provide a finer classification of the economic mechanisms that lead to a rise in the rate of investment. In this sense, the Rivera-Batiz-Romer and the Grossman-Helpman classification schemes are complete but uninformative. Furthermore, many economic mechanisms that would lead a reallocation effect were not explored. One reason for this is that much of the literature limited itself to extreme forms of integration - viz. the shift from free trade to autarky - or infinitesimal changes in trade barriers in the neighborhood of free trade. An exception is Rivera-Batiz-Romer (1991b), which considers ad valorem tariffs and finds a non-monotonic relationship between openness and growth.

The Grossman-Helpman-Rivera-Batiz-Romer trade and growth models rely on extremely unrealistic simplifying assumptions to keep the analysis tractable. While simplifying assumptions are inevitable in theoretical models, they often have the undesired by-product of shutting off important economic mechanisms. One of the most objectionable simplifying assumptions was the extremely simplistic nature of the I-sector production technology and market structure. Namely, the Grossman-Helpman-Rivera-Batiz-Romer models assumed that new units of knowledge capital were produced from labour alone according to private constant returns and perfect competition. Real-world innovation, however, is a much more complex process. For instance, the production of physical capital especially - but also of human and knowledge capital - involves factors other than labour. In particular it involves traded intermediates inputs, so trade barriers on such goods will affect the equilibrium price of capital.
Furthermore, a great deal of innovation is undertaken by imperfectly competitive firms. Given this, international integration may have a pro-competitive effect that lowers equilibrium markups and thereby lowers the price of capital.

In the rest of this section we use the q-approach to formally show the effects of different kind of tariff reduction on growth, and the effects of trade liberalization on the price of capital when the I-sector tecnology is enriched to allow for traded intermediates. The section ends with an informal treatment of the procompetitive effect of trade liberalization in the investment sector.

3.1 Protection and Growth

Here we consider the relationship between various forms of protection and the endogenous growth rate \(^8\).

3.1.1 Introducing Protection

Before proceding, recall that the q-approach enables us to take an enormous shortcut. As shown in Section 2:

\[
q = \frac{\pi}{F} \frac{1}{\rho + g}
\]  

(3-1)

and anything that leads to an incipient rise (fall) in q will lead to a pro-growth (anti-growth) effect. In this subsection, we stick with the simplest formulation of (2-1) where \(\lambda = 1\), \(\eta = 1\) and by symmetry of countries \(K = K^*\). All this means that \(F = 1/2\beta K\). As a consequence, none of the policies considered in this subsection affect \(F\) directly. Thus the only way a policy can have an incipient impact on \(q\) - and this is the same as saying that the only way it can have a growth effect - is by altering \(\pi\) when \(K\) and \(L_I\) are held constant. This is a shortcut since it means that we only have to consider the impact of various forms of protection on \(\pi\). We do not have to re-work

\(^8\)Baldwin and Forslid (1996b) provides a similar analysis in the context of a process innovation model
the whole model. While this may seem too easy to be true, it is not. It is an illustration of the usefulness of the $q$ approach. We turn now to introducing three types of protection into the model.

We return to the Section 2 model, and assume that trade in $X$ is subject to three types of import barriers: an ad valorem tariff (which we write as $\tau=1+t$, where $t$ is the tariff rate), a specific (i.e. per unit) tariff of $T$ units of the numeraire per unit of $X$ imported, and a frictional (i.e. iceberg) barrier such that $\phi \geq 1$ units must be shipped to sell one unit in the foreign market. Tariff revenue is either returned lump-sum to consumers or destroyed, depending upon the scenario studied.

In the $X$ sector, an $X$-firm chooses $x$ and $x^*$ to maximize:

\[(p-1)x + \left( \frac{p^*}{\tau} - \phi - T \right) x^* \tag{3-2} \]

where $x$ and $x^*$ are the firm's sales to consumers in the local market and the export market respectively. Recall that $w=1$ by choice of numeraire and the $X$-sector unit input coefficient is unity by choice of units. Since each firm's capital is variety-specific and therefore firm-specific, capital's reward is the Ricardian surplus, i.e. operating profit. For this reason, $K$'s reward is linked to barriers via the operating profit function. It can be shown that the operating profit function is:

\[\pi = \frac{(S + S^*)}{\sigma} \left( \frac{E}{\tau \sigma K} \right) = \frac{E \Psi}{\sigma K} \tag{3-3} \]

\[= (S + S^*) \frac{1}{\tau \Gamma^{\sigma-1}}, \quad \Gamma \equiv \tau(\phi + T), \quad S + S^* = S^* \]

where $S^*$ is the import penetration ratio in a typical market and $\Gamma$ a measure of overall protection.

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\[^9\]These are, for instance, the three types considered in Helpman and Krugman (1989). A quota is typically equivalent to a specific tariff under Cournot conjectures.
(it is the ratio of imported to local varieties). Notice that \( \pi \) is sales (at consumer prices) times the equilibrium profit rate in each market (viz. \( 1/\sigma \) in the local market and \( 1/\sigma \tau \) in the export market). Below we shall see that protection can affect both sales and profit rates.

Finally we need an expression for tariff revenue from ad valorem and specific tariffs. For ad valorem tariffs, total revenue in a typical country is \((1-1/\tau)p^*x^*\), which simplifies to \(E(\tau-1)/[\tau(1+\Gamma\sigma-1)]\) and for specific tariffs, total revenue is \(Tx^*\), which simplifies to \(TE(1-1/\sigma)/[(1+\Gamma\sigma-1)(\tau(\phi+T))]\). Combining these we have

\[
R = E\Theta, \quad \Theta = \frac{\tau-1 + \frac{T(1-1/\sigma)}{\phi+T}}{\tau(1+\Gamma^{\sigma-1})}
\]

Notice that when we have free trade, or only frictional barriers, \( \tau=1 \) and \( T=0 \). It is also useful to note that \( \Theta \) can never exceed unity, so \( 0 \leq \Theta < 1 \). Of course, in scenarios where the tariff revenue is assumed to be thrown away, \( \Theta=0 \).

The only significant change from the Section 2 model concerns \( E \). When the tariff revenue is returned lump-sum to consumers, \( E=L+R+K\pi-L_1 \). Using \( \pi=\Psi E/K\sigma \) and \( R=E\Theta \), we have:

\[
\Psi E = \frac{L-L_1}{1/\Psi - \theta/\Psi - 1/\sigma}
\]

Thus the expression for \( \pi \) is:

\[
\pi = \frac{(L-L_1)/K}{\sigma/\Psi - 1 - \theta/\Psi}
\]

3.1.2 Growth Effects of Protection

Using our shortcut, we know that the impact on \( \pi \) of changes in the three types of protection is proportional to the growth effect of the same changes in protection. Our task
therefore is to see how $dt$, $d\phi$ and $dT$ affect $\pi$.

Intuition is served by isolating effects. Thus we consider only one form of protection at a time, starting with frictional barriers. When $T=0$ and $\tau=1$, $\Psi=1$, so $\phi$ drops out of the expression for $\pi$, (3-6). This means that a change in the level of frictional barriers (on a symmetric basis) has no effect on $\pi$; It therefore has no effect on $q$ and this tells us that it has no growth effect.

Next consider a specific tariff in isolation. Apart from issues of tariff revenue, we can see from (3-6) and the definition of $\Psi$, that $T$ acts exactly in the same way as frictional barriers $\phi$. In particular when $\tau=1$, $\Psi=1$. It is worth highlighting this fact using the following thought experiment. Suppose that governments impose only specific tariffs but that the tariff revenue is not returned to consumers (e.g., the revenue is destroyed or wasted by the government). In this case, $\theta=0$, so again $\pi$ is unaffected by $T$. This, of course, means that growth is unaffected by specific tariffs when the tariff revenue is wasted. Returning to the more standard assumption that tariff revenue is returned to consumers, we see that $T$ can affect the growth rate. However, since the relationship between $\theta$ (the tariff revenue factor) and $T$ is bell-shaped (think of the Laffer curve), the relationship between $T$ and $\pi$ is also bell-shaped. Using the shortcut once again, this means that the relationship between $T$ and growth is bell-shaped.

Finally consider ad valorem tariffs in isolation (i.e. $T=0$, $\phi=1$). In this case there is a U-shaped relationship between $\tau$ and $\pi$. To show this start with differentiate $\pi=E(S+S^*/\tau)/K\sigma$ with respect to $\tau$:

\[
\frac{\partial \pi}{\partial \tau} = \left( \frac{E}{K} \right) \left( \frac{\partial S}{\partial \tau} \frac{1}{\sigma} + \frac{\partial S^*}{\partial \tau} \frac{1}{\tau \sigma} - \frac{S^*}{\tau^2} \right)
\]  (3-7)

The three terms reflect the three ways in which a reciprocal change in the ad valorem tariff affects $\pi$. The first term shows how extra protection of the local market boosts local firms' $\pi$ by raising their sales in the local market (since $S=1-S^*$, this is accomplished by diminishing the import penetration ratio $S^*$). The second reflects the fact that a rise in the foreign protection lowers local firms' export sales, thereby lowering $\pi$ holding the profit margin $1/\sigma \tau$ constant. The
third captures the negative impact that a foreign ad valorem tariff has on local firms' profit margins on export sales.

While the two sales-shifting effects are intuitively obvious, the negative export-market profit-margin effect is not. Thus before proceeding to demonstrate the U-shape of the $π-τ$ curve, we present the Helpman and Krugman (1989 page 65) intuition for the third effect. When an exporter lowers her price by one dollar, foreign consumers see the price drop by $1+\tau=\tau$ dollars. The reason is simply that the foreign customs authorities lower the tax in line with the producer price drop. The increase in sales therefore is $τσ$ rather than $σ$. For this reason the exporter perceives the foreign demand more elastic than it really is.

Now we are finally ready to demonstrate the U-shaped relationship between $π$ and $τ$. Consider first the impact of protection on $π$ when starting from free trade (i.e. $τ=1$). Since $S=1-S^*$ and the local-market and export-market profit margins are identical, the two sales-shifting effects cancel out. The partial derivative $\partial π/∂τ$ is therefore negative since only the negative export-market profit-margin effect remains. Next consider the impact of liberalization when starting from autarky (i.e. $τ$ is prohibitive, so $S^*=0$). In this case, the negative export-market profit-margin effect has no importance since $S^*=0$. The sign of $\partial π/∂τ$ therefore depends only on the sum of two sales shifting effects. This sum is positive, since the profit margin on local sales exceeds the profit margin on export sales. In other words, liberalization lowers $π$ since the loss in $π$ from the liberalization-induced decrease in local sales is greater than the gain in $π$ from the liberalization-induced increase in export sales. To summarize, the slope of the $π-τ$ curve is initially negative, zero at only one point and positive after that point. All of this tells us that the $π-τ$ curve is U-shaped.

Figure 2

Growth Rate

Specific Tariff
Frictional Barriers
ad valorem tariffs

Protection Rate

21
Using our shortcut one last time, we know that the relationship between ad valorem tariffs and growth is U-shaped. Rivera-Batiz and Romer (1991b) were the first to demonstrate this is a more complex model.

Figure 3-1 summarizes the relationship between growth and various forms of protection.

3.2 Traded-Intermediates and the Price of Capital

As argued at the beginning of this section, it seems quite reasonable to assume that the I-sector production function includes traded intermediate inputs as well as labour. As it turns out, doing so opens up an interesting trade and growth link. The basic logic of this link is straightforward. When the manufacture of human, knowledge or physical capital involves traded intermediate inputs, the price of traded goods enters the I-sector's cost function. Trade barriers affect these prices, so $P_K=F$ becomes a function of these barriers. Consequently, liberalization affects the denominator of Tobin's $q$. For instance, global liberalization can lower $P_K$ in both countries, thereby creating an incipient increase in $q$ that is translated into a rise in $r$. The rise in the real interest rate will triggers faster capital accumulation and faster growth in transition to the new steady state in an exogenous growth model or permanently faster growth in the endogenous growth version. To show an application of Tobin's $q$ approach to an exogenous growth model we develop this channel in that framework; This simply means to set $\eta = 0$ in (2-1). The $q = 1$ condition now will deliver the steady state capital stock ratio instead of the growth rate.

While this link has been only recently modelled formally, it has long been thought important by practical men. For instance, it is related to the literature on imported capital goods, which have long played an important role in the trade and growth literature. See, for instance, Cairncross (1962) and more recently Lee (1992, 1994). Although the analysis is sketchy, this link also seems to be the focus of the so-called lab equipment version of Rivera-Batiz and Romer (1991a). The illustration here is based on Baldwin and Seghezza (1996).

To illustrate this link as simply as possible, we modify the basic model by assuming that
the I sector employs L and intermediate inputs (specifically, a CES composite of all X varieties with an elasticity of substitution \( \sigma \)) to produce new capital under constant returns. Assuming a constant rate of depreciation \( \delta \), the marginal cost and net investment functions are:

\[
\dot{K} = \frac{wL_I + P_X X_I}{F}
\]

where \( P_X \) is the standard CES price index, and \( X_I \) and \( L_I \) are the X-sector composite and L employed. The second expression for \( F \) follows from the first using the definition of the standard CES price index and \( w = 1 \).

Since X-firms now sell to consumers and I-firms alike, the expression for steady-state \( \pi \) is somewhat different. In particular, a typical X-firm earns operating profit equal to \( E/\sigma K \) on sales to consumers and \( I/\sigma K \) on sales to I-firms. The expression for steady-state \( q \) is

\[
q = \frac{\mu L - K \delta (\mu F - 1) + \sigma}{K (\sigma - \mu) \rho F^2}
\]

By inspection, \( q \) is diminishing in both \( K \) and \( F \). From (3-8) \( F \) is increasing in \( \tau \). Thus a drop in \( \tau \) will necessarily increase the steady-state capital stock and thereby produce a medium-run growth effect. Baldwin and Forslid (1996a) show that this same link works in an endogenous growth model.

### 3.3 Procompetitive Effects and Growth

This section considers the procompetitive effect of trade liberalization in the investment sector. Since the procompetitive effect is involved, we briefly exposit its main economic logic before turning to the trade and growth link.

To illustrate this mechanism, consider a simple partial equilibrium example of two symmetric countries engaged in Brander-Krugman reciprocal-dumping trade and imposing identical import barriers \( \tau \). That is, we assume 'n' Cournot oligopolists (in each country) are
selling to the two segmented markets. The equilibrium in either market is characterized by two key conditions: (i) the first order condition for local sales by local firms \( p = a/(1-s/e) \), where \( s \) is the market share of local firms in the local market, \( a \) is marginal cost and \( e \) is the constant demand elasticity, and (ii) the free entry condition. Now it is easy to show - and intuitively obvious - that 's' depends negatively on the number of symmetric firms (more competitors means a lower market share per firm) and positively on the tariff (more protection raises the market share of local firms). From the first order condition, this means that \( p \) is increasing in \( \tau \) and decreasing in \( n \). It is also clear that raising \( n \) lowers operating profit while raising \( \tau \) increases operating profit.\(^{12}\)

Consider the impact of lowering import barriers on a reciprocal basis. On impact, lowering \( \tau \) reduces \( s \) and thereby prices in both markets. However, the two-way liberalization also reduces operating profit. Since all firms were just breaking even even before the \( \tau \) reduction, liberalization creates incipient negative profits. The result, of course, is the exit of firms up to the point where pure profits are restored to zero. The third round of effects concerns the impact of this industrial restructuring on prices. Since lowering \( n \) tends to raise \( s \), the restructuring tends to raise prices back up towards the preliberalization level. This mitigating effect on the first round price drop cannot, however, push prices back up to the preliberalization level. The reason is simply that if it did, firms would be making positive pure profits (the same price with fewer competitors implies pure profits). Because the post-liberalization \( n \) must equate pure profits to zero, we know that post-liberalization prices are lower than preliberalization prices.

Finally, we turn to the implication of fewer firms and lower \( \tau \)'s and prices. First with home and foreign prices lower, total output in each country must rise. With fewer firms this means increased exploitation of scale economies and reduced markups. Furthermore, the difference between \( s \) (local firms share in the local market) and a local firm's share in the export market is decreased by the liberalization. This de-fragmentation of international markets means that though the total number of firms drops, the degree of effective competition has increased.
3.3.1 Imperfect Competition in the Investment Sector

A popular interpretation of the basic model views K as knowledge capital and the I sector firms as R&D labs. Under this interpretation, however, the assumption of constant private return in the I sector is rather objectionable. Developers must invest a good deal of time in learning about the state of the art before being able to come up with their own advancements. To allow for this, we enrich the basic model by introducing scale economies and imperfect competition into the I sector. I-sector firms (think of them as a research laboratories) sell designs in the home and foreign markets. We assume, however, that trade in innovations is hindered by a range of cost-raising barriers. The cost is intended to capture a wide range of common real-world barriers, but standards provides a concrete example. Most new product needs to be certified as meeting industrial, health, safety and/or environmental standards. The certifying boards are typically influenced by local industries (directly, when the board has industry representatives, or indirectly via political pressure on the national government) for whom the new product constitutes a threat. It is quite common, therefore, for standards to provide de facto discrimination against foreign varieties.

Designs can be thought of as a homogenous good. That is to say, when sold by a lab, they are perfect substitutes (given the symmetry of varieties) even though they produce a unique variety after they are sold. With this homogeneity, the most natural market structure assumption for the I sector is a Cournot oligopoly with segmented markets, so two-way trade in designs occurs as long as trade barriers are not too high.

This trade in intellectual property rights is analogous to the reciprocal dumping trade of Brander and Krugman (1983). Once this analogy has been established, the outcome of reciprocal liberalization is obvious given the analysis of the procompetitive effect by Smith and Venables (1988). Heuristically, reciprocal liberalization defragments the markets, thereby raising the degree of competition (e.g., the Herfindahl index of concentration falls in both markets). This reduces the average markup of $P_k$ over $F$, thereby lowering prices and creating incipient I-sector losses. The incipient losses force exit, partially offsetting the competition
increase. With a lower number of firms, however, the remaining firms are better able to exploit scale economies, making a lower equilibrium $P_k$ possible. Since $P_k$ is the replacement cost of capital as far as the X-sector is concerned, the procompetitive effect in the I sector leads to an incipient rise in Tobin q's in both economies. Higher output growth is the results. A formal derivation of this link in the context of endogenous growth model is in Baldwin and Forslid (1996); Baldwin and Seghezza (1996) shows how this works in an exogenous growth model. A similar line of reasoning apply to the procompetitive effect in the financial sector and in the X sector. A formal derivation of the procompetitive effect in the financial sector is in Baldwin and Forslid (1996), Baldwin and Seghezza (1996) provides a formal derivation of the procompetitive effect in the X sector.

4. Policy Implications

The basic message of Section 3 is that reciprocal liberalization of trade is generally good for growth. However, since growth model very frequently involve non-Walrasian aspects, laissez-faire is usually not the socially optimal policy. For instance in the product innovation model discussed in Section 2.1, the innovative sector faces a sector-wide learning curve. This production externality implies that private agents investment too little in innovation since they cannot appropriate the full marginal benefit of their actions. This suggests that governments should support innovative activities via production subsidies, tax breaks, etc. Virtually all governments do exactly this, however the process innovation models discussed in Section 2.2 suggest that too much innovation activity may occur. Since innovators do not account for the value of the knowledge they destroy when they introduce new innovations, it is theoretically possible that private agents develop new technology at too fast a pace. While this may seem empirically irrelevant at the economy-wide level, it surely happens in some specific sectors. For instance, each time Microsoft introduces a new operating system, it destroys an incalculable amount of human capital around the world. Since Microsoft does not percieve the full cost of
this, it may innovate too quickly from the social welfare point of view.

This is just one example of how difficult it is to draw firm policy conclusions from non-Walrasian models. One line of reasoning that seems to be somewhat more robust concerns the disadvantages of direct government involvement in innovative activities. Understanding this point requires some background on creative destruction and political economy.

4.1 The Political Economy of Creative Destruction

The notion of creative destruction is extremely important in understanding how an economy directed by selfish motives can year after year produce more and more from the same primary factors. Each time a new product or process is developed, the part (or all) of the value of previous inventions is destroyed. Schumpeter wrote:

The fundamental impulse that sets and keeps the capitalist engine in motion comes from the new consumer goods, the new methods of production or transportation, the new markets, ... [This process] incessantly revolutionizes the economic structure from within, incessantly destroying the old one, incessantly creating a new one. This process of Creative Destruction is the essential fact about capitalism.

(Cited in Aghion and Howitt, 1990.)

Since most economists would agree with Schumpeter's assessment, it is worth considering the political economy of creative destruction. Many existing firms and industries wield political influence that far exceeds their numbers or even their weight in the economy. This is especially true of state-owned or state-controlled firms. Causal evidence for this abounds, but one that is often cited is the frequently adoption of import barriers that help the few at the expense of the many. Likewise, in Europe one often observes that state-owned firms find it much more difficult to undertake necessary restructuring since strikes by unions immediately

Note that literary styles do not seem to be governed by a quality ladder. Modern economist rarely approach the eloquence of the great neoclassical economists.
become political when the employer is the government in power. In developing countries the same sort of thing goes on more informally. The political power of the firm's managers and/or employees often allows state-owned firms to win protection and subsidies that support un-economic activities.

What we shall argue here is that this disproportional political power may very well be used to stifle technical progress. After all, technical progress threatens their status quo and this harms the current position of politically powerful agents. The current leaders have knowledge that gives them an edge over other firms. Like any good owner of capital, these firms will resist a move to increase competition. Unfortunately, resisting competition in this context means resisting the introduction of new technology. Thus selfish firms, left to their own devices, will attempt to stifle the economy's growth by suppressing the introduction of new technology. That is, the existing firm might be quite capable of matching the technical progress and thereby maintaining their positions, but clearly they would rather not have to do so. Thus just as competition was essential to an efficient static allocation of resources, competition or at least the threat of competition is a necessary condition for technical progress.

This point is especially important in both rich and poor countries. In an attempt to promote domestic industry, government may strive to protect existing firms from competition. The idea being that protection would raise the return to introducing productivity-enhancing technology and thereby promote technical progress and growth. The weak link in this chain of reasoning is the last one. Unless existing firms are threatened by new technologies, they have very little incentive to continually invest in new productivity-boosting knowledge.

5. Concluding Remarks

This paper presents three trade and growth links that are theoretically possible. Policy implications are hard to draw since the profession knows surprising little about which links are most relevant in the modern world. The conclusion, therefore, must be to intensify research into
the empirical relevance of trade and growth links already demonstrated. To date much of the empirical research has consisted of quite blunt tests involving growth regressions. However, the models presented above illustrate that more pointed empirical tests are possible. For instance, if openness is good for growth because it lowers the price of new capital, then we should observe a direct link between the cost of capital and openness.

Further work also needs to be done in investigating (more formally) the broad gamut of openness and growth links that are discussed informally, but have not yet been modelled. These includes most of the links between multinationals, foreign direct investment and growth. It also includes the growth effects of regional integration, of the brain drain, of international capital market integration, of monetary unions, of access to international capital markets, of international R&D cooperation, of antitrust policies, etc.

This rather unsatisfactory conclusion reminds us of the old joke told to generations of MIT graduated students. The story goes that Paul Samuelson was one day talking to President Kennedy about the pros and cons of some policy options. Professor Samuelson had just finished explaining the merits of undertaking the policy and launched into the usual, "On the other hand" closing statement. At this point President Kennedy interrupted to say: "What I needs is a one-handed economist!" To this, Samuelson is reputed to have said: "Yes, but in that case you would have to choose between a left-y and a right-y." In this vein, there is no shortage of economists purporting to know exactly what governments must do to encourage growth. The sad truth, however, is that these recommendations are not based firmly on a well-accepted body of scientific theory and evidence.
References


Mercenier and Yeldan (1997) See this volume.


Supplemental Guide to Calculations

1. That is \( \tau \) units must be shipped in order to sell one unit abroad.

2. This aspect of the Romer model is formally demonstrated in Grossman and Helpman (1991 Chapter 3).

3. Given that preferences are intertemporally separable and consumers take the path of prices as given, we can solve the utility maximization problem in two stages. The first is to determine the optimal path of consumption expenditure \( E \). To this end we set up the current valued Hamiltonian, which for this problem is:

\[
H[E,K,\lambda,t] = e^{-\rho t} \left( \ln \left( \frac{E}{P} \right) + \lambda \left( \frac{\pi K + wL - E}{P_K} \right) \right)
\]

where \( C=E/P \), \( P \) is the standard CES price index and the law of motion for the representative consumer's wealth is \( K=(Y-E)/F \) since \( K \) is the only store of value. The four standard necessary conditions for intertemporal utility maximization are:

\[
\frac{\partial H}{\partial E} = 0 \iff e^{-\rho t} \left( \frac{1}{E} - \frac{\lambda}{P_K} \right) = 0
\]

\[
d \frac{e^{-\rho t} \lambda}{dt} = -\frac{\partial H}{\partial K} \iff \rho - \frac{\dot{\lambda}}{\lambda} = \frac{\pi}{P_K}
\]

\[
\text{law of motion} \iff \dot{K} = (Y - E) / F
\]

\[
\text{transversality condition} \iff \lim_{t \to} \lambda(t)K(t) = 0
\]

The first three conditions characterizes the optimum path at all moments in time, while the transversality condition is only an endpoint condition. The total time derivative of the first expression can be used to eliminate \( \lambda \) from the second expression. The result reduces to:

\[
\dot{E} = \left( \frac{\pi}{P_K} + \frac{\dot{P}_K}{P_K} \right) E - \rho E
\]

The Euler equation is found by noting that the right-hand expression in parentheses is the rate of return to \( K \) (the first term is the 'dividend' component and the second is the 'capital gains' component) and that this is the rate of return to savings, viz. \( r \).
4. A typical firm's first order condition for pricing to the home market is \( p(1-1/\sigma) = w \). Thus operating profit in the home market, \((p-w)c\), equals \( pc/\sigma \), where \( c \) is home consumption. A similar expression gives operating profit in the foreign market. The expression in the text follows from the definition of \( E, E^*, s \) and \( s^* \).

5. Due to zero I-sector profits, the value of the sector's output equals the value of inputs. In equilibrium all of the sector's output is purchased for investment, thus \( wL_i = I \).

6. Here we use the facts that in steady state, \( L_i = 0 \), so from the I-sector production \( K \) grows at a constant rate \( g \). Thus:

\[
J(t) = \int_{s=t}^{\infty} e^{-r(s-t)} \pi(s) ds = \int_{s=t}^{\infty} e^{-\rho(s-t)} \frac{L - L_I}{(\sigma - 1)K(s)} ds = \frac{L - L_I}{(\sigma - 1)K(t)} \int_{s=t}^{\infty} e^{-\rho(s-t)} e^{-g(s-t)} ds
\]

Solution of the integral yields the formula in the text.

7. The standard CES price index is:

\[
P = \left( \sum_{i=1}^{K+K^*} p_i^{1-\sigma} \right)^{1/(1-\sigma)} = (2K)^{1-\sigma} p_i
\]

where symmetry of counties and varieties is used to derive the second expression. Since prices are time-invariant (recall that \( w = 1 \), and \( \tau \) and the markup are fixed), time differentiation of \( P \) yields the result in the text.

Alternatively, nominal income may be deflated with an output price index (something akin to the GDP deflator), i.e. an index that reflects the prices of the I-sector's and the X-sector's output. In using the consumption price index \( P \), we were guided by microeconomic theory. That is, the CES price index \( P \) was employed since \( E/P_X \) is the indirect utility function for a moment in time. In this way, real income is related to a welfare measure. Unfortunately, we do not have any microfoundations behind the production price index, so we are free to choose the exact specification. A natural (although arbitrary) specification is a Cobb-Douglas price index (this is the one chosen by Grossman and Helpman 1991) of \( P \) and \( P_K \), where the I and X sector value-added shares are the Cobb-Douglas power-coefficients. This is equivalent to a geometry average of the output prices in the I-sector (namely \( J \)) and the X sector (namely \( P_X \)). The value added shares are \( \zeta = (L_X + \pi K)/Y \) and \( 1 - \zeta = L_i/Y \); In steady-state they are time-invariant. Now since nominal income is time-invariant, GDP deflated by the production price index rises at a rate that is the weighted average of \( g/(\sigma - 1) \) (the rate at which \( P \) falls) and \( g \) (the rate at which \( P_K \) falls). The two GDP growth rate are, therefore:

\[
G_{GDP} = \frac{g}{\sigma - 1}, \quad G_{GDP}' = \zeta \frac{g}{\sigma - 1} + (1 - \zeta)g
\]

We cannot, \( a \ priori \), say which of which is greater. If \( \sigma > 2 \), then the consumption-price deflated GDP growth rate is faster than the output-price deflated rate, but if \( 1 < \sigma < 2 \), the reverse is true.
8. This model has no transitional dynamics since its only state variable - \( L_I \) - is a "jumper", i.e. it can be a discontinuous function of time. Pure profits appear unless \( q=1 \), so the fact that \( L_I \) can jump means that \( q \) always equals unity. This in turn means that there is never any transitional dynamics.

9. Given that each variety is produced by a single firm (a standard Dixit-Stiglitz result), a representative firm's sales to local-market and export-market consumers is equal to consumption by local-market and export-market consumers. Thus using the inverse demand function, the maximization problem of typical firm \( j \) is to choose \( x_j \) and \( x_j^* \), to maximize:

\[
\left( \frac{x_j^{1-\sigma}E}{\int_{i=1}^{K+K^*} x_i^{1-\sigma}} - x_j \right) + \left( \frac{1}{\tau} \frac{(x_j^*)^{1-\sigma}E^*}{\int_{i=1}^{K+K^*} x_i^{1-\sigma}} - (\phi+T)x_j^* \right) - \pi
\]

where \( p_i^* \) is the price charged for variety \( i \) in its export market (NB: given the assumed symmetry, it does not matter whether we work out the home firm's problem - in which case the export market is the foreign country - or the foreign firm's problem - in which case the export market is the home country). The first order conditions are:

\[
p_j(1-1/\sigma) = 1, \quad \frac{p_j^*}{\tau}(1-1/\sigma) = \phi + T
\]

Rearranging these, operating profit earned in the local and export markets are:

\[
(p_j-1)x_j = \frac{p_jx_j}{\sigma} = \frac{SE}{\sigma K}; \quad S = \frac{KP_jx_j}{E}
\]

\[
\frac{p_j^*x_j^*}{\tau} = \frac{p_j^*x_j^*}{\sigma \tau} = \frac{S^*E^*}{\sigma \tau K}; \quad S^* = \frac{KP_jx_j^*}{E^*}
\]

where \( S \) and \( S^* \) are the sum of market shares of local and nonlocal firms in a typical market (\( K \) is the number of firms in either of the symmetric countries, by full employment of knowledge capital). Using the prices implied by the first order conditions and the demand function, the equilibrium \( S^* \) (NB: \( S^* \) is the import penetration ratio and \( S+S^*=1 \)) is related to protection according to:

\[
S^* = \frac{KP_jx_j^*}{E} = \frac{(p^*)^{1-\sigma}}{p^{1-\sigma} + (p^*)^{1-\sigma}} = \frac{1}{1+(p^*/p)^{\sigma-1}} = \frac{1}{1+(\tau(\phi+T))^{\sigma-1}}
\]

Using this and the expressions for equilibrium operating profit in the two markets, total operating profit (which equals capital's reward \( \pi \)) is:

\[
\pi = \frac{SE}{\sigma A} + \frac{S^*E^*}{\sigma \tau A} = \left( S + \frac{S^*}{\tau} \right) \left( \frac{E}{\sigma A} \right)
\]
It is entirely possible to simplify this expression further, however doing so makes it more difficult to explain the impact of the various forms of protection on $\pi$. For instance, one particularly neat expression for $\pi$ comes from the fact that firms engage in "mill pricing" (i.e. they charge the same producer price on all sales, regardless of protection levels) is:

$$\pi = \frac{p(x+x*)}{\sigma} = \frac{\Psi E}{\sigma A}, \quad \Psi = \frac{1+\Gamma^{-\sigma}}{1+\Gamma^{1-\sigma}}, \quad \Gamma = \frac{P^*}{p} = \tau(\phi+T)$$

where we have used the inverse demand functions for local and imported varieties.

10. From the inverse demand function for imported varieties:

$$Kp^*x^* = \frac{(p^*)^{1-\sigma}}{p^{1-\sigma}+(p^*)^{1-\sigma}} E = \frac{E}{(p^*/p)^{\sigma^{-1}+1}} = \frac{E}{\Gamma^{\sigma^{-1}+1}}$$

Multiplication by $(\tau-1)/\tau$ yields the result in the text for *ad valorem* tariffs.

For specific tariffs $KTx^*$ is:

$$KTx^* = \frac{(p^*)^{1-\sigma}}{p^{1-\sigma}+(p^*)^{1-\sigma}} \frac{TE}{p} = \frac{TE}{\Gamma^{\sigma^{-1}+1}} \frac{1-1/\sigma}{\tau(\phi+T)}$$

11. We have:

$$J = \left( \frac{\mu(L-\delta FK)}{K(\sigma-\mu)} - \delta \right) \frac{1}{\rho F}$$

so

$$q = \frac{J}{F} = \left( \frac{\mu(L-\delta FK)}{K(\sigma-\mu)} - \delta \right) \frac{1}{\rho F^2} = \frac{\mu(L-\delta FK) - \delta K(\sigma-\mu)}{K(\sigma-\mu)\rho F^2}$$

$$= \frac{\mu L - \mu \delta FK - \delta K(\sigma-\mu)}{K(\sigma-\mu)\rho F^2} = \frac{\mu L - \delta K(\mu(F-1)+\sigma)}{K(\sigma-\mu)\rho F^2}$$

12. More formally the first order conditions are:

$$p = \frac{a}{1-s/\epsilon}, \quad p^* = \frac{a\tau}{1-s^*/\epsilon}$$

By symmetry $ns+ns^*=1$, so:

$$s = \frac{n\epsilon(\tau-1)+1}{n(1+\tau)}, \quad s^* = \frac{1}{n} - s$$

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Plainly $s$ is falling in $n$ and rising in $\tau$. Rearranging the first order conditions, we have:

$$\pi = (p-a)x + (p-a\tau)x = s\left(\frac{px}{\epsilon}\right) + s^*\left(\frac{p^*x^*}{\epsilon}\right) = \left(\frac{s^2 + s^{*2}}{\epsilon}\right)E$$

since $E=E^*$ by symmetry.