Review of Applied Economics, Vol. 2, No. 1, (2006) : 63-80

MARKET VALUATION AND RISK ASSESSMENT OF CANADIAN BANKS

***Ying Liu*[*], *Eli Papakirykos*[**] & *Mingwei Yuan*[***]**

This paper applies the asset valuation model developed by Rabinovitch (1989) to the six largest Canadian banks. The model is an extension of the Merton (1977a) option-pricing model with the incorporation of stochastic interest rates. We then introduce a measure of distance-to- default, Z-score. Our results indicate that the market value of bank assets is almost always below its book value and that Canadian banks have a very low insolvency risk over time, except for 1982 and 1983. We also find that both the market valuation of the bank assets and the z-score of these Canadian banks demonstrate similar regime switches in the late 1990s, which may be related to regulatory changes during the 1990s.

JEL classification: *G12, G21*

Keywords: *asset pricing, financial institution.*

INTRODUCTION

The insolvency risk of banks has important implications for policy makers and regulators. Bank failures are costly to an insurer of deposits and to other member institutions in a deposit insurance program. From the inception of the Canada Deposit Insurance Corporation (CDIC) in 1967 to March 2001, there were 43 failures of member institutions, with losses to CDIC of about $7 billion, or 15 per cent of the face value of the insured deposits of those institutions.[2] In more extreme situations, a failing bank could cause panic in the banking system and trigger runs in other financial institutions, creating a financial crisis, where the adverse impact on the financial system would be felt in other parts of the economy.

Regulators traditionally rely on accounting statements to monitor the financial health of banks. Accounting data, however, are not issued frequently, and they have a significant time lag. Moreover, there may be an incentive for a failing bank to disguise its true state from regulators and the financial market. For example, the external auditors of the two bank failures in recent Canadian history, Canadian Commercial Bank and the Northland Bank, were persuaded by management to accept accounting statements about which they had serious concerns (Binhammer and Sephton 1998). This is less likely to occur today, because regulation and supervision in the

* Bank of Canada

** TD Waterhouse, Toronto

*** MBIA Insurance Corporation, New York

financial service sector have improved. Nevertheless, accounting data are still prone to manipulation by the reporting institution and valuable information can be lost.

Information derived from market prices can be more accurate, frequent, and timely than that derived from other sources. Recent literature has argued that market price data should be used to assess the risk of bank failures. For example, Laeven (2002) uses market price data from banks in East Asia to estimate the costs of insuring a bank's deposits, and he uses this estimate as a measure of bank risk. Gropp et al. (2002) show that an unbiased equity-based fragility indicator, a Z-score, can be derived from a Black-Scholes (1973) type of option-pricing model and predict bank defaults more accurately than bank subordinated-debt spread and traditional CAMEL[3] type indicators. Giammarino et al. (1989) calculate the market value of assets for Canadian banks using an option-pricing model and find that there is a significant difference between a bank's market value and its book value. This difference is found to largely increase prior to a bank's bankruptcy.

This paper revisits the market valuation of Canadian banks and proposes a measure of distance-to-default, the Z-score, which can be used to assess the risk of bank failures on a timely basis. Our Z-score is a potential improvement over the one commonly used in the literature, because it takes into account the stochastic interest rate risk, which is an important risk that financial institutions face. The model is based on the Rabinovitch (1989) option-pricing model, which is an extension of the Merton (1977a) model. Duan et al. (1995) decompose the bank risk into interest rate risk and non-interest rate risk based on this model. We apply the model to the six largest Canadian banks over the period 1982–2002. Our study is the first we know of to apply such methods to Canadian banks.

We find that Canadian banks have a very high Z-score (very low insolvency risk), judging by the standard in the literature, except in 1982 and 1983. Moreover, asset volatility has decreased over time and most of this can be attributed to interest rate risk, except for the period 1998–2002. We also find that the market value of bank assets is almost always below its book value. The difference between the two narrowed considerably in the late 1990s. The evolution of the Z-score demonstrates a similar regime shift in that period—the insolvency risk significantly decreased. This market-perceived reduction in the banks' default risk might be due to the regulatory changes that took place during the 1990s.

The tools developed in our study are highly useful for regulators and policy makers. To gauge the financial conditions of individual banks, supervisors can monitor the Z-score of each bank in combination with existing ratios and metrics based on balance sheet data. From a financial stability point of view, policy makers can focus on the Z-score for the banking sector to sense the overall soundness of the banking system. The market valuation and the Z-score can also provide valuable insights for the evaluation and formulation of financial legislation and regulation. For example, there should be an improvement in the market valuation and the Z-score after the introduction of a safety-enhancing regulatory change.

The rest of this paper is organized as follows. Section 2 introduces the theoretical model and the Z-score. Section 3 provides empirical results. Section 4 offers some conclusions. The appendix solves the model for the asset value of a bank.

THEORETICAL MODEL

The theoretical model is based on Merton's (1977a) option-pricing model with stochastic interest rates. This section first describes how a bank's equity is modelled as a call option. Next, we introduce a stochastic interest rate process into the option-pricing model. We then show how the model can be used to distinguish interest rate risk from non-interest rate risk. After the model is solved, we introduce the Z-score, or distance-to-default.

Modelling bank equity as a call option

According to Black and Scholes (1973), it is possible to interpret a bank's equity as a call option on the bank's assets. Consider a bank with a planning horizon equal to $[0, T]$[4]. *The bank acquires an asset portfolio at time* $t = 0$, and finances it with a deposit liability of face value L, which matures at time $t = T$. Assuming a compounded interest rate, R, the liability to depositors is Le^{RT} at time $t = T$. When the liability matures, shareholders of the bank can either "repurchase" it from the depositors by making required interest and principal payments, or relinquish ownership to the depositors or other creditors of the bank. Thus, the equity of the bank *(E)* can be considered as a call option on the bank's assets with an exercise price of Le^{RT}, the payoff of which can be described as:

$$E = max\ (0, V - Le^{RT}), \tag{1}$$

where V is the market value of the bank's assets. With deposit insurance, R is equal to some risk-free government bond rate and equation (1) can be rewritten as

$$E = \max\ (0, V - \lambda Le^{RT}), \tag{2}$$

where λ is the closure rule of the regulatory authorities, and $0 < \lambda < 1$. When $\lambda = 1$, the regulatory authorities declare the bank bankrupt as soon as the value of its liabilities is higher than that of its assets. The lower λ is, the further regulators would allow the value of the bank's assets to fall below its liabilities before it is declared bankrupt.

Introducing a Stochastic Interest Rate

In many models of option pricing, a constant risk-free interest rate is assumed (e.g., see Black and Scholes 1973, Cox 1975, Geske 1979, Johnson and Shanno 1987, and Hull and White 1987). This assumption ignores the correlation between the risk-free rate and the value of the bank's assets and equity. In a model applied to banks, the balance sheets of which are highly leveraged, this assumption can be particularly problematic. During periods of volatile interest rates, banks are more likely to experience mismatches between the durations of their assets and liabilities, and thus are more likely to be insolvent.[5]

Merton (1973) assumes that the price of a default-free discount bond is a function of a stochastic interest rate. His model simplifies to the Black and Scholes (1973) model in the special case of a constant risk-free rate. Similar to Rabinovitch (1989) and Duan *et al.* (1995), we adopt Merton's approach and assume that the interest rate follows a stochastic process described by Vasicek (1977). The instantaneous interest rate is assumed to be mean-reverting, as follows,

$$dr_t = q(m - r_t)\, dt + vdZ_{r_t},$$

where r_t is the instantaneous risk-free rate of interest at time *t, m* is the long-run mean of the interest rate, *v* is the interest rate volatility, *q* is the converging speed of the interest rate towards its long-run equilibrium, and is a standard Wiener process.

Interest Risk and Non-interest Risk

Similarly, the total value of the bank's assets is specified as follows,

$$\frac{dV_t}{V_t} = \mu dt + \sigma_V dZ_{V_t}, \tag{4}$$

where V_t is the value of the bank's assets at time *t,* μ is the expected return on the bank's assets, σ_V is the total volatility of returns on the bank's assets, and Z_{V_t} is a Wiener process. σ_V is assumed to be influenced by the interest rate risk; thus, Z_{r_t} and Z_{V_t} are correlated. Duan et al. (1995) show that Z_{V_t} can be decomposed into a component relating to dZ_{r_t} and another orthogonal component, dW_t, through a projection exercise, where W_t is a Wiener process. More specifically,

$$dZ_{V_t} = \eta dZ_{rt} + (1 - \eta^2)^{\frac{1}{2}} dW_t, \tag{5}$$

where $\eta = \text{cov}(dZ_{V_t} dZ_{r_r}) / dt$.

Substituting equation (5) into (4) leads to

$$\frac{dV_t}{V_t} = \mu dt + \sigma_V \eta dZ_{r_t} + \sigma_V (1 - \eta^2)^{\frac{1}{2}} dW_r \tag{6}$$

Using the expression of based on equation (3), we have,

$$\frac{dV_t}{V_t} = [\mu - \phi_V q(m - r_t)]dt + \phi_V dr_t + \psi dW_t, \tag{7}$$

where $\phi_V = \sigma_V \eta / v$ and $\psi = \phi_V (1 - \eta^2)^{1/2}$. ϕ_V can be interpreted as the instantaneous interest rate elasticity of the bank's assets, and $\phi_V^2 v^2$ is the component of asset volatility that is contributed by the interest rate risk. Similarly, ψ is the volatility of the bank's assets caused by all risk that is orthogonal to the interest rate risk. The entire model is solved in the Appendix.

Distance-to-default

Next, we define a measure of the distance-to-default, the Z-score, which measures the market value of a bank's assets in relation to the book value of its liabilities. Our definition of the Z-score is similar to the one proposed by KMV Corporation (1993) and Gropp et al. (2002), based on an option-pricing model,[6] although our Z-score differs by taking into account the interest rate risk. It is therefore more relevant for banks than the existing Z-scores. Specifically,

$$z_t = \frac{(V_t - \lambda L_t)/V_t}{\sigma_V} = \frac{1 - \lambda(1 - E_t/V_t)}{\sigma_V}, \tag{8}$$

where

$$\sigma_V^2 = {\phi_V}^2 v^2 + \psi^2. \tag{9}$$

Equation (8) shows that the Z-score is determined by three variables: λ, σ_V, and E_t/V_t. The parameter λ is related to the deposit insurance program. A smaller value of λ means the deposit insurer would allow the bank's assets to go further below its liabilities before declaring it bankrupt. E_t/V_t can be considered a risk-based capital measure, as required by the market. The denominator σ_V can be decomposed into interest rate risk and non-interest rate risk. Equation (8) is the same as the distance-to-default measure from Gropp et al. (2002), who show that this measure satisfies the completeness and unbiasedness conditions for a good indicator of bank fragility.

Simply looking at the formula of the Z-score, we can see that a more forgiving closure rule (a smaller value of λ) can increase the value of Z, which for the market makes the bank appear further from insolvency. In the meantime, a higher E_t/V_t ratio has the same effect. This implies that a more stringent capital standard is qualitatively equivalent to a more lenient closure rule in regard to the market's perception of the insolvency risk of a financial institution. However, λ is exogenous to the bank; i.e., the regulator and all other agents in the economy have to bear the costs of a lenient rule, whereas the bank has control over its capital adequacy. More importantly, viewed in a dynamic context, a more lenient closure rule (a smaller λ) can generate a moral hazard, which leads to increased risk-taking and the increased probability of future failures. An increase in the total risk, s_V, leads to a lower value of Z, which means a higher risk of insolvency. Recall that s_V consists of interest rate risk and non-interest rate risk.

How does Interest Rate Volatility Affect the Z-score?

Before we apply the model to Canadian banks, we examine the properties of the model for a hypothetical bank, calibrated as follows. For the stochastic interest rate process, we use the parameter estimates from Ait-Sahalia (1996), where $q = 0.86$, $m = 0.089$, and $v^2 = 0.002154$. λ is set to 0.95, as in Giammarino et al. (1989). According to Duan et al. (1995), the interest rate elasticity of equity ϕ_E has a range of –1 to –6. Thus, we set the value of ϕ_E to be –2. The volatility of equity (σ_E) is set to be 0.25, close to the average value obtained in our empirical sample. We set $T = 1$, assuming that the bank's planning horizon is a year, consistent with the frequency of the publication of their annual report.

Our first sensitivity test studies the model's response to a change in the interest rate volatility. As Table 1 shows, a rise in v increases asset volatility, σ_V(total risk), and the interest rate elasticity of assets, ϕ_V, which implies that interest rate risk accounts for a disproportionately large part of the increase in total asset volatility. As a result, the non-interest rate risk decreases slightly. On the other hand, the interest rate elasticity of liability is unchanged, since, under the model's assumption, the default-free bond that represents the liability of the bank pays zero- coupon interest. Thus, the net impact of the mismatch in the interest rate risk exposure is a wider elasticity gap and duration gap. Not surprisingly, the Z-score is lower.

Table 1
Sensitivity of Z-score with Respect to Interest Rate Volatility, ν

Parameters	*Smaller values*	*Normal value*	*Larger value*
Interest rate volatility $\nu = 0.0464$	*0.9	*1.0	*1.1
Asset volatility σV	0.0395	0.0425	0.0456
Non-market risk ψ	0.0215	0.0212	0.0209
Interest rate elast. of liability $B(T)$	0.6707	0.6707	0.6707
Interest rate elast. of asset ϕ_V	0.7920	0.7922	0.7926
Elast. gap $\phi_V + B(T)$	0.1212	0.1215	0.1219
Prob. of insolvency $1 - N(h - \delta)$	0.0032	0.0052	0.0083
Market-based capital-adequacy ratio			
E/V	0.0909	0.0910	0.0910
Distance from insolvency			
Z-score	2.2952	2.1300	1.9788
Elast. of Z to market risk, Z_ν	33.6518	32.3228	30.9135
Elast. of Z to non-market risk, Z_ψ 27.6304	23.5516	20.1635	

How does the Degree of Forebearance Affect the Z-score?

The second sensitivity test focuses on the relationship between the forbearance of deposit insurance program and the capital adequacy ratio. According to Table 2, as the degree of forbear-ance increases (a lower l), non-interest rate risk of the bank, and consequently the total asset vol-atility of the bank, increase. The reason is intuitive. With a higher degree of forbearance, the value of the asset of the bank decreases[7]. Keeping interest rate volatility constant, a decrease in the level of a bank's asset leads to a higher interest rate elasticity of asset. This in turn results in a higher non-interest rate risk and therefore total risk. On the other hand, as the market value of asset decreases, the capital adequacy ratio *E/V* also increases, pulling the bank further away from insol-vency. Because the increase in the total risk is not large enough to offset the increase in the capital adequacy ratio of bank, the net effect of an increase in l on *z* is positive. Thus, the bank is left safer.

Table 2
Sensitivity of Z-score with Respect to the Degree of Forebearance, λ

Parameters	*Smaller values*	*Normal value*	*Larger value*
Interest rate volatility $\lambda = 0.95$	*0.9	*1.0	*1.1
Asset volatility σ_V	0.0448	0.0431	0.0418
Non-market risk ψ	0.0243	0.0221	0.0203
Interest rate elast. of liability $B(T)$	0.6707	0.6707	0.6707
Interest rate elast. of asset ϕ_V	-0.8099	-0.7973	-0.7868
Elast. gap $\phi_V + B(T)$	-0.1392	-0.1266	-0.1161
Prob. of insolvency $1 - N(h - \delta)$	0.0000	0.0000	0.0000
Market-based capital-adequacy ratio			
E/V	0.1047	0.0952	0.0873
Distance from insolvency			
Z-score	2.3392	2.2094	2.091
Elast. of Z to market risk, Z_ν	30.3867	31.7584	32.9404
Elast. of Z to non-market risk, Z_ψ	24.2608	23.7953	23.2418

EMPIRICAL RESULTS

We apply the above theoretical model to the "big six" Canadian chartered banks: the Royal Bank, the Bank of Montreal, Canadian Imperial Bank of Commerce, TD Bank Financial Group, the Bank of Nova Scotia, and the National Bank.[8] We set q equal to one, as suggested by Chan et al. (1992) and used by Duan et al. (1995), and we calculate v as the standard deviation of the daily 90-day treasury bill rate over the year prior to the end of the month.[9] The 90-day treasury bill rate is commonly adopted to proxy a risk-free interest rate. Following the calibrated model, we set $T = 1$ and $\lambda = 0.95$. E is the daily market capitalization of each bank, taken from Data Stream.[10] ϕ_E is estimated by regressing the daily return of the bank's equity on the change in daily 90-day treasury bill returns in the previous year. σ_{E_t} is the standard deviation of the market capitalization of each bank in the previous one year period and is calculated within a one-year moving window. Bongini et al. (2001) argue that equity volatility is accurately estimated for a specific time interval, as long as leverage does not change substantially over that period. L is the book value of the liability of the banks, taken from the quarterly report of each bank. The model is evaluated monthly, from June 1982 to December 2002.

Market Valuation of Banks

We calculate the market valuation of the assets of each bank. Figure 1 shows the average ratio of the market value to book value (market-to-book ratio) for the big six banks, weighted according to their asset size. We can see that the market value of a bank can be substantially

Figure 1: Weighted Average of Market-to-Book Ratio of Assets, Big Six Banks*

* Weighted according to asset size

lower than its book value. The market-to-book ratio typically lies between 94 and 99 per cent in our sample period of 1982 to 2002. This finding is similar to that by Giammarino et al. (1989), who show that the industry average market-to-book ratio is between 95 and 97 per cent in the period from 1981 to 1985. This seems to suggest that the market systematically discounts the book value of a bank's assets.

There seems to be some evidence of regime shifting in the evolution of the market-to-book ratio in the latter half of the 1990s. Prior to 1996, for example, the market-to-book ratio moves within the range of 94 to 96 per cent. The ratio rises to more than 100 per cent in about 18 months, and then it drops by three percentage points in August 1998, during a sharp increase in short-term interest rates and a general price decline in the stock market. This rapid decline in the market-to- book ratio may reflect the market's perception of an increased level of interest rate risk and a general decrease in the value of firms (a bursting of the "bubble"). Afterwards, the ratio steadily rises and stays in the 98 to 99 per cent range from the beginning of 1999 to 2002.

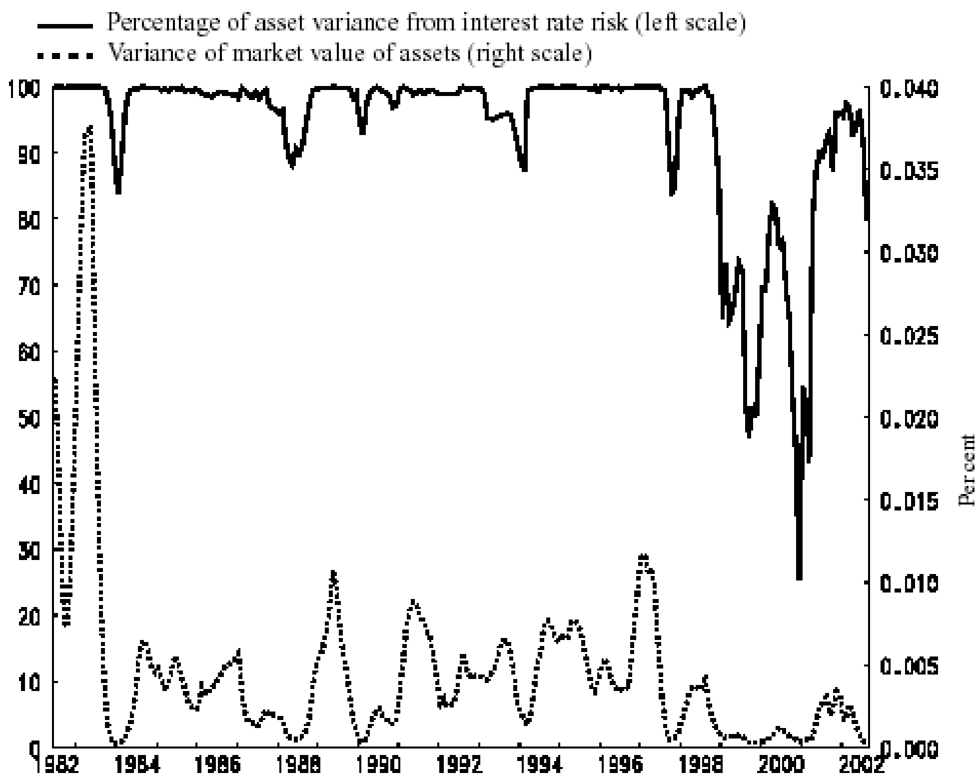
The possible regime shifts coincide with regulatory innovations in the financial safety net, which may have helped improve the market's perception of the banks' soundness. In 1996, the Office of the Superintendent of Financial Institutions (OSFI) was given the power (through amendments of the various financial-institution acts) to take control of the assets of an institution, without having to prove that it is insolvent.[11] When OSFI has taken control of an institution's assets, it may take all necessary measures to protect the interests of the institution's depositors and creditors. Other regulatory changes, such as the introduction of the Prompt Corrective Action in the mid-1990s, further formalized the supervisory options available to OSFI and CDIC to respond to a troubled institution. In 1999, OSFI introduced a procedural risk-based supervisory approach that focuses on evaluating an institution's material risks and the quality of its risk-management practices.

These steps towards improved supervisory incentives and increased supervisory powers may have sent a signal to market participants that banks are under closer supervision, and thus are making better decisions. Furthermore, the market may have believed that, if an institution was in trouble, remedial supervisory intervention would be more prompt. These signals may have helped improve the market valuation of banks' assets.

Interest Rate and Non-interest Rate Risks

Figure 2 shows the weighted average of the total asset volatility (total risk) of the big six banks and its decomposition into interest rate risk and non-interest rate risk. The highest asset volatility occurs over the period 1982–83, when interest rate volatility was at a record high. Indeed, our model shows that interest rate risk accounts for almost all asset volatility over the period 1982–83, beyond which time the total risk of banks seems to have continually decreased, particularly since 1998. The share of interest rate risk has also decreased since the end of 1998. On the one hand, the decrease in the total risk may be related to the market response to the regulatory changes discussed above. On the other hand, the increase in the share of the non-interest rate risk over the period 1998–2002 may be related to the banks' exposure to the stock market and to certain high-risk corporate sectors. The stock market experienced tremendous volatility over that period, creating large fluctuations in the banks' trading and transaction income as well as in some fee-based income related to market activities, such as underwriting fees and security commissions. The financial problems in certain corporate sectors over the period 1998–2002, notably in the high-tech sector, also may have contributed to a higher non-interest risk through a large amount of loan losses and loss provisions.

Figure 2: Big Six Banks: Decomposition of Asset Variance*

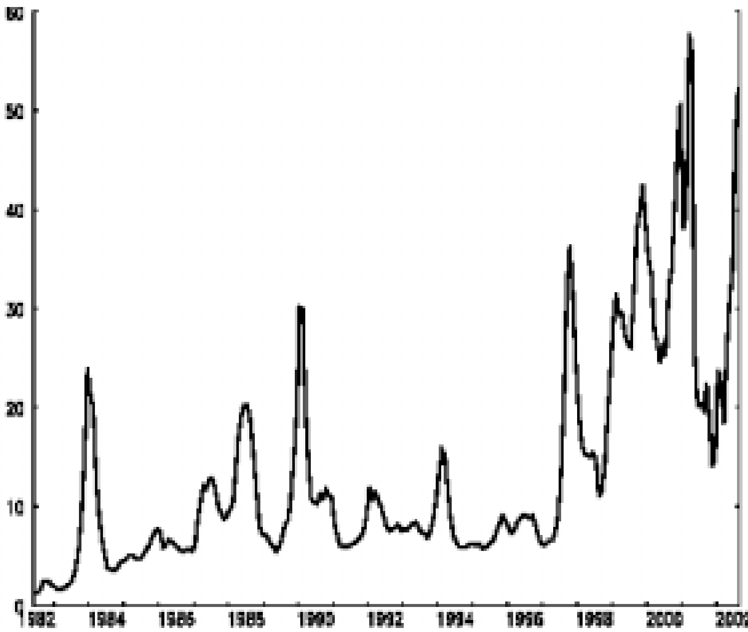


* Weighted according to asset size

Distance-to-default

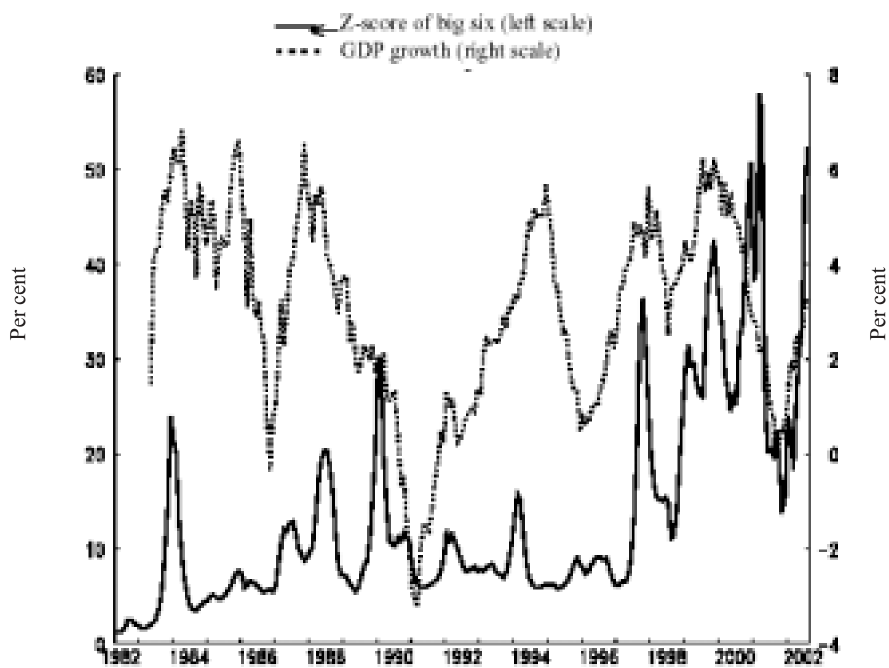
Figure 3 shows the weighted average of the Z-score of the big six banks. Overall, Canadian banks have a very high Z-score (or very low insolvency risk). Although the commonly adopted threshold for identifying troubled firms is between 1.5 and 2 (Calmès 2004, Altman 1977), the Z-score of Canadian banks is typically much higher than that. An exception occurs in 1982, when, on average, the big six produced a Z-score of around 2, a level considered high risk. This may reflect several adverse factors that the banks faced simultaneously: the recession, the fallout from the less-developed countries' (LDC) debt crisis, and the record high volatility of interest rates. Several other turning points of the Z-score also seem to correspond to important macroeconomic events: the economic slowdowns at the end of 1986, end of 1995, mid-1998,

Figure 3: Weighted Average of Z-score of the Big Six Banks*



* Weighted according to asset size

Figure 4: Z-score of Big Six vs. Output Growth

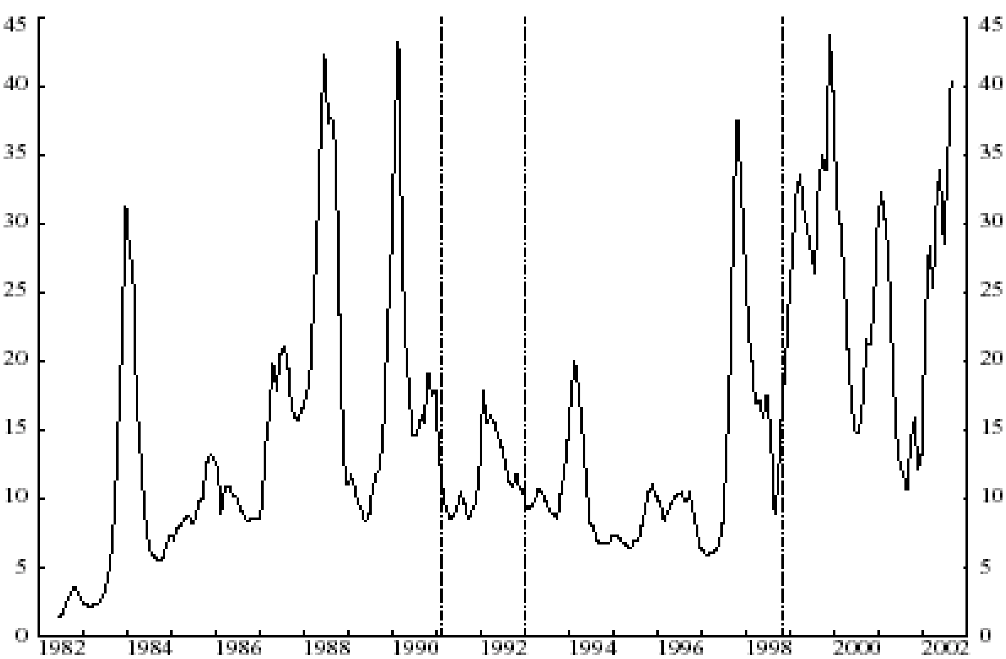


and end of 2001; the recession at the beginning of 1991; and the "Black Thursday" stock crash of 1987. Figure 4 plots the Z-score against real annualized quarter-over-quarter GDP growth. Although the two measures may not be highly correlated at the level, the Z-score seems to match well the turning points in the economy.

The regime shifts that we observed in the market valuation of bank assets also seem to be evident in the evolution of the Z-score. As stated in section 2, three factors contribute to the movements in the Z-score: λ, E/V, and σ_V. λ is assumed to be constant. The structural increases in the market-to-book ratio in 1996 and 1999 imply a higher market-based capital ratio (E/V). A decrease in the volatility of assets over time, particularly in the late 1990s, also contributes to a higher Z-score in that period.[12]

The usefulness of the Z-score is more evident when taking a closer look at the Z-score of individual banks. As shown on Figures 5 to 10, the insolvency risk of each bank varies from each other and across time. The dotted lines signify the bond rating downgrades/upgrades by Moody's. Interestingly, there were very few changes in the rating of Canadian banks in the last twenty years and they were all downgrades. As shown on the graphs, downgrades usually take place when the Z-score is low. However, the Z-score identifies more periods of higher insolvency risks than traditional bond rating.

Figure 5: Z-score, TD



* Dotted line represents a down-grade from Moody's.

Figure 6: Z-score, Bank of Montreal

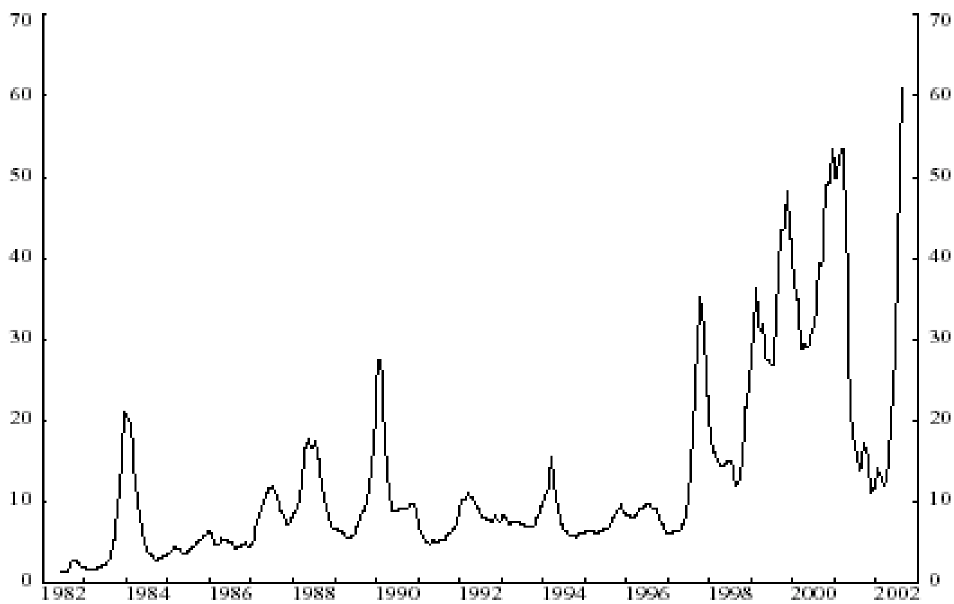
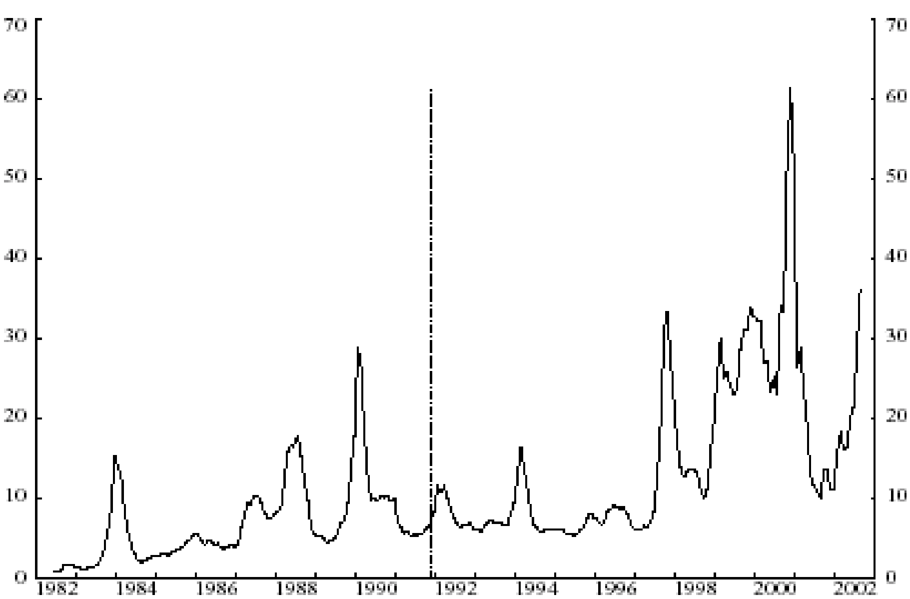
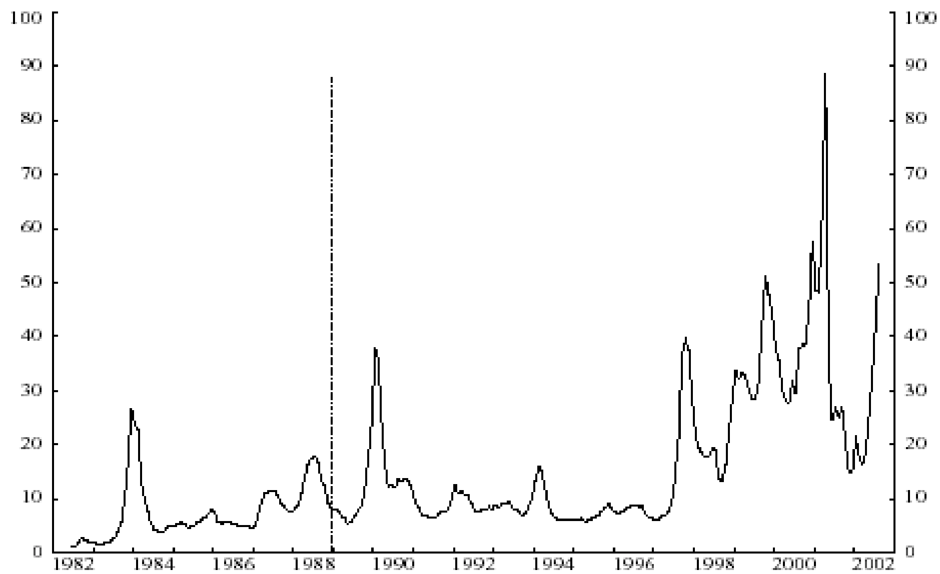


Figure 7: Z-score, CIBC*



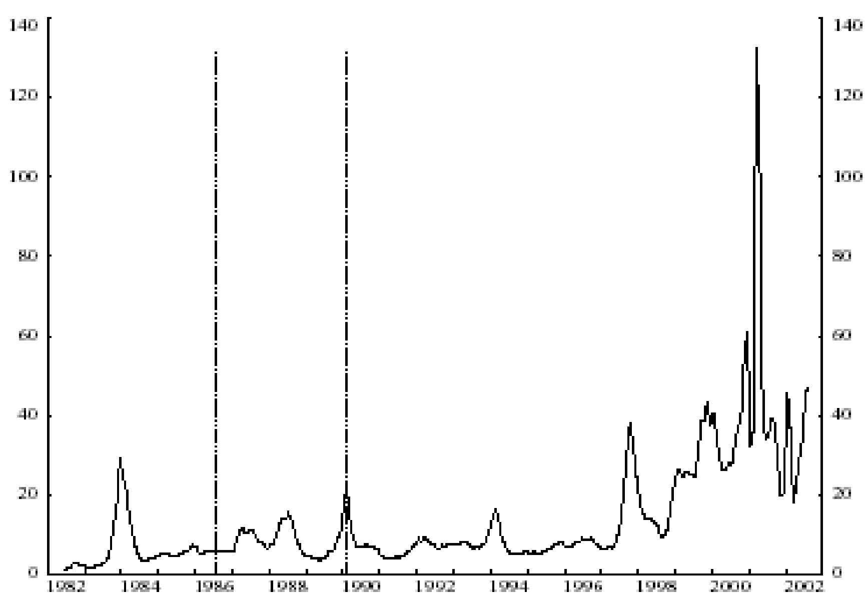
* Dotted line represents a down-grade from Moody's.

Figure 8: Z-score, Royal Bank of Canada*



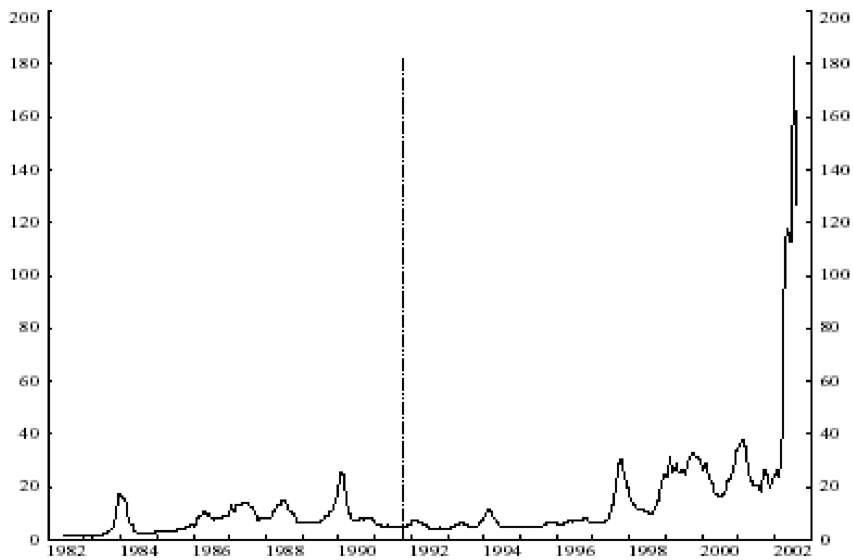
* Dotted line represents a down-grade from Moody's.

Figure 9: Z-score, Royal Bank of Nova Scotia*



* Dotted line represents a down-grade from Moody's.

Figure 10: Z-score, National Bank*



* Dotted line represents a down-grade from Moody's.

CONCLUSION

We have applied the Rabinovitch (1989) option-pricing model to the big six Canadian banks over the period 1982 to 2002. The market valuation of bank assets is derived from this model. The model also allows the bank risk to be decomposed into interest rate risk and non- interest rate risk. We introduced a measure of distance-to-default, the Z-score, which can be useful in assessing the financial strength of banks as well as the risk of bank failures.

We have found that the market value of a bank's assets almost always lies below its book value. This finding is similar to that by other researchers. The market-to-book ratio shows evidence of regime shifts in 1996 and 1999. These structural breaks correspond to regulatory innovations that occurred in those periods, which may have helped improve the market's perception of the insolvency risk of Canadian banks. We also find that Canadian banks have a very high Z-score (very low insolvency risk), judging by the standard in the literature, except over the 1982–83 period. Similar to the regime shifts observed in the market-to-book ratio, the industry Z-score improved significantly in the late 1990s. Moreover, asset volatility decreased over time and most of it can be attributed to the interest rate risk, except for the period 1998–2002, when the non-interest rate risk increased significantly.

There are many possible extensions to our work. For example, we have assumed that equity volatility is deterministic; more work can be done to extend the model to incorporate stochastic equity volatility, similar to that proposed by Hull and White (1987). One could also use a more forward-looking measure of equity volatility, such as the volatility derived from a Black- Scholes (1973) type option-pricing model.

NOTES

1. Based on CDIC annual reports.
2. CAMEL: Credit, Asset, Management, Equity, and Liquidity.
3. This time interval can be assumed to be one year, the frequency at which banks issue their annual reports.
4. Although banks are often well-hedged against interest risk.
5. There are many other ways to construct the Z-score. For example, Altman (1977, 1993) computes the Z-score based on working capital, total assets, earnings before interest and taxes, sales, and other financial variables.
6. This has to do with the fact that equity value is exogenous in this model. Thus, when the face value of liability is discounted, the value of asset must decrease, keeping equity value constant. In reality, it is likely to see an increase in the equity value when a higher degree of forbearance is introduced.
7. There are eight publicly traded banks in Canada. The other two are much smaller than the "big six" and only became listed in 1985 and 1994 respectively. Adding these two banks to the aggregated weight measures for the bank industry makes little difference from our results.
8. Ideally, we would like to use estimates of q and v based on equation (3). Using a conditional least-squares method, however, we obtain many non-converging results, in which $qv < 1$, a necessary condition for equation (3) to yield a mean-reverting process. This is not surprising, given that Chan et al. (1992) show that the Vasicek (1977) model performs poorly in matching real interest rate data. As a result, we follow Duan et al. (1995) and assume that interest rates are deterministic. Ronn and Verma (1986) show that their estimates of asset volatility are almost identical when using deterministic and stochastic interest rates.
9. Bank balance data is published quarterly. Market participants are likely to re-value the banks based on this new information. In addition, they are likely to change their valuation of banks more frequently based on non-balance sheet information, such as press release. As a result, the daily value of market capitalization is used instead of the quarterly.
10. See David and Pelly (1997), and Government of Canada (2001a, b).
11. We assume λ to be constant throughout the model. λ is likely to have increased, because the supervisors are given more power over troubled institutions at an earlier stage; i.e., a tighter closure rule may have been imposed. Implemented in a mechanical fashion, this could lead to a lower Z, or higher insolvency risk. This kind of regulatory change, however, establishes incentives that will generally reduce insolvency risk.

REFERENCES

Ait-Sahalia, Y., (1996), "Testing Continuous-Time Models of the Spot Interest Rate", *Review of Financial Studies*, 9: 385–426.

Altman, E.I., (1977), *The Z-Score Bankruptcy Model: Past, Present, and Future*. New York: John Wiley and Sons.

Altman, E.I., (1993), *Corporate Financial Distress and Bankruptcy*. New York: John Wiley and Sons.

Binhammer, H.H. and P.S. Sephton, (1998), *Money and Banking in the Canadian Financial Sys-tem*. Toronto: International Thomson Publishing Company.

Black, F. and M. Scholes, (1973), "The Pricing of Options and Corporate Liabilities", *Journal of Political Economy,* 3: 637–654.

Bongini, P., L. Laeven and G. Majnoni, (2002), "How good is the market at assessing bank fragil-ity? A horse race between different indicators", *Journal of Banking and Finance*, 26(15): 1011-1028.

Calmès, C., (2004), "Financial Market Imperfection, Overinvestment and Speculative Precaution", Bank of Canada Working Paper No. 2004-27.

Chan, K.C., G.A. Karolyi, F.A. Longstaff, and A.B. Sanders, (1992), "An Empirical Comparison of Alternative Models of the Short-Term Interest Rate", *Journal of Finance,* 47: 1209–1027.

Cox, J., (1975), "Notes on Option Pricing I; Constant Elasticity of Variance Diffusions" Working Paper, Stanford University.

David, G. and L. Pelly, (1997), *The 1997 Annotated Bank Act*.

Duan, J., A.F. Moreau, and C.W. Sealey., (1995), "Deposit Insurance and Bank Interest Rate Risk: Pricing and Regulatory Implications", *Journal of Banking and Finance,* 19: 1091–1108.

Geske, R., (1979), "The Valuation of Compound Options", *Journal of Financial Economics,* 7: 63–81.

Giammarino, R., E. Schwartz, and J. Zechner, (1989), "Market Valuation of Bank Assets and Deposit Insurance in Canada", *Canadian Journal of Economics,* 22: 109–127.

Government of Canada, (2001a), *Office of The Superintendent of Financial Institutions Act*, R. S. 1985, c. 18 (3rd supplement).

Government of Canada, (2001b), *Bank Act,* 1991, c.46.

Gropp, R., J. Vesala, and G. Vulpes, (2002), "Equity and Bond Market Signals as Leading Indica-tors of Bank Fragility", European Central Bank Working Paper, No. 150.

Hull, J. and A. White, (1987), "The Pricing of Options of Assets, with Stochastic Volatilities", *Journal of Finance*, 42: 281–300.

Johnson, H. and D. Shanno, (1987), "Option Pricing when the Variance is Changing", *Journal of Financial and Quantitative Analysis,* 22: 143–51.

KMV Corporation, (1993) *Credit Monitor Overview*. San Francisco, CA.

Laeven, L, (2002), "Bank Risk and Deposit Insurance", *World Bank Economic Review*, 16(1): 109–37.

Merton, R.C., (1973), "Theory of Rational Option Pricing", *Bell Journal of Economics and Man-agement Science*, 4: 141–183.

Merton, R.C., (1977), "On the Pricing of Contingent Claims and the Modigliani-Miller Theorem", *Journal of Financial Economics,* 5: 241–249.

Rabinovitch, R., (1989), "Pricing Stock and Bond Options when the Default-Free Rate is Stochas-tic", *Journal of Financial and Quantitative Analysis,* 24: 447–457.

Ronn, E.I. and A.K. Verma, (1986), "Pricing Risk-Adjusted Deposit Insurance: An Option-Based Model", *Journal of Finance,* 41: 871–895.

Vasicek, O.A., (1977), "An Equilibrium Characterization of the Term Structure", *Journal of Financial Economics,* 5: 177–188.

APPENDIX
SOLVING THE MODEL FOR THE ASSET VALUE OF A BANK

Vasicek (1977) shows that the price at time t of a zero-coupon bond that pays \$1 at time T is:

$$P(r(t),\, T-t) = A(T-t)e^{-B(T-t)r(t)}, \tag{A1}$$

where $A(T–t)$ and $B(T–t)$ are functions of the time to maturity of the bond,

$$B(T-t) = \frac{1-e^{-q(T-t)}}{q}. \tag{A2}$$

$$A(T-t) = \exp\left[\frac{B(T-t)-(T-t)\left(q^2 m - \frac{v^2}{2}\right)}{q^2} - \frac{v^2 B(T-t)^2}{4q}\right], \tag{A3}$$

where $-B(T–t)$ is the interest rate elasticity of the bond price. Recall that the liabilities of a bank can be modelled as a zero-coupon bond maturing at $t = T$. Thus, $-B(T–t)$ is the interest rate elasticity of bank liabilities. Recall that ϕ_V is the interest rate elasticity of the bank's assets. Thus, is $[\phi_V + \mathrm{B}(T-t)]$ the interest rate elasticity gap and the negative of this term is equivalent to the bank's duration gap.

Duan et al. (1995) show that, because the assets and liabilities of a bank have interest rate elasticities of ϕ_V and $-B(T–t)$, respectively, the interest rate elasticity of the bank's equity value is,

$$\phi_{E_t} = \Omega_t[\phi_V + B(T-t)] - B(T-t), \tag{A4}$$

where

$$\Omega_t \equiv N(h_t)\frac{V_t}{E_t} \tag{A5}$$

is the standard option elasticity.

Let σ_{E_t} denote the equity volatility at time t. Then,

$$\sigma_{E_t} = \sqrt{\phi_{E_t}^2 v + \Omega_t^2 \psi^2}. \tag{A6}$$

The bank's equity valuation is, therefore,

$$E = VN(h) - \lambda LN(h-\delta), \tag{A7}$$

where

$$h = \frac{1}{\delta}\, in\left[\frac{V}{\lambda L}\right] + \frac{\delta}{2}, \tag{A8}$$

$$\delta_t^2 = (\phi_V^2 v^2 + \psi^2)T + 2\phi_V v^2\left[\frac{T}{q} + \frac{1}{q^2}(e^{-qT} - 1)\right] + v^2\left[\frac{T}{q^2} + \frac{2}{q^3}(e^{-qT} - 1) + \frac{1 - e^{-2qT}}{2q^3}\right]. \tag{A9}$$

Given T, q, v, λ, ϕE, σ_{E_t} and L, the asset value of the bank, V_t, the interest rate elastic-ity of V, ϕ_V, and the measure of non-interest risk, ψ, can be obtained by solving the system of equations consisting of (A2), (A4), (A5), (A6), (A7), (A8), and (A9).