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Double Irreversibility and Environmental Policy Design

By **Aude Pommeret**, University of Lausanne and IREGE-University of Savoie
Fabien Prieur, IREGE-University of Savoie

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Summary

The design of environmental policy typically takes place within a framework in which uncertainty over the future impact of pollution and two different kinds of irreversibilities interact. The first kind of irreversibility concerns the sunk cost of environmental degradation; the second is related to the sunk cost of environmental policy. Clearly, the two irreversibilities pull in opposite directions: policy irreversibility leads to more pollution and a less/later policy while environmental irreversibility generates less pollution and a more/sooner policy. Using a real option approach and an infinite time horizon model, this paper considers both irreversibilities simultaneously. The model first is developed by paying particular attention to the option values related to pollution and policy adoption. Solving the model in closed form then provides solutions for both the optimal pollution level and the optimal environmental policy timing. Finally, the model is "calibrated" with the purpose of appraising which irreversibility has the prevailing effect and what is the overall impact of both irreversibilities on pollution and policy design.

Keywords: Environmental Policy, Environmental Irreversibility, Policy Irreversibility

JEL Classification: Q58, D81

We would like to thank Stéphanie Jamet. We are grateful to the participants at the Environmental Meetings CORE-EUREQua-ECONOMIX for their comments. We also thank the participants at the "Monte Verità Conference on Sustainable Resource Use and Economic Dynamics — SURED 2008" that took place on June 1-6th 2008 in Ascona (Switzerland) and the participants at the conference "Frontiers in Environmental Economics and Natural Resources Management — AFSE 2008 ", June 9-11, 2008, Toulouse, France.

Address for correspondence:

Aude Pommeret
IMUS, Université de Savoie
44 chemin de Bellevue
74940 Annecy-le-Vieux
France
E-mail: aude.pommeret@univ-savoie.fr

Corso Magenta, 63, 20123 Milano (I), web site: www.feem.it, e-mail: working.papers@feem.it

Double irreversibility and environmental policy design[1]

Aude Pommeret[2]   Fabien Prieur[3]

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[2] University of Lausanne and IREGE-University of Savoie. Postal address: IMUS, Université de Savoie, 44 chemin de Bellevue, 74940 Annecy-le-Vieux, France. E-mail address: aude.pommeret@univ-savoie.fr.

[3] IREGE-University of Savoie. Same address and e-mail: fabien.prieur@univ-savoie.fr.

Abstract

The design of environmental policy typically takes place within a framework in which uncertainty over the future impact of pollution and two different kinds of irreversibilities interact. The first kind of irreversibility concerns the sunk cost of environmental degradation; the second is related to the sunk cost of environmental policy. Clearly, the two irreversibilities pull in opposite directions: policy irreversibility leads to more pollution and a less/later policy while environmental irreversibility generates less pollution and a more/sooner policy. Using a real option approach and an infinite time horizon model, this paper considers both irreversibilities simultaneously. The model first is developed by paying particular attention to the option values related to pollution and policy adoption. Solving the model in closed form then provides solutions for both the optimal pollution level and the optimal environmental policy timing. Finally, the model is "calibrated" with the purpose of appraising which irreversibility has the prevailing effect and what is the overall impact of both irreversibilities on pollution and policy design.

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1 Introduction

The design of environmental policy typically takes place within a framework in which uncertainty over the future impact of pollution interacts with two different kinds of irreversibilities. This is particularly true for global warming. The impact of increased greenhouse gas (GHG) concentrations on the world's climate is uncertain. At the same time, we do not know whether a given climate change will cause economic damage or, if such damage does occur, the extent of its impact. Although recent studies have attempted to appraise the cost of controlling GHG emissions (for example, the Stern review, 2006), these costs remain a source of uncertainty.

Environmental problems involve two kinds of irreversibilities. The first pertains to the fact that damage caused to the environment is at least partly irreversible. Atmospheric concentrations of GHG are, for example, long lasting. Given that increases in temperatures depend on accumulated amounts of GHG in the atmosphere rather than on current emissions, even a sharp reduction in GHG emissions will not enable the world economy to instantaneously undo the damage wrought. Likewise, some evidence suggests that global warming may disrupt natural processes such as ocean currents and even cause thermohaline circulation to stop (see Keller et al. (2004)). This, together with the expected rise in seas levels, is a worrying illustration of the irreversibility of environmental damage.

Two distinct approaches to modelling environmental irreversibility (EI) have emerged in the literature. Some authors focus on the irreversibility of environmental damage (EI1). In deterministic frameworks, Forster (1975), Tavhonen and Withagen (1996), and Prieur (2008) assess the irreversibility at play in the pollution accumulation process. An ecosystem's natural capacity to assimilate pollution depends on the concentration of the pollutant. A critical threshold exists above which the assimilation capacity becomes permanently exhausted, thereby implying an irreversible concentration of pollution. In stochastic frameworks, discussion of the irreversibility of environmental damage appears in Pindyck (2002) and Fisher and Narain (2003), who simply set the parameter corresponding to the assimilation capacity equal to zero. The second approach (EI2) to modelling the problem of environmental irreversibility may be found in Kolstad (1996a) and Ulph and Ulph (1997). Rather than focusing on the irreversibility of environmental damage, these authors consider irreversibility in the decision making process. The basic concept is that in the presence of environmental externalities, if high levels of emissions result in excessive environmental damage, the damage cannot be instantaneously undone by reducing the stock of pollution. The irreversibility is introduced into the model as a non-negativity constraint on emissions. In this paper, we will focus on this second approach to modelling EI.

The second irreversibility involved in environmental problems, hereafter referred to as policy irreversibility (PI), is related to the sunk cost (irreversible) nature of most investments made to reduce GHG emissions. Again, two different definitions of PI may be found in the literature. The first (PI1) is provided by Kolstad (1996a), who defines policy irreversibility in terms of abatement capital durability while abatement investment is costless. The more durable the capital, the more sunk the policy. Assume, for example, that a firm decides at some point in time to invest in a free greener and/or abatement capital in response to a pollution control policy. The firm will not be able to remove this investment quickly because it is a long lasting investment. This implies that current emissions cannot go beyond a certain level because they are constrained by previous investments devoted to pollution control. Note that an upper boundary on emissions consequently is imposed, one that is exogenous to firms, no matter what the optimal level of production may be. The second definition of policy irreversibility (PI2), found in Pindyck (2000, 2002) and Fisher and Narain (2003), derives from literature on irreversible investment (Pindyck, 1991). A firm that invests in abatement capital will not be able to easily resell this capital in the short term since such an investment exhibits some degree of firm specificity. For example, a coal burning utility cannot uninstall a CO_2 scrubber and sell it on a second-hand market without incurring substantial costs. In other words, if a firm abates more than is later found to be optimal because environmental damages prove to be very limited, one cannot easily disinvest this abatement capital. In the most extreme case, disinvestment is completely impossible and investments in this kind of abatement capital are then fully irreversible. In this paper, we have chosen to focus on the second kind of PI because we have doubts concerning the way Kolstad introduces PI1 in his model. Even with abatement capital installed (lowering the emission-output ratio), if firms increase their production sufficiently, emissions should increase as well.

The two sources of irreversibilities, EI and PI, create a conflict in policy recommendations to control GHG emissions since the two clearly work in opposing directions. Due to uncertainty over the economic costs of global warming, an economy may prefer to under-invest in pollution abatement to avoid the sunk cost of taking action. Alternatively, the same uncertainty may be an incentive for the economy to under-emit today in order to prevent potentially irreversible damage. In this context, the following questions immediately arise: does either irreversibility have a significant effect on today's decisions? If both matter, what is the overall impact of irreversibilities on policymakers' actions?

Existing studies have focused primarily on the interaction between uncertainty and only one of the two irreversibilities at a time, not both simultaneously.[1] Arrow and Fisher (1974), Henry (1974), Freixas and Laffont (1984), and more recently Gollier and Treich (2003) have shown that the possibility of obtaining better information in the future about the future benefits and/or costs of current actions generates a quasi-option value.[2] When this value is incorporated in the decision making process, economic agents should undertake actions that involve a lower

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[1] Most focus on pollution and resource management issues.

[2] "Quasi-option value" is the formal terminology used in relation to learning and irreversibility (see Kolstad, 1996). However, following the common practice in the literature, we simply use the term "option value" in the rest of this paper.

level of irreversible commitment than when no learning has taken place. This is the well-known irreversibility effect that implies a higher current abatement of pollution in the presence of EI (Chichilnisky and Heal, 1993).[3]

More recent analyses have attempted to assess how the two kinds of irreversibilities interact. Kolstad (1996a) developed a two-period model of global warming in which uncertainty (and learning) deals with the economic costs of climate change and irreversibilities are defined according to the EI2 and PI1 definitions. In this setting, the author seeks to appraise the consequences of both irreversibilities on first period emissions. In general, it is difficult to draw a conclusion since the overall impact of the irreversibilities depends on various parameters that include the decay rate of GHG stocks and the depreciation rate of abatement capital. Numerical simulations are performed by Kolstad (1996b) in a stochastic model of growth where the same irreversibilities and learning are taken into account. For the parameterization chosen, the non negativity constraint on emissions (EI2) is never binding, which leads to the following result: uncertainty and irreversibility call for lower pollution controls and higher emissions. Using the definitions set by EI1 and PI2, Fisher and Narain (2003) obtain the same numerical result when both climate change occurrence and the possible related economic costs are uncertain. Finally, Pindyck (2000, 2002) examines the effect of policy irreversibility as defined by PI2 in an analytical model. Environmental damages are also assumed to be irreversible but again, this irreversibility does not have an impact in the sense that it does not generate any option value. As expected, Pindyck shows that the more uncertain the future cost of pollution, the greater is the incentive to wait rather than immediately adopt a policy. In sum, the literature concludes that by ignoring EI and PI, traditional cost/benefit analysis is biased toward policy adoption and thereby induces pollution levels that are unnecessarily low.

This paper attempts to challenge this conclusion by studying the effect of uncertainty and irreversibilities, as defined by EI2 and PI2, on the optimal level of pollution in a particular economy. Environmental irreversibility refers to a non-negativity constraint on emissions and policy irreversibility refers to abatement investment irreversibility. Moreover, the decision to emit involves a trade-off. On the one hand, emissions are viewed as a source of benefit since they enter the production process. On the other, emissions contribute to the accumulation of pollution which is costly for the economy. Uncertainty deals with the economic costs of pollution. It encompasses both the uncertainty regarding the exact impact of a given concentration of GHG and its related economic cost. Using a real option approach and an infinite time horizon model, this paper focuses first on environmental irreversibility. We show that this irreversibility is associated with an option value to pollute and induces a lower optimal level of pollution. The more uncertain the evolution of pollution costs borne by the economy, the lower the desired pollution stock since polluting more now rather than waiting involves a higher opportunity cost.

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[3] Note, however, that Epstein (1980) and Ulph and Ulph (1997) have found situations where this does not hold.

The second part of the analysis considers the two irreversibilities, EI and PI, together. The aim is to highlight the two distinct option values related to pollution and policy adoption. Once a policy is adopted, the study framework is reduced to the optimal pollution problem. What is more interesting is the situation before a policy is adopted when both irreversibilities play a role, provided that they work in opposite directions. Considering both irreversibilities prevents us from reaching an explicit solution. Note in particular that the EI requires comparing ex post and ex ante marginal value functions while the PI requires comparing ex post and ex ante value functions. However, solving the model numerically allows us to evaluate which irreversibility has the prevailing effect and what overall impact both irreversibilities have on pollution and policy design. The analysis notably stresses the importance of the sunk cost of environmental policy. If abatement occurs at least cost, then the optimal pollution stock is higher than the one that would prevail in the absence of irreversibilities. In fact, even if the irreversibility associated with the pollution decision forces the economy to reduce emissions, this effect is more than compensated by the opportunity to adopt a policy. In contrast, when environmental policy costs are sufficiently high, the option to adopt a policy becomes less attractive. In other words, the impact of the environmental irreversibility exceeds the impact of policy irreversibility. In this case, the economy reduces the optimal pollution stock when both irreversibilities are taken into account. In the calibrated model, the irreversibility associated with the adoption of a pollution policy prevails and optimal pollution is smaller than in the absence of any irreversibility. Our conclusion therefore differs sharply from that of existing literature: by ignoring irreversibilities, one may pollute too much.

Our model is described in the section that follows. The optimization problem first is solved in the presence of the environmental irreversibility alone. The policy irreversibility is added in Section 3. We calibrate the model and discuss the total effect of both irreversibilities in Section 4. Section 5 concludes.

2 Optimal pollution under environmental irreversibility

2.1 The basic set-up

Pollution M_t accumulates according to the following process:

$$dM_t = (E_t - \delta M_t)dt \tag{1}$$

with E_t the amount of emissions at time t, and $\delta > 0$ the natural rate of assimilation. The stock of pollution may be seen as the concentration of greenhouse gases in the atmosphere (GHG, such as carbon dioxide). Accordingly, emissions E_t refer to the anthropogenic emissions of GHG. Pollution is damaging to the economy. Through the greenhouse effect, emissions accelerate the rise in global temperatures. According to the IPCC (2007), the expected rise in temperature will lie in the range of $1.1 - 6.4°C$ by 2100. Among the most serious economic impacts of global warming are the multiplication of extreme weather events and the submersion of coastal regions due to rising oceans levels.

Economic models of climate change (grouped under the name, "integrated assessment models", such as the DICE model developed by Nordhaus, 1994) usually define damage as a quadratic function of the increase in temperature provided that this increase is mainly due to increasing concentrations of GHG. In this paper, as in Pindyck (2000; 2002), the relationship between economic damage and pollution is simplified by assuming that the former may be directly expressed as a function of the latter:

$$C(M_t, \theta_t) = -\theta_t M_t^2$$

where θ_t, that measures the intensity (or scale) of the impact of pollution, evolves according to a geometric Brownian motion:

$$d\theta_t = \alpha\theta_t dt + \sigma\theta_t dz_t \tag{2}$$

Through the formalization of θ_t, we aim to recognize that many uncertainties bear on the economic cost of pollution. For instance, uncertainty remains regarding the exact impact of a given concentration of GHG on temperature. In addition, for a given rise in temperature, it is difficult to give precise estimations of the economic cost.[4] One also should note the possibility of catastrophic consequences such as the complete shutdown of thermohaline circulation that would drastically modify appraisals of the impact of climate change.

Choosing an emission level involves the following trade-off: if emissions contribute to the accumulation of the pollutant, they also are used as an input in the production sector. The economic benefits of emissions are represented in the simplest way by considering a linear function production function, with $B > 0$,

$$\pi(E_t) = BE_t$$

thus, the economy's instantaneous net payoff writes as follows:

$$C(M_t, \theta_t) + \pi(E_t) = BE_t - \theta_t M_t^2$$

Finally, we assume pollution is irreversible, in the sense of Kostad (1996) and Ulph and Ulph (1997), meaning that the stock of pollution cannot be reduced overnight through negative emissions. More formally:

$$E_t \geqslant 0 \tag{3}$$

The irreversibility of pollution should be understood as an irreversibility in the decision process. It has nothing to do with either the irreversibility of pollution accumulation (see notably Tavhonen and Withagen, 1996; Prieur, 2008), nor with the irreversibility of environmental damage related to catastrophic phenomenon.[5]

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[4] In fact, many studies attempt to estimate these costs (see Tol, 2005 for a survey) but their results differ greatly.

[5] Consider again the shutdown of thermohaline circulation (Keller et al., 2004).

2.2 The central planner program

The central planner objective is to determine the emission policy that maximizes the expected present value of net payoffs. The value function writes:

$$V \equiv \max_{\{E_t\}} E \left\{ \int_0^{+\infty} e^{-\rho t} \left[C(\theta_t, M_t) + \pi(E_t) \right] dt \right\} \tag{4}$$

with $\rho > 0$, the discount rate.

This optimization problem looks like a traditional irreversible decision problem (Pindyck, 1988; Abel and Eberly, 1994). Using standard techniques, it is possible to show that the Hamilton-Jacobi-Bellman equation associated with the optimization problem (4) is:

$$\rho V(\theta_t, M_t) = C(\theta_t, M_t) + \frac{E(dV)}{dt}$$

Applying Itô's lemma:

$$\rho V(\theta_t, M_t) = C(\theta_t, M_t) - \delta M_t V_M(\theta_t, M_t) + \alpha \theta_t V_\theta(\theta_t, M_t) + \frac{1}{2}\sigma^2 \theta^2 V_{\theta\theta}(\theta_t, M_t) \tag{5}$$

where the Kuhn-Tucker conditions for the maximization are:

$$V_M(\theta_t, M_t) \leq 1, E_t \geq 0, \left[V_M(\theta_t, M_t) + B \right] E_t = 0, \forall t \geq 0.$$

The left-hand side (LHS) of equation (5) reflects the required rate of return. The right-hand side (RHS) is the expected change in the value in the region for the state variable θ where there are no emissions *i.e.* in the continuation region. Equation (5) holds identically in M. Therefore, the partial derivative of the LHS with respect to M equals the partial derivative of the RHS with respect to M. Performing this differentiation yields:

$$\rho V_M(\theta_t, M_t) = \left\{ \begin{array}{c} C_M(\theta_t, M_t) - \delta \left[V_M(\theta_t, M_t) + M_t V_{MM}(\theta_t, M_t) \right] + \\ +\alpha \theta_t V_{\theta M}(\theta_t, M_t) + \frac{1}{2}\sigma^2 \theta^2 V_{\theta\theta M}(\theta_t, M_t) \end{array} \right. \tag{6}$$

We define $\phi_t = 1/\theta_t$ for ease of presentation. It implies that the higher the intensity of pollution damage, the smaller ϕ_t. As a result of this new notation, the instantaneous profit function becomes homogenous of degree one in ϕ and M. Moreover, we can deduce using Itô's lemma that ϕ follows a geometric Brownian motion:

$$\begin{aligned} d\phi_t &= (\sigma^2 - \alpha)\phi_t dt - \sigma \phi_t dz_t \\ &\Leftrightarrow d\phi_t = \mu \phi_t dt - \sigma \phi_t dz_t \quad \text{with} \quad \mu = \sigma^2 - \alpha \end{aligned}$$

To solve equation (6), we use the fact that since the instantaneous payoffs are homogeneous of degree one in ϕ and M, the value function V is homogeneous of degree one in ϕ and M. This property of the value function implies that the marginal valuation of pollution V_M is homogeneous of degree zero in ϕ and M and, hence, can be written simply as a function of y_t; the ratio of M_t

to ϕ_t. This ratio increases with both the pollution stock M_t and the pollution intensity θ_t. We define the marginal valuation of pollution by:

$$q(y_t) = V_M(\phi_t, M_t)$$

Differentiating this equation and using the definition of y yields expressions for the partial derivatives of the value-function. Substituting the definition of $q(y)$ and its partial derivatives in equation ((6)) yields the following differential equation for the marginal valuation of pollution $q(y)$:

$$(\delta + \rho)q(y_t) = -2y_t + q'(y_t)y_t(\mu + \delta - \sigma^2) + \frac{1}{2}q''(y_t)\sigma^2 y_t^2$$

The optimization problem (4) can be solved using this differential equation and appropriate boundary conditions. In particular, as pollution sensitivity increases, the marginal valuation of pollution remains finite:

$$\lim_{y \to +\infty} q(y) < \infty$$

The marginal value of pollution is then:

$$q(y) = \frac{-2}{\rho + 2\delta + \mu - \sigma^2} y + Ay^{-\beta_2} \leftrightarrow q(y) = \frac{-2}{\rho + 2\delta - \alpha} y + Ay^{-\beta_2}$$

with β_2 the positive root of the fundamental quadratic equation. $Ay^{-\beta_2}$ is the option to pollute more in the future where A is a constant to be determined.

The central planner exercises the emission option when the state variable y becomes low enough to reach a trigger value y^*. Let E be the amount of emission used. Pollution then increases by a marginal amount dM and the economy benefits from BE. This generates the following condition:

$$V(\phi, M) = V(\phi, M + dM) + BE$$

Dividing by the increment dM, this condition can be written as follows:

$$q(y^*) = -B$$

To ensure that investment occurs along the optimal path, we also require the smooth pasting condition to be satisfied:

$$q'(y^*) = 0$$

Solving the system composed of the two boundary conditions in the two unknowns $\{y^*, A\}$ yields:

$$\begin{aligned} y^* &= \frac{1}{2}(\rho + 2\delta - \alpha) B \left(\frac{\beta_2}{1 + \beta_2}\right) \\ &\Leftrightarrow \; M^* = \phi \frac{(\rho + 2\delta - \alpha) B}{2} \left(\frac{\beta_2}{1 + \beta_2}\right) \end{aligned}$$

and, the expression for the value of the option to pollute is:

$$Ay^{-\beta_2} = \frac{-B}{(\beta_2+1)}\left[\frac{2}{(\rho+2\delta-\alpha)\,B}\left(\frac{1+\beta_2}{\beta_2}\right)\right]^{-\beta_2} y^{-\beta_2}$$

The larger the natural assimilation rate δ, the higher the desired level of pollution. Obviously; the larger the return on emissions (B) or the lower the intensity of pollution ($1/\phi$), the larger the desired level of pollution.

Ignoring the joint effect of uncertainty and irreversibility by simply applying a standard NPV rule, the desired pollution would be:

$$M^*_{NI} = \phi\frac{(\rho+2\delta-\alpha)\,B}{2} \tag{7}$$

$$\Leftrightarrow \quad y^*_{NI} = \frac{1}{2}(\rho+2\delta-\alpha)\,B \tag{8}$$

Since $\left(\frac{\beta_2}{\beta_2+1}\right) < 1$, the pollution irreversibility leads to a smaller desired level of pollution. Note that the optimal level of pollution does not correspond to the desired one when the realizations of the stochastic variable ϕ (resp. θ) happen to decrease (resp. increase) in time. This means that the economy may be stuck with too much pollution given the current realization of the stochastic variable.

Uncertainty affects y^* and $M^*_t = \phi_t/y^*$ through the ratio involving β_2. The effect highlights the impact of the irreversibility of pollution on optimal decisions. It plays through the ratio involving β_2. For a given ϕ, an increase in σ, by reducing β_2, is accompanied by a decrease in M^*: the more uncertain the evolution of pollution costs to be borne by the economy, the lower the desired pollution stock. Polluting more now rather than waiting involves a higher opportunity cost. In other words, due to the irreversibility and the increasing uncertainty, the economy will pollute less.

3 Policy design under environmental and policy irreversibilities

In this section, we will consider pollution irreversibility together with policy irreversibility. By dealing with the two irreversibilities at the same time, the aim is on the one hand to stress their respective impact on decision making and, on the other, to understand which one prevails provided they play in opposite directions. Following an adaptation of Pindyck's approach (2000), the policy intervention consists of a once-and-for-all reduction in M_t. The policy is the opportunity to reduce pollution from its current level to its desired level and involves a sunk cost for society. Such a policy becomes of interest when sensitivity to pollution becomes too high given the stock of pollution. The policy cost, $K.M_t$, is assumed to be increasing in the stock of pollution at the moment of policy adoption. The value of the central planner program is now:

$$V^D \equiv \max_{\{E_t\},\tilde{T}} E_t\left\{\int_0^{+\infty} e^{-\rho t}\left[C(\theta_t, M_t) + \pi(E_t)\right]dt + \left[C(\theta_{\tilde{T}}, M^*_{\tilde{T}}) + \pi(E_{\tilde{T}}) - KM_{\tilde{T}}\right]e^{-\rho\tilde{T}}\right\} ,$$

subject to (1), (2) with $\tilde{T}$ the random time when the adoption occurs. $V^D(\theta_t, M_t)$ denotes the value function in the "dirty" region, i.e. before adoption; $M^*_{\tilde{T}}$ is the desired level of pollution at the adoption time.

This program combines:

- the optimal investment problem, laid out in section 1, that gives the desired pollution stock, and
- an optimal stopping problem that determines when it is optimal to spend KM_t to reduce M_t to its desired level.

We note M^{D*} the desired level of pollution in the "dirty" economy i.e. before policy adoption and M^{C*} the desired level in the "clean" economy *i.e.* after adoption.The resolution still mobilizes the tools of dynamic programming and requires the problem to be decomposed into two parts depending on whether or not the policy has been adopted.

3.1 Optimal pollution when the policy is adopted

The problem is formally similar to the one studied in section 1. Let us define $V^C(\theta_t, M_t)$ as the value function for the "clean" region where M_t^{C*} is the desired stock of pollution after the adoption of a policy. The value function $V^C(\theta_t, M_t)$ must satisfy the Bellman equation:

$$\rho V^C(\theta_t, M_t) = C(\theta_t, M_t) - \delta M_t V^C_M(\theta_t, M_t) + \alpha\theta_t V^C_\theta(\theta_t, M_t) + \frac{1}{2}\sigma^2\theta^2 V^C_{\theta\theta}(\theta_t, M_t) \tag{9}$$

Recall that $y_t = M_t\theta_t$ and $q^C(y) = V^C_M(\phi_t, M_t)$. The similarity with the problem in section 1 allows us to deduce:

$$q^C(y) = \frac{-2}{\rho + 2\delta - \alpha} y + A_2^{'}(\frac{1}{y})^{\beta_2} \ , \tag{10}$$

where the term $A_2^{'}(y)^{\beta_2}$ still corresponds to the option to pollute. Moreover:

$$y^{C*} = \frac{1}{2}(\rho + 2\delta - \alpha) B \left(\frac{\beta_2}{1+\beta_2}\right)$$

$$\text{and} \qquad A_2^{'} = \frac{-B}{(\beta_2 + 1)}\left[\frac{2}{(\rho + 2\delta - \alpha) B}\left(\frac{1+\beta_2}{\beta_2}\right)\right]^{-\beta_2}$$

For the remainder of the analysis, it is useful to derive from (10) the expression of the value function $V^C(\phi_t, M_t)$. Integrating $V^C_M(\phi_t, M_t)$ between the appropriate boundaries provides the solution:

$$V^C(\phi_t, M_t) = -\frac{1}{\rho + 2\delta - \alpha}\frac{(M_t)^2}{\phi_t} + \frac{A_2^{'}\phi_t^{\beta_2}}{1-\beta_2}(M_t)^{1-\beta_2} \ . \tag{11}$$

The next section deals with the case where both irreversible decisions must be taken.

3.2 Optimal pollution and optimal adoption

Let us denote $V^D(M_t, \theta_t)$ and $q^D(y_t)$ the value function and the marginal valuation of pollution in the "dirty" region *i.e.* before the adoption of a policy. Then, by making use of the same techniques as before, it is easy to show that $q^D(y_t)$ admits the following general form:

$$q(y) = \frac{-2}{\rho + 2\delta - \alpha} y + A_1 (y)^{-\beta_1} + A_2 (y)^{-\beta_2}$$

with $\beta_1 < 0$ and $\beta_2 > 1$, $A_1, A_2 > 0$.

The marginal valuation of pollution exhibits an additional term $A_1 \left(y^D\right)^{-\beta_1}$ that is the second option value related to policy adoption. $A_2 (y)^{-\beta_2}$ is the option value related to pollution, which also exists after policy adoption. Assume $\theta \to 0$, which means that the cost of pollution is almost nil, then the option to adopt the policy becomes negligible. This is precisely the basic property conveyed by the term $A_1 (y)^{-\beta_1}$ since it satisfies:

$$\lim_{y \to 0} A_1 (y)^{-\beta_1} = 0.$$

The associated value function writes:

$$V^D(\phi_t, M_t) = -\frac{1}{\rho + 2\delta - \alpha} \frac{(M_t)^2}{\phi_t} + \frac{A_1 \phi_t^{\beta_1}}{1 - \beta_1} (M_t)^{1-\beta_1} + \frac{A_2 \phi_t^{\beta_2}}{1 - \beta_2} (M_t)^{1-\beta_2} \,. \tag{12}$$

Four optimality conditions must be considered:

- the two conditions for the irreversible investment decision:

$$\left.\begin{array}{c} V^D_{M^D}(\phi, M^{D*}) = -B \\ q'(y^*) = 0 \end{array}\right\} \text{that provide } M^{D*}(\phi) \text{ and } A_2$$

- the two conditions for the optimal stopping problem:

$$\left.\begin{array}{c} V^D(\phi^*, M) = V^C(\phi^*, M^{C*}) - KM \\ V^D_\phi(\phi^*, M) = V^C_\phi(\phi^*, M^{C*}) \end{array}\right\} \text{that provide } \phi^*(M) \text{ and } A_1$$

Thus, we are left with a system of four equations (with $\chi = (\rho + 2\delta + \mu + \sigma^2)^{-1}$):

$$\begin{cases} -2\chi\theta M^{D*} + A_1 \left(\theta M^{D*}\right)^{-\beta_1} + A_2 \left(\theta M^{D*}\right)^{-\beta_2} = -B \\ -2\chi \left(\theta M^{D*}\right)^2 + A_1\beta_1 \left(\theta M^{D*}\right)^{1-\beta_1} + A_2\beta_2 \left(\theta M^{D*}\right)^{1-\beta_2} = 0 \\ M \left[-\chi\theta^* M + \frac{A_1}{1-\beta_1} (\theta^* M)^{-\beta_1} + \frac{A_2}{1-\beta_2} (\theta^* M)^{-\beta_2}\right] = M^{C*} \left[-\chi\theta M^{C*} + \frac{A_2'}{1-\beta_2} \left(\theta M^{C*}\right)^{-\beta_2}\right] - KM \\ \chi (\theta^* M)^2 + \frac{A_1\beta_1}{1-\beta_1} (\theta^* M)^{1-\beta_1} + \frac{A_2\beta_2}{1-\beta_2} (\theta^* M)^{1-\beta_2} = \chi \left(\theta^* M^{C*}\right)^2 + \frac{A_2\beta_2}{1-\beta_2} \left(\theta^* M^{C*}\right)^{1-\beta_2} \end{cases}$$

with four unknowns: M^{C*}, A_2, θ^* and A_1. These optimality conditions become more tractable when rewritten in terms of y^D and y^C. In fact, the following logic applies:

- Let y^{a*} be the trigger value at which the policy option is exercised. The value y^{a*} is the first unknown (corresponding to ϕ^*)
- The policy adoption allows the economy to "jump" to the post-adoption desired - and known - level y^{C*}
- As for the emission decision, let us denote y^{D*} the trigger level at which the option to pollute is exercised (*i.e.* the desired level before policy adoption). y^{D*} is the second unknown to be determined.

The last two unknowns remain A_1 and $A_{2,}$ that provide the option values (i) to adopt the environmental policy and (ii) to pollute more.

An intuitive representation of the behavior of $y = M\theta$ is provided by figure 1:

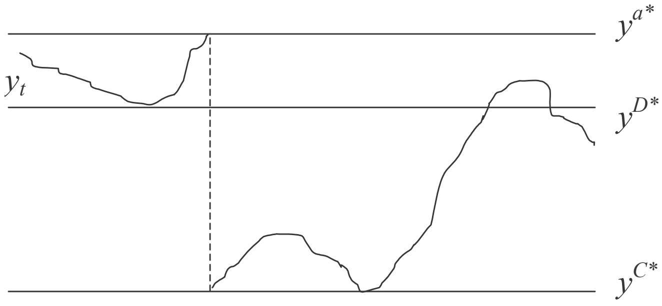

Figure 1

The system of optimality conditions is:

$$\begin{cases} y^{C*}\left(-\chi y^{a*} + \frac{A_1}{1-\beta_1}\left(y^{a*}\right)^{-\beta_1} + \frac{A_2}{1-\beta_2}\left(y^{a*}\right)^{-\beta_2} + K\right) = Z_1 \\ \chi(y^{a*})^2 + \frac{A_1\beta_1}{1-\beta_1}\left(y^{a*}\right)^{1-\beta_1} + \frac{A_2\beta_2}{1-\beta_2}\left(y^{a*}\right)^{1-\beta_2} = Z_2 \\ -2\chi(y^{D*}) + \frac{A_1}{1-\beta_1}\left(y^{D*}\right)^{-\beta_1} + \frac{A_2}{1-\beta_2}\left(y^{D*}\right)^{-\beta_2} = -B \\ 2\chi(y^{D*})^2 + \frac{A_1\beta_1}{1-\beta_1}\left(y^{D*}\right)^{1-\beta_1} + \frac{A_2\beta_2}{1-\beta_2}\left(y^{D*}\right)^{1-\beta_2} = 0 \end{cases}$$

where Z_1 and Z_2 are constant:

$$Z_1 = -\chi\left(y^{C*}\right)^2 + \frac{A_2'}{1-\beta_2}\left(y^{C*}\right)^{1-\beta_2}$$
$$Z_2 = \chi\left(y^{C*}\right)^2 + \frac{A_2'\beta_2}{1-\beta_2}\left(y^{C*}\right)^{1-\beta_2}$$

Due to the non linearities, this system cannot be solved analytically.

4 Numerical resolution

Based on a numerical resolution, we assess the impact of the striking parameters on optimal decisions.

4.1 Calibration

Numerical resolution is performed by making use of the following set of baseline parameters:

$$\{K, \delta, \rho, \alpha, B, \sigma\} = \{27.28, 0.005, 0.05, 0.03, 7.14, 0.2\}$$

Parameter K, corresponding to the abatement cost per metric ton of CO_2, is drawn from the 2007 DICE model (Nordhaus, 2008). It refers to the carbon tax induced by the optimal policy.[6] for 2005, expressed in 2005 US \$ per metric ton. The value for δ is the one reported by most climate change experts in surveys (see Pindyck, 2002). As for the discount rate, the value we set is close to what most western governments use for most long term investments (see Tol, 2005). Parameter α is chosen to account for the positive dependence of environmental concerns with respect to wealth. We interpret this relationship by setting the trend in intensity of damage equal to the mean economic growth rate in developed countries. Such an intensity might grow faster and could be mitigated by adaptation; in the absence of precise information about these two effects, our choice seems a reasonable first approximation. Parameter B reflects the benefits per unit of emissions. It is approximated using the (inverse of the) CO_2 emissions to GDP ratio in metric tons carbon per constant \$, as provided by the 2007 DICE model. The size of uncertainty is largely undetermined and has been chosen as large as possible provided that the existence conditions define an upper boundary for σ.

4.2 Effect of uncertainty

Both thresholds $y^{D*} = \theta M^{D*}$ and $y^{C*} = \theta M^{C*}$ providing the desired pollution levels before and after policy adoption are monotonically decreasing in σ, as shown by Figure 2. This means that the higher the uncertainty, the smaller the pollution for a given value of the stochastic variable θ. In contrast, the threshold y^{a*} governing policy adoption monotonically increases with uncertainty. Therefore, for a given amount of pollution, the larger the uncertainty, the later the policy is adopted. Since the value before adoption V^D and the value after adoption V^C both increase with uncertainty, the total effect on policy adoption is *a priori* ambiguous.

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[6] In the 2007 DICE model, the optimal policy is defined as the emissions trajectory that balances current abatement costs against future damage from global warming.

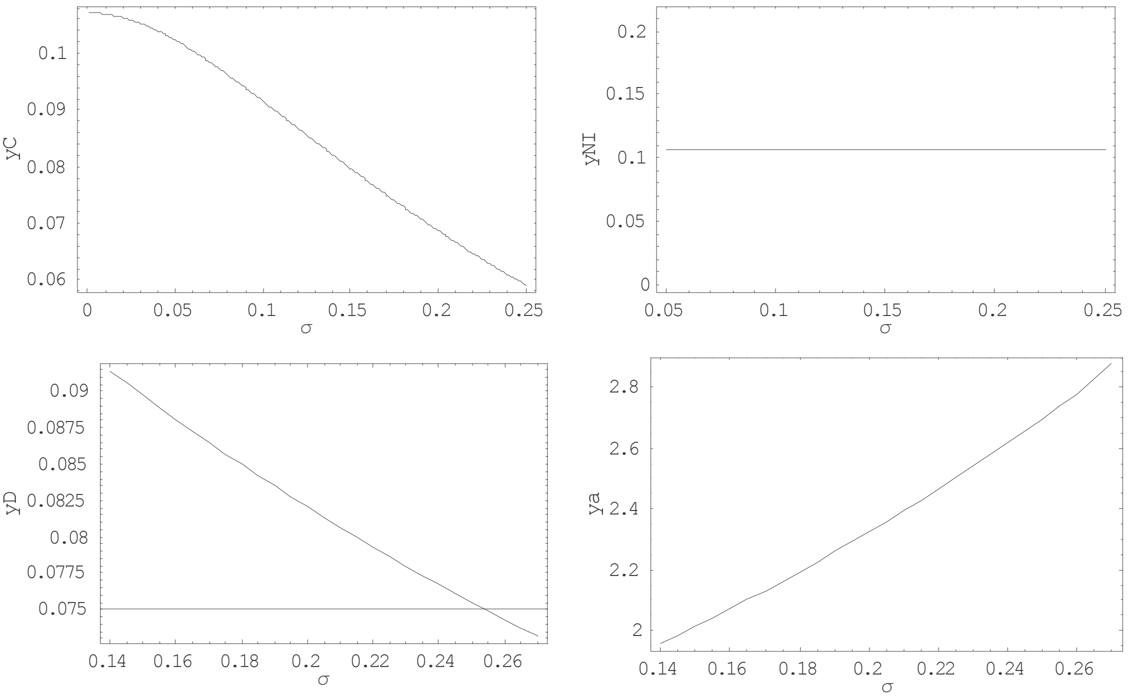

Figure 2

4.3 Effect of irreversibilities

In order the gain insight into the respective effects of both irreversibilities, we compare the respective evolution of the y^{D*}, which encompasses the two options, and y^{*}_{NI}, defined as the trigger value when ignoring the joint effect of irreversibility and uncertainty (see equation (8)). For low levels of environmental policy cost, we observe $y^{D*} > y^{*}_{NI}$, or equivalently, $M^{D*} > M^{*}_{NI}$ for a given realization of θ. The effect of the opportunity to adopt a policy prevails on the effect of the irreversibility associated with the pollution, leading to more pollution than when irreversibility is ignored.

In contrast, it appears that for high levels of environmental policy cost, $y^{D*} < y^{*}_{NI}$, or equivalently, $M^{D*} < M^{*}_{NI}$. This means that the impact of the pollution irreversibility exceeds the impact of the policy irreversibility. In this case, the economy pollutes less when both irreversibilities are taken into account. For an infinite level of K the economy never adopts the policy, thus y^{D*} tends towards y^{C*}.

When comparing Figures 2 and 3, it is clear that it is only for a particular level of abatement cost (around $K = 10$) that the effect of both irreversibilities perfectly offset each other. In such a case $y^{D*} = y^{*}_{NI}$ and it is not misleading to ignore the two irreversibilities. Considering the calibrated level of the abatement costs, namely $K = 27.28$, we therefore find that the optimal pollution level for a given realization of θ is smaller than the level that would prevail if both irreversibilities were ignored. In other words, by not considering pollution irreversibility and policy irreversibility, one may pollute too much. This is a striking result compared to the findings of other studies attempting to introduce the two irreversibilities (Kolstad, 1996; Fisher and Narain, 2003; Pindyck, 2000, 2002). Because these papers only take the policy irreversibility into effective account, their conclusions are that introducing irreversibility should lead to more pollution.

Our results probably would be weakened by allowing policy adoption to occur more than once since pollution would then become less problematic. Nevertheless, our results seem robust in the sense that, assuming a once-and-for-all policy, a policy cost valued at less than half the one we calibrated (from $K = 27.28$ to $K = 10$) is needed to reverse them.

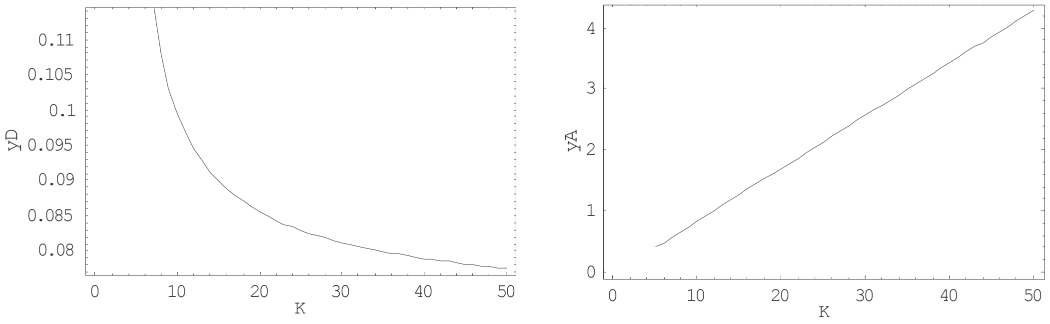

Figure 3

5 Conclusion

In this paper we deal simultaneously with the two irreversibilities that characterize pollution issues, namely the sunk cost of environmental policy and the sunk cost of environmental degradation. The first part of the paper concentrates on pollution irreversibility alone. We find that such an irreversibility generates a lower level of pollution when the damage of pollution is uncertain. Adding the environmental policy irreversibility leads to a situation where the two irreversibilities work in opposite directions. Solving the model provides implicit solutions for both the optimal pollution level and the optimal timing of environmental policy adoption. With the calibration, we find that the irreversibility associated with the adoption of a pollution policy prevails and optimal pollution is smaller than in the absence of any irreversibility. Further research will focus on the explicit modelling of the different sources of uncertainty. This should involve first decomposing uncertainty on the pollution-climate relationship and uncertainty on the climate-economic costs relationship. Second, uncertainty on the environmental policy should be introduced. Finally, the problem of optimal pollution under uncertainty and irreversibilities probably should be considered in a general equilibrium framework, therefore explicitly taking into account the consumption/environmental arbitrage.

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