Nonpoint pollution policy evaluation under ambiguity

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Abstract. Environmental policy evaluation is characterised by a paucity of information. Bounded sets may be more appropriate for representing this ambiguity than traditional probability distributions. A formal calibration method for regional policy models, positive mathematical programming, is thus extended to incorporate parameter definition using bounded sets through the novel method of robust non-linear programming. The resulting procedure identifies strong bounds on the range of abatement costs accruing to environmental policy and improves the relevance and value of modelling studies through not limiting conclusions to realisations of specific point estimates or probability distributions. Moreover, it may easily be solved using standard mathematical-programming algorithms. Empirical insights are provided in an application to a New Zealand inland lake threatened by nitrate pollution from dairy farming. Factor substitution could potentially be used to reduce the abatement costs accruing to regulation. However, such behaviour is shown not to be optimal at the parameter values used in this study. Accordingly, large reductions in nitrate leaching and concomitant improvements in water quality potentially bear a substantial cost to producers.

Keywords. Interval analysis, nonpoint pollution, robust optimisation.

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1. Introduction

Nonpoint pollution (NP) involves the diffuse entry of pollutants into water bodies. Pollution of the world's aquatic environments is now primarily attributable to NP (United Nations Environment Programme, 2008) since point sources are, by definition, generally more easily identified and regulated. A primary source of nonpoint pollution throughout the world is agricultural activity. Key agricultural pollutants are agricultural chemicals (e.g. pesticides); pathogens (e.g. *Escherichia coli*); nutrients, mainly nitrogen (N) and phosphorus (P); soil salts; and soil sediments. These can decrease biodiversity and impair electricity generation, human consumption, provision of inputs for industry (e.g. irrigation), and recreation (e.g. swimming). Moreover, maintaining water quality at reasonable levels can ensure the ongoing provision of nonmarket values, such as existence and option values. Eutrophication of lakes and rivers following nutrient enrichment is particularly widespread and damaging, with more than three-quarters of fresh water bodies in the United States exceeding safe thresholds for total N and P, imposing a cost of around 2.2 billion U. S. dollars annually (Dodds et al., 2009).

Efficient regulation of NP is necessary to sustain environmental quality at apposite levels, while minimising abatement costs. However, formulation of such policy instruments is often problematic as the relationship between agricultural management and pollutant concentration in a given water body is generally difficult to discern. This stems from the diffuse nature of pollution, high number of polluters, asymmetric information between producers and regulators, and temporal variation in pollutant concentration typical of NP problems. Prediction of abatement costs and agent behaviour is also complicated by the presence of complex production relationships, such as substitution between polluting and mitigating factors, and the response of producers to climate and market uncertainty. Furthermore, the benefits of regulation are typically costly and/or difficult to identify.

The stochastic processes that pervade NP-policy problems has motivated a substantial literature exploring the implications of risk (see Shortle and Horan (2001) and Kampas and White (2004) and references therein). However, the definition of specific density functions is difficult to justify for many important parameters, as (a) adequate information is commonly unavailable to guide their accurate estimation, (b) additional data can be costly or time-consuming to attain, (c) information-gathering can be complicated by measurement error, and (d) historical and future values (e.g. for market prices) may only be weakly correlated. Indeed, the evaluation of NP policies is characterised by severe uncertainty (i.e. ambiguity or Knightian uncertainty), which invalidates the use of standard expected-value analysis considering risk (Shaw and Woodward, 2008).

This factor appears to have received no attention in previous economic analysis, despite the availability of appropriate analytical tools. The most-prominent methodological approaches for dealing with ambiguity are found in robust optimisation (RO) (Ben-Tal et al., 2009), in which decision makers are assumed to know the bounded set of outcomes accruing to decisions, but cannot define specific probability distributions. RO is based on the maximin theory of Wald (1950), in which a decision maker is assumed to be constrained by the worst-case realisations of important parameters within a model.

Maximin models provide conservative solutions by construction, as the maximum payoff is determined for the “worst-case” model. However, this conservativeness can be valuable in the context of NP policy evaluation. First, irreversible degradation to environmental systems can bear a large cost, so a precautionary approach to management is often justified. Second, linear policy models typically respond highly elastically to small parameter perturbations. The endogenous stability provided by RO can provide greater realism when evaluating alternative policies. RO also has a number of additional benefits. First, it can strengthen policy evaluation by identifying strong bounds on the range of abatement costs. Second, it can help to prevent the identification of optimal plans that are infeasible or suboptimal in practice following the specification of inappropriate point estimates. Third, RO ensures that the general relevance of model output is not limited through the definition of specific probability distributions. Last, on a pragmatic level, the inclusion of bounded sets in optimisation problems and their subsequent solution is straightforward.

This analysis concerns the evaluation of NP policy instruments under ambiguity using RO. Positive mathematical programming (PMP) (Howitt, 1995; Henry de Frahan et al., 2007), a method commonly used for calibrating regional policy models, is extended to incorporate severe uncertainty using robust non-linear programming (RNLP) (Wu, 2008). This is the first application of RNLP outside illustrative examples and offers economists an additional tool for policy analysis, particularly as such problems can be easily solved using standard non-linear programming (NLP) algorithms. This novel method is applied to a specific case study concerning the mitigation of nitrate enrichment of a New Zealand inland lake.

The paper is structured as follows. Section 2 describes RNLP, PMP, and their integration. Section 3 describes the model used to evaluate various policy options for the case study. Section 4 presents an empirical application of this model. Section 5 concludes.

2. Modelling approach

2.1 Robust non-linear programming

This section provides a short introduction to RNLP following Wu (2008). The closed interval C is a set of numbers in $\Re$ including its endpoints in its membership. This is denoted $C=[c^L,c^U]$, where c^L and c^U are respectively the lower and upper bounds of the interval. The midpoint of an interval is denoted $C^{mid}=0.5(c^L+c^U)$, while its range (a measure of its spread) is defined $C^{ran}=0.5(c^U-c^L)$. A point estimate contains a single point such that $c=[c,c]$ and $C^{ran}=0$. Elementary mathematical operations may be performed on two intervals, say $B=[b^L,b^U]$ and $C=[c^L,c^U]$. Standard rules relevant to this study are $B+C=[b^L+c^L,b^U+c^U]$ and $B-C=[b^L-c^U,b^U-c^L]$.

An interval-valued function $F(\mathbf{x})$ is a closed interval in $\Re$ for the vector $\mathbf{x}\in\Re^n$. This can also be written $F(\mathbf{x})=[F^L(\mathbf{x}),F^U(\mathbf{x})]$, where F^L and F^U are functions in $\Re^n$ that satisfy $F^L(\mathbf{x})\leq F^U(\mathbf{x})$ for $\mathbf{x}\in\Re^n$. This function is differentiable at $\mathbf{x}_0\in\Re^n$ if and only if F^L and F^U are differentiable at $\mathbf{x}_0$. The functions $F^L(\mathbf{x})$ and $F^U(\mathbf{x})$ may have the same functional form, but possess different parameters, or may be entirely disparate. Thus, RNLP can incorporate both parametric and functional uncertainty.

An RNLP problem (**RNP1**) can be defined as: $\max_{\mathbf{x}} J=[\pi^L(\mathbf{x}),\pi^U(\mathbf{x})]$, subject to

$[g^L(\mathbf{x}), g^U(\mathbf{x})] \le 0$ and $\mathbf{x} \ge 0$, where J is the interval-valued objective function and $g(\mathbf{x})$ is the interval-valued constraint functions.

In standard NLP, solutions belong to $\Re$ and thus may be ordered using inequality notation. Closed intervals may not be ordered equivalently; thus, an alternative method of ranking must be specified. Wu (2008) introduces the concept of Pareto optimality from multiobjective programming. Assume $B = [b^L, b^U]$ and $C = [c^L, c^U]$. Then $B \succ C$ if and only if $b^L > c^L$ and $b^U \ge c^U$; $b^L \ge c^L$ and $b^U > c^U$; or $b^L > c^L$ and $b^U > c^U$. B dominates C if any of these sets of conditions hold. A Pareto-optimal maximising solution x^* is one for which no solution $\bar{x} \in X$ exists such that $F(\bar{x}) \succ F(x^*)$.

Wu (2008) prescribes a method of solving **RNP1** through redefining it as (**RNP2**): $\max_{\mathbf{x}} F^L(\mathbf{x}) + F^U(\mathbf{x})$, subject to $g^L(\mathbf{x}) \le 0$, $g^U(\mathbf{x}) \le 0$, and $x \ge 0$. The Karush-Kuhn-Tucker (KKT) conditions characterising a Pareto-optimal solution to **RNP2** are as follows.

Theorem 1 (Wu, 2008, p. 309-310). Assume that x^* is a Pareto-optimal solution to **RNP2** and F and g are differentiable at x^*. Then, there exist multipliers $\boldsymbol{\mu} \ge 0$ and $\boldsymbol{\lambda} \in \Re$ such that $\nabla \pi^L(\mathbf{x}^*) + \nabla \pi^U(\mathbf{x}^*) + \boldsymbol{\mu}' g^L(\mathbf{x}^*) + \boldsymbol{\lambda}' g^U(\mathbf{x}^*) = \mathbf{0}$, $\boldsymbol{\mu}' g^L(\mathbf{x}^*) = \mathbf{0}$, and $\boldsymbol{\lambda}' g^U(\mathbf{x}^*) = \mathbf{0}$. This result only holds provided that a standard Kuhn-Tucker constraint qualification is satisfied at $\mathbf{x}^*$.

Thus, a Pareto-optimal solution to an RNLP may be identified through transcription of **RNP1** to **RNP2** and solution using a standard non-linear programming algorithm. For convenience, a Pareto-optimal solution to a mathematical programming (MP) problem involving interval-valued uncertainty is henceforth referred to as “optimal”.

2.2 Positive mathematical programming

Policy analysis conducted using MP is generally more acceptable to regulators when the standard solution replicates or closely resembles current management over a range of key variables. This is inherently difficult to achieve in regional LP models because optimal solutions typically respond highly elastically within some feasible range and data limitations restrict the accurate depiction of the nonlinearities (e.g. risk aversion) that help describe production choices (Howitt, 1995). LP models can be tightly constrained to reflect the baseline situation; however, this decreases the feasible set of solutions for subsequent policy analysis. An alternative is to use positive mathematical programming (PMP). This method of calibrating MP models is based on the notion that descriptive LP models often fail to converge to baseline situations because the true objective function is non-linear in a subset of the decision variables. PMP is based on the assertion that observed levels are consistent with optimal production behaviour. The following description is based on Howitt (1995) and Henry de Frahan et al. (2007).

PMP requires three stages. Consider a standard LP model (**PMP1**): $\max_{\mathbf{x}} J = \boldsymbol{\pi}'\mathbf{x}$, subject to $\mathbf{Ax} \leq \mathbf{b}$ and $\mathbf{x} \geq \mathbf{0}$, where J is total profit, $\boldsymbol{\pi}$ is a $(n \times 1)$ vector of gross margins, $\mathbf{x}$ is a $(n \times 1)$ vector of decision variables, $\mathbf{A}$ is a $(m \times n)$ matrix of technical coefficients, and $\mathbf{b}$ is a $(m \times 1)$ vector of resource endowments.

The first step involves the addition of calibration constraints to **PMP1** to force the optimisation to return the observed situation, provided it is feasible. This model (**PMP2**) is: $\max_{\mathbf{x}} J = \boldsymbol{\pi}'\mathbf{x}$, subject to $\mathbf{Ax} \leq \mathbf{b}$, $\mathbf{x} \leq (\mathbf{x}^{ob} + \boldsymbol{\varepsilon})$, and $\mathbf{x} \geq \mathbf{0}$, where $\mathbf{x}^{ob}$ is a $(n \times 1)$ vector of baseline activity levels and ε is a $(n \times 1)$ vector of small numbers introduced to prevent degeneracy. The shadow price vector for the calibration constraints is denoted $\boldsymbol{\rho}$.

The second step involves using the dual variables $\boldsymbol{\rho}$ to estimate the parameters of a non-linear function to incorporate in *J*. Most applications of PMP involve specification of a quadratic variable-cost function $C(\mathbf{x}) = \mathbf{d}'\mathbf{x} + \mathbf{0}.5\mathbf{x}'\mathbf{Qx}$, where $\mathbf{d}$ is a vector of cost parameters and $\mathbf{Q}$ is a $(n \times n)$ positive semi-definite matrix of cost parameters. This functional form is simple and consistent with the stylised facts of production theory. The unknowns $\mathbf{d}$ and $\mathbf{Q}$ are estimated by identifying those terms that validate $\nabla C(\mathbf{x})' = \mathbf{d} + \mathbf{Qx} = \mathbf{c} + \boldsymbol{\rho}$, as this ensures exact calibration in the third stage (Howitt, 1995).

The estimation of $\mathbf{d}$ and $\mathbf{Q}$ is underdetermined, but can be overcome through various means (Heckelei and Wolff, 2003; Henry de Frahan et al., 2007). A common and pragmatic approach is to assume (1) $\mathbf{d} = \mathbf{c}$, where $\mathbf{c}$ is the accounting cost contained in $\boldsymbol{\pi}$ in **PMP2** and is recovered through simple decomposition; (2) the marginal cost accruing to one activity is independent of other activity levels; and (3) $q_{ii} = \rho_i / x_i^{ob}$, where q_{ii} is a diagonal term in $\mathbf{Q}$ and ρ_i is the shadow price accruing to the calibration constraint for activity level x_i^{ob}.

The third step involves the specification of the quadratic programming problem (**PMP3**): $\max_{\mathbf{x}} J = \boldsymbol{\pi}'\mathbf{x} - 0.5\mathbf{x}'\mathbf{Q}\mathbf{x}$, subject to $\mathbf{Ax} \le \mathbf{b}$ and $\mathbf{x} \ge \mathbf{0}$. The optimal solution to this problem will calibrate exactly to the defined values of $\mathbf{x}^{ob}$, without the definition of calibration constraints, provided $\nabla C(\mathbf{x})' = \mathbf{d} + \mathbf{Qx} = \mathbf{c} + \boldsymbol{\rho}$ has been satisfied in the second stage.

Heckelei and Wolff (2003) omit the first step and use Generalised Maximum Entropy to estimate model parameters. However, Howitt (2005) has shown that PMP, as described here, is superior when a regional model is to be calibrated in the presence of minimal data, such as that faced in the application described in Section 3.

2.3 Robust positive mathematical programming

The standard PMP process can easily be extended to incorporate interval-valued uncertainty. The following application uses two years of calibration data to establish lower and upper bounds for the non-linear cost function.[1] Exact calibration is not guaranteed given the definition of two cost functions and interval-valued constraints. Nonetheless, practical experience shows that solutions will often be very similar to baseline values. Moreover, this is of secondary importance to more accurately estimating abatement costs through bounding the unknown cost function.

A practical sequence for robust positive mathematical programming (RPMP) is:

1. Construct a robust LP model for the lowest baseline activity level ($\mathbf{x}^{ob,L}$) (**RPMP1**): $\max_{\mathbf{x}} J(\mathbf{x}) = \boldsymbol{\pi}^{L}{}'\mathbf{x}$, subject to $\mathbf{A}^L\mathbf{x} \le \mathbf{b}^L$, $\mathbf{A}^U\mathbf{x} \le \mathbf{b}^U$, and $\mathbf{x} \ge \mathbf{0}$. Parameters in $\boldsymbol{\pi}^L$ are those for the year in which $\mathbf{x}^{ob,L}$ is observed.

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[1] A single year of calibration data may be used, but this will not bound the estimated q_{ii} parameter(s). If three or more years of calibration data are available, lower and upper baseline activity levels can be selected.

2. Construct a robust LP model for the highest baseline activity level ($\mathbf{x}^{ob,U}$) (**RPMP2**): $\max_{\mathbf{x}} J(\mathbf{x}) = \boldsymbol{\pi}^{U}{}'\mathbf{x}$, subject to $\mathbf{A}^{L}\mathbf{x} \leq \mathbf{b}^{L}$, $\mathbf{A}^{U}\mathbf{x} \leq \mathbf{b}^{U}$, and $\mathbf{x} \geq \mathbf{0}$. Parameters in $\boldsymbol{\pi}^{U}$ are those for the year in which $\mathbf{x}^{ob,U}$ is observed.
3. Add calibration constraints to **RPMP1** to obtain (**RPMP3**): $\max_{\mathbf{x}} J(\mathbf{x}) = \boldsymbol{\pi}^{L}{}'\mathbf{x}$, subject to $\mathbf{A}^{L}\mathbf{x} \leq \mathbf{b}^{L}$, $\mathbf{A}^{U}\mathbf{x} \leq \mathbf{b}^{U}$, $\mathbf{x} \leq (\mathbf{x}^{ob,L} + \boldsymbol{\varepsilon})$, and $\mathbf{x} \geq \mathbf{0}$. Calculate the dual values ($\boldsymbol{\rho}^{L}$) associated with the calibration constraints.
4. Add calibration constraints to **RPMP2** to obtain (**RPMP4**): $\max_{\mathbf{x}} J(\mathbf{x}) = \boldsymbol{\pi}^{U}{}'\mathbf{x}$, subject to $\mathbf{A}^{L}\mathbf{x} \leq \mathbf{b}^{L}$, $\mathbf{A}^{U}\mathbf{x} \leq \mathbf{b}^{U}$, $\mathbf{x} \leq (\mathbf{x}^{ob,U} + \boldsymbol{\varepsilon})$, and $\mathbf{x} \geq \mathbf{0}$. Calculate the dual values ($\boldsymbol{\rho}^{U}$) associated with the calibration constraints.
5. Estimate $\mathbf{d}^{L}$ and $\mathbf{Q}^{L}$ from $C(\mathbf{x}) = \mathbf{d}^{L}{}'\mathbf{x} + \mathbf{0}.5\mathbf{x}'\mathbf{Q}^{L}\mathbf{x}$ using $\nabla C(\mathbf{x})' = \mathbf{d}^{L} + \mathbf{Q}^{L}\mathbf{x} = \mathbf{c}^{L} + \boldsymbol{\rho}^{L}$.
6. Estimate $\mathbf{d}^{U}$ and $\mathbf{Q}^{U}$ from $C(\mathbf{x}) = \mathbf{d}^{U}{}'\mathbf{x} + \mathbf{0}.5\mathbf{x}'\mathbf{Q}^{U}\mathbf{x}$ using $\nabla C(\mathbf{x})' = \mathbf{d}^{U} + \mathbf{Q}^{U}\mathbf{x} = \mathbf{c}^{U} + \boldsymbol{\rho}^{U}$.
7. Include the estimated cost functions in an interval-valued quadratic programming model (**RPMP5**): $\max_{\mathbf{x}} J(\mathbf{x}) = (\boldsymbol{\pi}^{L}{}'\mathbf{x} - \mathbf{0}.5\mathbf{x}'\mathbf{Q}^{L}\mathbf{x}) + (\boldsymbol{\pi}^{U}{}'\mathbf{x} - \mathbf{0}.5\mathbf{x}'\mathbf{Q}^{U}\mathbf{x})$, subject to $\mathbf{A}^{L}\mathbf{x} \leq \mathbf{b}^{L}$, $\mathbf{A}^{U}\mathbf{x} \leq \mathbf{b}^{U}$, and $\mathbf{x} \geq \mathbf{0}$.

A number of comments are pertinent:

1. A profit function specific to each baseline activity level ($\mathbf{x}^{L,ob}$, $\mathbf{x}^{U,ob}$) is used in steps 1–4, as these are specifically related to market conditions and the parameters are usually straightforward to obtain anyway. In contrast, it can be difficult to identify specific annual values for technical coefficients, so these are bounded. Nonetheless, this assumption may be relaxed.
2. Construction of **RPMP1–RPMP4** is efficient as they share a common objective-function structure and constraint set.
3. **RPMP1–RPMP5** are often large as the constraints containing uncertain parameters are replicated for each bound. However, restraint linearity and the efficiency of modern solution algorithms promote rapid solution (see below).
4. Linearity of the constraints in **RPMP5** renders Theorem 1 as appropriate given their

satisfaction of standard Kuhn-Tucker constraint qualifications (Bazaraa et al., 2006; Wu, 2008).

5. Replication of constraints reflecting parametric bounds reflects that a significant proportion of the constraint set will be redundant. This feature is commonly associated with degeneracy; however, this has never occurred in practice.
6. It is necessary to state all objective-function coefficients in terms of a base year to aid interpretation and ensure that **RPMP5** output is consistent. For example, all monetary values in the following application are stated in 2008 amounts.

The inherent conservatism of a RPMP model is directly related to the breadth of the uncertainty sets incorporated within it. Such uncertainty sets can easily be removed and their midpoint used as a point estimate. This can be useful to examine the degree of conservatism present in the model. The deterministic analogue of a RPMP can be recovered as follows. For each interval-valued parameter $[c^L, c^U]$, define in the model $[c^L + \vartheta\Lambda(c^U - c^L), c^U - \vartheta\Lambda(c^U - c^L)]$, where $\vartheta = 0.5$ and Λ is a proportionality factor representing the degree of robustness, with $\Lambda = 0$ defining full robustness, $0 < \Lambda < 1$ representing different degrees of certainty, and $\Lambda = 1$ defining the deterministic analogue. Varying Λ allows one to examine the effects of differing degrees of certainty. Alternative values of Λ may be specified for different parameters to investigate the implications of relative certainty among coefficients.

3. Application

3.1 Nitrate pollution of New Zealand freshwater resources

The New Zealand dairy industry is now the dominant agricultural industry in this nation, with dairy products valued at \$7.5 billion comprising 21 per cent of total merchandise exports in the year ending June 2007 (Statistics New Zealand, 2007). The high prices received for dairy products over the last decade have promoted significant intensification of what traditionally has been a low-input, pasture-based system. In fact, between 1997 and 2007, national milk production increased by 33 per cent and stocking rates and per cow production both increased by 12.5 per cent (Livestock Improvement Corporation, 2008). Augmented production intensity follows increasing use of supplementary feeds, particularly maize silage, and nitrogenous fertiliser. Indeed, mean use increased by more than 375 and 300 per cent, respectively, in the study region between 1997 and 2007 (Environment Waikato, 2008a). However, intensification has promoted nitrate leaching and subsequent nutrient enrichment of freshwater bodies (see Monaghan et al. (2007) and references therein).

Lakes Karapiro and Arapuni are hydroelectric dams on the Waikato River, New Zealand's longest watercourse. These lakes are important for electricity generation, recreation, tourism, and have cultural value to local Maori. Algal blooms have been observed in recent years, as nitrate discharges from dairy farms in the surrounding catchment have decreased water quality (Environment Waikato, 2008a, 2008b). N concentrations determine the severity of eutrophication in these lakes since the underlying soils of the catchment possess high native levels of phosphorus.

Dairy farming currently covers 46,984 ha of the catchment, comprising nearly three-quarters of agricultural land in this area. Accordingly, there is an urgent need for Environment Waikato (EW), the regional environmental agency, to establish appropriate regulatory tools to minimise ongoing nutrient enrichment. This analysis contributes to this goal through using RPMP to identify the potential costs of different policy instruments. Though applied to a single catchment, this study is also of national importance given the strong link between production intensity and nitrate leaching in New Zealand dairy systems.

3.2 Model description

This section presents an interval-valued LP (consistent with **RPMP1** and **RPMP2**) calibrated using RPMP. It extends the model of McCall et al. (1999) to include environmental impacts and uncertainty. A detailed description of the model and the source and estimation of parameter values is available in Doole (2009). Lakes Karapiro and Arapuni are henceforth referred to collectively as the "lake".

The model describes a management year consisting of 26 fortnightly periods ($i = [1,2,...,26]$), beginning on 1 July. The first time period follows the last time period in a cyclical fashion. Feed supplies are measured using tonnes of dry matter (DM).

New Zealand dairy farms are typically rotationally grazed. This involves the delineation of a farm into multiple paddocks and the rotation of a herd between individual fields. Intermittent grazing at high stocking rates improves pasture quality, utilisation, and usually production. Producers may spell fields from grazing during periods of substantial pasture growth and harvest them for grass silage.

The regulator is assumed to manage a catchment, or proportion of a catchment, consisting of a fixed area of a hectares. The area of pasture grazed at time *t* that has not been grazed since period *i* is represented by $A_{i,t}^{G}$. Similarly, $A_{i,t}^{SM}$ denotes the area harvested for silage production (i.e. ensiled) at time *t* that has not been grazed since period *i*. In addition, $A_{i,t}^{X}$ represents the area of pasture grazed at time *t* that was ensiled in period *i*. Total land use at time *t* is described by:

$$a \geq \sum_{i=1}^{26}(A_{i,t}^{G} + A_{i,t}^{SM} + A_{i,t}^{X}) + \sum_{i}\sum_{t\#}(A_{i,t\#}^{G} + A_{i,t\#}^{SM} + A_{i,t\#}^{X})_{\forall i\neq t, t>i, t\#>t} + \sum_{i}\sum_{t\#}(A_{i,t\#}^{G} + A_{i,t\#}^{SM} + A_{i,t\#}^{X})_{\forall i\neq t, i>t, t\#>t} \,. \tag{1}$$

Grazing and silage production require pasture biomass to be between given bounds. (These bounds are deterministic and do not represent uncertain coefficients.) Minimum biomass levels (α_t) ensure adequate regrowth and cow intake. Maximum biomass levels (β_t) define thresholds at which senescence and decay reduce pasture growth and digestibility. Grazing ceases at a residual biomass (r_t) to ensure pasture persistence and improve regrowth.

Total feed production in period *t* (P_t^{j} for $j = \{G, SM, X\}$) is:

$$[P_t^{j,L}, P_t^{j,U}] = \sum_{i=1}^{26} A_{i,t}^{j}(r_i^{j} + \sum_{g=i+1}^{t}[b_g^{L}, b_g^{U}] - r_t^{j}) \;\; \forall t \neq i, \tag{2}$$

where b_g represents pasture biomass growth in period *g*.

Eq. 2 is conditioned by the bounds:

$$A^{j}_{i,t}\alpha^{j}_{t} \le A^{j}_{i,t}(r^{j}_{i} + \sum_{g=i+1}^{t}[b^{L}_{g}, b^{U}_{g}]) \le A^{j}_{i,t}\beta^{j}_{t} \quad \forall t \ne i. \tag{3}$$

Pasture growth may be promoted using nitrogen fertiliser:

$$P^{N}_{t} = \sum_{i=1}^{26} f_{i,t} F_{i}, \tag{4}$$

where P^{N}_{t} is the pasture biomass (t ha^{-1}) produced through nitrogen fertilisation in period *t*, $f_{i,t}$ is the yield response (t DM ha^{-1}) in time *t* following application of one tonne of nitrogen fertiliser in period *i*, and F_{i} is the amount of nitrogen fertiliser (t ha^{-1}) applied during period *i*.

Use of nitrogen fertiliser is constrained to represent agronomic and environmental constraints (McCall et al., 1999; Monaghan et al., 2007). The maximum annual application of urea fertiliser is 0.4 t ha^{-1}. An upper limit of 0.1 t ha^{-1} is defined for any six-week period. Also, a maximum of 0.05 t ha^{-1} is specified for each fortnightly period.

Possible herd configurations differ by calving date, lactation length, herd status, and productivity. Calving can begin on July 1, July 15, and August 1. 29, 38, 22, and 11 per cent of each herd is assumed to calve each fortnight following the start of calving. There are five possible lactation lengths: 180, 210, 240, 270, and 300 days. There are two herd classifications: cull or standard. Cull herds can be milked for any of the five lactation lengths, with all cows culled at the end of lactation. In contrast, standard herds can only be milked for 240, 270, and 300 days. There are three possible productivity levels: low, medium, and high. The number of cull herds (45) is the product of 5 lactation lengths, 3 calving dates, and 3 productivity levels. The number of standard herds (27) is the product of 3 lactation lengths, 3 calving dates, and 3 productivity levels. The total number of cows in cull herds is constrained to be no greater than typical levels (17 per cent of total herd size) (Chaston, 2008).

Metabolisable energy (ME) is that available for livestock growth and maintenance after the digestion of feed. Temporal demand for energy depends on the characteristics of the herd. Milk production increases with productivity level (represented by bodyweight) and lactation length for a given calving date. However, the cost of increased production is additional energy demand.

Feed supply is represented as a pool of ME. Energy may be obtained from grazed pasture, grass silage, maize silage, and concentrates. Grass silage is produced on-farm, but maize silage and concentrates are purchased.

The demand and supply of energy is calculated for each fortnightly period through:

$$\begin{gathered}\sum_{h=1}^{72} D_h E_{h,t} \le ([P_t^{G,L}, P_t^{G,U}] + [P_t^{X,L}, P_t^{X,U}] + [P_t^{N,L}, P_t^{N,U}]) u_t^P q_t^P \\ + [P_t^{SF,L}, P_t^{SF,U}] u^S q_t^S + V_t u^V q^V + K_t u^K q^K\end{gathered}, \qquad (5)$$

where D_h represents the number of cows in herd *h*, $E_{h,t}$ represents the energy requirement (measured in MJ of ME per fortnightly period) of a cow in herd *h* at time *t*, u_t represents the proportion of the feed that is consumed by livestock (e.g. u_t^P represents pasture utilisation), q_t is the energy content of each feed at time *t* (MJ ME per t DM), V_t is the amount of maize silage (t DM) fed to cows at time *t*, and K_t is the amount of concentrate (t DM) fed to cows at time *t*.

The total amount of grass silage fed to cows is $[P_t^{SF,L}, P_t^{SF,U}]$. The total amount of grass silage produced is $[P_t^{SM,L}, P_t^{SM,U}]$. It is required that $[P_t^{SM,L}, P_t^{SM,U}] \succ [P_t^{SF,L}, P_t^{SF,U}]$.

The feed intake of cows is constrained so herds do not consume an unrealistic quantity through:

$$\begin{gathered}\sum_{h=1}^{72} D_h I_t^P \ge ([P_t^{G,L}, P_t^{G,U}] + [P_t^{X,L}, P_t^{X,U}] + [P_t^{N,L}, P_t^{N,U}]) u_t^P \\ + [P_t^{SF,L}, P_t^{SF,U}] \Gamma^S u^S + \Gamma^S V_t u^V + \Gamma^K K_t u^K\end{gathered}, \qquad (7)$$

where I_t^P is the maximum per cow intake of pasture dry matter at time *t* (t DM cow^{-1}), Γ^S is the substitution rate of pasture to forage supplements (grass and maize silage), and Γ^K is the substitution rate of pasture to concentrate.

Production impacts nitrate levels in the lake through:

$$[N^L, N^U] = [N_0^L, N_0^U]\Phi_{lake} + \begin{bmatrix} \omega^L\left(\chi^L + \phi^L a^{-1}\sum_{t=1}^{26} F_t + \eta^L a^{-1}\sum_{h=1}^{72} D_h - \tau^L a^{-1}\sum_{t=1}^{26} V_t\right), \\ \omega^U\left(\chi^U + \phi^U a^{-1}\sum_{t=1}^{26} F_t + \eta^U a^{-1}\sum_{h=1}^{72} D_h - \tau^U a^{-1}\sum_{t=1}^{26} V_t\right) \end{bmatrix} \Omega\Phi_{farm}, \qquad (8)$$

where N is the total concentration of nitrate in the lakes (g m^{-3}), N_0 is the current nitrate concentration in the lakes (g m^{-3}), Φ_{lake} is the proportion of total water volume not arising from farm drainage, Φ_{farm} is the proportion of total water volume arising from farm drainage, $\Phi_{lake} + \Phi_{farm} = 1$, and Ω is the inverse of mean annual drainage per hectare on-farm (mm yr^{-1}). The terms in the large closed interval on the RHS of eq. 8 represent linear relationships between production decisions and nitrate leaching loads (kg ha^{-1} yr^{-1}). Within each equation, ω is an attenuation factor representing losses of nitrate between leaching and entry into the lake, χ is a constant term, and $\{\phi, \eta, \tau\}$ are slope coefficients representing the relationship between nitrate leaching and nitrogen fertiliser application, stocking rate, and maize silage use, respectively. Multiplication with Ω converts these loads to concentrations. Φ and Ω can also be defined as intervals, but this requires corresponding information.

Stocking rate is the primary driver of nitrate leaching in New Zealand dairy-farming systems since grazed pastures typically provide more nitrogen than cows require and this is excreted in urine (Monaghan et al., 2007). Nitrogen fertiliser plays an indirect role under standard management, increasing pasture production and hence stocking rate. In contrast, the low N content of maize silage decreases the N excreted by cows, helping to reduce nitrate leaching.

The linear objective function is defined as:

$$\begin{aligned} \max \pi = {} & p^{milk}\sum_{h=1}^{72} D_h z_h + p^{cull}\sum_{h=1}^{45} D_h + p^{calf}\left(\sum_{h=1}^{72} D_h \psi - \sum_{h=1}^{45} D_h \omega\right) - c^D \sum_{h=1}^{72} D_h \\ & - c^S \sum_{i=1}^{26} P_i^{SM} - c^V \sum_{i=1}^{26} V_i - c^K \sum_{i=1}^{26} K_i - c^F \sum_{i=1}^{26} N_i - c^{FC} a \end{aligned}, \qquad (9)$$

where p^{milk} is the price received for milk solids (MS) (\$ t^{-1}), z_h is annual milk production (t cow^{-1}) of a cow in herd *h*, p^{cull} is the price received for one cull cow (\$ cow^{-1}), p^{calf} is the price received for one calf (\$ calf^{-1}), ψ is the calving rate, ω is the replacement rate, c^D is the variable cost associated with a single cow (\$ cow^{-1}), c^S is the cost of conserving grass silage (\$ per t DM), c^V is the cost of maize silage (\$ per t DM), c^K is the cost of concentrate (\$ per t DM), c^F is the cost of nitrogen fertiliser (\$ t^{-1}), and c^{FC} is the fixed cost of production (\$ ha^{-1}). Eq. 9 is maximised subject to the constraints listed above, with all decision variables constrained to be non-negative.

3.3 Parameter values

This section concisely describes the origin of model parameters. A full description is available in Doole (2009).

The area of the catchment consisting of dairy farming is 46,984 ha (AsureQuality, 2008). Nitrogen fertiliser responses and minimum, maximum, and residual pasture masses are taken from McCall et al. (1999). Feed energy, substitution, and utilisation rates are taken from Dexcel (2008), McCall et al. (1999), and Hedley (2007).

Energy demands and milk production in each herd are estimated as follows. The herd is assumed to consist of Holstein-Fresian cows, the dominant breed in the study area (Livestock Improvement Corporation, 2008). Temporal milk production in each herd is described using the widely-used gamma function. Shape parameters from Johnson (2008) are used, while the coefficient specifying maximum daily milk production is determined for each herd using data from McCall et al. (1999) and a Generalised Reduced Gradient (GRG) method (Bazaraa et al., 2006) to perform root-finding. Energy demand as a function of grazing, milk production, and pregnancy is taken from Dexcel (2008).

Pasture production for 1986–2006 is determined using meteorological data from the New Zealand Climate Database (NZCD) (http://cliflo.niwa.co.nz) and a variant of the model of Moir et al. (2000). Parameter values in the model are estimated using a GRG method, with the goal of minimising the squared difference between recorded mean production in the study region (DairyNZ, 2008) and average pasture production over the 20-year period. This achieves an excellent fit, with a sum of squared errors of 0.049. Only production values between the 15th and

85th percentile are defined in the RPMP model, as the extremes are too restrictive to allow feasibility of the calibrated activity levels. Relaxation of the bounds defining pasture growth also improves practical relevance, as production systems designed to withstand the most-extreme conditions will seldom be profitable over the long-term (e.g. because of high levels of supplementary feeding).

The relationship between production decisions and nitrate leaching is determined from the OVERSEER model (Wheeler et al., 2006). Leachate burdens are calculated for multiple combinations of nitrogen fertiliser, stocking rate, and maize silage for a typical farm in the study region. These are regressed using SHAZAM econometrics software (White, 1997). The response surfaces are bounded to represent spatial differences in soil type. Lake volumes are taken from Brown (2005). Annual drainage is determined from the NZCD. Ranges for nitrate concentration in the lake are taken from Environment Waikato (2008b). Attenuation factors are taken from McKergow et al. (2007).

The response surfaces relating primary production choices to leaching load for an allophanic and pumice soil in the study region are displayed in Figs. 1a and 1b, respectively. These figures show the strong relationship between stocking rate and nitrate leaching. Figs. 1c and 1d display the lower and upper bounds, respectively, of the functions relating production intensity to the nitrate concentration of the lake. (The level of maize silage feeding is held at zero to allow the depiction of Figure 1.) The relationships shown in Figs. 1a and 1b are somewhat similar in strength; however, those determining nitrate concentration in the lake (Figs. 1c and 1d) are more disparate given better information describing temporal variation in nitrate concentration in the waterway.

[Insert Figure 1 near here]

Prices for calibration are taken from Livestock Improvement Corporation (2008). The value of supplementary feeds, calves, and cull cows are drawn from different editions of the New Zealand Financial Budget Manual (e.g. Chaston, 2008). Variable and fixed costs are calculated from the Economic Survey (ES) of New Zealand Dairy Farmers (e.g. Dexcel, 2006). Nitrogen fertiliser prices are taken from fertiliser company records. The standard milk price used in the following analysis is \$6000 t^{-1} MS, the schedule price in December 2008.

3.4 Solution of model with Robust Positive Mathematical Programming

Estimates of the lower and upper bound for the total dairy cow population in the catchment are 138,603 (Livestock Improvement Corporation, 2006) and 141,104 (AsureQuality, 2008) in 2005 and 2008 respectively. Unsurprisingly, the linear model does not naturally calibrate to either of these magnitudes. It is hypothesised that a reasonable instrument for calibration is a convex quadratic variable-cost function associated with herd size. These costs may increase with herd size *ceteris paribus* due to the greater need for supplementary feeds, which exhibit substantial price variation due to supply and demand fluctuations; inefficiencies associated with fixed capital (e.g. milking sheds); and soil compaction.

Models **RPMP1**-**RPMP4** are solved using the COIN CLP solver in GAMS Distribution 22.8 (Brooke et al., 2008). These each incorporate 6,500 variables and 13,600 constraints, and are solved in 0.1 seconds. **RPMP5** contains 6,508 variables and 13,607 constraints and is solved in 2.6 seconds using the CONOPT3 solver in GAMS Distribution 22.8. **RPMP5** is slightly larger because it involves the RNLP analogues of eq. 8 and takes longer to solve given its nonlinearity. The GAMS program containing **RPMP1**-**RPMP5** and subsequent policy analysis is available from the authors on request.

The dual variables $\boldsymbol{\rho}^L = \$644.48$ and $\boldsymbol{\rho}^U = \$1316.31$ are identified from **RPMP3** and **RPMP4**, respectively; thus, $Q^L = 0.0047$ and $Q^U = 0.0093$. The only set defined in π in this application is $[Q^L, Q^U]$. This corresponds with the use of RPMP to bound the unknown cost parameter and a focus on determining useful ranges for abatement costs.

3.5 Model scenarios

The base solution presents output for the standard parameter values used in the model. EW currently uses emissions standards elsewhere to improve water quality in a lake. In this vein, the implications of nitrate-leaching reductions of 0–50 per cent from those present in the base solution are investigated. These scenarios are enforced using constraints on the upper bound of leaching load, consistent with a precautionary approach to environmental management. The New Zealand Ministry of Agricultural and Forestry (2008) predicts milk prices between \$5000 t^{-1} MS and \$7000 t^{-1} MS over the next five years. These thresholds are used to explore the implications of different output prices on the range of abatement costs accruing to these emissions standards.

The impacts of achieving 0–50 per cent reductions in nitrate concentration in the lake are also explored. Each scenario is investigated using constraints on the upper bound of nitrate concentration. These improvements are related to indicators specified by Environment Waikato (2008b). These state that total nitrogen (TN) of 0.1–0.5 g m^{-3} indicates a satisfactory level of nutrient enrichment and a TN value less than 0.1 g m^{-3} denotes excellent water quality. (Values for total Kjedahl nitrogen and nitrite, the components of TN apart from nitrate, are sourced from Environment Waikato (2008b).) Moreover, these nitrate concentrations are related to trophic-level indices defined in the model through incorporating equations from National Institute for Water and Atmospheric Research (2006).

The key role of stocking rate in determining leaching loads suggests that restricting livestock density may be an effective policy response, particularly as herd manipulation may help to minimise the abatement costs accruing to these policies. Indeed, stocking-rate restrictions have been introduced in various European nations (e.g. Denmark) following the 1991 European Commission Nitrate Directive. Therefore, the implications of restrictions of 0–50 per cent are explored below.

Use of RO suggests that model output may be conservative in comparison to the specification of point estimates. The implications of specifying (1) all bounded sets as point estimates, and (2) defining just pasture production as uncertain are therefore investigated. These scenarios are explored through manipulation of proportionality factors defined for each uncertain component.

4. Results and Discussion

4.1 Base solution

The optimal solution determined for the standard parameters of the model closely describes production behaviour in the Waikato region. The optimal stocking rate is 2.89 cows ha^{-1}, 1.7 per cent lower than the 2007/08 stocking rate in this region (Livestock Improvement Corporation, 2008). In addition, milk production is 0.354 t cow^{-1}, 0.8 per cent lower than mean production on

177 Waikato farms in the 2006/07 season (DairyBase, 2007). Also, lactation length is 281 days, only 5 per cent longer than mean days in production in the 2006/07 season (Livestock Improvement Corporation, 2008). Moreover, the proportion of an individual cow's diet consisting of imported feed is [11,13] per cent; thus, this model represents the most typical production system (DairyBase, 2007). Farming activity incurs nitrate leaching of [32.6, 36.6] kg N ha^{-1} yr^{-1}, which are within the range of plausible loads arising from New Zealand dairy production (Monaghan et al., 2007). The nitrate concentration in the lake is defined over the interval [0.064, 0.179].

Many of these results highlight a close association between model output and reality. The stocking-rate result obviously arises from the use of formal calibration; however, the results for many key variables (e.g. milk production) do not. Model output should be conservative given the use of bounded parameter estimates in RPMP. In sharp contrast, its rather accurate description of current production levels, even with a broad range describing pasture production, highlights that RO could provide a realistic description of farmer behaviour in some settings.

4.2 Restriction of nitrate emissions

Table 1 presents model results for restrictions on nitrate emissions. Surprisingly, lower-bound profit increases marginally at the lowest N reductions. The definition of intervals in an objective function allows different responses to occur for each bound following a given perturbation. In this instance, lower-bound profit improves marginally as variable cost declines with a decrease in stock numbers. These increases are marginal, so are ignored in subsequent figures.

[Insert Table 1 near here]

The stocking rate and the level of nitrogen fertilisation decrease linearly with the specified leaching restrictions (Table 1). In contrast, the level of maize silage fed to cows fluctuates, but increases markedly at the highest N restrictions. Thus, although low-protein feeds are useful to decrease leaching load, their overall impact is insufficient to warrant significant factor substitution for environmental mitigation. This extends earlier research that reports that the environmental impact of low-N supplementary feeds is also magnified once leaching losses from crop land where the forage was produced are accounted for (Basset-Mens et al., 2009).

An interval-valued function delineating the trade-off between optimal profit and the decrease in nitrate leaching is shown in Figure 2. For example, a 50 per cent decrease in nitrate leaching lowers optimal profit by 37–49 per cent. This range arises from the specification of the bounded quadratic-cost function and represents the uncertainty that constrains a practitioner's capacity to accurately describe the catchment in an analytical framework. (However, of course, this range will depend on other factors also if uncertainty sets are defined for additional parameters in the objective function.)

[Insert Figure 2 near here]

The trade-off between environmental improvement and producer profit is sufficiently strong to take a cautious approach to policy formulation. However, given the inelastic relationship between lower-bound profit and environmental improvement (Figure 2), small but significant enhancements may be implemented without substantial cost. For example, a 20 per cent reduction in nitrate leaching will lower profit by 1–13.5 per cent. This improves the lower (upper) bound of nitrate concentration in the lake by 14 (11) per cent (Table 1). In contrast, greater improvements involve higher decreases in profit. These investments may therefore be difficult to justify as appropriate targets for policy, unless the lakes are regarded as primary assets. The overall cost associated with regulation will depend on the specific goal for the nitrate concentration of the lake specified by EW. The RPMP approach is of considerable benefit in this context because decision-makers may specify a target range, rather than a single value.

Figure 3 presents potential profit losses accruing to different levels of nitrate regulation for different milk prices. For \$5000 t^{-1} MS, a 20 per cent reduction in nitrate leaching will have an abatement cost of [0, 127], representing a decrease in profit by 0–15.8 per cent. In comparison, for \$7000 t^{-1} MS, a 20 per cent reduction in nitrate leaching will have an abatement cost of [81, 416], representing a decrease in profit by 4–13.6 per cent. The lower bound of potential profit loss remains at zero up to a 35 per cent reduction in leaching load at the lower output price (Figure 3). This follows a decrease in marginal revenue product.

[Insert Figure 3 near here]

In comparison, a higher milk price has a number of effects:

- It inflates the value of production losses accruing to nitrate regulation; hence, increasing abatement cost.
- It increases profit. At \$7000 t^{-1} MS, the lower (upper) bound of profit increases above its base value by 98 (61) per cent *ceteris paribus*.
- It inflates the marginal value of maize silage, which promotes its use. This reduces nitrate leaching and allows the maintenance of a higher stocking rate *ceteris paribus*.
- It boosts total revenue, dampening the effect that variable costs specified per cow have on overall profit. This reduces the breadth of the range of potential profit loss (Figure 3), which arises from the uncertain bounds on the quadratic-cost function.

4.3 Improvement of nitrate concentration within the lake

Percentage reductions in the nitrate concentration are more costly than leaching reductions given attenuation and the dilution of drainage water with that present in the lake (Table 2). In fact, profit becomes negative when the nitrate concentration is to be reduced by more than 40 per cent. Accordingly, stocking rate and nitrogen fertiliser application must fall by greater amounts than in the emissions scenario (Section 4.2) to achieve the same proportional environmental improvement. Also, in contrast to the use of emissions standards, maize silage use increases monotonically with the intensity of regulation, as the forage is used strategically to enhance carrying capacity, while helping to offset the impact of this behaviour on water quality.

[Insert Table 2 near here]

The trade-off function for nitrate concentration in Figure 4 is steeper than that depicted in Figure 2 given the attenuation and dilution effects outlined above. This reinforces that achieving large improvements in water quality in the lake will incur substantial costs within this catchment. However, this relationship is of insufficient strength for either bound to ever achieve excellent water quality, as defined by the 0.1 g m^{-3} standard for TN. (In fact, a 50 per cent decline in nitrate concentrations achieves a TN value of [0.214,0.35].) It is difficult to achieve such a rigourous standard since an “excellent” classification is more commonly attributed to pristine waterways, the lake studied here is 129 km downstream from the source of the river (Brown, 2005), and a large proportion of the upstream catchment is used for agriculture.

[Insert Figure 4 near here]

The trophic level index for the lake in the base model is [3.63, 4.12], where a score of 3 signifies a lake with medium nutrient enrichment (i.e. mesotrophic) and a score of 4 signifies a lake with high nutrient enrichment (i.e. eutrophic). It is infeasible at the parameter values specified in the model to transition to a state of low nutrient status. However, ensuring that the lake is never eutrophic (i.e. only mesotrophic) can be achieved with an abatement cost of [\$141, \$606] ha^{-1} or a 15.4 (31.9) per cent reduction in income for the lower (upper) bound, respectively. The regulated solution involves a stocking rate of 2.02 cows ha^{-1}, 99.8 kg N ha^{-1} yr^{-1}, and 0.8 t cow^{-1} of maize silage. This transition is obviously costly; nonetheless, it represents a substantial improvement in water quality.

There is an obvious disparity between the output of existing water quality indicators used by EW and the trophic level index used here. The eutrophic status of the lake indicated by the index calculated here is consistent with the observation of algal blooms in recent years; thus, the TN indicator currently used by EW should ideally be reviewed.

4.4 Manipulation of milk production

Improving the productivity of individual cows or extending lactation length could minimise abatement costs given the strong relationship between nitrate leaching and stocking intensity on New Zealand dairy farms (Figure 1). However, there is no evidence of such behaviour in this model for nitrate restrictions between 0–50 per cent. For example, lactation length and milk production have coefficients of variation of only 0.0027 and 0.0016, respectively, in the emissions scenarios. In addition, experiments with a broad range of plausible stocking-rate restrictions indicate coefficients of variation of only 0.0043 and 0.0023 for productivity and lactation length, respectively. Production on most dairy farms in New Zealand is constrained by the inability of more-productive cows to derive sufficient nutrition from pastures (Clark, 2005). Likewise, in the model, the retention of a predominantly pasture-based diet prevents such increases in production to offset the costs of environmental regulation.

4.5 Comparison between RPMP and deterministic models

RPMP is conservative by construction, so could yield abatement costs that are widely dissimilar from those computed using standard deterministic MP. However, abatement costs computed for different sets of deterministic parameters in the model are intuitively contained within the range specified by the RPMP (Figure 5). Regulation is less costly assuming complete certainty (dashed white line in Figure 5) than when considering only uncertain pasture production (solid white line in Figure 5), particularly when emissions restrictions are more stringent. The area between these two white lines represents the cost accruing to instigating a production plan that is immune to temporal variation in pasture growth. The RPMP solution provides a favourable lower bound given its inclusion of a relationship between variable costs and stocking rate.

[Insert Figure 5 near here]

Figure 5 displays the close relationship between deterministic and robust NLP. As discussed above, the former enters as a special case where the range of bounded coefficients is zero. The breadth of intervals defining uncertainty in the objective function directly determines the width of the "shadow" functions depicted in Figures 2–5. In contrast, defining intervals within the constraint set controls their placement in the co-ordinate space. Optimal solutions derived from RPMP retain their feasibility and optimality for all perturbations of uncertain parameters within their defined bounds. Therefore, as long as their point-estimate analogues are contained within this bounded set, the deterministic abatement-cost relationship will be subsumed in the interval-valued abatement-cost function, as displayed in Figure 5.

5. Conclusions

Economic modelling has a key role to play in environmental policy analysis given its predictive capacity without requiring the extensive data sets required by econometric approaches. Nonetheless, practitioners still face much ambiguity given the cost of information acquisition, measurement error, and often a weak correlation between historical and future states. Failure to consider this uncertainty correctly can promote environmental degradation through misinforming policy evaluation. This analysis extends positive mathematical programming, a pragmatic method for calibrating regional policy models, to incorporate interval-valued parameters where imperfect information complicates the determination of coefficients. This improves the descriptive ability of a regional model and permits the explicit treatment of severe uncertainty.

Robust positive mathematical programming is of value in environmental policy evaluation given (a) the possibility of irreversible degradation, (b) the optimistic solutions that may arise from deterministic mathematical programmes, (c) the subsequent capacity to bound the range of abatement costs accruing to a given policy instrument, (d) the chance to identify robust plans that are immune to parametric variation within the specified bounds, and (e) the straightforward algorithmic solution of these problems. Nonetheless, though closed intervals for parameters are generally straightforward to generate, the identification of appropriate bounds can be time-consuming. There is a direct relationship between the range of interval estimates and the conservativeness of the optimal solution. Accordingly, these bounds should be carefully constructed if the model is to accurately describe a given policy problem. This is particularly pertinent in robust positive mathematical programming, where a calibration constraint could be infeasible given a model's inherent conservatism.

This method is applied to an illustrative example involving regulation of nitrate enrichment of two New Zealand lakes. New Zealand dairy producers possess a number of management options to reduce abatement costs. Use of low-protein supplementary feed can reduce nitrate emissions and the negative impact of reducing livestock density, the primary driver of leaching in these systems, can be buffered through switching to high-producing animals and/or extending lactation length. This analysis highlights that factor substitution is of little value in offsetting the financial impact of nitrate regulation for various reasons. Subsequently, large reductions in nitrate leaching are associated with high levels of abatement cost. Nonetheless, the range of these costs may be favourably broad at lower output prices. Moreover, an inflated output price reduces the range and proportion of income lost when nitrate bounds are defined more restrictively.

A number of extensions of this analysis are worthy of further research. First, specification of bounded environmental goals may arguably be more practical than the use of mean values. Robust positive mathematical programming seems of direct relevance, so it is worthwhile to examine the issues involved with its application in this context. Second, abatement costs may be reduced through the spatial differentiation of environmental policy. Extending robust positive mathematical programming to calibrate individual farms within a microsimulation context would be a practical means of investigating this issue. Third, the capacity of RO to explicitly describe the conservative decision making of many producers could help better represent their behaviour in farm-planning models. The extent to which this contentious hypothesis is true is ultimately an empirical question, and would be an interesting area for further work.

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Table 1. Key model output for proportional reductions in nitrate leaching load.

N leaching reduction (%)	**Profit ($\$$ ha^{-1})**	**Stocking rate (cows ha^{-1})**	**N fertiliser (kg N ha^{-1} yr^{-1})**	**Maize silage (kg cow^{-1})**	**N conc. in lake (g m^{-3})**
0	[985,1906]	2.89	184	689	[0.064,0.179]
5	[987,1874]	2.84	167	748	[0.062,0.175]
10	[988,1811]	2.74	154	766	[0.06,0.017]
15	[984,1737]	2.62	142	774	[0.057,0.165]
20	[975,1650]	2.48	131	764	[0.055,0.160]
25	[960,1543]	2.3	123	718	[0.053,0.155]
30	[925,1437]	2.16	114	714	[0.051,0.151]
35	[867,1317]	2.02	104	744	[0.049,0.146]
40	[775,1219]	2.01	87	948	[0.047,0.141]
45	[705,1081]	1.85	80	950	[0.045,0.136]
50	[625,935]	1.68	72	952	[0.043,0.132]

Table 2. Key model output for proportional reductions in nitrate concentration in the lake.

N conc. reduction (%)	**Profit ($\$$ ha^{-1})**	**Stocking rate (cows ha^{-1})**	**N fertiliser (kg N ha^{-1} yr^{-1})**	**Maize silage (kg cow^{-1})**	**N leaching load (kg ha^{-1} yr^{-1})**
0	[985,1906]	2.89	184	689	[32.6,36,6]
5	[987,1822]	2.76	155	768	[29.5,33.1]
10	[978,1675]	2.52	134	768	[26.5,29.8]
15	[938,1485]	2.23	117	732	[23.5,26.4]
20	[824,1274]	2.02	96	843	[20.5,23]
25	[681,1036]	1.80	77	951	[17.5,19.5]
30	[514,756]	1.48	63	953	[14.5,16.1]
35	[301,462]	1.21	47	1060	[11.5,12.7]
40	[44,164]	1.04	21	1261	[8.4,9.3]
45	[-233,-184]	0.66	13	1306	[5.4,5.9]
50	[-576,-562]	0.35	0	1721	[2.45,2.5]

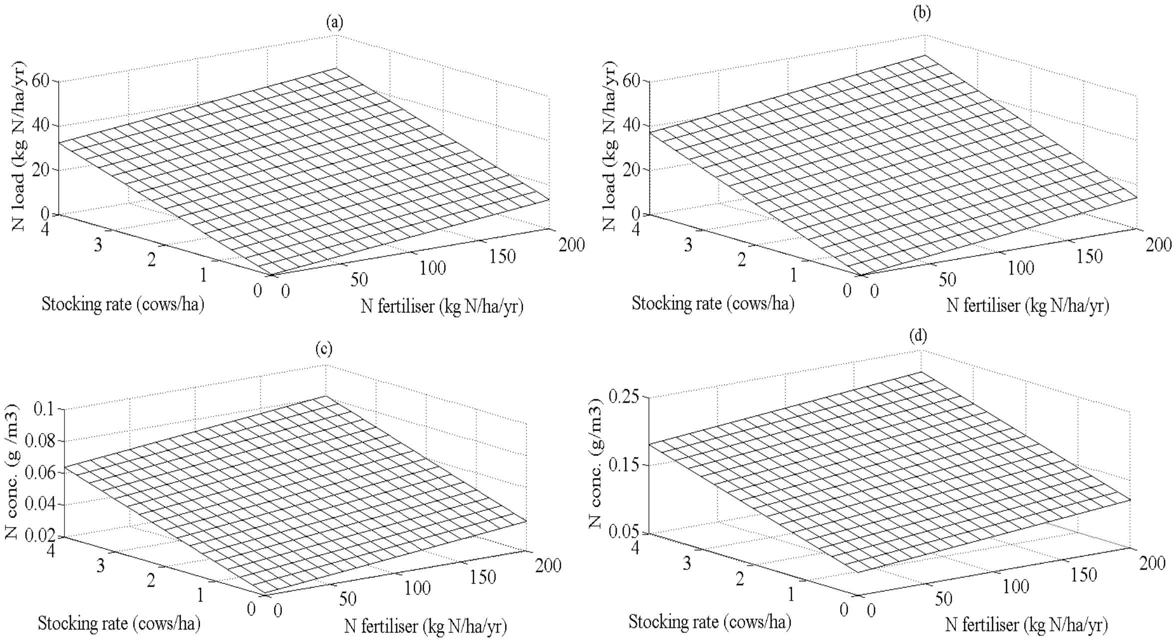

Figure 1. Response surfaces determining nitrate leaching load (kg N ha^{-1} yr^{-1}) as a function of production intensity for an (a) allophonic soil (lower bound) and (b) pumice soil (upper bound). (c) Lower and (d) upper bound relationships describing nitrate concentration (g m^{-3}) in the lake as a function of production intensity.

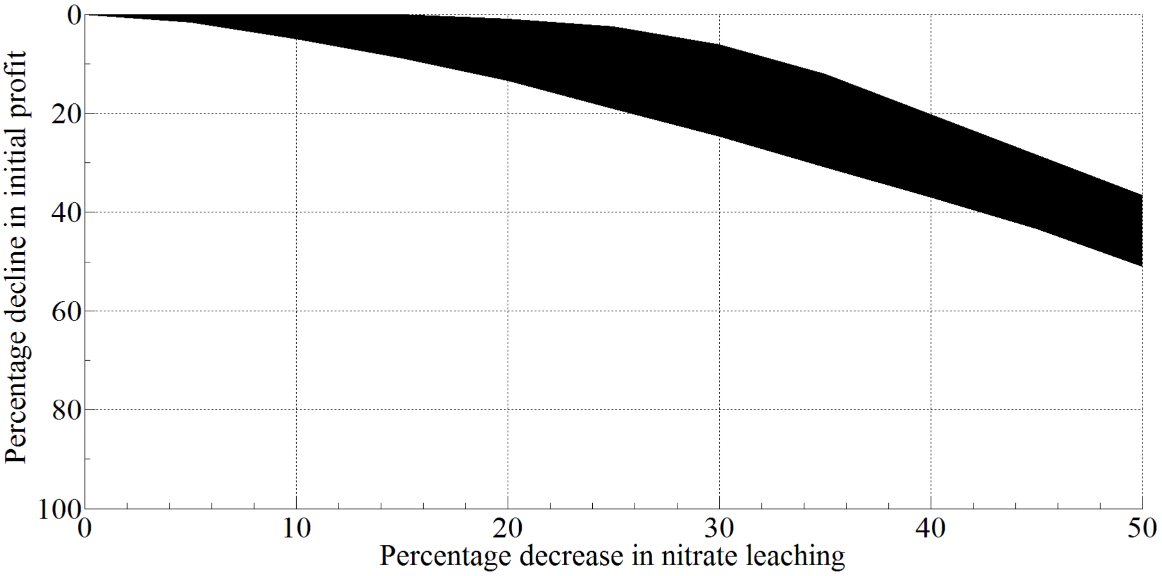

Figure 2. The range of decreases in initial profit accruing to a reduction in nitrate emissions for the standard parameter values.

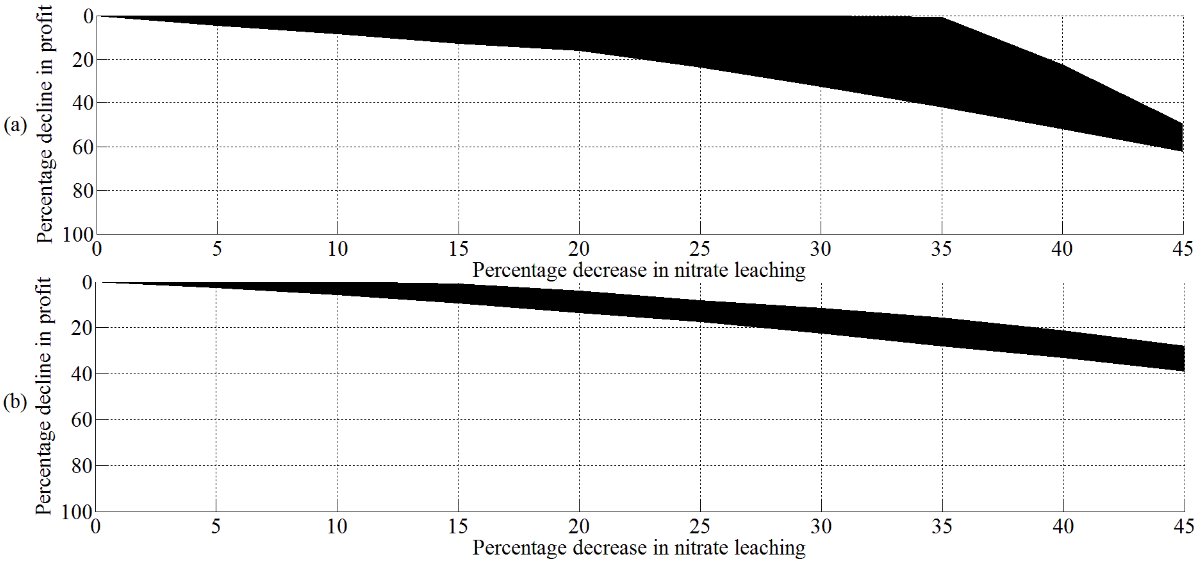

Figure 3. The range of decreases in initial profit accruing to a reduction in nitrate emissions for milk prices of (a) \$5000 t^{-1} MS and (b) \$7000 t^{-1} MS.

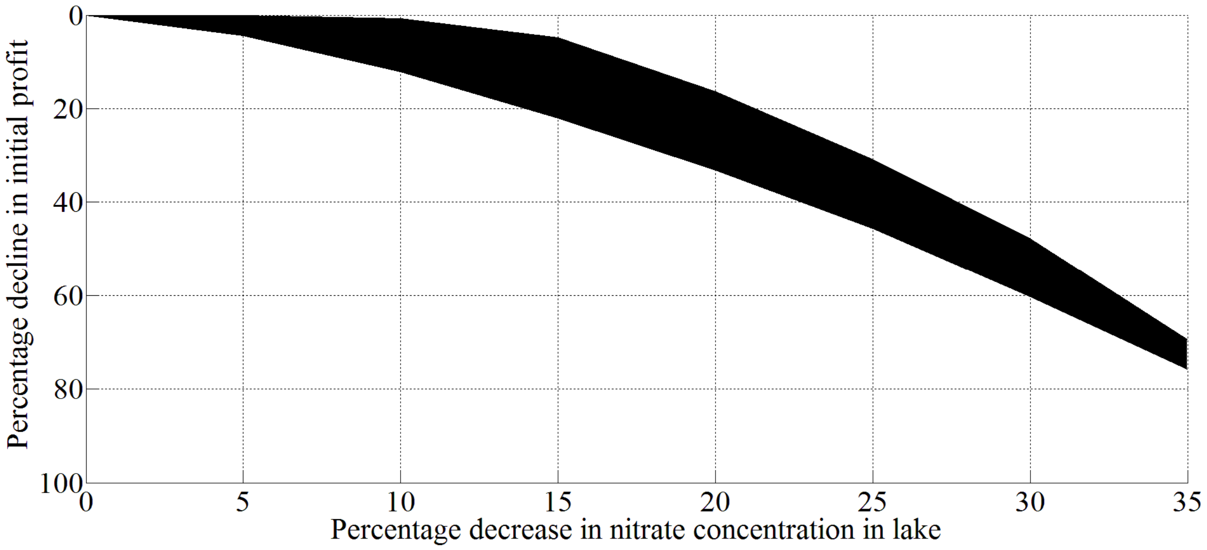

Figure 4. The range of decreases in initial profit accruing to a reduction in the nitrate concentration of the lake for the standard parameter values.

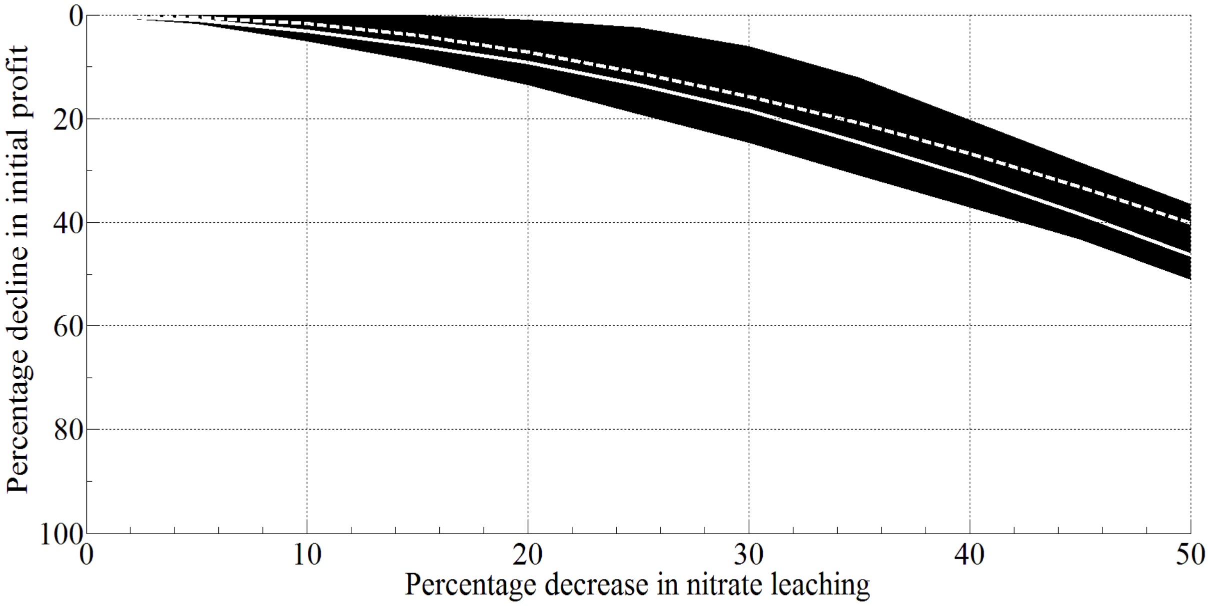

Figure 5. The range of decreases in initial profit accruing to a reduction in nitrate emissions for the standard parameter values. The dashed white line denotes an abatement-cost function when all variables are deterministic. The solid white line denotes an abatement-cost function when pasture growth remains uncertain, but all other variables are deterministic.