Simulating the Effects of Supply and Demand Elasticities on Political-Economic Equilibrium

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* Prepared for the World Bank's Conference on the Political Economy of Distortions to Agricultural Incentives, 23-24 May 2008. While I was writing this paper, two giants of the agricultural economics profession, Bruce Gardner and Ed Schuh, passed away. I am grieved by their loss. They were gentlemen, scholars, and mentors to us all. They made important contributions to the subject matter of this article, as well.

Simulating the Effects of Supply and Demand Elasticities on Political-Economic Equilibrium

Wallace's (1962) pioneering study brought to agricultural economics a focus on the role of supply and demand elasticities in determining the efficiency with which agricultural policy transfers income among interest groups. His discussion provides key insights into what I call the "consequences of policy" side of current models of political economy. Becker's (1983) and Grossman and Helpman's (1994) models, both famously seminal in the political economy literature, have such "consequences" sides, and therefore when agricultural policy is examined in their models' frameworks, market elasticities play key roles. These two models have also "causes of policy" sides, which describe interest groups' abilities to create political pressure (by controlling members' free-riding, etc.). My aim is to provide a theoretical review and exposition of the effects of elasticities on redistributional efficiency, and in turn on distortional policies in political-economic equilibrium. I demonstrate how models of actual political economies might be built and used to derive theoretical predictions about how market elasticities affect policy and income transfers. Applying similar models to examine real-world political economies can provide testable hypotheses about how market parameters affect distortional policies.

The Literature on the Political-economic Effects of Redistributive Efficiency

Becker made interesting and provocative claims about his model's results:

> PROPOSITION 2. An increase in deadweight cost reduces the equilibrium subsidy. (p. 381)
>
> CORROLARY. Political policies that raise efficiency are more likely to be adopted than policies that lower efficiency. (p. 384)
>
> … in the political sector … investments in human or physical capital specific to a firm, industry, or even region reduce the short-run elasticity of supply, and the deadweight costs of "distortions" are lower when supply (and demand) is less elastic. (p. 383)

Grossman and Helpman (G&H) made related statements:

> … All else equal, industries with higher import demand or export supply elasticities (in absolute value) will have smaller ad valorem deviations from free trade. This is true for two reasons. First, the government may bear a political cost for creating deadweight loss… To the extent that this is so, all else equal, it will prefer to raise contributions from sectors where the cost is small. Second, the members of lobbies as a group will share in any deadweight loss that results from trade policy. (p. 842)

I analyze and critique these claims, and suggest how models of political economies might be developed to empirically test the above hypotheses about the policies' underlying causes.

An "Economy" for Two Models of Political Economy

Following the basics of Gardner (1983), I develop a simple economic model of typical agricultural policies, used later as part of two models of political economy. Assume an economy with two goods: agricultural good, and composite good numeraire q_2. There are two groups of economic agents: agricultural producers ("farmers") and consumers-taxpayers. The consumer price of the agricultural good is p^d. The producer price is p^s. Markets are competitive. $D(p^d, \mathbf{b})$ is the demand function for the agricultural good is:

(1) $q^d = \underbrace{Max\{0, a_0 + a_1 p^d\}}_{D(p^d, \mathbf{b})}$, defined for $p^d \geq 0$, where $a_0 > 0$, $a_1 < 0$.[i]

The demand curve is illustrated in figure 1 for particular parameter values a_0' and a_1'.

Farmers' supply function $S^*(p^s, \mathbf{b})$ is,

(2) $q^s = \underbrace{Max\{0, Min\{b_0 + b_1 p^s, q^H\}\}}_{S^*(p^s, \mathbf{b})}$, where $b_0 < 0$, $b_1 > 0$.

The supply function is illustrated in figure 1, with parameter values b_0' and b_1' in the upper panel and b_0'' and b_1'' in the lower.

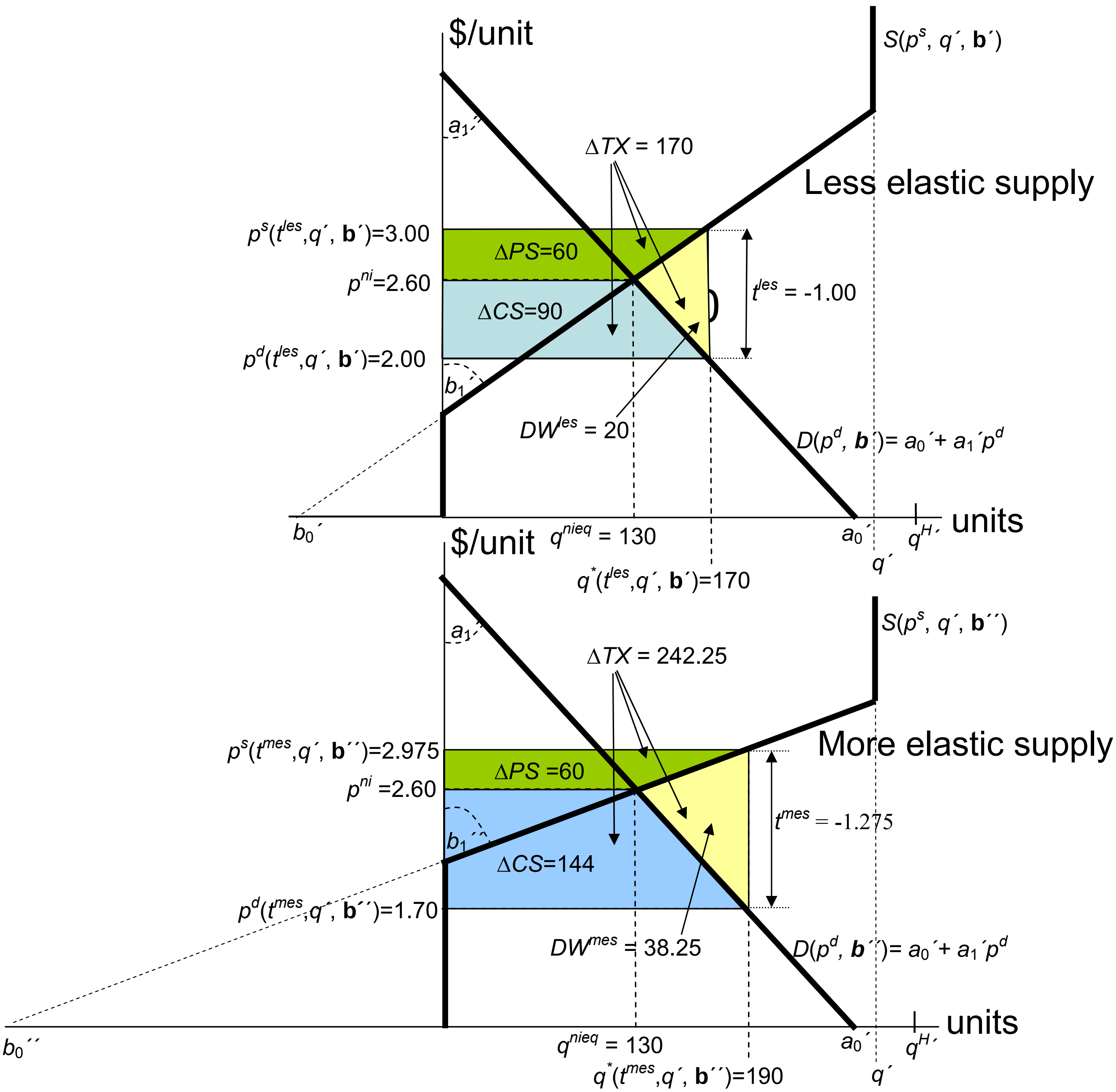

Figure 1. A higher supply elasticity decreases a production subsidy's redistributive efficiency.

I close the economic model by assuming that markets must clear in economic equilibrium, and that the price that demanders pay equals the price that suppliers receive:

(3) $q^s = q^d$,

(4) $p^s = p^d$.

Equations (1)-(4) define economic equilibrium when there is no government intervention. From these we can find the equilibrium non-intervention price and quantity as functions of market parameters: $p^{nieq}(\mathbf{b}) = (a_0\text{-}b_0)/(b_1\text{-}a_1)$, and $q^{nieq}(\mathbf{b}) = b_0 + b_1 p^{ni}(\mathbf{b})$.

The "Policy Consequences" Side of Two Models of Political Economy

I define political-economic equilibrium in terms of market parameters, political parameters, and government policy.

Economic Equilibrium under an Arbitrary Government Strategy

Both Becker's and G&H's models are game-theoretical models of interest group competition for government transfers. An interest group in my models is government, indexed by 0. Government has two strategy variables: a per-unit production tax (or subsidy if negative) t, and a production quota q. Government chooses t from $[t^{vn}, t^{ch}]$, where $t^{ch} = b_0/b_1 - a_0/a_1 > 0$ is the "choke tax," high enough to drive the quantity consumed and produced to zero; t^{vn} is some very negative number representing a subsidy larger than the government would ever set in political-economic equilibrium.

Generalizing (2) and (4), the quota's impact on supply is shown in (5). (When no quota is used, $q = q^H$, which makes (5) identical to (2).)

(5) $$q^s = \underbrace{Min\left\{q, \underbrace{Max\{0, Min\{b_0 + b_1 p^s, q^H\}\}}_{S^*(p^s,\mathbf{b})}\right\}}_{S(p^s,q,\mathbf{b})}.$$

From (4), the equilibrium producer price is the consumer price less the tax/subsidy:

(6) $p^s = p^d - t$.

When government does not use the tax/subsidy, (4) and (6) are the same condition. For an arbitrary value of the vector of market parameters $\mathbf{b} = (a_0, a_1\ b_0, b_1\ q^H)$ and of government policy (t, q), economic equilibrium is determined by (1), (3), (5), and (6), which can be solved for prices and quantities in economic equilibrium, in terms of policy variables t and q, and market parameters $\mathbf{b}$. Call these solutions $p^s(t, q, \mathbf{b})$, $p^d(t, q, \mathbf{b})$,

$q^s(t, q, \mathbf{b})$, and $q^d(t, q, \mathbf{b})$. Since markets clear, re-label $q^s(t, q, \mathbf{b})$, and $q^d(t, q, \mathbf{b})$, calling them both $q^*(t, q, \mathbf{b})$. In the upper panel of figure 1, the market parameter vector takes a particular value $\mathbf{b}' = (a_0', a_1', b_0', b_1', q^{H\prime})$, and policy levels are $t^{les} < 0$ and $q' < q^{H\prime}$. Resulting equilibrium prices and quantity are $p^s(t^{les}, q', \mathbf{b}')$, $p^d(t^{les}, q', \mathbf{b}')$, and $q^*(t^{les}, q', \mathbf{b}')$.

Government's Redistribution of Interest Group Welfare under a Production Subsidy

I use consumer surplus plus production tax/subsidy revenues (negative if a subsidy) to measure group 1 welfare. In equilibrium, u_1 depends on policy and market parameters:

$$(7)\quad u_1 = \underbrace{u_1^{ex} + \int_{p^d(t,q,\mathbf{b})}^{\infty} D(z,\mathbf{b})dz + t \cdot S\big(p^s(t,q,\mathbf{b}),q,\mathbf{b}\big)}_{u_1(t,q,\mathbf{b})},$$

where $u_1^{ex} > 0$ is an exogenous utility level and z is a dummy variable of integration.

I use producer surplus to measure the welfare of farmers:

$$(8)\quad u_2 = \underbrace{u_2^{ex} + \int_{0}^{p^s(t,q,\mathbf{b})} S(z,q,\mathbf{b})dz}_{u_2(t,q,\mathbf{b})},$$

where $u_2^{ex} > 0$ is exogenous utility and z is a dummy variable of integration. The vector of non-governmental interest group welfare functions in economic equilibrium is $\mathbf{u}(t, q, \mathbf{b}) = (u_1(t, q, \mathbf{b}), u_2(t, q, \mathbf{b}))$.

Effects of Supply and Demand Elasticities on Redistributive Efficiency

Following Wallace (1962) and Gardner (1983, 1987), I show how changes in market elasticities affect a production subsidy's redistributive efficiency. This framework will aid the analysis of the effects of elasticity changes on political-economic equilibrium.

An Increased Supply Elasticity Decreases the Efficiency of a Production Subsidy

The supply curve in the top panel has an elasticity of 2.0 around the point $(q^{nieq}, p^{ni}) = (130, 2.60)$; the bottom panel's supply curve has elasticity 3.2 around that point. In the case of less elastic supply, the policy shown is $(t^{les}, q^{ni}) = (-1.00, q^H)$, where q^H is some non-binding quota. In the top panel this policy leads to a \$3 supply price, a \$2 demand price, and 170 units produced. Producer surplus rises by ΔPS = \$60, consumer surplus rises by ΔCS = \$90, and taxes rise by ΔTX = \$170. Deadweight loss is DW = -(ΔPS + ΔCS - ΔTX) = \$20. Given the more elastic supply in the lower panel, to transfer \$60 to producers as was done in the top panel requires a production subsidy of $1.275, resulting in a $2.975 producer price and a $1.70 demand price. Consumer surplus rises by $144, but taxes rise by $242.25. Deadweight is $38.25. Thus, with more elastic supply, for any given-sized transfer to producers the subsidy instrument becomes less efficient.

The intuition behind the figure 1's results is straightforward. Deadweight results when a wedge is driven between the marginal benefit of consumption and marginal production cost. A production subsidy lures into the sector resources more valuable outside the sector. As resources enter, their marginal opportunity cost increases to the new price level. The more elastic is supply, the easier resources enter. Since the demand curve slopes downward, the greater production leads to a bigger distortional wedge, which must grow because the consumer price must drop to increase consumption.

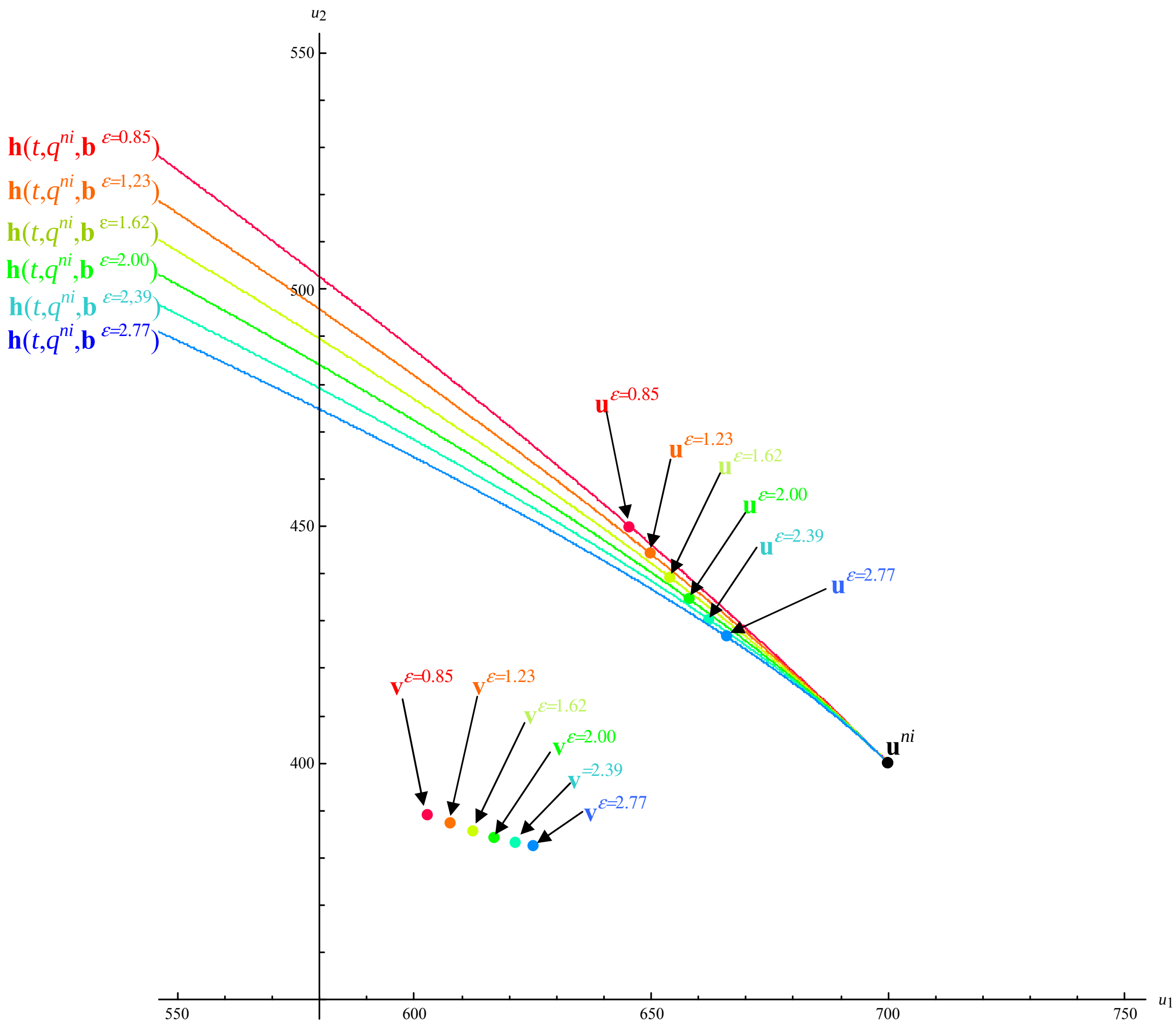

Figure 2. An increased supply elasticity decreases the redistributive efficiency of a production subsidy and decreases the equilibrium transfer to producers in the Becker model, as shown in welfare-space. (A similar diagram from the G&H model appears in the on-line appendix.)

Redistributive efficiency can also be illustrated in welfare-space, with u_1 and u_2 on the axes, as in figure 2, which reflects elements of Gardner (1983), Bullock (1994, 1995, 1996), and Bullock and Salhofer (2003). The curves are surplus transformation curves, introduced by Josling (1974). For supply elasticities of $\varepsilon' = 0.85$, 1.23, 1.62, 2.00, 2.39, and 2.77, surplus transformation curves labeled $\mathbf{h}(t,q^{ni},\mathbf{b}^{\varepsilon=\varepsilon'})$ show the welfare outcomes from a continuum of subsidy levels, assuming that the production quota is not employed, that parameters b_0 and b_1 are set at levels leading to a supply elasticity of ε', and that a_0 and a_1 are set at their baseline levels, implying a demand elasticity of -4/3. For any $\varepsilon'' > \varepsilon'$, and for any point on $\mathbf{h}(t,q^{ni},\mathbf{b}^{\varepsilon=\varepsilon''})$, there exists a Pareto superior point on $\mathbf{h}(t,q^{ni},\mathbf{b}^{\varepsilon=\varepsilon'})$. That is, a lower supply elasticity increases the efficiency of the subsidy instrument.

More Elastic Demand Increases the Efficiency of a Production Subsidy

More elastic demand raises the redistributive efficiency of a subsidy. (See the appendix for a detailed discussion). The subsidy increases production, which must be consumed. The more elastic is demand, the more readily consumers substitute the good for the other. Thus the marginal benefit of consuming the good does not change greatly as more of the good is consumed, and the wedge between marginal cost and marginal benefit remains small.

Intuitive Discussion

The results above suggest an explanation of why we might see agricultural price supports more often than price supports for industrial goods. Land limitations inherently cause the supply elasticity of agricultural goods as a whole to be low. But for certain industrial products—pencils, for example—there are few limiting factors. Because new firms can enter to copy the practices of existing firms, then the production technology has nearly constant returns to scale in the relevant neighborhood. Supporting the price of a pencil would lead to far greater pencil production. For most people, the marginal value of owning yet another pencil is low once they already have some number of pencils. So a pencil price support would lead to a large wedge between marginal costs and marginal benefits of pencils, and thus a large deadweight loss. But agriculture is special because land is a prominent limiting factor of production. This is a possible explanation of why subsidies tend to be provided for crops for which it is difficult to expand acreage.

The "Policy Causes" Sides of Two Models of Political Economy

Next I present two political-economy models, one like Becker's (1983), the other like G&H's (1994). In both models, consumers-taxpayers have one political strategy variable, political expenditures x_1. They use x_1 to influence government's choices.[ii] Assume that it is technically feasible for group 1 to make political expenditures within $X_1 = [0,\ x_1^H]$. Similarly, producers have $X_2 = [0,\ x_2^H]$.

A Becker Model

Becker presents a one-shot, non-cooperative Nash game in political expenditures. "Government" in Becker's model is not quite an economic agent, but a type of "machine" with no real independent will or objective. Government policies are determined by the "political pressures" of non-governmental interest groups, which in turn depend on their strategies.[iii] This dependence is characterized by function $I(x_1, x_2, \mathbf{k})$, where $\mathbf{k}$ is a vector of influence parameters, to be discussed later.[iv] I assume the following functional form:

$$I(x_1,x_2,\mathbf{k}) = k_0 + k_1x_1 + k_2x_2 + k_{11}[x_1]^2 + k_{22}[x_2]^2 + k_{12}x_1x_2.$$

When I is positive, government transfers utility from group 1 to group 2 and vice-versa when I is negative. In the baseline version of the Becker model, I set $\mathbf{k}$ = (0, -1.7, 2.5, 0.01, -0.01, -0.0025), which, letting subscripts denote partial derivatives, results in $I_1 > 0$, $I_{11} < 0$, $I_2 < 0$, and $I_{22} > 0$, implying satisfaction of the second-order conditions in Becker's (1983) appendix,[v] and leading to an a transfer group 1 to group 2.

I can conveniently and without loss of generality define the "size of government," Z, as group 1's utility loss (negative if group 1 gains) due to the government's policy:

(9) $$Z = \underbrace{u_1^{ni}(\mathbf{b}) - u_1(t,q,\mathbf{b})}_{Z(t,q,\mathbf{b})}.$$

With the influence function in (9) and the government-size function in (10), now I can link the economic sub-model with the political sub-model. This is accomplished by assuming that in political economic equilibrium *the size of the government that is called for by politics must equal the size of the government that is brought about by policy*:

(10) $$I = Z.$$

To apply Becker's political model to an economic model of agricultural policy, I assume that government can only use the tax/subsidy. So in (1) and (5)-(11), q is replaced by q^{ni}. This forms a system of eight equations with ten unknowns: p^s, p^d, q^s, q^d,

u^1, t, Z, I, x_1, and x_2. Under the conditions of the implicit function theorem, in principal the first eight variables can be solved in terms of x_1, x_2, and (**b**, **k**). For f being p^s, p^d, q^s, q^d, u_1, t, Z, and I, denote these solutions $f^\#(x_1, x_2, \mathbf{b}, \mathbf{k})$. It can be shown that,

(11) $$t^\#(x_1,x_2,\mathbf{b},\mathbf{k}) \equiv \frac{-\beta}{2\alpha} - \frac{\sqrt{\beta^2 + 4\alpha I(x_1,x_2,\mathbf{k})}}{2\alpha}, \text{ for } I(x_1,x_2,\mathbf{k}) \geq \frac{\beta^2}{4\alpha},$$

where $\alpha = b_1 a_1(a_1 - b_1/2)/(b_1 - a_1)^2$, and $\beta = a_1 q^{ni}/(b_1 - a_1)$. Substituting this function for t throughout the model, we can obtain economic-equilibrium functions: $p^{s\#}(x_1, x_2, \mathbf{b}, \mathbf{k})$, $p^{d\#}(x_1, x_2, \mathbf{b}, \mathbf{k})$, $q^{s\#}(x_1, x_2, \mathbf{b}, \mathbf{k})$, $q^{d\#}(x_1, x_2, \mathbf{b}, \mathbf{k})$, $u_1^\#(x_1, x_2, \mathbf{b}, \mathbf{k})$, and $u_2^\#(x_1, x_2, \mathbf{b}, \mathbf{k})$.[vi]

Comment [OU1]: $-\frac{\partial I(x_1,x_2,\mathbf{b}^1)}{\partial x_2}$ $= \frac{}{\partial S(p^s(t(x_1,$

We can write consumer-taxpayer welfare and producer welfare (both net of political expenditures) as explicit functions of interest group political expenditures:

(12) $$v_1(x_1,x_2,\mathbf{b},\mathbf{k}) = u_1(t^\#(x_1,x_2,\mathbf{b},\mathbf{k}),q^{ni},\mathbf{b}) - x_1$$

$$= u_1^{ex} + \int_{p^d(t^\#(x_1,x_2,\mathbf{b},\mathbf{k}),q^{ni},\mathbf{b})}^{\infty} D(z)dz + t(x_1,x_2,\mathbf{b}) \cdot S(p^s(t^\#(x_1,x_2,\mathbf{b},\mathbf{k}),q^{ni},\mathbf{b})) - x_1.$$

(13) $$v_2(x_1,x_2,\mathbf{b},\mathbf{k}) = u_2(t^\#(x_1,x_2,\mathbf{b},\mathbf{k}),q^{ni},\mathbf{b}) - x_2 = u_2^{ex} + \int_0^{p^s(t^\#(x_1,x_2,\mathbf{b},\mathbf{k}),q^{ni},\mathbf{b})} S(z)dz - x_2.$$

In the non-cooperative equilibrium, each group each group i = 1, 2 solves

(14) $$\underset{x_i \in X_i}{Max}\ v_i(x_1,x_2,\mathbf{b},\mathbf{k}).$$

Assuming interior solutions, taking derivatives in (13) and (14) gives the following first order conditions that characterize the political-economic equilibrium:

(15) $$\frac{\partial v_1(x_1^e,x_2^e,\mathbf{b},\mathbf{k})}{\partial x_1} =$$

$$\underbrace{\left[-S(p^s(t^\#(x_1^e,x_2^e,\mathbf{b},\mathbf{k})q^{ni},\mathbf{b}^1)) + t^\#(x_1^e,x_2^e,\mathbf{b},\mathbf{k})\frac{\partial S(p^s(t^\#(x_1^e,x_2^e,\mathbf{b},\mathbf{k})q^{ni},\mathbf{b}^1))}{\partial p^s}\right]\frac{\partial p^s(t^\#(x_1^e,x_2^e,\mathbf{b},\mathbf{k})q^{ni},\mathbf{b})}{\partial t}}_{\frac{\partial u_1(t^\#(x_1^e,x_2^e,\mathbf{b},\mathbf{k})q^{ni},\mathbf{b}^1)}{\partial t}}$$

$$\bullet \frac{\partial t^\#(x_1^e,x_2^e,\mathbf{b},\mathbf{k})}{\partial x_1} - 1 = 0.$$

$$(16) \quad \frac{\partial v_2\left(x_1^e, x_2^e, \mathbf{b}, \mathbf{k}\right)}{\partial x_2} =$$

$$\underbrace{S\left(p^s\left(t^{\#}\left(x_1^e, x_2^e, \mathbf{b}, \mathbf{k}\right), q^{ni}, \mathbf{b}\right)\right)\frac{\partial S\left(p^s\left(t^{\#}\left(x_1^e, x_2^e, \mathbf{b}, \mathbf{k}\right), q^{ni}, \mathbf{b}^1\right)\right)}{\partial p^s}\frac{\partial p^s\left(t^{\#}\left(x_1^e, x_2^e, \mathbf{b}, \mathbf{k}\right), q^{ni}, \mathbf{b}\right)}{\partial t}}_{\frac{\partial u_2\left(t^{\#}\left(x_1^e, x_2^e, \mathbf{b}, \mathbf{k}\right), q^{ni}, \mathbf{b}\right)}{\partial t}}\frac{\partial t^{\#}\left(x_1^e, x_2^e, \mathbf{b}, \mathbf{k}\right)}{\partial x_2} - 1 = 0.$$

The system of equations (16) and (17) has two unknowns, and in principal can be solved for those unknowns as functions of the parameters. The solutions are political expenditures of the non-governmental interest groups in political-economic equilibrium: $x_1^e(\mathbf{b},\mathbf{k})$, $x_2^e(\mathbf{b},\mathbf{k})$. Substituting these functions into the economic equilibrium government policy function gets us the political-economic equilibrium government policy function: $t^e(\mathbf{b},\mathbf{k}) \equiv t^{\#}\left(x_1^e(\mathbf{b},\mathbf{k}), x_2^e(\mathbf{b},\mathbf{k}), \mathbf{b}, \mathbf{k}\right)$. Then substituting this function into the other economic equilibrium functions, we finish solve for the political-economic equilibrium: $f^e(\mathbf{b}, \mathbf{k}) \equiv f^{\#}(x_1^e(\mathbf{b},\mathbf{k}), x_2^e(\mathbf{b},\mathbf{k}), \mathbf{b}, \mathbf{k})$, for $f = p^s$, p^d, q^s,q^d, u_1 u_2, I, and Z.

Figure 3 illustrates political-economic equilibrium in what I will call the baseline Becker model. Using a superscript 0 to denote baseline values, the model's baseline parameter vector is $(\mathbf{b}^0, \mathbf{k}^0) = \left(a_0^0, a_1^0, b_0^0, b_1^0, k_0^0, k_1^0, k_2^0, k_{11}^0, k_{22}^0, k_{12}^0\right)$ = (910/3, -200/3, -130, 100, 910/3, -1.7, 2.5, 0.01, -0.01, -0.0025). These parameter values imply that in the non-intervention economic equilibrium, the elasticity of supply is 2, the demand elasticity is -4/3, price is \$1.30/unit, and 130 units are supplied and demanded. The values also ensure satisfaction of conditions for the function $I(\ )$ in Becker's appendix.

In the baseline model, the value of the political-economic equilibrium tax/subsidy is $t^e(\mathbf{b}^0, \mathbf{k}^0)$ = -\$0.6076 per unit. The welfare effect (gross of political expenditures) of the this subsidy is shown by the movement from the baseline non-intervention point[vii] $\mathbf{u}^{ni}$ = (700, 400) to the baseline Nash equilibrium welfare outcome (gross of political expenditures) of $\mathbf{u}^e(\mathbf{b}^0, \mathbf{k}^0)$ = (658.07, 434.55). The resultant consumer-taxpayer loss gross of political expenditures and relative to the non-intervention equilibrium is $\Delta u_2^e(\mathbf{b}^0, \mathbf{k}^0)$ = \$700.00 - \$658.07 = \$41.93, the gross producer gain is $\Delta u_1^e(\mathbf{b}^0, \mathbf{k}^0)$ = \$434.55 - \$400.00 = \$34.55. Deadweight loss[viii] (disregarding any political expenditures) is $DW^e(\mathbf{b}^0, \mathbf{k}^0) = -[\Delta u_1^e(\mathbf{b}^0, \mathbf{k}^0) + \Delta u_2^e(\mathbf{b}^0, \mathbf{k}^0)]$ = \$41.93 - 34.55 = \$7.38. Consumers-taxpayers spend $x_1^e(\mathbf{b}^0, \mathbf{k}^0)$ = \$41.27 on politics, and producers spend $x_2^e(\mathbf{b}^0, \mathbf{k}^0)$ = \$50.15.

Consumer-taxpayer welfare (net of political expenditures) is $u_1^e(\mathbf{b}^0, \mathbf{k}^0) - x_1^e(\mathbf{b}^0, \mathbf{k}^0) = v_1^e(\mathbf{b}^0, \mathbf{k}^0) = \$616.80$, and producer welfare net of political expenditures is $u_2^e(\mathbf{b}^0, \mathbf{k}^0) - x_2^e(\mathbf{b}^0, \mathbf{k}^0) = v_2^e(\mathbf{b}^0, \mathbf{k}^0) = \$384.39$. The net welfare outcome is $\mathbf{v}^e(\mathbf{b}^0, \mathbf{k}^0) = (616.80, 384.39)$.

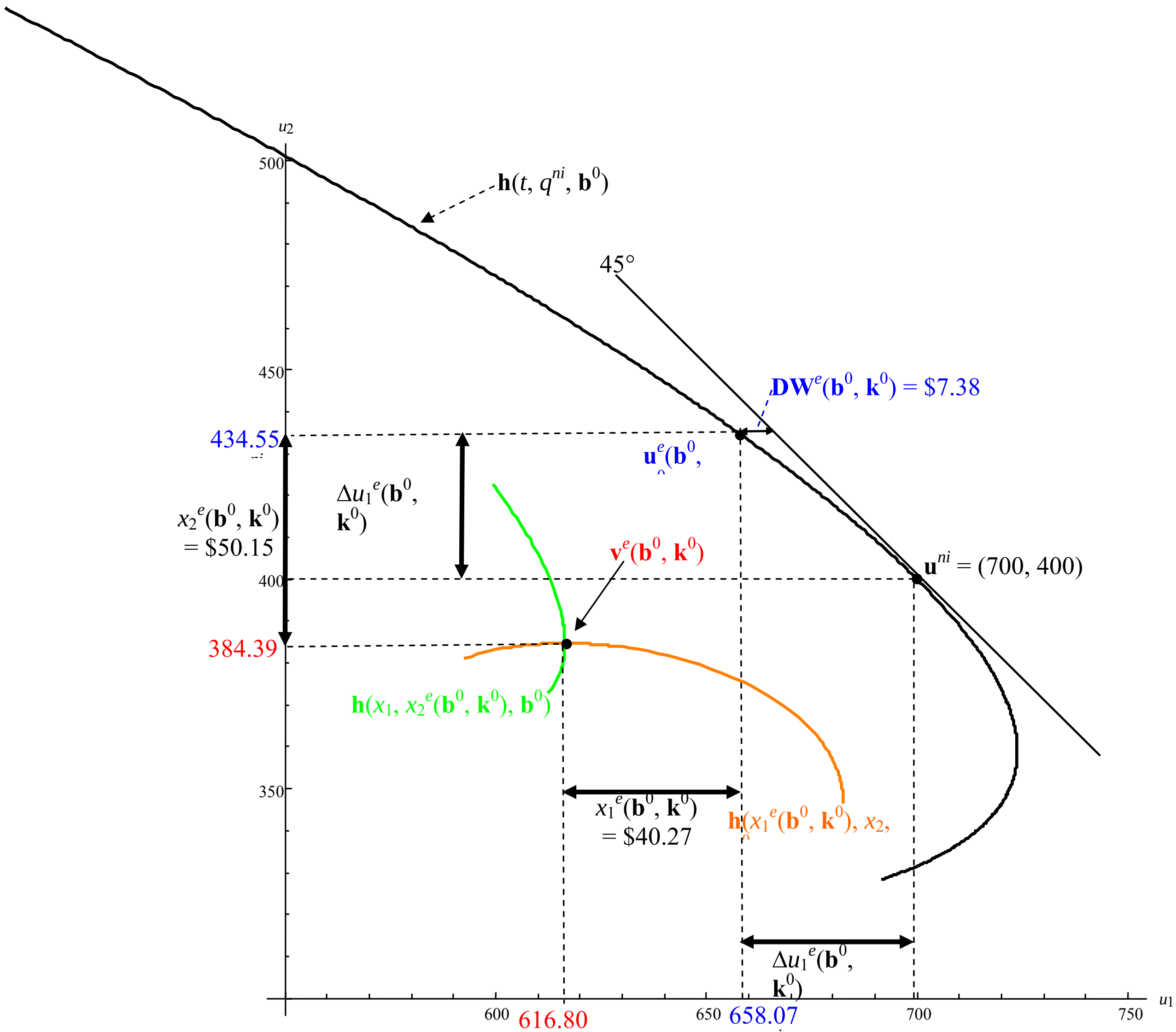

Figure 3. Nash equilibrium in the baseline Becker model of a production tax/subsidy.

A Grossman and Helpman Model

G&H's (1994) model assumes two types of interest groups, lobbying and non-lobbying. A political parameter, here denoted γ, establishes a weight that is used to characterize the degree to which lobbying influences government. L denotes the set of lobbying groups. The political influences on government cause it to set the policy that it would when maximizing "political preference function" that grants a weight of γ to non-lobbying groups and a weight of 1 + γ to lobbying groups (see their (11)):

$$\max_t \left\{ \gamma \sum_{j \notin L} h_j\left(t, q^{ni}, \mathbf{b}\right) + (1+\gamma) \sum_{j \in L} h_j\left(t, q^{ni}, \mathbf{b}\right) \right\}.$$

Assuming that consumers-taxpayers do not lobby but farmers do, the problem reduces to

$$\max_t \left\{ \gamma h_1\left(t, q^{ni}, \mathbf{b}\right) + (1+\gamma) h_2\left(t, q^{ni}, \mathbf{b}\right) \right\}.$$

Calling t^{GHe} the political-economic equilibrium policy, the maximization problem above implies that it will depend on the market parameters **b** and the lone political parameter, γ.

$$\text{(18)} \qquad \frac{\dfrac{\partial h_2\left(t^{GHe}(\mathbf{b},\gamma), q^{ni}, \mathbf{b}\right)}{\partial t}}{\dfrac{\partial h_1\left(t^{GHe}(\mathbf{b},\gamma), q^{ni}, \mathbf{b}\right)}{\partial t}} = -\frac{\gamma}{1+\gamma}.$$

That is, the equilibrium policy takes the welfare outcome (gross of political expenditures) to that place on the surplus transformation curve with slope $-\gamma/(1 + \gamma)$.

Two other equations are necessary to define equilibrium. A focal policy in the model is t^{Not2}, that policy that would come about in political-economic equilibrium were group 2 never to lobby. Policy t^{Not2} must satisfy (see G&H's (16)):

$$\text{(19)} \qquad \frac{\dfrac{\partial h_2\left(t^{Not2}(\mathbf{b}), q^{ni}, \mathbf{b}\right)}{\partial t}}{\dfrac{\partial h_1\left(t^{Not2}(\mathbf{b}), q^{ni}, \mathbf{b}\right)}{\partial t}} = -1.$$

In the particular model presented here, $t^{Not2}(\mathbf{b})$ equals the non-intervention value, 0. This G&H model is closed by (20), which determines the lobbying group's political expenditures, x_2^{GHe} (see G&H's (17)): [ix]

$$(20)\ x_2^{GHe} = h_2\left(t^{GHe}(\mathbf{b},\gamma),q^{ni},\mathbf{b}\right)$$

$$-\gamma\left(\left[h_1\left(t^{Not2}(\mathbf{b}),q^{ni},\mathbf{b}\right)+h_2\left(t^{Not2}(\mathbf{b}),q^{ni},\mathbf{b}\right)\right]-\left[h_1\left(t^{GHe}(\mathbf{b},\gamma),q^{ni},\mathbf{b}\right)+h_2\left(t^{GHe}(\mathbf{b},\gamma),q^{ni},\mathbf{b}\right)\right]\right)$$

$$= h_2\left(t^{GHe}(\mathbf{b},\gamma),q^{ni},\mathbf{b}\right)-\gamma\underbrace{\left(\left[h_1\left(0,q^{ni},\mathbf{b}\right)+h_2\left(0,q^{ni},\mathbf{b}\right)\right]-\left[h_1\left(t^{GHe}(\mathbf{b},\gamma),q^{ni},\mathbf{b}\right)+h_2\left(t^{GHe}(\mathbf{b},\gamma),q^{ni},\mathbf{b}\right)\right]\right)}_{DW}.$$

(By assumption, group 1 does not lobby, so $x_1^{GHe} = 0$.)

Comment [OU2]: $a\left[W_1\left(p^{Not2}\right)\right.$ $B_2^0 = W_2\left(p^0\right.$ $B_2^0 = W_2\left(p^0\right.$

The economic side of the G&H model at its baseline is identical to that of the Becker model at its baseline, as presented in equations (1), (3), and (5)-(8), where the market parameter values are $\mathbf{b}^0 = \left(a_0^0, a_1^0, b_0^0, b_1^0\right)$ = (910/3, -200/3, -130, 100). The G&H model has only a single political parameter, γ. Given the nature of our simulative model, the value set for γ is relatively arbitrary. For ease of comparison with the Becker model, from (19) I calibrate γ by setting $-\gamma/(1+\gamma)$ equal to the slope of the surplus transformation curve when the elasticity of supply is 2.00 and the elasticity of demand is -4/3. (This slope is -0.717488, the slope at point $\mathbf{u}^{\varepsilon=2.00}$ in figure 2. The resulting γ is γ^0 = 2.53967, and so $(\mathbf{b}^0, \mathbf{k}^0) = \left(a_0^0, a_1^0, b_0^0, b_1^0, \gamma^0\right)$ = (910/3, -200/3, -130, 100, 2.53967.) Given these values, the per-unit subsidy and gross transfers in the baseline G&H model are equal to those of the baseline Becker model.

The Comparative Statics of Changes in Market Elasticities

With the economic model formulated and the effect of elasticities on redistributive efficiency illustrated, I can ask my central question in an abstract and compact form: *What is* $\nabla t^e(\mathbf{b}, \mathbf{k})$*?* That is, what are the partial derivatives of the equilibrium distortional policy with respect to the model's parameters? Related to this question is *what is* $\nabla\Delta\mathbf{u}^e(\mathbf{b}, \mathbf{k})$? That is, what are the partial derivatives of the equilibrium transfers with respect to the model's parameters? Answering these questions of theory may be a useful step in developing testable hypotheses about why various nations use various distortional policies and make different levels of transfers among interest groups.

Effects of Market Elasticities on the Total Subsidy

The redistributional consequences (in the Becker model's political-economic equilibrium) of changes in the supply elasticity are shown in figure 2, where six "gross-of-political-expenditures" welfare outcomes are illustrated. These are labeled $\mathbf{u}^{\varepsilon=0.85}$,

$\mathbf{u}^{\varepsilon=1.23}$, $\mathbf{u}^{\varepsilon=1.62}$, $\mathbf{u}^{\varepsilon=2}$, $\mathbf{u}^{\varepsilon=2.38}$, and $\mathbf{u}^{\varepsilon=2.77}$, where $\mathbf{u}^{\varepsilon=\varepsilon'} \equiv \mathbf{u}(t^e(q^{ni},\mathbf{b}^{\varepsilon=\varepsilon'}), q^{ni}, \mathbf{b}^{\varepsilon=\varepsilon'})$. As the supply elasticity falls, the equilibrium gross welfare outcome moves northwest. Since the transfer to producers when the supply elasticity is ε' is the vertical distance between $\mathbf{u}^{\varepsilon=\varepsilon'}$, and $\mathbf{u}^{ni}$, then the gross transfer to producers increases as demand becomes less elastic. The relationship between the total subsidy and the supply elasticity is also shown in the lower right-hand panel of figure 4, where as the supply elasticity rises, the total subsidy falls. Similar comparative static results come about in the G&H model: as supply becomes more elastic, the total subsidy falls. These results are in keeping with the tenor of the literature, which is that when redistributional efficiency decreases, total transfers fall.

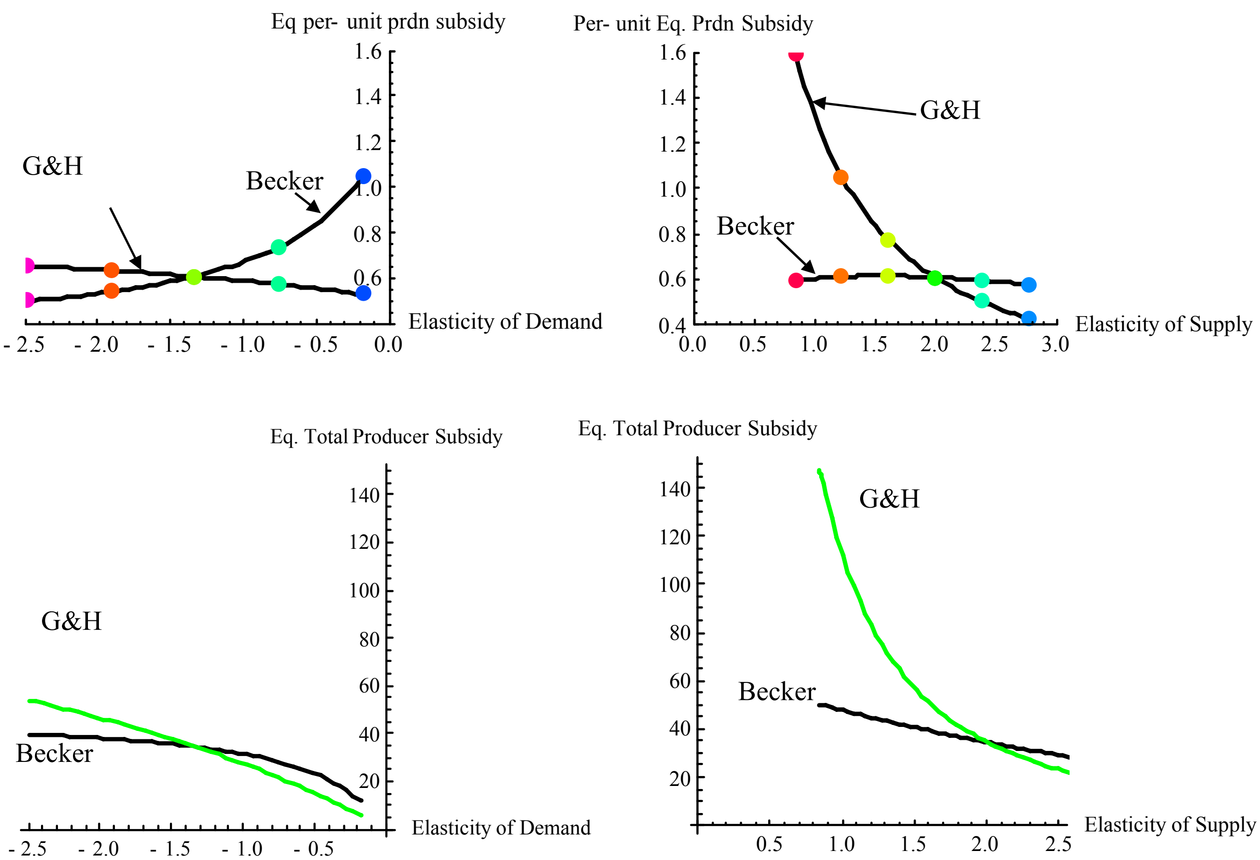

Figure 4. In the Becker model's equilibrium, as demand becomes more elastic, the per-unit subsidy decreases, whereas in G&H's model, it decreases. Raising the supply

elasticity lowers the equilibrium per-unit subsidy in the G&H model, but the effect in the Becker model depends on the elasticity's initial value. In both models' equilibria, the total producer subsidy rises as demand becomes more elastic, but falls as supply becomes more elastic.

How net-of-political-expenditures welfare outcomes in the Becker model change with the elasticity of supply is illustrated in figure 4 by points labeled $\mathbf{v}^{\varepsilon=\varepsilon'}$, where $\mathbf{v}^{\varepsilon=\varepsilon'} \equiv \mathbf{v}(t^e(q^{ni},\mathbf{b}^{\varepsilon=\varepsilon'}), q^{ni}, \mathbf{b}^{\varepsilon=\varepsilon'})$ for the elasticity of supply being $\varepsilon' = 0.85$, 1.23, 1.62, 2.00, 2.38, and 2.77. Whether the question is about producer welfare gross-of- or net-of-political-expenditures, the producers' welfare outcome in political-economic equilibrium improves as supply becomes less elastic and redistributive efficiency improves. This result is also in line with the general tenor of the literature. A figure in the appendix shows that the same holds true for changes in the demand elasticity; redistributive efficiency improves as demand becomes more elastic, and producers are better off in political-economic equilibrium. In every case, the net-of-political-expenditures welfare outcome is Pareto inferior to the non-intervention outcome. That is, it would be better for everybody if it were impossible for everybody to make political expenditures. This is very much in line with Tullock's (1967, 1980) discussion of rent dissipation and the literature that has followed (e.g., Krueger 1974, Hillman 1989, Coggins 1995, Bullock and Rutström 2007).

Effects of Market Elasticities on the Per-unit Subsidy

I have reported above that in both political-economy models, the comparative static results of how changes in demand and supply elasticities affect total subsidies is consistent with predictions in the literature. This type of result also holds for the per-unit subsidy in the political-economic equilibria of the G&H model. As illustrated in the upper-left hand panel of figure 4, as demand becomes more elastic (making redistribution more efficient), the G&H model's equilibrium per-unit subsidy rises. Similarly, as supply becomes more elastic and makes redistribution less efficient, G&H's equilibrium per-unit subsidy falls, as shown in figure 4's upper right-hand panel. But in the Becker model, the direction of the effect of changes in elasticities on the *per-unit* subsidy is much less in keeping with the literature's predictions. The upper left-hand panel of figure 4 shows how in Becker's model the per-unit subsidy depends on the initial value of the supply elasticity. Beginning at a low supply elasticity, raising the elasticity actually raises the per-unit subsidy, which is contrary to the discussion in the literature. Similarly, the upper left-hand panel shows that as demand becomes less elastic (implying that redistributive efficiency improves), the per-unit subsidy for the Becker model falls. This result runs contrary to the discussion in the literature, which anticipates that the extent to which a policy instrument is used should increase with increases in its redistributive efficiency.

Using figure 2, an intuitive explanation can be provided of the Becker model's unexpected result that a policy may be used less after its redistributive efficiency rises. Note that an increase in redistributive efficiency does increase the total transfer. For example, the point labeled $\mathbf{u}^{\varepsilon=0.85}$ is the equilibrium welfare outcome (gross of political expenditures) when the elasticity of supply is 0.85. This point lies further to northwest of the non-intervention point $\mathbf{u}^{ni}$ than does $\mathbf{u}^{\varepsilon=1.62}$, the equilibrium welfare (gross of political expenditures) point when the supply elasticity is 1.62. That is, when redistribution is relatively efficient, the total subsidy is larger, and the total transfer from consumers-taxpayers is larger. However, the increased efficiency makes it possible to transfer any amount to producers by using a smaller per-unit subsidy; and this is exactly what happens in equilibrium: more is transferred in total, but with a smaller per-unit subsidy. As in figure 4, when $\varepsilon = 0.85$ a total subsidy of \$49.98 is generated by a per-unit subsidy of \$0.593; when $\varepsilon = 1.62$, producers receive \$39.20 from a per-unit subsidy of \$0.614.

Conclusions and Implications for Empirical Work

My analysis highlights a number of points regarding Becker's and G&H's statements. First, when discussing the effects of supply and demand elasticities on distortions and transfers, it is important to distinguish carefully between per-unit transfers and total transfers, as well as between per-unit distortions (per-unit deadweight loss) and total distortions (total deadweight loss). Second, Becker's statement that the deadweight costs of distortions are lower when supply (and demand) is less elastic is not completely correct and must be interpreted with care. How an elasticity change effects the deadweight costs of distortions depends very much on the type of policy instrument(s) used. (Gardner 1983, 1987 makes this point clear). I have shown an example in which a production subsidy is the policy instrument used, and a lower supply elasticity does indeed lead to less deadweight loss for any amount of income transferred to producers. In turn, a greater *total* subsidy is provided in Becker's political-economic equilibrium.

The results just discussed and summarized in table 1 reveal challenges that remain as economists try to determine the effects of market parameter changes on a political-economy's choices. The theoretical results are not as cut-and-dry as some of the literature has anticipated. This ambiguity in theoretical results implies that some of the answers to the questions being asked about the effects of redistributive efficiency on policy in political-economic equilibrium must be answered empirically. Hypotheses can be tested. Do higher supply elasticities lead to less use of production subsidies, or greater use? Results in this article illustrate that no unambiguous theoretical answer has been reached, and therefore there is a need for empirical analysis to guide us towards a better understanding of which models of political economy are the more realistic. Of course, the simulations reported here come from a very simple economic model, with two interest groups, and one policy instrument used. Real political economies have many interest groups, who have many strategy variables, and interact with governments that can use multiple policy instruments simultaneously. It would not have been possible in this brief space to report the simulation results from models of the many "real" political economies. But I have provided general guidelines for how an economic model and a political model can be joined to create a model of political economy, which then can be used as a theoretical basis for empirical work. The next step is to build such a model for a real-world political economy. Such a model might include several interest groups, multiple producer groups from different regions of a nation, an urban consumer/laborer group, and a group of government bureaucrats. The model might also include several linked markets. Simulations could be run with the developed model of political economy, to begin to generate ideas about which economic and political parameters might be key in the political-economic process. Then empirical work, estimating the economic model, the size of observed transfers, and the values of observed policies, would be conducted to test the specific simulative results that come out of the model

Table 1. The effects of elasticity changes on total subsidization and per-unit subsidization in the equilibria of two models of political economy

Elasticity change	Effect on the efficiency of redistribution from consumer/taxpayers to producers	Becker Model		G&H Model	
		Total subsidy	Per-unit subsidy	Total subsidy	Per-unit subsidy
Supply more elastic	Less efficient	Smaller	Ambiguous—depends on the initial value of the elasticity	Smaller	Smaller
Demand more elastic	More efficient	Larger	Smaller	Larger	Larger

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Appendix: Simulating the Effects of
Supply and Demand Elasticities on Political-Economic Equilibrium

1. Finding *t* as a function of Δu_1.

From equations (8) and (11), we have

$$\text{(A.1)}\quad -\Delta u_1(t,q,\mathbf{b}) = u_1^{ni} - u_1(t,q,\mathbf{b}) = \left(\int_{p^{ni}}^{\infty} D(z)dz + t\cdot S(p^{ni})\right) - \left(\int_{p^d(t,q,\mathbf{b})}^{\infty} D(z)dz + t\cdot S\big(p^s(t,q,\mathbf{b})\big)\right).$$

When government can only use the production tax/subsidy policy instrument, then the quota is held constant at q^{ni}. Then from above we have

$$\text{(A.2)}\quad \begin{aligned} -\Delta u_1(t,q^{ni},\mathbf{b}) &= u_1^{ni} - u_1(t,q^{ni},\mathbf{b}) = \left(\int_{p^{ni}}^{\infty} D(z)dz + \underbrace{t^{ni}}_{0}\cdot q^{ni}\right) - \left(\int_{p^d(t,q^{ni},\mathbf{b})}^{\infty} D(z)dz + t\cdot S\big(p^s(t,q^{ni},\mathbf{b})\big)\right) \\ &= \int_{p^{ni}}^{p^d(t,q^{ni},\mathbf{b})} D(z)dz - t\cdot S\big(p^s(t,q^{ni},\mathbf{b})\big) \end{aligned}$$

Given (1) and (5)-(7), the equilibrium price functions can be found to be

$$\text{(A.3)}\quad \begin{aligned} p^s(t,q^{ni},\mathbf{b}) &= p^{ni} + \frac{a_1}{b_1-a_1}t \\ p^d(t,q^{ni},\mathbf{b}) &= p^{ni} + \frac{b_1}{b_1-a_1}t \end{aligned}$$

Substituting (1), (5), and (A.3) into (A.2), in can be shown that

$$\text{(A.4)}\quad \underbrace{\frac{b_1 a_1\left(a_1 - \frac{b_1}{2}\right)}{(b_1-a_1)^2}}_{\alpha>0} t^2 + \underbrace{\frac{a_1 q^{ni}}{b_1-a_1}}_{\beta<0} t + \Delta u_1(t,q^{ni},\mathbf{b}) \equiv 0.$$

Using the quadratic formula,

$$\text{(A.5)}\quad t \equiv \frac{-\beta \pm \sqrt{\beta^2 - 4\alpha\Delta u_1(t,q^{ni},\mathbf{b})}}{2\alpha}.$$

Comment [OU3]:

$-\Delta u_1(t,q^{ni},\mathbf{b}$

$= a_0 \frac{b_1}{b_1-a_1} t$

$= a_0 \frac{b_1}{b_1-a_1} t$

$-\Delta u_1(t,q^{ni},\mathbf{b}$

$-\Delta u_1(t,q^{ni},\mathbf{b}) = \left(\frac{b_1 a_0}{b_1-a_1} + \frac{a_1 b}{b_1 -}\right.$

$= \left(\frac{b_1}{b_1-a_1}(a_0 + a_1 p^{ni}) - (b_0 + b\right.$

$= \left(\frac{b_1}{b_1-a_1}\underbrace{(a_0 + a_1 p^{ni})}_{q^{ni}} - \underbrace{(b_0 + b}_{q^{n}}\right.$

$= \left(\frac{b_1}{b_1-a_1} - 1\right) q^{ni} t + \frac{b_1 a_1}{b_1-a_1}\left(\frac{1}{2}\right.$

$= \frac{a_1 q^{ni}}{b_1-a_1} t + \frac{b_1 a_1}{b_1-a_1}\left(\frac{a_1 - \frac{b_1}{2}}{b_1-a_1}\right) t^2$

$-\Delta u_1(t,q^{ni},\mathbf{b}) = \underbrace{\frac{b_1 a_1\left(a_1 - \frac{b_1}{2}\right)}{(b_1-a_1)^2}}_{\alpha>0} t$

From (A.5) it can be shown that the tax $t^* = -\beta/2\alpha$ corresponds to a point like E in figure 3. That is, for any greater than t^*, not only producers but also consumers-taxpayers are harmed by a further increase in *t*. We assume that no *t* higher than t^* can be part of a political economic equilibrium. This implies that the only one of the two solutions to (A.5) that can be part of an economic equilibrium is:

$$\text{(A.6)} \quad t \equiv \frac{-\beta}{2\alpha} + \frac{\sqrt{\beta^2 - 4\alpha\Delta u_1\left(t, q^{ni}, \mathbf{b}\right)}}{2\alpha}.$$

Therefore in political economic equilibrium t depends on $-\Delta u_1$ as follows:

$$\text{(A.6)} \quad t\left(-\Delta u_1\right) \equiv \frac{-\beta}{2\alpha} + \frac{\sqrt{\beta^2 - 4\alpha\Delta u_1}}{2\alpha}, \text{ for } \Delta u_1 \leq \frac{-\beta^2}{4\alpha}$$

2. Mappings from Policy-space to Welfare-space.

The vector function **u** can be used to map the feasible policy set X_0 or a subset of it into welfare space (defined here as the nonnegative orthant of two-dimensional Euclidean space, with u_1 on the horizontal axis and u_2 on the vertical axis). Such mappings will prove important in my presentation of political economy models, and so I present them in some detail here. Given that the economy is characterized by some a value **b**′ of the vector of parameters **b**, using the welfare vector function $\mathbf{u}(t, q, \mathbf{b}')$ to map all of the feasible policy set into welfare space, we obtain the set of feasible welfare outcomes $\{\mathbf{u}(t, q, \mathbf{b}'): (t, q) \in X_0\}$. We will give this set the name $\mathbf{h}(t, q, \mathbf{b}')$, where no superscripts on the policy variables imply that each can take on any feasible value. $\mathbf{h}(t, q, \mathbf{b}')$ is the set of all the welfare outcomes that government can bring about with its available policies, given that the economy is always in economic equilibrium. This mapping is illustrated in figure A.3.1, where the shaded feasible policy set X_0 is shown mapped into the shaded set of feasible welfare outcomes $\mathbf{h}(t, q, \mathbf{b}')$. Because two policy instrument variables are allowed to change in this mapping, $\mathbf{h}(t, q, \mathbf{b}')$ is a two-dimensional manifold in welfare space.[x]

Another such mapping occurs when a single policy, that is a single point in X_0, is mapped into a point welfare space. This is illustrated in figure 2, where the non-intervention policy shown as (t^{ni}, q^{ni}) in the lower panel is mapped by function $\mathbf{u}(t, q, \mathbf{b}')$ to the set $\mathbf{h}(t^{ni}, q^{ni}, \mathbf{b}')$ in the upper panel.[xi] Because no policy instruments vary in this mapping, the result is a manifold of degree zero (a point) in welfare space.

Another such mapping takes the dashed line segment running from triangle A to triangle B in the lower panel of figure 2, and results in the dashed surplus transformation curve[xii] running from triangle A to triangle B in the upper panel. Note that because the dashed line segment runs through the point (t^{ni}, q^{ni}), then the dashed surplus transformation curve **h**(t, q^{ni}, **b**′) must contain the single element of **h**(t^{ni}, q^{ni}, **b**′).[xiii]

Just as we drew a surplus transformation curve for the tax/subsidy, we can also draw one for the production quota. The function **u**(t, q, **b**′) takes the solid line segment running from circle C to circle D in the lower panel of figure 2, and results in the solid curve running from circle C to circle D in the upper panel. Note that in places this production quota surplus transformation curve is positively sloped. The intuition is that once the production quota is less than the quantity that would be chosen by a profit-maximizing monopolist (which would take itself to the "top" of **h**(t^{ni}, q, **b**′), further reduction in quantity would not only hurt consumers, but also producers.

The concavity (relative to the horizontal axis) of the surplus transformation curves shown in figure 2 relates to the dead weight costs that often accompany government policy that redistributes welfare. Taking an arbitrary point, say (t', q') in X_0 and mapping it to the corresponding welfare outcome **h**(t', q', **b**′) in the upper panel of figure 2, the horizontal (or vertical) distance from a **h**(t', q', **b**′) to the 45-degree line through the non-intervention welfare outcome **h**(t^{ni}, q^{ni}, **b**′) is a measure of dead weight loss. This distance is denoted $DW(t', q', \mathbf{b}')$. (Gardner (1983) provides a discussion.) The concavity of the surplus transformation curves presented relates to how dead weight loss (think of the Harberger triangle) not only grows as a tax, subsidy, or production quota further distorts an economy, but grows at a greater rate as the distortion is increased. Thus, moving "northwest" along a surplus transformation curve, the distance from the curve to the 45-degree line through **h**(t^{ni}, q^{ni}, **b**′) increases.

3. Mappings from Policy-space to Welfare-space

The vector function **u** can be used to map the feasible policy set X_0 or a subset of it into welfare space (defined here as the nonnegative orthant of two-dimensional Euclidean space, with u_1 on the horizontal axis and u_2 on the vertical axis). Such mappings are important in my presentation of political economy models, and so I present them in some detail here. Given that the economy is characterized by some arbitrary value $\mathbf{b}'$ of the vector of market parameters **b**, using the welfare vector function $\mathbf{u}(t, q, \mathbf{b}')$ to map all of the feasible policy set into welfare space, I obtain the set of feasible welfare outcomes $\{\mathbf{u}(t, q, \mathbf{b}'): (t, q) \in X_0\}$. I give this set the name $\mathbf{h}(t, q, \mathbf{b}')$, where no superscripts on the policy variables imply that each can take on any feasible value. This is the set of all the welfare outcomes that government can bring about with its available policies, given that the economy is always in economic equilibrium. This mapping is illustrated in figure A.3.1, where the shaded feasible policy set X_0 is shown mapped into the shaded set of feasible welfare outcomes $\mathbf{h}(t, q, \mathbf{b}')$. Because two policy instrument variables are allowed to change during this mapping, $\mathbf{h}(t, q, \mathbf{b}')$ is a two-dimensional manifold in welfare space.

Another such mapping occurs when a single policy, that is a single point in X_0, is mapped into a point welfare space. This is illustrated in figure A.3.1, where the non-intervention policy shown as (t^{ni}, q^{ni}) in the lower panel is mapped by function $\mathbf{u}(t, q, \mathbf{b}')$ to the set $\mathbf{h}(t^{ni}, q^{ni}, \mathbf{b}')$ in the upper panel.[xiv] Because no policy instruments vary in this mapping, the result is a manifold of degree zero (a point) in welfare space.

Another such mapping takes the dashed line segment running from triangle A to triangle B in the lower panel of figure A.3.1, and results in the dashed curve running from triangle A to triangle B in the upper panel. Josling (1974) was the first to discuss policy outcomes using such a curve, and Gardner (1983) popularized the analytic tool, naming such curves *surplus transformation curves*, since they illustrate how a single policy instrument can be used to "transform" one interest group's welfare into another's. Surplus transformation curves are one-dimensional manifolds in welfare space, created by varying one policy instrument's level between its lower and upper limits while holding constant all other policy instruments' levels.[xv] Note that because the dashed line segment runs through the point (t^{ni}, q^{ni}), then the dashed surplus transformation curve $\mathbf{h}(t, q^{ni}, \mathbf{b}')$ must contain the single element of $\mathbf{h}(t^{ni}, q^{ni}, \mathbf{b}')$.

Figure A.3.1 also shows a surplus transformation curve for a production quota. The function $\mathbf{u}(t, q, \mathbf{b}')$ takes the solid line segment running from circle C to circle D in the lower panel of figure A.3.1, and results in the solid curve running from circle C to circle D in the upper panel. Note that in places this production quota surplus transformation curve is positively sloped. The intuition is that once the production quota is less than the quantity that would be chosen by a profit-maximizing monopolist (which would take itself to the "top" of $\mathbf{h}(t^{ni}, q, \mathbf{b}')$, further reduction in quantity would not only hurt consumers, but also producers.

The concavity (relative to the horizontal axis) of the surplus transformation curves shown in figure A.3.1 relates to the dead weight costs that often accompany government policy that redistributes welfare. Taking an arbitrary point, say (t', q') in X_0 and mapping it to the corresponding welfare outcome $\mathbf{h}(t', q', \mathbf{b}')$ in the upper panel of figure 2, the horizontal (or vertical) distance from a $\mathbf{h}(t', q', \mathbf{b}')$ to the 45-degree line through the non-intervention welfare outcome $\mathbf{h}(t^{ni}, q^{ni}, \mathbf{b}')$ is a measure of dead weight loss. This distance is denoted $DW(t', q', \mathbf{b}')$. (Gardner (1983) provides a discussion.) The concavity of the surplus transformation curves presented relates to how dead weight loss (think of the Harberger triangle) not only grows as a tax, subsidy, or production quota further distorts an economy, but grows at a greater rate as the distortion is increased. Thus, moving "northwest" along a surplus transformation curve, the distance from the curve to the 45-degree line through $\mathbf{h}(t^{ni}, q^{ni}, \mathbf{b}')$ increases.

4. The Effects of Changes in Market Elasticities

The Effect of Changes in Market Elasticities on the Redistributive Efficiency of the Production Subsidy Policy Instrument

It was shown in the main text that increasing the supply elasticity decreases the redistributive efficiency of the production subsidy policy instrument. The effects of changes in the demand elasticity on the subsidy instrument's redistributive efficiency are detailed here. A rise in the (absolute value of) the elasticity of demand raises the redistributive efficiency of the subsidy policy instrument. Again, this can be shown in (quantity, price)-space or in welfare-space.

As Seen in (Quantity, Price)-Space

The effect of the elasticity of demand on redistributive efficiency of the subsidy is illustrated in (quantity, price)-space in figure A.4.1. The intuitive explanation is reasonably straightforward. One of the main determinants of elasticity in demand is the substitutability of the good with other goods. The subsidy increases production, and that production must be consumed. If demand is relatively elastic, then consumers readily substitute the good in question for other goods. Thus the marginal benefit of consuming the good does not change greatly as more of the good is consumed, and the wedge between marginal cost and marginal benefit remains small. In figure A.4.1, the same amount of surplus, $\Delta PS = 60$, is transferred to producers in both panels. But in the bottom panel, demand becoming less elastic leads to a greater loss in consumer-and-taxpayer surplus and greater deadweight loss.

As Seen in Welfare-Space

Figure A.4.2 illustrates in welfare-space the effects of changes in the elasticity of demand on the redistributive efficiency of a production subsidy. As demand becomes more elastic, the subsidy's surplus transformation curve shifts northeast, implying for any level of transfer

Effects of Changes in Market Elasticities on Welfare, the Total Subsidy, and the Per-unit Subsidy in Political-economic Equilibrium

Changes in the Supply Elasticity

In the Becker Model

The effects of changes in the supply elasticity on welfare, the total subsidy, and the per-unit subsidy in the Becker model's political-economic equilibrium are discussed in detail in the main text.

In the Grossman and Helpman Model

The effects of changes in the supply elasticity in the Grossman and Helpman model's political-economic equilibrium differ from those in the Becker model in some important aspects. The effects on the total subsidy in the G&H model are shown in figure A.4.3.

As with the Becker model, an increased supply elasticity decreases the redistributive efficiency of a production subsidy and decreases the total subsidy. But unlike in the Becker model, the effects on the per-unit subsidy are uniformly consistent with the discussion in the literature: as the supply elasticity increases, the per-unit subsidy declines.

Changes in the Demand Elasticity

The comparative static effects of a change in the demand elasticity depend on which political-economy model is used. In the following I first discuss the effects in the Becker model, and then in the G&H model after that.

In the Becker Model

Figure A.4.2 illustrates the comparative static effects of changes in the demand elasticity on welfare in the Becker model. Redistributive efficiency improves as demand becomes more elastic, and producers become better off in political economic equilibrium as this redistributive efficiency improves. Note also that in any of these programs, the net-of-political-expenditures welfare outcome is Pareto inferior to the non-intervention outcome. That is, it would be better for everybody if it were impossible for everybody to make political expenditures. This is very much in line with Tullock's (1967, 1980) discussion of rent dissipation and literature that has followed it (e.g., Krueger 1974, Hillman 1989, Coggins 1995, Bullock and Rutström 2007).

When demand becomes more elastic, income can be more efficiently transferred to producers through the production subsidy instrument, in the sense that for any amount of income transferred to producers, the loss to consumers/taxpayers is lower under a more elastic demand than under a less elastic demand. In the equilibrium of the Becker model, his increased efficiency results in a greater total subsidy to producers. This is the result largely anticipated in the literature. However the effect of a more elastic demand on the per-unit subsidy is the opposite of what is predicted in the literature: as seen in figure as demand becomes more elastic, the per-unit subsidy decreases. When demand is more elastic, it is possible to transfer more to producers while driving a smaller wedge between the marginal cost of producing the good and the marginal benefit of demanding it.

In the Grossman and Helpman Model

The effects of changes in the demand elasticity on the total subsidy in the Grossman and Helpman model are shown in the lower left-had panel of figure 4 and in figure A.4.4. Results are similar to those from the Becker model: as demand grows more elastic, the transfer efficiency increases, and the size of the total subsidy increases with it. In the upper left-hand panel of figure 4 it is shown that as demand becomes more elastic, the per-unit subsidy increases in the G&H equilibrium. This result is in keeping with the general tone of the existing literature, whereas Becker model's result for the per-unit subsidy is not.

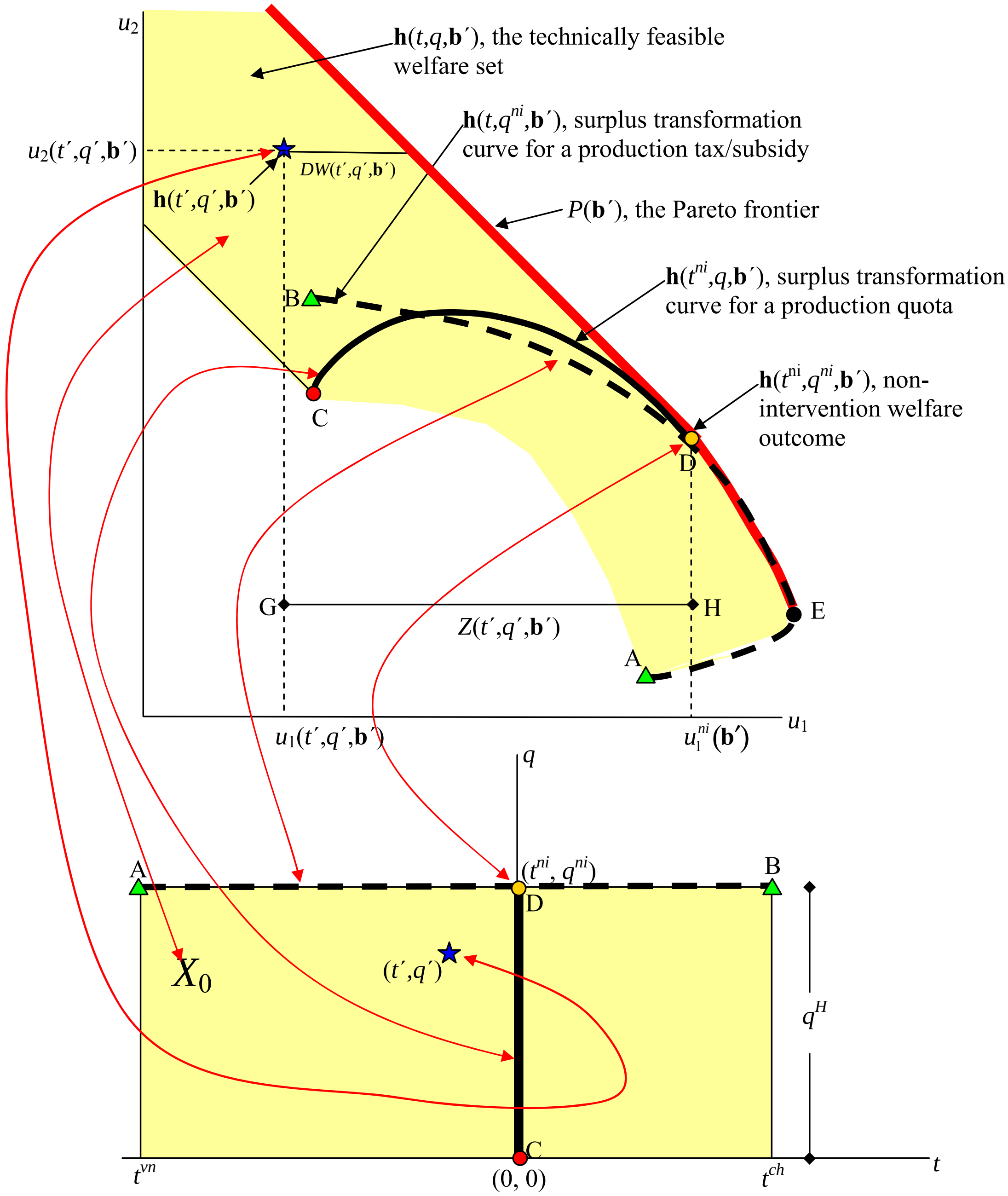

Figure A.3.1. Mappings from policy instrument space into welfare space[xvi]

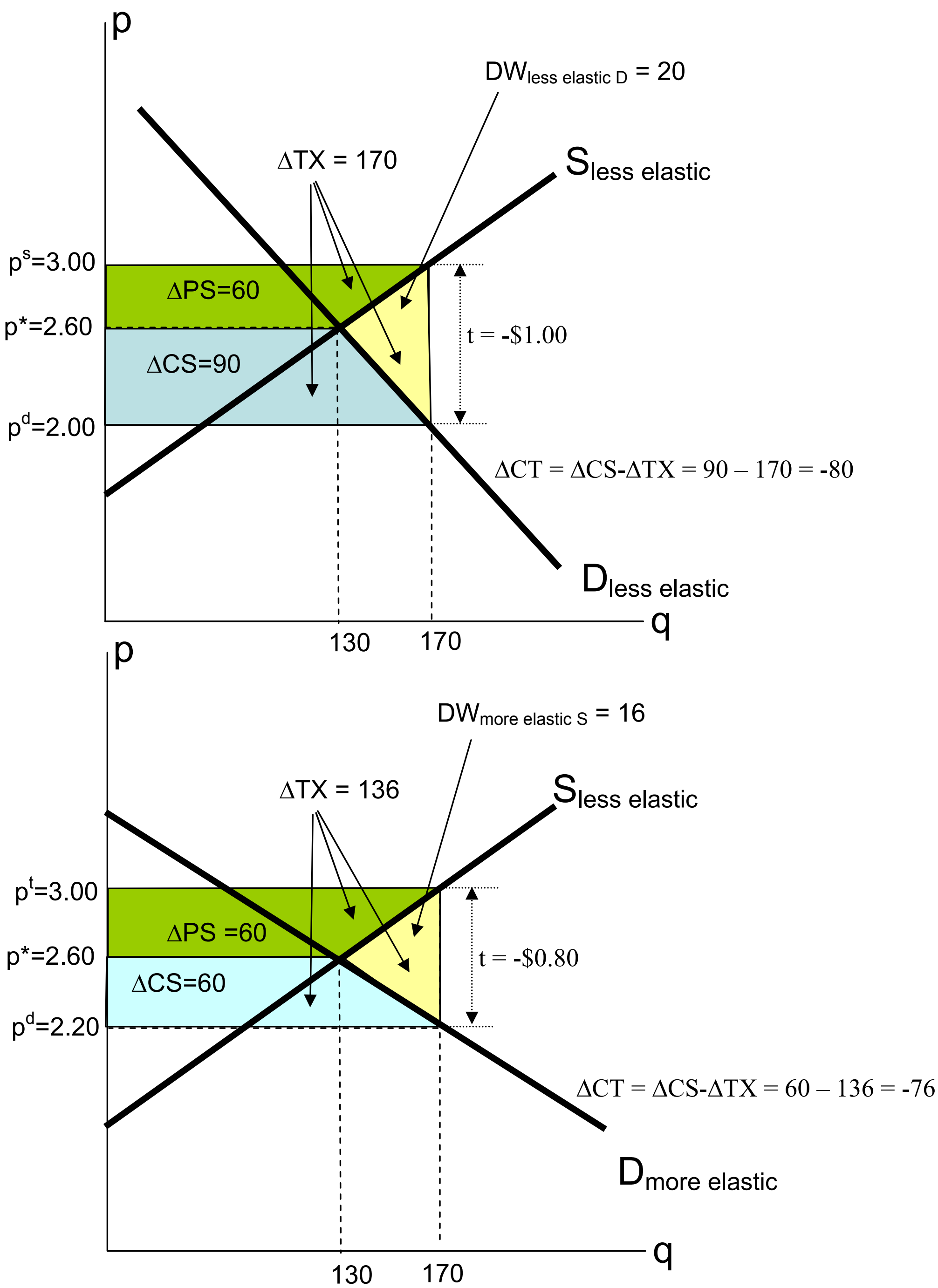

Figure A.3.2. As demand becomes more elastic, the redistributive efficiency of a production subsidy increases

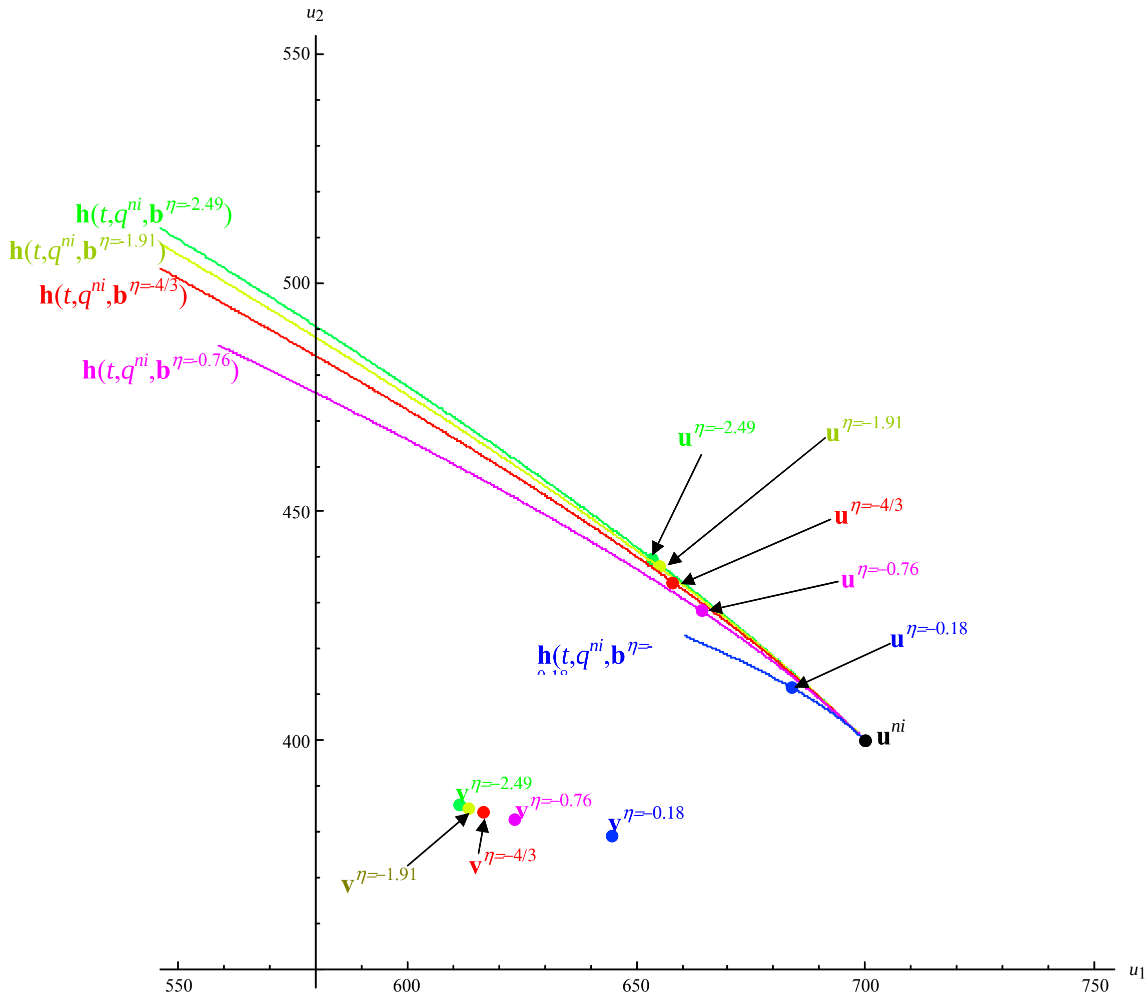

Figure A.3.3. Demand becoming more elastic increases the redistributive efficiency of a production subsidy and increases the equilibrium transfer to producers in the Becker model, as shown in Welfare space..

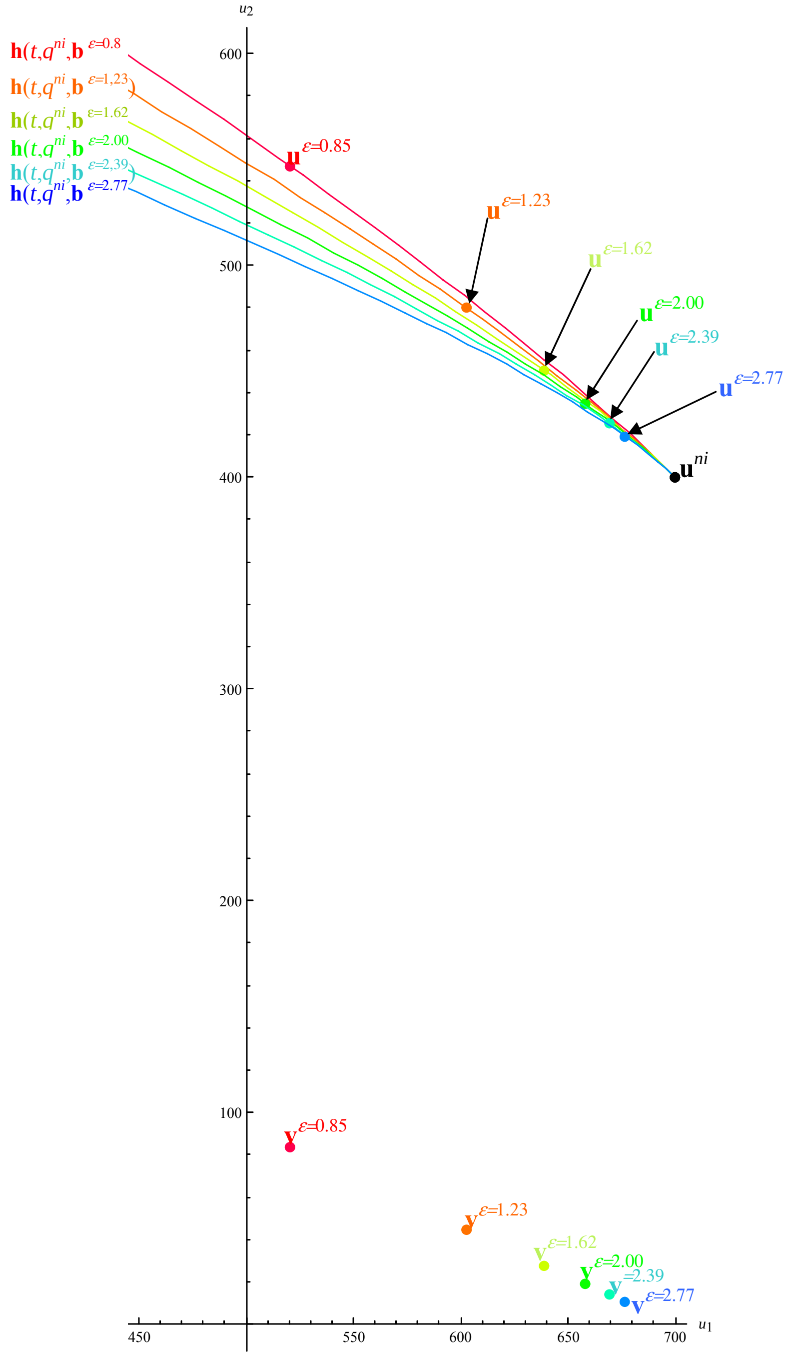

Figure A.3.4. An increased supply elasticity decreases the redistributive efficiency of a production subsidy and decreases the equilibrium transfer to producers in the Grossman and Helpman model, as shown in Welfare space.

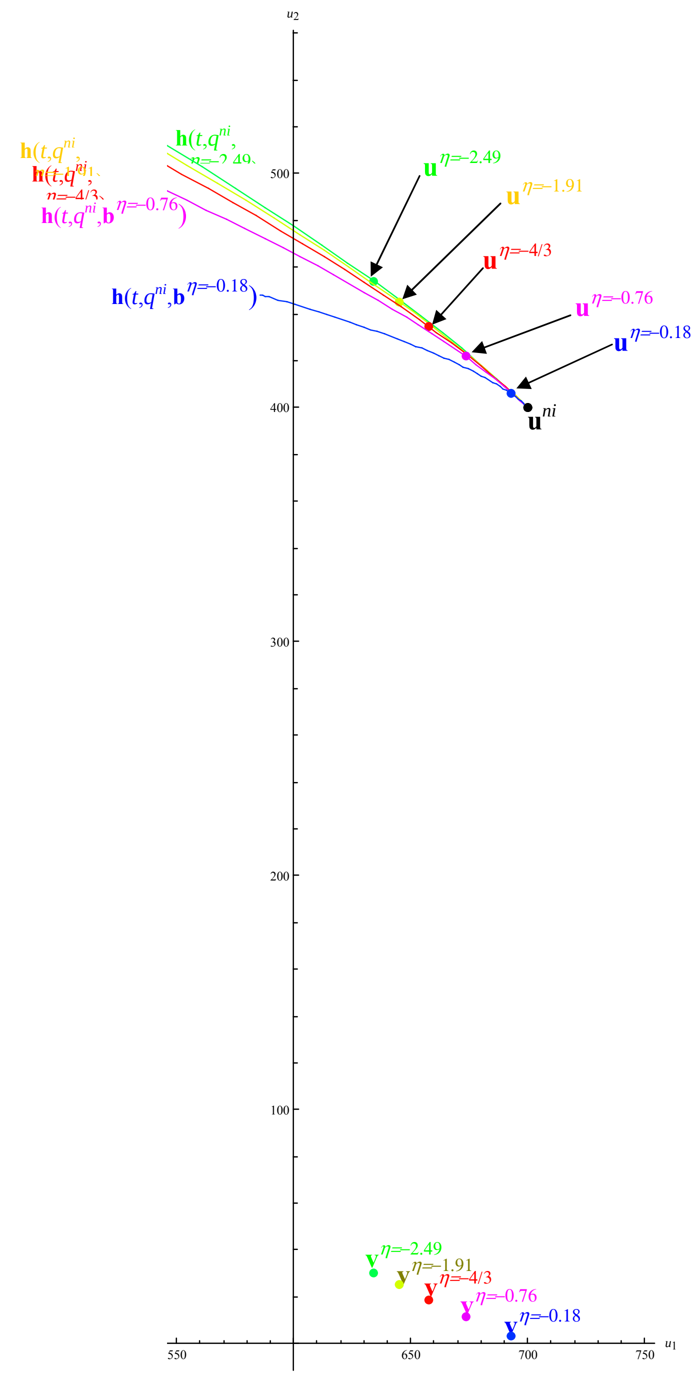

Figure A.3.5. In the Grossman and Helpman model's equilibrium, as demand becomes more elastic, redistributive efficiency increases, and the size of the transfer increases.

[i] The complete economic sub-model will have seven parameters, which are the elements of vector **b** = (a_0, a_1, b_0, b_1, q^H, u_1^{ex}, u_2^{ex}). For notational convenience, we write any function that depends on any of these seven parameters as dependent upon **b**.

[ii] Formally, in Becker interest groups choose political contribution schedules, which are not real numbers but rather real-valued functions of government policy. But in essence, by choosing a schedule they simultaneously choose a level of contribution.

[iii] Nonetheless, I treat government as a "player," because it makes it easier to compare and contrast the Becker model with the G&H-type model that I present later in the paper.

[iv] Though Becker's model actually presents a separate influence function for each group, eventually in the model he sets one identically equal to the negative of the other, so we can achieve the same results using only one influence function in the model. Becker's model only deals with I being positive, but it is easy to slightly modify the framework letting I be positive or negative.

[v] I also set **k** such that each group's best-response curve slopes upward and such that the relative slopes of the groups' best-response curves are like those pictured in Becker's figure I (p. 379), with the taxed group's (group 1's) best-response curve being steeper than the subsidized group's, given that taxed group's strategy variable is on the horizontal axis.

[vi] For notational convenience we can refer to the function $I(x_1, x_2, \mathbf{k})$, as $I^{\#}(x_1, x_2, \mathbf{b}, \mathbf{k})$, though these represent the same function.

[vii] It can be seen that in non-intervention the equilibrium utility level for consumers/taxpayers changes as the supply and demand parameters change. But none of the important elements of my discussion depends on the constant u_1^{ex}. Therefore, in order to keep the non-intervention equilibrium welfare point unchanged, I alter the u_1^{ex} term (and similarly u_2^{ex}), so that the non-intervention equilibrium welfare outcome is always $\left(u_1^{ni}, u_2^{ni}\right)$ = (700, 400). This makes it easier to use surplus transformation curves to visually compare the relative redistribution efficiencies of policies, since then all the surplus transformation curves pass through (700, 400).

[viii] Deadweight is shown as the horizontal (or vertical) distance between the gross welfare outcome (658.07, 434.55) and the 45-degree line passing through $\mathbf{u}^{ni}$ (Gardner 1983).

[ix] That is, when applied to a two-group model in which only one group lobbies, the G&H framework implies that the lobbying group is willing to spend on lobbying an amount the size of their equilibrium gross-of-political-expenditures welfare, less γ times the deadweight loss resultant from the intervention. The result is that producers' political expenditures are almost as large as their gross welfare, and so net welfare for the lobbying group in equilibrium is almost zero, as is illustrated in the appendix.

[x] An intuitive explanation of the shape of the surplus transformation curve of the production tax/subsidy is as follows. We have stated that in principal t can be either positive (a tax) or negative (a subsidy). Increasing the production subsidy (making a negative t more negative) always increases producer welfare and decreases consumer welfare (The supply price rises. But the increases in taxes paid by consumers taxpayers dominates the increase in consumer surplus when the demand price falls.) Increasing a production tax is always bad for producers. Starting from a zero tax and increasing it marginally, consumers-taxpayers must gain, because the increase in tax revenues paid out to them dominates the decrease in consumer surplus caused by the rise in the demand price. As the per-unit tax is further increased, however, eventually tax revenues can fall (a Laffer-effect). Thus, when the production tax is sufficiently high, increasing can harm both producers and consumers-taxpayers. In figure 3, when the tax rises beyond the point that results in welfare outcome E, both consumers-taxpayers and producers are harmed by a further tax increase.

[xi] Our notation distinguishes between the point $\mathbf{u}(t^{ni}, q^{ni}, \mathbf{b}')$ and the set whose single element is that point: $\mathbf{h}(t^{ni}, q^{ni}, \mathbf{b}') = \{\mathbf{u}(t, q, \mathbf{b}'): (t, q) = (t^{ni}, q^{ni})\}$. In figure 3, we label

the set $\mathbf{h}(t^{ni}, q^{ni}, \mathbf{b}')$, not the point $\mathbf{u}(t^{ni}, q^{ni}, \mathbf{b}')$. It so happens that because our model is one of competitive equilibrium in a closed economy with externalities, the conditions of the first theorem of welfare economics are met, and non-intervention by government is Pareto efficient. Thus, the non-intervention welfare outcome is on the Pareto frontier in figure 3. Of course, non-intervention need not be Pareto efficient in economies in which the conditions of the first fundamental theorem are not met. It also so happens that in our model the Pareto frontier is linear with a slope of -1. This happens because the production quota and production tax/subsidy can be combined to transfer income without deadweight loss. (See Just (1984) and Bullock and Salhofer (2003).)

[xii] Josling (1974) was the first to discuss policy outcomes using such a curve, and Gardner (1983) popularized the analytic tool, using the term *surplus transformation curves*, since they illustrate how a single policy instrument can be used to "transform" one interest group's welfare into another's. Surplus transformation curves are one-dimensional manifolds in welfare space, created by varying one policy instrument's level between its lower and upper limits while holding constant all other policy instruments' levels.

[xiii] We are abusing notation slightly, letting $\mathbf{h}(t, q^{ni}, \mathbf{b}')$ represent both a set and a curve.

[xiv] My notation distinguishes between the point $\mathbf{u}(t^{ni}, q^{ni}, \mathbf{b}')$ and the set whose single element is that point: $\mathbf{h}(t^{ni}, q^{ni}, \mathbf{b}') = \{\mathbf{u}(t, q, \mathbf{b}'): (t, q) = (t^{ni}, q^{ni})\}$. In figure A.3.1, I label the set $\mathbf{h}(t^{ni}, q^{ni}, \mathbf{b}')$, not the point $\mathbf{u}(t^{ni}, q^{ni}, \mathbf{b}')$. It so happens that because the model is one of competitive equilibrium in a closed economy with externalities, the conditions of the first theorem of welfare economics are met, and non-intervention by government is Pareto efficient. Thus, the non-intervention welfare outcome is on the Pareto frontier in figure A.3.1. Of course, non-intervention need not be Pareto efficient in economies in

which the conditions of the first fundamental theorem are not met. In this particular model the Pareto frontier is linear with a slope of -1. This happens because the production quota and production tax/subsidy can be combined to transfer income without deadweight loss. (See Just (1984) and Bullock and Salhofer (2003).)

[xv] I are abusing notation slightly, letting $\mathbf{h}(t, q^{ni}, \mathbf{b}')$ represent both a set and a curve.

[xvi] An intuitive explanation of the shape of the surplus transformation curve of the production tax/subsidy is as follows. I have stated that in principal *t* can be either positive (a tax) or negative (a subsidy). Increasing the production subsidy (making a negative *t* more negative) always increases producer welfare and decreases consumer welfare (The supply price rises. But the increases in taxes paid by consumers-taxpayers dominates the increase in consumer surplus when the demand price falls.) Increasing a production tax is always bad for producers. Starting from a zero tax and increasing it marginally, consumers-taxpayers must gain, because the increase in tax revenues paid out to them dominates the decrease in consumer surplus caused by the rise in the demand price. As the per-unit tax is further increased, however, eventually tax revenues can fall (a Laffer-effect). Thus, when the production tax is sufficiently high, increasing it can harm both producers and consumers-taxpayers. In figure A.3.1, when the tax rises beyond the point that results in welfare outcome *E*, both consumers-taxpayers and producers are harmed by a further tax increase.