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Risk, Overconfidence and Production in a Competitive Equilibrium

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Risk, Overconfidence and Production in a Competitive Equilibrium

Abstract

Previous studies have found underestimation of risk, or overconfidence, to be pervasive. In this paper, we model overconfidence as a reduction in perceived variance. We generalize the analysis of Sandmo and examine the effects of competition on firms displaying overconfidence. Cases for both competitive equilibrium and imperfect competition are investigated. We show that overconfidence may strictly dominate rationality in a competitive market by leading risk averse producers to invest greater amounts and produce more. This leads to a higher average profit, and greater variance of profits, leaving the producer a greater probability of surviving competitive pressures. Despite the greater variance of profits, if enough producers underestimate their risk, they should collectively drive more rational decision makers from the market. Our results suggest that overconfidence may be as important a determinant of market behavior as diminishing marginal utility of wealth.

Key Words

Overconfidence, Misperception, Production, Competition

1. Introduction

The expected utility (EU) framework has become the primary analytical tool economists have used to analyze choices of economic agents under risk. A substantial literature investigates behavior of risk-averse producers and its implication for market pricing, resource allocation and welfare (for example Sandmo, 1971; Feder, 1977). But the favored status of the EU model has been credibly challenged by behavioral findings. Subjects regularly exhibit loss aversion (Kahneman and Tversky, 1979) and overconfidence (Alpert and Raiffa, 1982) in behavioral experiments involving risk and information. An overconfident producer will recognize the level of risk she faces when making production decisions. Thus if overconfidence is pervasive, it may be important to determine the extent to which overconfidence alters the conclusions of the EU-based literature on producer choices under risk.

Overconfidence occurs when individuals do not recognize the extent of the risk they face in the act of decision-making. Thus individuals may act as if they have greater certainty about the possible outcomes than they truly do. We model overconfidence as a pure change in the moments of outcomes. This model has the advantage of allowing us a relatively simple measure of overconfidence, similar to the simple Arrow-Pratt measures of risk aversion prevalent in the literature.

We generalize the analysis of Sandmo, applying our model to examine the effects of competition on firms displaying overconfidence or loss aversion. Our study is divided into two types of overconfidence. Type-1 overconfidence is defined as a simple additive shift in the distribution resulting in a higher mean but a true

perception of all central moments of the risk. Type-2 overconfidence is typified by a diminished perception of variance.

In order to clarify the effects of overconfidence on competitive behavior, we identify the decision rules of the economic agents in the competitive market. We find that the expected profit is higher for the firms displaying Type-2 overconfidence, and thus, overconfident decision makers will be the first into the market and the last to shut down. Thus, the market may necessarily be dominated by “irrational” actors. Market arguments are among the primary reasons economists generally dismiss irrationality as unimportant. This paper provides one rationale as to why some sets of behavioral anomalies may be prevalent, persistent, and important drivers of behavior generally.

In addition to the analysis of behavior under competition, we investigate the welfare impacts of overconfidence. Because overconfidence leads to greater production for all levels of expected prices, consumers must benefit from the resultant lower equilibrium prices. On the other hand, the ex post producer surplus cannot be subject to the same behavioral anomaly. Once the profits are realized, the producer no longer considers variation. In equilibrium, the variance is misperceived at a rate determined by the degree of risk aversion. Thus, overconfidence in the marketplace will lead to an average welfare loss for particular producers displaying overconfidence. We solve for an explicit analytical threshold under which the decline in producer surplus would be less than the increase in the consumer surplus, resulting in societal benefits of overconfidence. This work has implications for government

policies on providing decision information (such as extension work or market reports) and the provision of subsidized business revenue insurance.

The remaining paper is organized in the following way: section 2 is literature review about effects of risk-attitude and overconfidence on competition. Section 3 introduces the model of overconfidence, the model of production in a competitive equilibrium and under imperfect competition respectively. Optimal decision rules for risk-averse and overconfident producers are developed. We will also examine how overconfidence may drive rational producers out of the market. Section 4 develops welfare measures under equilibrium. In section 5, we discuss the effect of overconfidence on production. Section 6 concludes the paper.

2. Literature Review

In classic production theory, the firm is assumed to maximize expected profits. In a static model of production, this assumption rules out the possibility of adjusting production to changes in risk. Sandmo (1971) argues that this model is an unsatisfactory representation of firm decisions, as even casual observation of the marketplace seems to indicate a prevalence of risk aversion.

Sandmo introduced risk aversion into the production story assuming that firms maximize expected utility, showing the impacts of diminishing marginal utility of wealth under price risk. Assuming a competitive market, he specifically outlines the impacts of risk aversion on production, welfare, and competition. Among Sandmo's most prominent results is that risk-averse firms unambiguously produce less output than risk-neutral firms when faced with price risk. Thus, risk-averse firms are at a

competitive disadvantage. For this reason, many have supposed that those displaying severe risk preferences would be sifted from the market through competition, eliminating the need to model risk in many circumstances.

However, risk-aversion is not the only influential factor that may affect production behaviors when dealing with uncertainty. Many other behavioral patterns are also found consistently among decision-makers facing risk. Here, we hope to generalize this model of competitive production by allowing a class of behavioral anomalies found both within the lab (Alpert and Raifa, 1982) and in the field (Odean, 1995, Ausubel, 1991).

Overconfidence, or a favorable misperception of the risk involved in choice, is among the most prevalent behavioral anomalies. As found in the psychology literature, most people are overconfident about their own relative abilities, and unreasonably optimistic about their futures (e.g., Weinstein 1980; Taylor and Brown, 1988). When assessing their position in a distribution of peers on almost any positive trait – like prospects, or longevity – a vast majority of people say they are above the average. These misperceptions are generally classified into one of two types. Those suffering from this first type of overconfidence may perceive the expected outcome of a venture to be much better than current information would suggest.

A second type of overconfidence, in contrast, is found in agents that are “too certain” about some event. For example, an agent may believe that the stock price for tomorrow will be between \$55 and \$60 with probability 90%, while in reality the 90% confidence interval is found between \$35 and \$90. On average, individuals produce

confidence intervals that contain the truth much less often than expected (e.g. Alpert and Raiffa, 1982, find the truth is contained in respondents' 90% confidence intervals only about one-third of the time). These two types of overconfidence have been widely investigated in different fields.

A classic example of type 1 overconfidence is found in Svensson (1981), who asked individuals whether they were a better than average driver. Nearly all were. Camerer and Lovo (1999) found overconfidence in entry decisions in games of skill using an experimental approach. Ausubel (1991) found that credit card users were overconfident regarding their future ability to make payments.

Type 2 overconfidence has been widely explored in the finance literature (e.g., De Long et al. 1999, Kyle and Wang 1997, and Daniel et al. 1998). Further evidence of overconfidence has been found in a wide variety of studies targeting specific professional fields. This includes clinical psychologists in training (Oskamp 1965), lawyers (Wagenaar and Keren 1986), entrepreneurs (Cooper, Woo, and Dunkelberg 1988), managers (Russo and Schoemaker 1992), security analysts and economic forecasters (e.g., Staël von Holstein 1972, Ahler and Lakonishok 1983, Elton et al. 1984, Froot and Frankel 1989, De Bondt and Thaler 1990, and De Bondt 1991). For a complete overview of the overconfidence literature, see Odean (1997).

Given such broad evidence for both types of overconfidence, it is natural to ask whether there is any logical relation between the two types. First, there is no clear relation between the two types of overconfidence. We cannot induce one from the other. Actually, type-1 overconfidence relates to an upward biased first moment of a

probability distribution regarding one's own abilities, while type-2 overconfidence relates to a downward biased second central moment of a probability distribution forecast for some external event.

Second, type-2 overconfidence may simply be due an aggressive form of Bayesian updating and a lack of experience with the tails of a distribution. We discuss this extensively in the following section of the paper. Alternatively, type-1 overconfidence is hard to explain without appealing to some form of bounded rationality. We focus primarily on type-2 overconfidence. However, based on the results we obtain for type-2 overconfidence, we can make some inferences regarding the prevalence and impact of type-1 overconfidence.

Regarding the impact of overconfidence on production and competition, existing literature argues different conclusions with different approaches. Camerer and Lovo (1999) consider the hypothesis that business failure is a result of managers acting on optimism about their own relative skill. Using an experimental setting with basic features of business entry situation, they linked economic decisions to type-1 overconfidence. In the experiment, the success of entering subjects depends on their skill level relative to other entrants. Most subjects who enter think the total profit earned by all entrants will be negative, but their own profit will be positive. The findings are consistent with the prediction that overconfidence leads to excessive business entry.

Alternatively, Hvide (2000) showed with a game-theoretic model of job hunting that if agents form beliefs pragmatically, overconfidence can be the equilibrium outcome and further interpreted overconfidence as a way for the player to obtain a first mover advantage. In Hvide's model of job hunting, the take-it or leave-it offer made by the firm will in equilibrium depend partly on the worker's productivity in the firm, and partly on the agent's beliefs about his outside opportunity, which is commonly known between the firm and the worker. The model confirmed that if agents form beliefs pragmatically, then in equilibrium these beliefs will be inflated compared to the true distribution of the outside opportunity. Thus overconfidence may be the result of a game theoretic equilibrium.

Kyle and Wang (1997) investigate type-2 overconfidence in a financial context. Using a duopoly trade model of informed speculation, they showed that overconfidence may strictly dominate rationality. An overconfident trader may not only generate higher expected profit and utility than his rational opponent, but also higher than if he himself were rational. The implication behind this is that overconfidence can act like a commitment device in equilibrium, allowing them to credibly trade larger quantities. Further, they show that for some parameter values the Nash equilibrium of a two-fund game is a Prisoner's Dilemma in which both funds hire overconfident managers. Thus overconfidence can persist and survive in the long run.

3. Modeling Overconfidence

While individuals tend to underestimate the variance associated with any variable, their perceptions do adjust for changes in reality. Here, we represent overconfidence decrease in the variance of the distribution. Thus, individual beliefs regarding a variable, such as price, can be described by a simple single parameter.

Suppose an individual faces a gamble with wealth outcomes distributed with a two parameter probability density $f(s|\mu, \sigma^2)$, where s represents wealth outcome, μ and σ^2 are parameters representing the true mean and variance of the distribution. Then, we can represent an overconfident producer as perceiving the distribution $f(s|\mu, \sigma_g^2)$ given $\sigma_g^2 < \sigma^2$. Given that this is a two parameter distribution, these two parameters should completely determine the perceived distribution. If the distribution is uni-modal, reducing the variance will have the impact of decreasing the height of the distribution further from the mode and increasing the distribution closer to the mode. Thus we will refer to the overconfidence parameter σ_g^2 as the parameter of diminishing distance perception (DDP) – as it represents the degree to which the individual does not perceive outcomes the further one moves from the center of the distribution.

We assume that the objective of the firm is to maximize the expected utility of profits given the DDP parameter. Let the utility function of the firm's decision maker be a concave, continuous, and differentiable function of profits,

$$u'(\pi) > 0, u''(\pi) < 0 \quad (1)$$

The cost function of the firm is given by

$$F(x) = C(x) + B \quad (2)$$

where x is output, $C(x)$ is the variable cost function, with $C(0) = 0, C'(x) > 0$, and B

is the fixed cost. The firm's profit function is thus given by

$$\pi(x) = px - C(x) - B \quad (3)$$

where p is the price of output, assumed to be random with true density $f(p | \mu, \sigma^2)$.

The firm thus maximizes

$$E[u(px - C(x) - B) | \mu, \sigma_g] = \int_0^{\infty} u(px - C(x) - B) f(p | \mu_g, \sigma_g) dp \quad (4)$$

To proceed, we will use a Taylor-series approximation of the utility function,

$$u(\pi(x, p)) = u(\pi(x, \mu)) + u'(\pi(x, \mu)) \cdot x \cdot (p - \mu) + \frac{1}{2} u''(\pi(x, \mu)) \cdot x^2 \cdot (p - \mu)^2 \quad (5)$$

Thus, the maximization problem can be written as

$$\begin{aligned} \max_x E \left[u(\pi(x, \mu)) + u'(\pi(x, \mu)) \cdot x \cdot (p - \mu) + \frac{1}{2} u''(\pi(x, \mu)) \cdot x^2 \cdot (p - \mu)^2 \mid \mu_g, \sigma_g \right] \\ = u(\pi(x, \mu)) + u'(\pi(x, \mu)) \cdot x \cdot (\mu_g - \mu) + \frac{1}{2} u''(\pi(x, \mu)) \cdot x^2 \cdot \sigma_g^2. \end{aligned}$$

(6)

The first-order condition associated with (7) can be written as

$$\begin{aligned} \frac{\partial EU}{\partial x} = u'(\pi(x, \mu))(\mu - C'(x)) + u''(\pi(x, \mu))(\mu - C'(x)) \cdot x \cdot (\mu_g - \mu) \\ + u'(\pi(x, \mu))(\mu_g - \mu) + \frac{1}{2} u'''(\pi(x, \mu))(\mu - C'(x)) \cdot x^2 \cdot \sigma_g^2 + u''(\pi(x, \mu)) x \sigma_g^2 \end{aligned}$$

(7)

or, dividing by marginal utility

$$(\mu - C'(x)) + (\mu_g - \mu) - R_A \cdot [x \sigma_g^2 + (\mu - C'(x)) \cdot x \cdot (\mu_g - \mu)] + \frac{1}{2} P_A (\mu - C'(x)) \cdot x^2 \cdot \sigma_g^2 = 0$$

(8)

where $R_A = -\frac{u''}{u'}$ is the coefficient of absolute risk aversion and $P_A = \frac{u'''}{u'}$ is the coefficient of absolute prudence.

We will consider here the simple case where $f(p | \mu_g, \sigma_g^2)$ is a symmetric distribution. In this case, we can totally differentiate to derive the comparative static result

$$\frac{dx}{d\sigma_g^2} = \frac{-\left[\frac{1}{2} P_A (\mu - C'(x)) \cdot x^2 - R_A x\right]}{SOC} < 0 \quad (10)$$

To see this note that dividing (8) by the perceived variance yields

$$\frac{\mu - C'(x)}{\sigma_g^2} - R_A x + \frac{1}{2} P_A (\mu - C'(x)) \cdot x^2 = 0. \text{ The model implies risk aversion on average}$$

(note risk neutrality obtains if $\sigma_g^2 = 0$, and the decision maker acts as if he knows with certainty that the price will be μ), thus $\mu > C'(x)$. Therefore, the firm displaying DDP will use more inputs on average and produce more on average. We can couple this with Sandmo's result showing that the greater is R_A , the less will be produced to find the tension between DDP and risk aversion in behavior under uncertainty.

3.1 Competitive Equilibrium

We will now turn our attention to the implications of DDP for competitive equilibrium. According to the classical model of competition, firms will enter the market if they can make a profit by doing so. In our model, entry will occur if the firm perceives that they will earn expected utility greater than $u(0)$, the profit earned

prior to committing fixed costs. Further, a firm in the industry will shut down when $E(u(\pi)) < u(-B)$. Differentiating with respect to σ_g^2 obtains

$$\frac{\partial EU(\pi)}{\partial \sigma_g^2} = \frac{1}{2} u''(\pi(x, \mu)) \cdot x^2 < 0 \quad (10)$$

Thus, firms with greater DDP (smaller σ_g^2) will enter the market, while more rational firms that perceive correctly the risks they face would consider the expected profit too small considering the risk involved. This result further supports the result found by Camerer and Lovallo (1999) that overconfidence leads to greater rates of entry. However, here we show this in the case of type-2 overconfidence rather than type-1. Further, this result is well supported by the entrepreneurship literature (e.g., Das and Teng 1997; Barron 2000) which has uniformly found that entrepreneurs are not more inclined to take risks, but rather less inclined to take notice of the risks they face. Thus, as expected profit increases from zero, overconfident decision makers will be the first into the market and, as expected profits decline below zero, overconfident decision makers will be the last to shut down.

3.2 Imperfect Competition under Overconfidence

In order to evaluate the effects of DDP on competition, it is necessary to describe the market. Suppose the inverse demand is given by

$$P = P(X) + \varepsilon \quad (11),$$

with $P'(X) < 0$ and $\varepsilon \sim (0, \sigma^2)$, where $X = \sum_{i=1}^N x_i$ is the total production level in the market, i is the index of (potential) firms, and N is the number of firms producing.

Differentiating from perfect competition, here price is a function of the total market

production X and a random variable ε which captures the price shocks in the market.

Firms with identical risk-aversion but different levels of overconfidence simultaneously make production decision in the market, taking into account that each firm optimally chooses production level based on its own overconfidence level $\sigma_i^2 \leq \sigma^2$ and the total amount of production determines the expectation of the market price.

In equilibrium, the more overconfident the firm, the more it is going to produce and hence the greater the profit it obtains on average. The overconfident firms will also necessarily face greater variance in profits, having a higher probability of substantial success, and a higher probability of spectacular failure. To see this, note first that the standard equilibrium conditions dictate that

$$E(U_i(\pi) | P(X), \sigma_g) < U_i(0) \quad (12)$$

for all firms i that are not producing, and

$$E(U_i(\pi) | P(X), \sigma_g) > U_i(-B) \quad (13)$$

for all firms producing.

From the previous discussion (and from Sandmo's result), we can specify

$x_i^* = x(R_A, \sigma_i^2)$, and $\frac{\partial x_i}{\partial R_A} < 0$, $\frac{\partial x_i}{\partial \sigma_i^2} < 0$. We will represent perfectly rational (EU)

behavior as resulting from $\sigma_i^2 = \sigma^2$.

The profit function for each firm is given as

$$\Pi_i(X, P) = (P(X) + \varepsilon) \cdot x_i - C(x_i) - B \quad (14)$$

where B is fixed cost. Using a 2-dimensional Taylor expansion, along the production and price axis, for the utility function, we have

$$\begin{aligned}
u[\Pi(X, P)] &= u[\Pi(\bar{X}, P(\bar{X}))] \\
&+ u_1 [\Pi(\bar{X}, P(\bar{X}))] \cdot [P'(\bar{X}) \cdot \bar{x} + P(\bar{X}) - C'(\bar{x}) + \varepsilon] \cdot (x - \bar{x}) \\
&+ u_2 [\Pi(\bar{X}, P(\bar{X}))] \cdot \bar{x} \cdot (P - P(\bar{X})) \\
&+ \frac{1}{2} u_{11} [\Pi(\bar{X}, P(\bar{X}))] \cdot [P'(\bar{X}) \cdot \bar{x} + P(\bar{X}) - C'(\bar{x}) + \varepsilon]^2 (x - \bar{x})^2 \\
&+ \frac{1}{2} u_{22} [\Pi(\bar{X}, P(\bar{X}))] \bar{x}^2 (P - P(\bar{X}))^2 \\
&+ u_{12} [\Pi(\bar{X}, P(\bar{X}))] \cdot [P'(\bar{X}) \cdot \bar{x} + P(\bar{X}) - C'(\bar{x}) + \varepsilon] \cdot \bar{x} \cdot (x - \bar{x}) \cdot (P - P(\bar{X}))
\end{aligned}
\tag{15}$$

For simplicity, we omit the subscript i letting X denote the total amount of production in the market and x denote the production level of any arbitrary firm. The value \bar{x} is the risk neutral production level and $P(\bar{X})$ is the expectation of the market price at the risk neutral production level.

Thus, each firm i solves:

$$\begin{aligned}
\max_{x_i} E(u[\Pi(X, P)]) \\
&= u[\Pi(\bar{X}, P(\bar{X}))] \\
&+ u_1 [\Pi(\bar{X}, P(\bar{X}))] \cdot [P'(\bar{X}) \cdot \bar{x}_i + P(\bar{X}) - C'(\bar{x}_i)] \cdot (x_i - \bar{x}_i) \\
&+ \frac{1}{2} u_{11} [\Pi(\bar{X}, P(\bar{X}))] \cdot \left\{ [P'(\bar{X}) \cdot \bar{x}_i + P(\bar{X}) - C'(\bar{x}_i)]^2 + \sigma_i^2 \right\} \cdot (x_i - \bar{x}_i)^2 + \frac{1}{2} u_{22} [\Pi(\bar{X}, P(\bar{X}))] \bar{x}_i^2 \sigma_i^2 \\
&= u_1 \cdot [P'(\bar{X}) \cdot \bar{x}_i + P(\bar{X}) - C'(\bar{x}_i)] \cdot (x_i - \bar{x}_i) + \frac{1}{2} u_{11} \cdot \left\{ [P'(\bar{X}) \cdot \bar{x}_i + P(\bar{X}) - C'(\bar{x}_i)]^2 + \sigma_i^2 \right\} \cdot (x_i - \bar{x}_i)^2 + K
\end{aligned}
\tag{16}$$

Note that since we are dealing exclusively with Type-2 overconfidence, $E(\varepsilon) = 0$ and $E(P) = P(\bar{X})$. Thus the expectation for the third and the last terms of (15) are 0. Further, the first and the fifth term are merely constants, which we represent with K , yielding equation (16) above.

The first order condition with respect to x_i can be written as

$$u_i \cdot \left[P'(\bar{X}) \cdot \bar{x}_i + P(\bar{X}) - C'(x_i) \right] + u_{11} \cdot \left\{ \left[P'(\bar{X}) \cdot \bar{x}_i + P(\bar{X}) - C'(x_i) \right]^2 + \sigma_i^2 \right\} \cdot (x_i - \bar{x}_i) = 0 \quad (17)$$

Solving for the optimal output yields

$$x_i^* = \frac{1}{R_A} \cdot \frac{\left[P'(\bar{X}) \cdot \bar{x}_i + P(\bar{X}) - C'(x_i) \right]}{\left\{ \left[P'(\bar{X}) \cdot \bar{x}_i + P(\bar{X}) - C'(x_i) \right]^2 + \sigma_i^2 \right\}} + \bar{x}_i \quad (18)$$

The level of production x_i^* is a function of risk-aversion and the level of overconfidence, $x_i^* = x(R_A, \sigma_i^2)$, and $\frac{\partial x_i}{\partial R_A} < 0$, $\frac{\partial x_i}{\partial \sigma_i^2} < 0$ (19).

Expected profit given DDP can be written

$$E\Pi_i(X^*) = P(X^*) \cdot x_i^* - C(x_i^*) - B \quad (20)$$

and thus

$$\frac{\partial E\Pi_i}{\partial x_i^*} = P'(X^*) \cdot x_i^* + P(X^*) - C'(x_i^*) \geq 0 \quad (21).$$

Equation (21) follows from Sandmo (1971), i.e. production under uncertainty is always smaller than that under certainty. First order condition under certainty implies $P'(X) \cdot x_i + P(X) - C'(x_i) = 0$, and second order conditions requires that (21) will hold.

Next, the variance of profit given DDP is given by $Var(\Pi_i^*) = \sigma_i^2 \cdot (x_i^*)^2$, which is also an increasing function of x_i^* . Hence, in equilibrium, the more overconfident the firm, the greater the firm produces and the greater the profit it obtains on average. The overconfident firms will also necessarily face greater variance in profits, having a

higher probability of substantial success, and a higher probability of spectacular failure, as well.

Based on the above, we can derive the following proposition.

Proposition 1 Let $F \subseteq R^+ \times R^+$ be the set of potential firms, and $F_c \subseteq F$ the set of firms producing under competitive equilibrium. Then, for any (R_A, σ_g^2) , $R_A > 0$ with $(R_A, \sigma_g^2) \in F_c$, it must be the case that every firm with $(R_A, \sigma_{g'}^2) \in F_c$ where $\sigma_{g'}^2 < \sigma_g^2$.

Proof The result follows directly from (19), (20) and (21). Differentiating (20) with respect to DDP yields

$$\frac{\partial E\Pi_i^*}{\partial \sigma_i^2} = \frac{\partial E\Pi_i^*}{\partial x_i^*} \cdot \frac{\partial x_i^*}{\partial \sigma_i^2} \leq 0. \quad (22)$$

The result in the above proposition suggests that as long as each decision maker displays some level of risk aversion, at any level of risk aversion for which a rational actor produces, every actor with that level of risk aversion (or less) that meets some minimum level of misperception will operate. If all actors had identical levels of risk aversion, but varied by DDP, the market would necessarily be dominated by irrational actors. Rational actors would have a competitive disadvantage in being averse to risk, and recognizing the level of risk. Alternatively, those who could not see the risk would invest more heavily and drive more rational investors from the market.

3.3 *Ex Post Profits*

A possibly more interesting question is what will happen when those with misperceptions begin to realize their results. Equation (18) can be useful in exploring the answers to this question. Recall our definition for \bar{x} as the production level the

overconfident firm chooses when they think price will be realized at the expected level $E(P) = P(X)$ with certainty. If a firm is risk-neutral or if there is no uncertainty, this firm will choose \tilde{x} such that

$$P'(\tilde{X}) \cdot \tilde{x}_i + P(\tilde{X}) - C'(\tilde{x}_i) = 0, \text{ where } \tilde{X} = \sum_{i=1}^N \tilde{x}_i.$$

Since the firms are risk-averse and they also misperceive risk, given any guess of the production choice vector $\bar{x} = (\bar{x}_1, \bar{x}_2, \dots, \bar{x}_N)$, a firm's final choice x_i^* will be adjusted by a term which depends on its own risk-aversion and level of overconfidence corresponding to the first term of the right hand side of equation (18),

$$\frac{1}{R_A} \cdot \frac{[P'(\bar{X}) \cdot \bar{x}_i + P(\bar{X}) - C'(\bar{x}_i)]}{\left\{ [P'(\bar{X}) \cdot \bar{x}_i + P(\bar{X}) - C'(\bar{x}_i)]^2 + \sigma_i^2 \right\}}.$$

This term could be interpreted as *production shortfall*, which is the amount the firm produces below the risk neutral level \tilde{x} . This production shortfall is negatively related to risk-aversion R_A and DDP level σ_i^2 . This means the less risk-averse and/or the more overconfident, the more the final choice of production x_i^* approaches the risk neutral level \tilde{x} .

It is only when a firm behaves as if it is risk-neutral, that the adjustment term is equal to zero and what we guess for the production level is exactly what the firm may choose optimally, i.e. $x_i^* = \bar{x}_i = \tilde{x}_i$. A risk averse firm will behave as if risk neutral if the firm is so confident as to believe the world is certain, i.e. $\sigma_i^2 = 0$.

Alternatively, we can interpret the situation this way, given the individual is not risk neutral, the perceived variance that maximizes expected profit is $\sigma_i^2 = 0$, or that resulting from the most overconfident (least rational) DDP. Thus, we can conclude

again the more overconfident (or loss averse) the firm, the greater the profits obtained on average. This will necessarily result in a lower ex post expected utility, as the individual has taken on more risk than they would prefer. Nonetheless their average profit will be higher.

Alternatively, the ex post variance of profit is given by $\sigma^2 \cdot (x_i)^2$. Thus, firms displaying overconfidence, which invest more heavily when operating, will necessarily face greater variance in profits due to a higher level of production. This implies, under a symmetric distribution, both a higher probability of substantial success, and a higher probability of failure. Finally, the skewness of profits is given by $\tau_i x_i^3$, where τ_i is the skewness of price. Thus, overconfidence (as we have defined it) will not alter the perceived direction of skew in the profit distribution, but can substantially increase the skewness through increased investment. This potentially increases upside risk in the profit distribution over those with true perceptions of the price distribution.

So far, we assume that overconfidence doesn't change the first moment of the distribution. If the overconfidence alters the perceived mean, only a few of the preceding results differ. If overconfidence increases the mean, it will reinforce the results of reducing the variance, so long as it does not lead the firm to produce more than the risk-neutral level of production. Firms begin to be at a competitive disadvantage once they produce more than the risk-neutral amount. Alternatively, if decision makers perceive a mean price that is below the true mean, this perception

will work against the reduction in perceived variance, reducing the amount produced, and placing the firm at a competitive disadvantage.

4. Welfare Analysis

Finally, one may wonder about the welfare effects of overconfidence. This is easiest to consider by comparing equilibria consisting of identical actors. Clearly, because overconfidence leads to greater production for all levels of expected prices, consumers must benefit from the resultant lower equilibrium price. On the other hand, producers necessarily obtain lower utility of profit on average than they anticipate, meaning they could be made better off. The ex post producer surplus must disregard overconfidence, calculating the true average net benefit. This necessarily declines as variance is misperceived at a rate determined by the degree of risk aversion. If actors were truly risk neutral, misperceptions of variance would not matter to producers. Alternatively, if producers are very risk averse, misperceptions of variance could reduce producer surplus by more than the increase in consumer surplus leading to a market failure. Thus, if firms are only mildly risk averse, there may exist some socially optimal level of overconfidence. On the other hand, if firms were severely risk averse, the government may have a role in reducing overconfidence (through education, market publications, etc.) or reducing risk (through disaster relief) to improve welfare of producers.

From the previous section we have

$$\Pi_i(X, P) = (P(X) + \varepsilon) \cdot x_i - C(X) - B, \text{ where } X = \sum_{i=1}^N x_i \text{ and } \varepsilon \sim (0, \sigma^2).$$

Define $\Pi_i^* = E\Pi_i(X^*) = P(X^*) \cdot x_i^* - C(x_i^*) - B$ to be the expected profit at production level x_i^* . We can use the certainty equivalence, CE , to represent producer welfare. Note that for a risk averse (neutral or loving) producer, CE is smaller than (equal to or larger than) the expectation of the profit Π_i^* given accurate perception. We define the ex ante certainty equivalent as $EU(\Pi(x^*(\sigma_i^2)) | \sigma_i^2) = U(CE^{ex\,ante})$, and the ex post certainty equivalent as $EU(\Pi(x^*(\sigma_i^2)) | \sigma^2) = U(CE^{ex\,post})$. Thus the anticipated certainty equivalent, or ex ante CE, is affected by overconfidence through both the selected production level and the misperception of the distribution. Alternatively, the ex post certainty equivalent, or realized certainty equivalent, is impacted by overconfidence only through the choice of production level. For a risk averse producer, misperception of risk affects the ex ante CE in a way such that the more overconfident the firm, the larger the CE it anticipates.

We want to solve for a threshold where the ex post CE of the overconfident firm could be larger than those with rational perception of risk, thus, implying together with the increased consumer surplus, an increase in total welfare.

By the definition of the certainty equivalent, we have

$$u(CE_i) = Eu[\Pi_i(X^*)]$$

Using the Taylor-expansion at Π_i^* for both sides and solving, we have

$$\begin{aligned} CE_i^{ex\,post} &= \frac{1}{2} \frac{u''(\Pi_i^*)}{u'(\Pi_i^*)} \cdot (x_i^*)^2 \cdot \sigma^2 + \Pi_i^* \\ &= -\frac{1}{2} R_{A_i} \cdot (x_i^*)^2 \cdot \sigma^2 + \Pi_i^* \end{aligned} \quad (23)$$

Three points can be made using equation (23). First, for a risk-averse producer, the ex post CE is smaller than his expected profit Π_i^* and the difference $-\frac{1}{2}R_{A_i} \cdot (x_i^*)^2 \cdot \sigma^2$ is determined by the absolute risk-aversion R_{A_i} (evaluated at Π_i^*), the level of the actual production (a function of risk aversion and overconfidence level) and the real risk σ^2 . The level of production, x_i^* influences CE by increasing the variance of profit and thus, decreasing CE of a risk-averse producer. Thus, in every case, greater overconfidence will decrease ex post welfare of the producer.

Second, if the producer displays overconfidence with $\sigma_i^2 \leq \sigma^2$, the ex post CE is always smaller than their own ex ante level

$$CE_i^{ex\ post} = -\frac{1}{2}R_{A_i} \cdot (x_i^*)^2 \cdot \sigma^2 + \Pi_i^* \leq -\frac{1}{2}R_{A_i} \cdot (x_i^*)^2 \cdot \sigma_i^2 + \Pi_i^* = CE_i^{ex\ ante}.$$

Third, this producer may not necessarily be worse off when compared with his rational counterparts because we proved in the previous part that both $E\Pi_i^*$ and x_i^* are greater than their counterparts under rational perception. And thus, the final welfare status may be highly variable though sub optimal from a policy perspective.

To see this, let firm j be the less overconfident producer (or to the extreme, be the perfectly rational producer), so that we have $\sigma_i^2 \leq \sigma_j^2$, $x_i^* \geq x_j^*$ and $E\Pi_i^* \geq E\Pi_j^*$.

Thus, for the overconfident firm being better off, we need $CE_i^{ex\ post} \geq CE_j^{ex\ post}$.

That is,

$$CE_i^{ex\ post} = -\frac{1}{2}R_{A_i} \cdot (x_i^*)^2 \cdot \sigma^2 + \Pi_i^* \geq -\frac{1}{2}R_{A_j} \cdot (x_j^*)^2 \cdot \sigma^2 + \Pi_j^* = CE_j^{ex\ post}$$

(24)

Supposing both firms display the same level of risk aversion, we can solve for:

$$R_A \leq \frac{2 \cdot (\Pi_i^* - \Pi_j^*)}{\sigma^2 \cdot [(x_i^*)^2 - (x_j^*)^2]} \quad (25)$$

Equation (25) tells us that overconfidence may make producers with modest risk-aversion being better off, whereas making those with severe risk-aversion worse off. The threshold between those with severe or less severe risk aversion is given by the right hand side of (25).

Furthermore, the threshold depends on both the real risk σ^2 and the relative difference of the expected profits (scaled by the difference of their production levels) between two (types) of firms $\frac{(\Pi_i^* - \Pi_j^*)}{[(x_i^*)^2 - (x_j^*)^2]}$. On the one hand, if the real risk σ^2 is relatively small, the threshold will be moved to the right, meaning a larger tolerance of risk-aversion. On the other hand, if the difference between the expected profits is very small, a firm displaying overconfidence could be better off only when it maintains a very low level of risk-aversion. If we have constant marginal cost, then the right hand side of (25) can be written as

$$\frac{2(\Pi_i^* - \Pi_j^*)}{\sigma^2 [(x_i^*)^2 - (x_j^*)^2]} = \frac{2(P(X^*) - MC) \cdot (x_i^* - x_j^*)}{\sigma^2 (x_i^* + x_j^*) \cdot (x_i^* - x_j^*)} = \frac{2(P(X^*) - MC)}{\sigma^2 (x_i^* + x_j^*)}. \text{ In this case, the}$$

threshold is increased as individuals behave more risk averse (so that price exceeds marginal cost) and production levels are low.

From a market evolution perspective, at the very beginning, when overconfident firms are competing with perfectly rational firms, the difference between profits could be high, thus, even firms with larger risk-aversion could survive, as long as they display some level of overconfidence. But as rational producers are driven away from

the market, the gap between profits shrinks, since all firms are now overconfident, and they differ only by the levels of their misperception. And hence, firm with a low level of risk-aversion is better off. Or, for a certain level of risk-aversion, a firm can only survive the market by displaying even greater level of overconfidence and performing as if he is as close to risk-neutral as possible. This may explain how overconfidence could persist in the long run.

5. Model Impacts of Overconfidence on Production

Previous work has shown that overconfidence may persist in financial trading, as well as cause entry. Our results suggest that overconfidence may be a natural result of market pressures, and may thus persist in a competitive production market. While all would agree that starting a new business is an extremely risky venture, there is little evidence that entrepreneurs are more risk tolerant than other individuals (Palich and Bagby (1995)). In fact, Low and MacMillan (1988) find specifically that propensity to take on risk does not differentiate entrepreneurs from nonentrepreneurs. Rather, many have discovered that entrepreneurs differ in the process by which they evaluate opportunities and assess the risks involved (Das and Teng (1997), Cooper, Woo, and Dunkelberg (1988), Forlani and Mullins (2000)). For example, Baron (2000) finds that entrepreneurs are less likely to engage in counterfactual thinking, not recognizing the possibility for alternative outcomes of their venture.

Many have found an empirical link between the underestimation of risk and entrepreneurship activity (see, for example, Simon, Houghton, and Aquino (2000)). Camerer and Lovallo (1999) use economic theory to argue that such overconfidence

should lead to excess entrepreneurial activity. Despite an increasingly evident link between overconfidence and entrepreneurship, little is known of the effects of overconfidence on business performance under competition. In this paper, we follow the analysis of Sandmo (1971), applying the principles of DDP and EU maximization to examine the effects of competition on firms displaying overconfidence. We show that overconfidence may not only lead to excessive entry, but also give entrepreneurs a competitive edge not achieved by more rational decision makers. Our result may also explain the recent results of Bogan and Just (forthcoming) suggesting that CEOs are display greater confirmation bias and overconfidence than other populations. They show that this may be behind excessive merger activity. Overconfidence can create a competitive advantage in production decisions. But the same behavioral anomaly that makes a CEO desirable for competitive production may make them the wrong person for the job when it comes to merger decisions.

6. Conclusion

While many have published proofs that competition forces rationality (see, for example, Green (1987)), this paper provides a rationale for why non-rational models may be relevant even in highly competitive industries. In fact, it seems clear that DDP, while irrational, creates a competitive advantage, and thus markets may be dominated by this particular brand of irrationality. The fact that competition may encourage such behavior in the face of risk aversion makes it a little more understandable why such behavior may pop up in experimental settings. Further, empirical assessments in the entrepreneurship literature suggest that behavioral phenomena such as DDP may play

a larger role in entry decisions than factors like DMUW that are more commonly considered. There is little reason to believe that competition will sort DDP from the market, and thus DDP may also play a large role in production level decisions and exit from a competitive industry. The work in this paper provides a neoclassical economic argument for why this patently non-classical phenomenon should exist and persist and why behavioral effects may be important. Those who underestimate risk are likely to invest more, increasing their chances for greater success (or failure) than can be realized with a rational view of the world.

References

- [1] D. Ahler and J. Lakonishok, A study of economists' consensus forecasts, *Management Science*, 29 (1983), 1113-1125.
- [2] A.M. Allais, Le comportement de l'homme rationnel devant le risque, critique des postulats et axiomes de l'école Américaine, *Econometrica* 21 (1953), 503–546.
- [3] M. Alpert, H. Raiffa, A progress report on the training of probability assessors, in: D. Kahneman, P. Slovic, A. Tversky (Eds.), *Judgment Bias under Uncertainty: Heuristics and Biases*, Cambridge University Press, New York, 1982.
- [4] K. Arrow, *Essays on the Theory of Risk Bearing*. Markham Press, New York, 1971.
- [5] L. M. Ausubel, The failure of competition in the credit card market, *American Economic Review* 81 (1991), 50-81.
- [6] R.A. Baron, Counterfactual thinking and venture formation: The potential effects of thinking about 'what might have been,' *J. Bus. Venturing* 15 (2000), 79–91.

- [7] C. Camerer, D. Lovallo, Overconfidence and excess entry: An experimental approach, *Amer. Econ. Rev.* 89 (1999), 306–318.
- [8] S.H. Chew, A generalization of the quasilinear mean with applications to the measurement of income inequality and decision theory resolving the Allais Paradox, *Econometrica* 51 (1983), 1065–1092.
- [9] S.H. Chew, W. Waller, Empirical tests of weighted utility theory, *J. Math. Psych.* 30 (1986), 55–72.
- [10] A.C. Cooper, C.Y. Woo, W.C. Dunkelberg, Entrepreneurs' perceived chance of success, *J. Bus. Venturing* 3 (1988), 97–108.
- [11] T. K. Das, B.S. Teng, Time and entrepreneurial risk behavior, *Entrepreneurship Theory and Practice* 22 (1997), 69–88.
- [12] W. F. M. De Bondt and R. H. Thaler, Do security analysts overact? *American Economic Review* 80 (1990), 52-57.
- [13] W. F. M. De Bondt, What do economists know about the stock market? *Journal of Portfolio Management* 17 (1991), 84-91.
- [14] E. J. Elton, M. J. Gruber and M. N. Gültekin, Professional expectations: Accuracy and diagnosis of errors, *Journal of Financial and Quantitative Analysis* 19 (1984), 351-365.
- [15] G. Feder, The impact of uncertainty in a class of objective functions, *J. Econ. Theory* 16 (1977), 504–512.
- [16] D. Forlani, J.W. Mullins, Perceived risks and choices in entrepreneurs' new venture decisions, *J. Bus. Venturing* 5 (2000), 305–322.

- [17] K. A. Froot and J. A. Frankel, Forward discount bias: Is it an exchange risk premium? *Quarterly Journal of Economics* 104 (1989), 139-161.
- [18] J. Green, Making book against oneself, the independence axiom, and nonlinear utility theory, *Quart., J. Econ.* 102 (1987), 785–796.
- [19] J.D. Hey, C. Orme, Investigating generalizations of expected utility theory using experimental data, *Econometrica* 62 (1994), 1291–1326.
- [20] H.K. Hvide, Pragmatic beliefs and overconfidence, *Journal of Economic Behavior and Organization* 48 (2002), 15–28.
- [21] D. Kahneman, A. Tversky, Prospect theory: An analysis of decision under risk, *Econometrica* 47 (1979), 263–292.
- [22] A. Kyle, F. A. Wang, Speculation duopoly with Agreement to Disagree: Can overconfidence survive the market test? *Journal of Finance* 52 (1997), 2073-2089.
- [23] M.B. Low, I.C. MacMillan, Entrepreneurship: Past research and future challenges, *Journal of Management.* 14 (1988), 139–161.
- [24] W. Nielson, Calibration results for rank-dependent expected utility, *Econ. Bulletin* 4 (2001), 1–5.
- [25] T. O’Dean, Volume, volatility, price and profit when all traders are above average, working paper No. RPF-266 (1997), University of California at Berkeley..
- [26] T. O’Dean, B.M. Barber, Boys will be boys: Gender, overconfidence, and common stock investment, *Quart. J. Econ.* 115 (2001), 261–283.

- [27] S. Oskamp, Overconfidence in case-study judgments, *J. Consulting Psych.* 29 (1965), 261–265.
- [28] L.E. Palich, D.R. Bagby, Using cognitive theory to explain entrepreneurial risk-taking: Challenging conventional wisdom, *J. Bus. Venturing* 10 (1995), 425–438.
- [29] J.W. Pratt, Risk aversion in the small and in the large, *Econometrica* 32 (1964), 122–136.
- [30] M. Rabin, Risk aversion and expected-utility theory: A calibration theorem, *Econometrica* 68 (2000), 1281–1292.
- [31] J. Russo and P. Schoemaker, Managing overconfidence, *Sloan Management Review*, 33 (1992), 7-17.
- [32] A. Sandmo, On the theory of the competitive firm under price uncertainty, *Amer. Econ. Rev.* 61 (1971), 65–73.
- [33] M. Simon, S. M. Houghton, K. Aquino, Cognitive biases, risk perception and venture formation: How individuals decide to start companies, *J. of Bus. Venturing* 15 (2000), 113–134.
- [34] C. Staël von Holstein, Probabilistic forecasting: An experiment related to the stock market, *Organizational Behavior and Human Performance*, 8 (1972), 139-158.
- [35] C. Starmer, Developments in non-expected utility theory: The hunt for a descriptive theory of choice under risk, *J. of Econ. Lit.* 38 (2000), 332–382.

- [36] O. Svenson, Are we all less risky and more skillful than our fellow drivers?, *Acta Pshchologica*, 47 (1981), 143-148.
- [37] S. Taylor and J. Brown, Illusion and well-being: A social psychological perspective on mental health, *Psychological Bulletin*, 103 (1988), 193-210.
- [38] A. Tversky, D. Kahneman, Advances in prospect theory: Cumulative representation of uncertainty, *J. of Risk and Uncertainty* 5 (1992), 297–323.
- [39] A. Tversky, D. Kahneman, Judgment under uncertainty: Heuristics and biases, *Science* 185 (1974), 1124-1131.
- [40] W.K. Viscusi, Prospective reference theory: Toward an explanation of the paradoxes, *J. of Risk and Uncertainty* 2 (1989), 235–264.
- [41] W. Wagenaar and G. Keren, Does the expert know? The reliability of predictions and confidence ratings of experts, In eds. E. Hollnagel, G. Maneini, and D. Woods, *Intelligent Decision Support in Process Environments*, Berlin: Springer, 87-107.
- [42] P. Wakker, D. Deneffe, Eliciting von Neumann-Morgenstern utilities when probabilities are distorted or unknown, *Manage. Science* 42 (1996), 1131-1150.
- [43] N. Weinstein, Unrealistic optimism about future life events, *Journal of Personality and Social Psychology*, 49 (1980), 806-820.