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THE OPTIMAL SUPPRESSION OF A LOW-COST TECHNOLOGY BY A DURABLE-GOOD MONOPOLY

by

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Abstract

If a durable-good monopoly can use either of two technologies whose properties are known to consumers, the monopoly uses only the technology with the lowest average cost at low levels of production. If consumers only know about technologies in use, the monopoly may use an inferior technology initially to increase its profits, keeping the new, efficient technology secret and switching later. Thus, in either case, an inferior technology may be used; however, switching between technologies occurs only if consumers are not fully informed about both technologies.

The Optimal Suppression of a Low-Cost Technology by a Durable-Good Monopoly

1. Introduction

We ask two questions about a durable-good monopoly. First, would a durable-good monopoly ever use an inferior technology that has a higher average cost at a given output level than another technology? Second, under what conditions would a durable-good monopoly switch from one known technology to another? Although a static monopoly never uses an inferior technology, we show that a durable-good monopoly may. We also derive a "super Coase Conjecture" result that shows that a shift in technology will not occur if consumers could anticipate it.

Our questions are reminiscent of the widely-held, paranoid story that a major corporation buys the rights to a superior product or technology, suppresses it, and continues to use its inferior technology (or sell its inferior product). Many economists are skeptical of these claims, arguing that such behavior is not profit maximizing. However, even paranoiacs can have enemies, and a durable-good monopoly may suppress a superior technology in equilibrium.

To make our point as starkly as possible, we assume a durable-good monopoly does not fear entry, its product is infinitely durable, and it can choose between two known technologies. Corresponding to each technology is an upward-sloping marginal cost curve. Unless otherwise noted, we assume there are no associated fixed costs. A technology is unambiguously superior only if its marginal cost lies below that of the other technology for all rates of production. If the marginal cost curves cross, each technology is superior for some rates of production. We also assume that the monopoly cannot commit to a particular output path.

Coase (1972) observed that a durable-good monopoly that can only commit for an infinitesimal period may not be able to convince buyers that its future production will be low. As a result of its inability to commit, the firm cannot exploit its monopoly position. Indeed, if that monopoly has a constant marginal cost curve, it earns zero (competitive) profits.[1] A monopoly, however, may credibly commit (and thereby reduce the Coase problem) by renting, through planned obsolescence (including introducing new products too soon), by honoring buy-back provisions, or by developing a reputation for not cutting price.[2] By assumption, our monopoly cannot use such techniques.

Our question is whether the monopoly can choose an inefficient technology as a means of credibly committing to a lower level of output. Such an approach is similar in spirit to using a production process that has a binding capacity constraint. For example, an artist destroys a lithograph plate after making a specified number of copies.[3]

Similarly, an automobile company manufactures a luxury car by hand, so that consumers believe it would be very expensive to increase the rate of production. If the firm used mass production techniques, consumers would expect a higher rate of production (flatter marginal cost curve). By analogy, a durable good manufacturer might use a relatively high marginal cost technology to convince consumers that it will produce less output in the future.

In general, a durable-good monopoly benefits from having an upward-sloping marginal cost curve. Kahn (1986) shows that a monopoly earns positive profits, even if it can only commit for an infinitesimal period, if it has an upward sloping marginal cost curve. Its sales policy has a lower trajectory of stock (cumulative sales) than the socially optimal level, but the monopoly stock asymptotically approaches the socially optimal long-run solution.

Kahn takes the choice of technology as given, whereas we assume it is endogenous. Moreover, we assume that the monopoly cannot make a binding choice of technology in the initial period (or we would be back in Kahn's world). Instead, the monopoly can switch technologies at any time in the future.

We start by assuming that consumers know of the existence of both technologies and that the monopoly can costlessly switch from one technology to the other. We show that the monopoly is faced with a problem similar to that described by Coase: The inability to rule out certain types of future behavior (a switch in technology) constrains current behavior. The monopoly never switches technology in equilibrium. Consumers' knowledge that a monopoly in equilibrium will eventually use the technology associated with the lowest marginal cost at low levels of output, forces the monopoly to use that technology always. That is, we obtain a "super Coase" result similar to the Coase Conjecture. As in the standard Coase Conjecture model, consumers' expectations lead to an erosion of market power. Where the monopoly has a choice of technologies, however, these efficiency gains in distribution may be more than offset by losses of efficiency in production. For example, if the two marginal costs cross, the technology associated with the lower-intercept marginal cost may be inferior in the sense that it yields lower present values of monopoly profits and consumer surplus.

We investigate how robust our results are by relaxing two assumptions. First we suppose that the monopoly has a cost of switching technologies (technology-specific start-up costs). If the switching costs are sufficiently great, switching is not equilibrium strategy. Perversely, it is possible that, with a specific, moderate switching cost, the firm will switch technologies (even though it would not switch if the cost were zero). It is extremely unlikely, however, that such a switching cost would ever be observed.

Next, we examine switching behavior if consumers only know about technologies that are currently in use or have previously been used. With consumer ignorance, switching may occur. Moreover, the monopoly may choose to use an inferior technology initially to increase its profits. As a result, the monopoly wants to keep a new technology secret. Further, the monopoly has little incentive to engage in research to find lower-cost technologies.

The next two sections describe the model and present the results under zero switching costs and full information. The following two sections show, respectively, how positive switching costs and one-sided information alter the model. Our conclusions are in the last section.

2. Full-information, zero cost of switching

A monopoly sells an infinitely durable good. Consumers know that the monopoly may use either of two technologies. The "high-intercept" (h) technology is described by the marginal cost function $C^h(q) = \gamma + \eta q$, where q is the rate of production and sales, $\gamma > 0$, and $\eta > 0$ unless otherwise stated. The "low-intercept" (l) technology is $C^l(q) = \gamma - \varepsilon + \delta\eta q$, with $\varepsilon > 0$ and $\gamma - \varepsilon > 0$. Figure 1 graphs C^h and two examples of C^l: one with $\delta > 1$ and one with $0 < \delta < 1$. If $\delta \leq 1$, C^l is more efficient (lower marginal costs) than C^h for all levels of production. If $\delta > 1$, C^l is the less-expensive technology only for low levels of production. As the stock of the durable good approaches its steady state and the rate of production approaches 0, the monopoly prefers producing using C^l.

Given Kahn's (1986) results concerning the value of increasing marginal costs, one might expect that the monopoly would want to use C^h during the initial production phase and then switch to C^l. This strategy is particularly plausible if δ is close to 0, so that the Coase Conjecture holds approximately, and monopoly profits are very low when C^l is used.[4] We show, however, that under full information and zero switching costs, it is never an equilibrium strategy to switch. Moreover, provided that $\delta > 0$, the monopoly uses only C^l.

We make the following assumptions about the equilibrium:

Assumption 1: The monopoly has an infinitesimal period of commitment (so we use a continuous time model).

Assumption 2: Buyers condition their expectations about future sales on only the current stock of the durable good, Q, and the technology currently in use.

The second assumption says that we are restricting attention to Markov equilibria (where agents condition their beliefs on information that is directly payoff-relevant, such as the current stock and tehcnology, rahter than less tnagible factors such as reputation).[5] These two assumptions imply that, if an equilibrium exists, there is an endogenous price function, $p^i(Q)$, $i = h, l$, that determines how much buyers are willing to pay for a unit of the good given the current state, Q, and the technology in use.[6] The monopoly takes this function as given. The function $p^i(Q)$ depends on the inverse demand for services, $F(Q)$, which is exogenous.

For expositional simplicity, we make

Assumption 3: The monopoly is able to switch costlessly from C^h to C^l, but cannot switch from C^l to C^h. That is, the monopoly may initially use C^h and then switch to C^l later, or start with C^l and remain with that technology.[7]

We explain below why our main result also holds if Assumption 3 is replaced by the weaker assumption:

Assumption 3': The monopoly can switch technologies at most a finite number of times, but those changes in technology are costless.

Optimality Conditions

We now derive the optimality conditions for the monopoly's problem under Assumptions 1 through 3. Let $J^h(Q)$ and $J^l(Q)$ be the monopoly's present discounted value of future profits, where the current stock of the durable good is Q and the current technology is, respectively, C^h or C^l. Given Assumption 3 (costless switching from C^h to C^l), $J^h(Q)$ must be at least as great as $J^l(Q)$. The value function $J^h(Q)$ is

$$J^h(Q_0) = \max_{q,T} \int_0^T e^{-rt}\,[p^h(Q) - AC^h(q)]\,q\,dt + e^{-rT} J^l(Q_T), \qquad (1)$$

where AC^h is the average cost associated with marginal cost C^h; T (which may be zero, finite, or possibly infinite) is the time at which the monopoly switches to C^l; the initial stock is Q_0; and q is the rate of production and sales ($q \equiv dQ/dt$, $q \geq 0$). Given the Markov assumption, the amount consumers are willing to pay for a unit of the good depends only on the current stock of the resource and (possibly) on the technology currently in use. Because the monopoly is not able to commit to future behavior, it takes the price functions as given.

The dynamic programming equation for (1) is

$$rJ^{h}(Q) = \max_{q \geq 0} \left[p^{h}(Q) - \left(\gamma + \frac{\eta}{2}q\right) + J_{Q}^{h}(Q)\right]q. \tag{2}$$

and the first-order condition for q is

$$p^{h}(Q) - \gamma - \eta q + J_{Q}^{h}(Q) = 0. \tag{3}$$

Substituting Equation (3) into (2) implies that

$$rJ^{h} = \frac{\left(p^{h} - \gamma + J_{Q}^{h}\right)^{2}}{2\eta}. \tag{4}$$

Similarly, $J^{l}(Q)$ is the solution to

$$J^{l}(Q) = \max_{q \geq 0} \int_{0}^{\infty} e^{-rt} \left[p^{l}(Q) - AC^{l}(q)\right] q \, dt. \tag{5}$$

The first-order condition for Equation (5) is

$$p^{l}(Q) - \gamma + \varepsilon - \eta\delta q + J_{Q}^{l}(Q) = 0. \tag{6}$$

Substituting Equation (6) into (5) implies that

$$rJ^{l} = \frac{\left(p^{l} - \gamma + \varepsilon + J_{Q}^{l}\right)^{2}}{2\eta\delta}. \tag{7}$$

If a switch from technology C^h to C^l is to occur at some T between 0 and ∞, three conditions must hold. In order to derive these conditions, we start by assuming that there is a Markov Perfect Equilibrium (MPE) that involves switching at the stock $Q^* > Q_0$ so that $T > 0$. From the definition of J^h it must be the case that

$$J^{h}(Q^{*}) = J^{l}(Q^{*}) \equiv J^{*}. \tag{8}$$

Given that consumers have rational expectations (perfect foresight), the equilibrium price must be continuous, even when there is a switch in technology. Therefore, if the monopoly changes technologies when the stock is Q^*, the prices must be equal at the switch:

$$p^{h}(Q^{*}) = p^{l}(Q^{*}) \equiv p^{*}. \tag{9}$$

If the value functions are differentiable,[8] the first derivatives must be equal at the switch as well:

$$J_Q^h(Q^*) = J_Q^l(Q^*) \equiv J_Q^* . \tag{10}$$

Equation (10) is the optimality condition for the value of Q^* that solves Equation (1).

We now derive five results for a durable-good monopoly that has no cost to switching between linear technologies and where consumers have perfect information. In Proposition 1, we show that, for a general demand for services, a necessary condition for the monopoly to switch from C^h to C^l is that $\delta > 1$. Next, we show that, for a linear demand for services, a necessary condition for such a switch is that $\delta < 1$. Because both of these conditions cannot hold, we conclude that there is no MPE in which the monopoly switches when demand and costs are linear (Proposition 2). We then explain, in Proposition 3, why the only MPE involves using C^l always for $\delta > 0$. Next, we point out that Assumption 3 can be replaced by Assumption 3'.

We then discuss the intuition for these results. An example is used to illustrate our intuition. Then we establish that social welfare can be higher when the monopoly is able to commit to using C^h (Proposition 4). Last, we show that for $\delta = 0$ there are two MPE, one of which involves using always C^h and the other of which involves using always C^l (Proposition 5).

Three Results about Switching

Switching technologies will not be an equilibrium in general, as we show using Equations (4) and (7) - (10). If we evaluate Equations (4) and (7) at any candidate for a switch and impose the continuity conditions, Equations (8) - (10), we have two equations in one unknown, Q. As a result, there may not be a value of Q that solves both equations. We now show that, with linear rental demand, there is no solution and thus no switch from one technology to another.

Proposition 1. Under Assumptions 1 - 3, for linear marginal costs and general demand for services, a necessary condition for the monopoly to switch from C^h to C^l is that $\delta > 1$.

Proof: Evaluating Equations (4) and (7) at Q^* and using the continuity conditions, Equations (8) - (10), we find that

$$\delta = \frac{\left(p^* - \gamma + \varepsilon + J_Q^*\right)^2}{\left(p^* - \gamma + J_Q^*\right)^2} > 1. \tag{11}$$

The inequality in Equation (11) holds because, from Equation (4), $p^* - \gamma + J_Q^h = q \geq 0$ and the numerator in Equation (11) is greater than the denominator. Thus, if there exists some Q^* at which it is an equilibrium strategy to switch from C^h to C^l, $\delta > 1$. ■

Next we show that there is no MPE in which a switch from C^h to C^l occurs if the demand for services is linear: $F(Q) = a - bQ$. For a general demand curve, we can solve Equations (4) and (7) to obtain

$$J_Q^h = \gamma - p^h \pm \sqrt{2\eta r J^h}, \tag{12}$$

and

$$J_Q^l = \gamma - \varepsilon - p^l \pm \sqrt{2\delta\eta r J^l}. \tag{13}$$

With a general demand curve, we cannot determine whether the correct differential equations for the value functions use positive or negative signs (the "positive or negative parts") on the last terms of Equations (12) and (13). By restricting rental demand to be linear, however, we are able to show that it is correct to use the negative parts of Equations (12) and (13), as shown in the proof of Proposition 2 below.

For now, suppose we knew that we should use the negative parts of Equations (12) and (13). Then, evaluating Equations (12) and (13) at Q^* and using the continuity conditions, Equations (8) - (10), implies that

$$\varepsilon = (1 - \sqrt{\delta})\sqrt{2\eta r J^*}. \tag{14}$$

Because $\varepsilon > 0$ and $J^* \geq 0$, Equation (13) implies that $\delta < 1$. However, from Proposition (1) we know that a necessary condition for there to be a switch in technology is that $\delta > 1$.

Therefore, in order to show that there can be no MPE that involves a switch in technology, it is sufficient to show that it is correct to use the negative part of Equations (12) and (13). To establish this result, it is necessary only to show that we must take the negative part of (13).[9] If demand is linear, we can obtain closed-form expressions for J^l and carry out the steps described above. We now state

<u>Proposition 2</u>. Under Assumptions 1 - 3 and given that the demand for services is linear, there is no MPE that involves a switch from C^h to C^l.

The proof of Proposition 2 is in Appendix 2.

From Proposition 2, we obtain the chief result of this section:

Proposition 3: Under Assumptions (1) - (3), for linear marginal costs and rental demand, if $\delta > 0$, using only C^l is the only MPE.

Proof: From Proposition 2 we know that no MPE involves a switch in technologies. Using only C^h cannot be an equilibrium, as we can show using a proof by contradiction.

If the monopoly were to use C^h always, the stock would approach the level Q_h given by $F(Q_h)/r = \gamma$ (which is less than the level Q_l given by $F(Q_l)/r = \gamma - \varepsilon$). Under this regime, as Q approaches Q_h, the present discounted value of future profits approaches 0.

The monopoly, however, would want to switch as Q approaches Q_h because the monopoly could earn positive future profits using the low-intercept technology. That is, the present discounted value of future profits would be strictly positive: $J^l(Q_h) > 0$. This inequality follows from the assumption that $\delta > 0$. To establish the inequality, we substitute the necessary condition Equation (6) into Equation (7), which gives $J^l(Q) = \delta\eta q^2/(2r)$. Thus, $J^l(Q)$ is positive if the equilibrium rate of production, q, is positive.

If C^l is used, the equilibrium rate of production, q, must be positive at Q_h, because Q_h is less than Q_l. We can rule out the possibility that, on the equilibrium trajectory, q is 0 over a finite interval. By the Markov assumption, it would not be optimal to stop producing for an interval of time and then to resume. Such behavior would simply defer future profits without achieving any benefit. Thus, if the monopoly produces near Q_l, it must also produce at Q_h. Stopping production forever when the stock is less than Q_l cannot be an equilibrium because additional profits could be earned from additional production. If the monopoly does not produce more, the equilibrium price would be strictly greater than the marginal cost of producing another unit by definition of Q_l.

Thus, we have a contradiction: Producing only on C^h is not an equilibrium strategy. Therefore, the only remaining possibility is to always use C^l. Kahn (1986) shows how to construct the equilibrium when only C^l is used. A sketch of this construction is contained in the proof of Proposition 2. ■

The monopoly must eventually use the low-intercept technology because it is not credible that it stops selling when there are still opportunities for profit. Because the monopoly eventually has to use the low-intercept technology and no switch is possible, it must always use the low-intercept technology.

If Assumption 3 (zero cost of switching from C^h to C^l) is replaced by Assumption 3' (the monopoly can costlessly switch technologies a certain finite number of times), Proposition 3 still holds. Suppose that it is possible for the monopoly to switch at most n times. Along any equilibrium path, the final part of the trajectory must involve the use of C^l for the reasons given above. Therefore, if there is at least one switch, the last switch must be from C^h to C^l. We have shown, however, that this final switch cannot occur. Thus, there cannot be $n > 1$ switches either.

Proposition 3 (and the generalization obtained by replacing Assumption 3 with Assumption 3') is in the spirit of the Coase Conjecture. Coase's insight was that the monopoly's inability to convince buyers that it would not make future sales eliminates its incentive to restrict current sales. In our model, the monopoly's inability to convince buyers that the low-intercept technology will never be used eliminates the monopoly's incentive to delay using the technology.

Intuition

The intuition for the result that the monopoly only uses the low-intercept technology is particularly clear where δ is less than or equal to one. If δ is less than or equal to one so that the marginal cost of the low-intercept technology lies everywhere below that of the high-intercept technology, it is always cost-effective to use the low-intercept technology. Will the monopoly want to use the high-intercept technology nonetheless? As Kahn (1986) showed, a technology with a steeper slope of the marginal cost has greater commitment value in the sense that the monopoly will produce less output in each time period. If δ equals one, the high-intercept technology is less efficient and has no additional commitment value, hence the monopoly never uses it.

If δ is less than one, the high-intercept technology has a steeper-sloped marginal cost curve, and hence would have greater commitment value if the monopoly could convince consumers that it would use only that technology. The monopoly, however, cannot convince consumers that it will never use the low-intercept, flatter-sloped technology. Eventually the monopoly can make positive profits using the low-intercept technology but not the high-intercept one. Thus, the high-intercept technology does not have any commitment value. Because it is also less efficient, the monopoly never uses it.

If δ is greater than one, the low-intercept technology has the steeper slope, and therefore greater commitment value. Because the monopoly will eventually use the low-intercept technology, the steeper-slope provides commitment value. Why not use the high-intercept technology when it is cheaper, however? Our intuition is that only the consumers benefit from using the more efficient technology, not the monopoly. This intuition is clearest in the extreme case where the high-intercept technology has no slope — marginal cost is constant — but that the low-intercept technology has a slope.

An Example

To help develop our intuition about why only the low-intercept technology is used, we consider the special case where the high-intercept marginal cost curve is flat, $C^h = \gamma$, the low-intercept marginal cost curve is steeply upwardly sloping, and the rental demand is linear (though linearity can be replaced with weaker assumptions about curvature). We already know what happens if the monopoly must choose only one of these technologies. If the monopoly uses only the flat, high-intercept technology, it earns zero profits (as Coase conjectured). If the monopoly uses only the steep low-intercept technology, it makes positive profits (because the steep slope gives it commitment value, as Kahn showed), but these profits are small because production costs are large.

Suppose that the monopoly can switch between the technologies. We know that the monopoly will use the low-intercept technology eventually. We, thus, consider two possible strategies. Either it always uses the low-intercept technology, or it starts on the high-intercept technology and then switches.[10]

If the monopoly could *commit* to switching at a particular stock Q^*, there are choices of Q^* such that it could increase its profit by switching. The monopoly, however, *cannot commit* to switch at a particular stock by merely announcing its intention. Consumers will not believe the monopoly unless it is in the monopoly's best interest to switch at that point.

What we will show is that, because the monopoly is not free to choose its switch point, it always uses the low-intercept technology. The monopoly's inability to choose its switch point is analogous to the credibility problem in Coase's Conjecture.

To show these results, let the monopoly's initial stock be Q_0, and suppose that the monopoly announces that it will switch at $Q^* > Q_0$. If consumers believed its announcement, its payoff would be

$$L(Q_0, Q^*) = [p(Q^*) - \gamma](Q^* - Q_0) + J^l(Q^*),$$

where $[p(Q^*) - \gamma](Q^* - Q_0)$ is the amount the monopoly earns from producing and selling $(Q^* - Q_0)$ units using the constant-cost, high-intercept technology,[11] $J^l(Q^*)$ is the value function after the switch when the monopoly uses the low-intercept technology, and $p(Q^*)$ is the price at the switch.[12] Because both the value function $J^l(Q)$ and the price function $p(Q)$ depend on behavior after the switch, the monopoly takes those functions as given in deciding at which point to switch.

The monopoly would like to choose Q^* to maximize $L(Q_0, Q^*)$. The first-order condition for a $Q^* > Q_0$ is

$$G(Q_0, Q^*) \equiv \frac{\partial L}{\partial Q^*} = p'(Q^*)(Q^* - Q_0) + p - \gamma + J^{l'} = 0. \qquad (15)$$

With linear rental demand, we can show that L is concave in Q^*, so this first-order condition is sufficient. Equation 15 gives the monopoly's optimal switch point as an implicit function of the initial stock, which we write as an explicit relation: $Q^* = g(Q_0)$.

The credibility problem arises because, when the monopoly has produced $Q^* - Q_0 = g(Q_0) - Q_0$ and then reconsiders its options, it will want to produce more output before switching to the low-intercept technology.[13] It is unable to commit to switching at an arbitrary value of Q^*, or at an arbitrary time in the future. Unless it announces a value of Q^* at which it is *ex post* optimal to switch, buyers will not believe the announcement, and the monopoly will not receive the price $p(Q^*)$. Thus, because of the inability to commit, the monopoly is not free to choose whatever value of Q^* that it wants This reduces the value of using the high-intercept, low-average cost technology.

In order to determine whether the monopoly ever switches, or uses only the low-intercept technology, the correct comparison is not between profits when only the low-intercept technology is used, and profits when the switch is made at the first-best level of Q^* under full commitment. The correct comparison is between profits when only the low-intercept technology is used, and profits when the switch is made at the *ex post* optimal (credible) level.

An announced switch point $\hat{Q}$ is credible if and only if, when $Q_0 = \hat{Q}$, the optimal value of Q^* also equals $\hat{Q}$, so $\hat{Q}$ must solve[14]

$$G(\hat{Q}, \hat{Q}) = p(\hat{Q}) - \gamma + J^{h\prime}(\hat{Q}) = 0. \tag{16}$$

If this condition holds, the announcement is *ex post* optimal: Once the stock reaches $\hat{Q}$, the announced switch point, the monopoly actually wants to make the switch.

In order to complete the argument, we need to show that the monopoly prefers to start with the low-intercept technology, because the only credible alternative is to switch at $\hat{Q}$. To do so, we need to show that for all initial values Q_0, the value function with a switch at $\hat{Q}$,

$$L(Q_0, \hat{Q}) = [p(\hat{Q}) - \gamma](\hat{Q} - Q_0) + J^l(\hat{Q}),$$

is less than or equal to the payoff from always using the low-intercept technology, $J^l(Q_0)$, as is illustrated in Figure 2. For $Q_0 \geq \hat{Q}$, it is optimal to begin immediately with the low-intercept technology because $L(Q_0, \hat{Q}) = J^l(Q_0)$ for $Q_0 \geq \hat{Q}$. We now show that, when $Q_0 < \hat{Q}$, $L(Q_0, \hat{Q})$ lies below $J^l(Q_0)$, as shown in Figure 2. Because $L(Q_0, \hat{Q})$ is a linear function of Q_0, $\partial L(Q_0, \hat{Q})/\partial Q_0 = -[p(\hat{Q}) - \gamma]$ is constant. From Equation 16, which defines $\hat{Q}$, we know that, $-[p(\hat{Q}) - \gamma] = J^{l\prime}(\hat{Q})$, so L and J^l are tangent at $\hat{Q}$. Thus, because rental demand is linear, J^l is convex, so $L(Q_0,\hat{Q})$ must approach J^l from below as shown in Figure 2.[15]

To summarize, with a flat high-intercept marginal cost curve and a steeply upward sloping low-intercept marginal cost curve, the monopoly would like to produce using the high-intercept technology and then switch to the low-intercept technology <u>if it could credibly commit to making the switch at the stock of its choosing</u>. Because it cannot make such a commitment credibly, its only alternatives are to begin with the low-intercept technology, or begin with the high-intercept technology and switch at the ex post optimal level $\hat{Q}$. It prefers to begin with the low-intercept technology.

Welfare

Although our results are in the spirit of the Coase Conjecture, the welfare implications are different. The Coase Conjecture implies that the inefficiency resulting from monopoly may be negligible. Proposition 3 does not have this positive welfare implication. The Proposition states that C^l will be used even if $\delta > 1$. Therefore, social welfare may be higher if the monopoly were able to commit to using C^h. We state this as:

Proposition 4: Consumer welfare and monopoly profits may be higher if only the high-intercept technology is used than if only the low-intercept technology is used.

Proof: If ε is small and δ very large, the C^l technology is only slightly less expensive than the C^h technology for low levels of output, but is much more expensive for moderate or high levels of output. By making δ sufficiently large, the equilibrium rate of sales can be kept arbitrarily close to 0, and the equilibrium present value of profits and of consumer surplus is also arbitrarily close to 0 under C^l. With the C^h technology, consumer surplus and monopoly profits (because $\eta > 0$) are bounded away from 0. ■

For completeness, we now consider the implications of $\delta = 0$. We make one more assumption:

Assumption 4: $F(Q)$ is decreasing and continuously differentiable and there exists a $\bar{Q}$ such that $F(Q)/r < \gamma - \varepsilon$ for $Q > \bar{Q}$.

Proposition 5: Under Assumptions 1 - 4, for $\delta = 0$ and general demand, there are two Markov equilibrium strategies: Always use C^h or always use C^l.

Proof: We first show that always using C^l is an equilibrium. If the buyers expect the low-intercept technology will be used, then they expect the future price to be constant at $\gamma - \varepsilon$. Given these expectations, the equilibrium price function under either technology is constant: $p^h(Q) \equiv p^l(Q) \equiv \gamma - \varepsilon$. Because buyers are unwilling to pay more than $\gamma - \varepsilon$ under any circumstances, the monopoly would produce nothing if it had to use its high-intercept technology, and there cannot be an equilibrium. The only possible equilibrium (given the hypothesized expectations of the buyers) is to use the low-intercept technology and immediately produce Q_l. This strategy confirms buyers' expectations and is an equilibrium outcome of the type described in the Coase Conjecture.

We now show that there is a MPE in which only C^h is used. By Assumption 3, the monopoly is not able to switch from C^l to C^h. If ever the monopoly begins to use C^l, then, given Assumption 4, there exists a unique MPE in which its future profits are 0 (Gul, Sonnenschein, and Wilson, 1986). Kahn (1986) shows that, if the monopoly uses only C^h, its profits are strictly positive. Therefore the monopoly strictly prefers to continue to use C^h, and confirms buyers' expectations. ■

3. Full information, positive switching costs

We now relax Assumption 3 so that there is a positive cost, S, of switching technologies, such as technology-specific installation costs. We maintain the assumption that consumers have full information about the two technologies.

If technology C^h is in use, the cost of switching to C^l is the installation cost of the latter technology. By choosing the installation costs appropriately, we can insure that either technology is chosen initially. By choosing the switching cost to be sufficiently large, we can insure that it is never an equilibrium policy to change technologies. Therefore, for $S = 0$ or S very large, no switch occurs.

The question remains, however, whether switching can occur in equilibrium for moderate values of S. With linear rental demand and a given set of parameters, we show in Proposition 6 that the set of values of S such that a switch would occur is of measure zero. Thus, it would take an amazing coincidence for a switch to actually occur. Even for a small S, we show in Proposition 7 that there is an equilibrium in which the monopoly always uses the inefficient technology provided that δ is sufficiently small.

We first describe the model if a switch occurs. If the monopoly is using the C^h technology, its problem is

$$J^h(Q_0) = \max_{q,\,T} \int_0^T e^{-rt} \left[p^h(Q) - AC^h(q)\right] q \, dt + e^{-rT} \left[J^l(Q_T) - S\right], \qquad (1')$$

where J^l is now the remaining payoff after the switching cost has been incurred. We investigate whether it is ever an equilibrium policy to switch at T between 0 and ∞. If a switch does occur at a finite T, then, at the corresponding value Q^*,

$$J^h(Q^*) + S = J^l(Q^*) \equiv J^*. \tag{8'}$$

The continuity conditions (9) and (10) must still hold, so the necessary conditions for an interior switch are (4), (7), (8'), (9), and (10).

Equations (11) and (14) are replaced by

$$\delta = \frac{\left(p^* - \gamma + \varepsilon + J_Q^*\right)^2 - 2\delta\eta r S}{\left(p^* - \gamma + J_Q^*\right)^2} \tag{17}$$

and

$$\varepsilon = \sqrt{2\eta r}\left(\sqrt{J^* - S} - \sqrt{\delta J^*}\right). \tag{18}$$

The introduction of S does not affect the value function $J^l(Q)$, so, for linear rental demand, we can calculate this function as was done in Proposition 2. Equations (12) and (13) are unchanged by the introduction of a positive S [because Equations (4) and (7) are unchanged], so it is still correct to take the negative parts. Thus, Equation (18) must hold at a switch, so a necessary condition for a switch is that $\delta < 1$. Equation (17) does not necessarily imply that $\delta > 1$, however, so we are unable to apply the logic of the previous section to show that a switch is impossible.

Nonetheless, Equations (17) and (18) are informative. If we fix all parameter values other than S, we can view these two equations as defining S as functions of Q that solve (17) and (18), which we denote as $S_1(Q)$ and $S_2(Q)$. For a given S, a necessary condition for a switch to occur is that both Equations (17) and (18) are satisfied by a stock Q^*. This condition can be written as

$$S = S_1(Q^*) = S_2(Q^*). \tag{19}$$

By analyzing (19), we now show that a switch can occur in equilibrium only for a "knife-edge" configuration of parameter values:

> <u>Proposition 6:</u> Suppose Assumptions (1) and (2) hold and the rental demand is linear. For any given parameter values (not including S), the set of S such that a switch occurs in equilibrium is of measure zero.

Proof: The proposition states that, if $S_1(Q)$ and $S_2(Q)$ intersect, they intersect only at isolated points: That is, $S_1(Q)$ and $S_2(Q)$ are not coincident on a nondegenerate interval of Q. Suppose $S_1(Q) = S_2(Q)$ did hold over a nondegenerate interval $\mathbf{I}$ of Q. Let $\mathbf{S} = \{S\colon S = S_i(Q);\ Q \in \mathbf{I}\}$ be the set of S where the necessary condition, Equation (19), is met for a switch. That is, for $S \in \mathbf{S}$, which is not of measure zero, there exists a Q that satisfies both Equations (17) and (18). Thus, it is sufficient to show that no such nondegenerate interval $\mathbf{I}$ exists.

Suppose, to the contrary, that there does exist a nondegenerate interval $\mathbf{I}$ such that Equation (19) is satisfied for $Q \in \mathbf{I}$. Then it must be the case that $dS_1/dQ = dS_2/dQ$ for all Q in the interior of $\mathbf{I}$. Differentiating (17) and (18) on $\mathbf{I}$, we obtain

$$\frac{dS_1}{dQ} = \frac{g'(Q)[2g(Q)(1-\delta) + 2\varepsilon]}{2\delta\eta r}, \tag{20}$$

where $g(Q) \equiv p^l(Q) - \gamma + J_Q^l$, and

$$\frac{dS_2}{dQ} = J_Q^l\left[(1-\delta) - \frac{\varepsilon\delta}{2\eta r}\sqrt{\delta J^l}\right]. \tag{21}$$

As shown in the proof of Proposition 2, J^l is quadratic and p^l is linear, so $g(Q)$ is a linear function of Q, and, therefore, dS_1/dQ is a linear function of Q. However, dS_2/dQ is a nonlinear function. Consequently, the equality $dS_1/dQ = dS_2/dQ$ cannot hold identically over a nondegenerate interval **I**. ■

Because the set of S such that a switch could occur is of measure zero, the chance of a switch actually occurring is negligible.[16] We can now show that, if δ is small so that the high-intercept technology is inefficient, the monopoly will always choose it:

Proposition 7: Under the conditions assumed in Proposition 6, for arbitrary $S > 0$, the monopoly always uses the high-intercept technology provided that δ is sufficiently close to 0.

Proof: As δ approaches 0, the profits from the low-intercept technology approach 0, by Kahn's (1986) argument. The high-intercept technology has an increasing marginal cost ($\eta > 0$), hence, if it can be used, the monopoly earns positive profits. Thus, if we can show that an equilibrium path involving the high-intercept technology exists, the monopoly prefers it.

Suppose the monopoly begins with the high-intercept technology. The direct cost of switching is S. The opportunity cost is the profit foregone by not staying with the high-intercept technology. The sum of these two costs is strictly positive. The benefit from the switch is the profit from using the low-intercept technology, which is negligible. Therefore, beginning with, and always using, the high-cost technology is a credible strategy. ■

The possibility that the monopoly will always use the inefficient technology, for arbitrary $S > 0$, requires that δ be small. To understand why, suppose that we fix δ, and let S become small. From Proposition 6, we know that there is no switch (except possibility for a set of parameter values of measure zero), and, following the reasoning in the previous section, we know that the final part of the equilibrium trajectory must involve the technology C^l for sufficiently small S. Therefore, for fixed $\delta > 0$ and sufficiently small S, the monopoly always uses technology C^l in equilibrium.

In particular, if S is sufficiently large that no switch will occur, the monopoly may credibly use either technology. The monopoly may use an inefficient technology if it has greater commitment value. This result does not turn on linearity of either the demand curve or the technologies.

4. Asymmetric Information

Now suppose buyers are imperfectly informed about the technologies characterized by $C^h(q)$ and $C^l(q)$, switching costs are again zero, and rental demand is linear. We make the following assumption about the buyers' information: Buyers believe that the technology the monopoly chose at the initial time is the only technology available; however, if the monopoly deviates from the equilibrium sales trajectory corresponding to that choice of technology, buyers become perfectly informed about the alternative technology. [With a less extreme assumption about how information is revealed, we expect that the monopoly would be even more likely to take advantage of the buyers' ignorance.] If the monopoly begins using C^h, then in order to keep buyers ignorant of the existence of C^l, it must behave as if C^h is the only technology available.

In contrast to the full-information case, where there is imperfect information, a switch may occur. We illustrate this possibility using an example. With asymmetric information the price need not be continuous at the time of the switch, unlike with full information. That is, Equation (9) and Propositions (1) - (3) no longer hold.

We showed that, if buyers are perfectly informed, there are no costs of switching, and $\delta > 0$, in the MPE the firm uses only C^l. As a result, with limited information (of the type we assumed above), the monopoly will never begin using C^l with the intention of switching later. If it were to do so, buyers would be perfectly informed at the time of the switch, and thereafter the monopoly would have no alternative but to use C^l. Therefore, the only interesting possibility is for the monopoly to begin with C^h, and at some time switch to C^l. If the monopoly does begin with C^h, then eventually it will be optimal to switch. We show that it may be optimal to begin with C^h, and we show how the value of the switching point depends on exogenous parameters.

In Appendix 3, we derive the equation used in our simulation. Using the parameters $r = .1$, $\varepsilon = .1$, $a = 10$, $b = 1$, $\gamma = 1$, and $\eta = 1$, we plot the switch point, Q^*, as a function of δ, in Figure 3. For these parameter values, the maximum stock that would be produced usingonly C^h, Q_h, equals 9.9. If δ is close to zero, the monopoly does not want to switch unless the price is close to the marginal cost at a nearly zero rate of production, which means that Q is close to Q_h. For δ close to one, the monopoly wants to switch almost immediately.[17]

These results are consistent with our earlier intuition concerning Proposition 3. When δ is small, profits are low when the low-intercept technology is used. Therefore, the monopoly will only switch when stock is close to Q_h, so that there are virtually no additional profits to be had from continuing to use the high-intercept technology. When δ is close to one, however, both technologies have essentially the same commitment value, so that the monopoly may as well use the more efficient technology for virtually the entire path.

5. Conclusions

A durable-good monopoly with access to two technologies wants to use the technology or combination of technologies that produce the highest profit. Its profit depends on its instantaneous costs of production and on consumers' beliefs about its future production.

If consumers know that two linear technologies exist and that the monopoly has no cost to switching, there is no benefit to the monopoly from using the technology with the higher marginal cost intercept. Consumers are not fooled into believing that the monopoly's future production is constrained, so we have a "super Coase Conjecture" result. The monopoly chooses the technology with the lower marginal cost intercept and does not switch technologies. As a result, if the marginal costs corresponding to the two technologies cross, the monopoly uses the inefficient technology for some range of output.

At one particular positive cost of switching, the monopoly may switch technologies even if consumers have full-information. Because this result depends on a knife-edge set of parameters, however, for all practical purposes, switching does not occur if consumers know about both technologies.

If consumers do not know about the second technology, as long as the monopoly behaves as though it has access to only one technology, the monopoly may find it profitable to switch. Again, for certain ranges of parameters, the monopoly uses the inefficient technology. The monopoly may start with an inefficient technology and later switch to an efficient one, effectively suppressing the efficient technology for a while.

Thus, whether or not consumers have full information, the monopoly may use an inefficient technology. The monopoly is only likely to switch technologies, however, if consumers do not have full information.

Appendix A

Alternative Proof of Proposition 1

We provide a proof of Proposition 1 which does not require $J^h(Q)$ to be differentiable at the switch, Q^*. This proof requires that $J^l(Q)$ be differentiable at Q^*, however, we know that $J_l(Q)$ is differentiable. By Assumption 3, we know that once the monopoly begins to use the low-intercept technology, it must continue to use that technology. Therefore, given the assumption of linear demand for services, once the low-intercept technology is used, we have the continuous time version of Kahn's model. For that model, we obtain explicitly the quadratic value function $J^l(Q)$, which is differentiable.

Let H^h be the current-value Hamiltonian associated with the control problem in Equation (1), and let λ^h be the costate variable associated with that problem. The first-order condition to this control problem is

$$p^h(Q) - \gamma - \eta q + \lambda^h = 0. \tag{A1}$$

If we substitute the solution to Equation (A1) into the Hamiltonian H^h we obtain

$$H^h = \frac{(p^h - \gamma + \lambda^h)^2}{2\eta}. \tag{A2}$$

The optimality conditions at the switch are

$$H^h(T) = \left[p^h - (\gamma + \frac{\eta}{2}q) + \lambda^h\right]q = rJ^l(Q). \tag{A3}$$

$$\lambda^h(T) = \frac{dJ^l(Q^*)}{dQ}. \tag{A4}$$

Denote the Hamiltonian for the control problem once the switch has been made as H^l, and denote the costate variable for that control problem as λ^l. The first-order condition for q in this problem is

$$p^l(Q) - \gamma + \varepsilon - \eta\delta q + \lambda^l = 0. \tag{A5}$$

If we substitute the solution to Equation (A5) into the Hamiltonian H^l we obtain

$$H^l = \frac{(p^l - \gamma + \varepsilon + \lambda^l)^2}{2\eta\delta}. \tag{A6}$$

Because J^l is differentiable and given theorem 3.10 of Seierstad and Sydsaeter (1987), we know that $H^l = rJ^l$ and $dJ^l(Q_T)/dQ = \lambda^l(T)$. Substituting these two equations in Equations (A3) and (A4), we obtain

$$H^h(Q^*) = H^l(Q^*) \equiv H^*. \tag{A7}$$

$$\lambda^h(Q^*) = \lambda^l(Q^*) \equiv \lambda^*. \tag{A8}$$

By evaluating Equations (A2) and (A6) at Q^* and using the continuity conditions (A7), (A8) and (9), we obtain Equation (11).

Appendix B

Proof of Proposition 2

The outline of the proof was given above the statement of the proposition. It remains only to show that the correct root of Equation (13) is the negative part. That is, we need to show that

$$J_Q^I = \gamma - \varepsilon - p^I - \sqrt{2\delta\eta r J^I}\,. \tag{B1}$$

For notational simplicity, we define $\bar{\gamma} = \gamma - \varepsilon$ and $\bar{\eta} = \delta\eta$ and suppress the superscript "I" because there is no ambiguity, so that we can rewrite Equation (B1) as

$$J_Q = \bar{\gamma} - p - \sqrt{2\bar{\eta} r J}\ . \tag{B2}$$

There are three steps to the proof. First, we obtain a differential equation for price. Second, we show how to construct the solution to the differential equations for the price and value (Equation 7) functions. Third, using these solutions, we conclude by showing that the value function has to satisfy Equation (B2).

Step 1: Given rational expectations (perfect foresight), the price function must satisfy

$$p(Q_t) = \int_t^\infty e^{r(t-\tau)} F(Q_\tau)\mathrm{d}\tau\,.$$

Differentiating this expression with respect to time, we obtain

$$\dot{p} = rp - F(Q). \tag{B3}$$

That is, in equilibrium, the capital losses of owning a unit of the stock must equal the interest costs of buying the unit less the implicit rent (the "dividend"). Because the equilibrium price is a function of Q, Equation (B3) implies

$$\dot{p} = p'(Q)q = rp - F(Q). \tag{B4}$$

Step 2: Equations (7) and (B4) are a system of ordinary differential equations for J and p, the value and price functions. The boundary conditions for this system are $p(Q_l) = \bar{\gamma}$ and $J(Q_l) = 0$, where Q_l is the solution to $F(Q_l)/r = \bar{\gamma}$. In a MPE, the stock must converge to Q_l, the level at which the steady state price equals the cost of producing another unit. The monopoly cannot commit to stopping production where the price exceeds the production costs because it leaves unexploited opportunities for profit, and it will never produce where costs exceed the price. By definition, at stock Q_l, future profits are 0.

In order to find the equilibrium, we need to solve Equations (7) and (B4) subject to these boundary conditions. Kahn (1986) has solved this problem and shown how to derive the unique equilibrium, in which the value function is quadratic and the price function is linear.[18] Thus, the solution to Equations (7) and (B4) may be written as $J(Q) = \alpha + \beta Q + ½(\rho Q^2)$ and $p(Q) = A - BQ$. Using this linear-quadratic solution, we show that Equation (B2) must be satisfied by deriving two conditions that ρ and B must satisfy.

We now obtain the first condition on ρ and B. We substitute the functions for $J(Q)$ and $p(Q)$ in Equation (7). The resulting equation is of the form $\omega_0 + \omega_1 Q + \omega_2 Q^2 = 0$, where the ω_i are functions of the parameters α, β, ρ, A, and B. This equation in Q must hold for all values of Q, implying that $\omega_0 = \omega_1 = \omega_2 = 0$. Our first condition is that $\omega_2 = 0$, which can be written as

$$(B - \rho)^2 = \bar{\eta} r \rho. \tag{B5}$$

We now obtain our second condition on ρ and B. We substitute the quadratic and linear expressions for $J(Q)$ and $p(Q)$ into Equation (B4). Next, we substitute for q using the first-order condition, Equation (6). The resulting equation is a linear function, $\lambda_0 + \lambda_1 Q = 0$, where λ_i is a function of the parameters α, β, ρ, A, and B. Because this condition must hold for all values of Q, we know that $\lambda_0 = \lambda_1 = 0$. The restriction $\lambda_1 = 0$ can be written as

$$(B - \rho)B = -\bar{\eta}(rB - b). \tag{B6}$$

Equations (B5) and (B6) are two equations in two unknowns, ρ and B. Using $\theta \equiv \rho - B$, we can rewrite Equations (B5) and (B6) as

$$\theta^2 = \bar{\eta} r (\theta + B), \tag{B7}$$

and

$$\theta = \frac{\bar{\eta}(rB - b)}{B} \equiv f(B). \tag{B8}$$

Equation (B7) has two roots, $\frac{1}{2}\left(\bar{\eta}r \pm \sqrt{\bar{\eta}^2 r^2 + 4\bar{\eta}rB}\right)$, which we denote $\theta_+(B)$ and $\theta_-(B)$. It is straightforward to verify that $\theta_+(B) \geq \bar{\eta}r$ and $\theta_+(B)$ is strictly increasing, whereas $\theta_-(B)$ is nonpositive and strictly decreasing. The function $f(B)$ defined in Equation (B8) is strictly increasing, $f(B) < \bar{\eta}r$, and the limit as $B \to 0$ is $-\infty$. Therefore the solution to Equations (B7) and (B8), $\tilde{\theta} < 0$, is unique and given by the intersection of $\theta_-(B)$ and $f(B)$, as illustrated in Figure B1.

<u>Step 3</u>: We now use this result to show that J is the solution to Equation (B2). That is, we show that it is correct to take the negative part of Equation (13). Suppose to the contrary that it was correct to take the positive part of Equation (13). Then, substituting the linear-quadratic functions for J and p in Equation (13) gives

$$\beta + \rho Q = \bar{\gamma} - (A - BQ) + \sqrt{2\bar{\eta}r\,[\alpha + \beta Q + \tfrac{1}{2}\rho Q^2]}\ . \tag{B9}$$

Because Equation (B9) must hold for all Q, it also holds as $Q \to \infty$. Dividing both sides of Equation (B9) by Q and taking the limit as $Q \to \infty$ implies

$$\rho = B + \sqrt{\bar{\eta}r\rho}\ , \tag{B10}$$

which, using the definition of θ, gives $\theta = \rho - B = (\bar{\eta}r\rho)^{1/2} > 0$. This inequality contradicts the earlier result that the equilibrium value of θ is negative. Therefore we conclude that Equation (B2) is the correct equation. That is, it is correct to use the negative part of Equation (13), which is all that we needed to show. ■

Appendix C

Asymmetric Information

When consumers do not initially know that the low-intercept technology exists, the monopoly may want to start on the high-intercept technology and later switch to the low-intercept technology. We now derive necessary conditions for a switch to occur.

To fool consumers that it will continue to use the high-intercept technology indefinitely, the monopoly must produce initially according to the equilibrium control rule corresponding to C^h,

$$q^h \equiv \frac{(A^h + \beta^h - \gamma) - (B^h - \rho^h)\, Q}{\eta} \equiv \frac{\mu_0 - \mu_1 Q}{\eta}, \tag{C1}$$

where the parameters A, B, ρ, and β are as defined in Appendix B, and the superscript h indicates that these parameters correspond to technology C^h. The endogenous price corresponding to this technology is $p^h(Q) \equiv A^h - B^h Q$.

The problem of choosing the stock at the switching time, Q^*, is equivalent to choosing the time, T, of the switch. The monopoly's problem can be written as

$$J^h(Q_t) = \max_T \int_t^T e^{r(t-\tau)}[p^h(Q) - C^h(q^h)]q^h d\tau + e^{r(t-T)}J^l(Q). \tag{C2}$$

subject to $dQ/dt = q^h$. Because p^h and q^h are linear functions of Q, and C^h is linear in q^h, this problem can be written as

$$J^h(Q_t) = \max_T \int_t^T e^{r(t-\tau)}\left[\sigma_1 + \sigma_2 Q + \sigma_3 \frac{Q^2}{2}\right] d\tau + e^{r(t-T)} J^l(Q), \qquad \text{(C3)}$$

where σ_i are obtained by substituting p^h and q^h into Equation (C2) and collecting terms.

The first-order condition for T is

$$-r\left(\alpha^l + \beta^l Q + \frac{\rho^l Q^2}{2}\right) + \sigma_1 + \sigma_2 Q + \frac{\sigma_3 Q^2}{2} + (\beta^l + \rho^l Q)\frac{\mu_0 - \mu_1 Q}{\eta} = 0. \qquad \text{(C4)}$$

This quadratic expression has two roots. In all the simulations we performed, the roots are real, and one is greater than Q_h, the maximum stock that would ever be produced using technology C^h. Therefore, in these simulations, there is only one feasible candidate for Q^*. In order to show that this Q^* is indeed a (local) maximum, it is necessary and sufficient to show that $\partial^2 J^h(Q^*)/\partial Q^2 > \rho^l$. That is, the curve J^h intersects J^l from above, so switching earlier reduces the present value of the monopoly's profits. In our simulations, we verify that this inequality is satisfied.

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FOOTNOTES

[1] See, for example, Stokey (1981), Bulow (1982), and Gul, Sonnenschein, and Wilson (1986) for proofs of the Coase conjecture under various conditions.

[2] See, e. g., Swan (1972), Bulow (1982, 1986), Ausubel and Deneckere (1989), and Waldman (1993).

[3] Similar points about planned obsolescence are made by Bulow (1986) and Waldman (1993).

[4] This statement holds if the equilibrium is continuous in δ at $\delta = 0$, which is true in the linear demand and marginal cost example we examine below. It may be possible, however, to construct counter-examples where the equilibrium is not continuous.

[5] Without the Markov assumption, almost any outcome can be supported as an equilibrium. Welfare analysis is difficult in models with a multiplicity of equilibria.

[6] We assume existence. Because our chief result is based on linear primitive functions for which we explicitly construct the endogenous price function $p^j(Q)$, and because we show that a switch cannot occur, Kahn's proof of existence applies here.

[7] We show below that the monopoly never wants to switch in the other direction.

[8] In Appendix 1, we provide a proof that does not require differentiability of J^h at the switch. We also explain why we know that J^l is differentiable for linear rental demand, which is the special case we use for our main results.

[9] This conclusion stems from the continuity conditions, Equations (7) - (9). These conditions imply that, if the correct root is the negative part of Equation (13), at Q*, the correct root of Equation (12) must also be the negative part.

[10] Using assumption 3 or 3', we can ignore the possibility of several switches.

[11] When the monopoly is producing using constant costs, it's inability to commit prevents it from raising the price by slowing production. Therefore, in equilibrium, it produces the amount $Q^* - Q_0$ immediately, which is the Coase Conjecture result. The switch, Q^* is endogenous.

[12] In the discussion above Equation (9), we showed that price is continuous at a switch point.

[13] Using the first-order condition, $dg/dQ_0 = p'(Q)/[\partial^2 L/\partial Q^{*2}] > 0$ when L is concave in Q^*.

[14] For linear rental demand, $\hat{Q}$ exists and is unique. The following argument requires existence, but does not require uniqueness.

[15] If rental demand is linear, J^l is quadratic, and, because it is bounded below, it must be convex. Our results hold more generally where J^l is convex and L is concave.

[16] We ran a number of simulations to verify that there may be a single solution to (17). For example, for $\delta = .5$, $r = .1$, $\varepsilon = .1$, $a = 10$, $b = 1$, $\gamma = 1$, and $\eta = 1$, $Q^* = 8.88$ and $S^* = .575$.

[17] We were unable to find a set of parameters for which a switch occurred when $\delta > 1$. If no switch occurs, the monopoly uses the low-intercept technology for the reasons given above.

[18] Kahn shows, in the discrete stage, finite horizon problem with linear rental demand and linear marginal cost, the value function is quadratic and the price function is linear. In the limit as the horizon becomes infinite and the length of each stage approaches zero (the problem becomes continuous), there is a unique linear-quadratic equilibrium.

Figure 1

Marginal Cost Curves of the Two Technologies

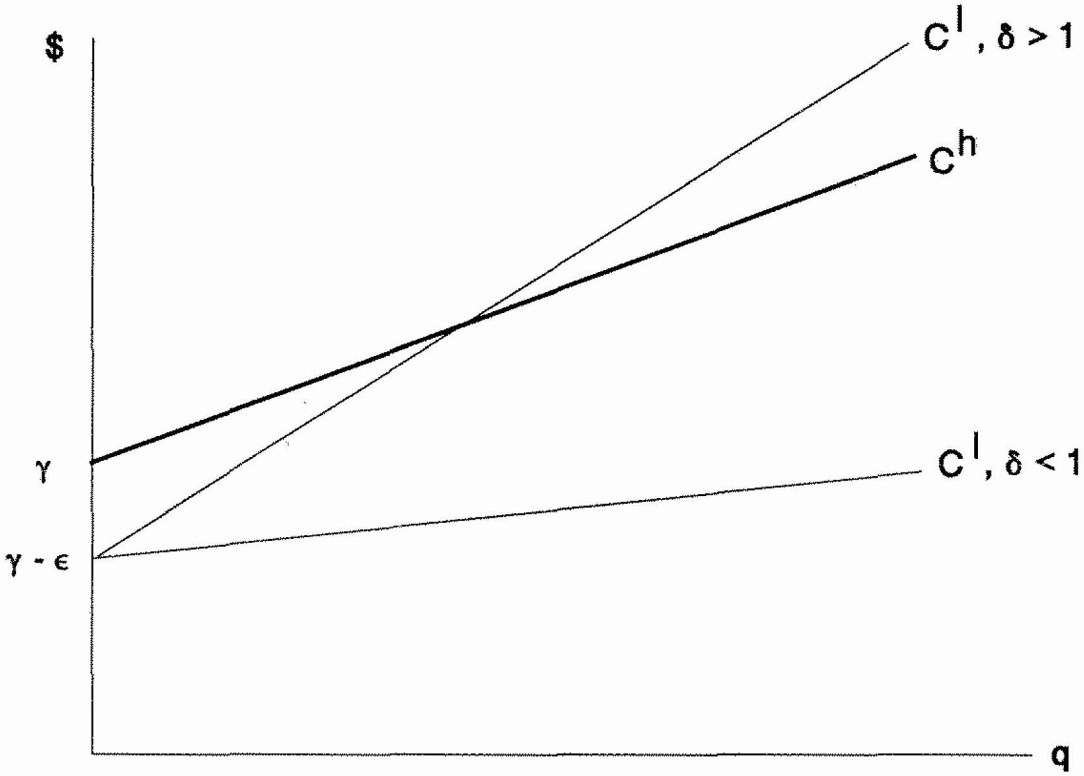

Figure 2

Payoffs from Switching and from Using only the Low-Intercept Technology

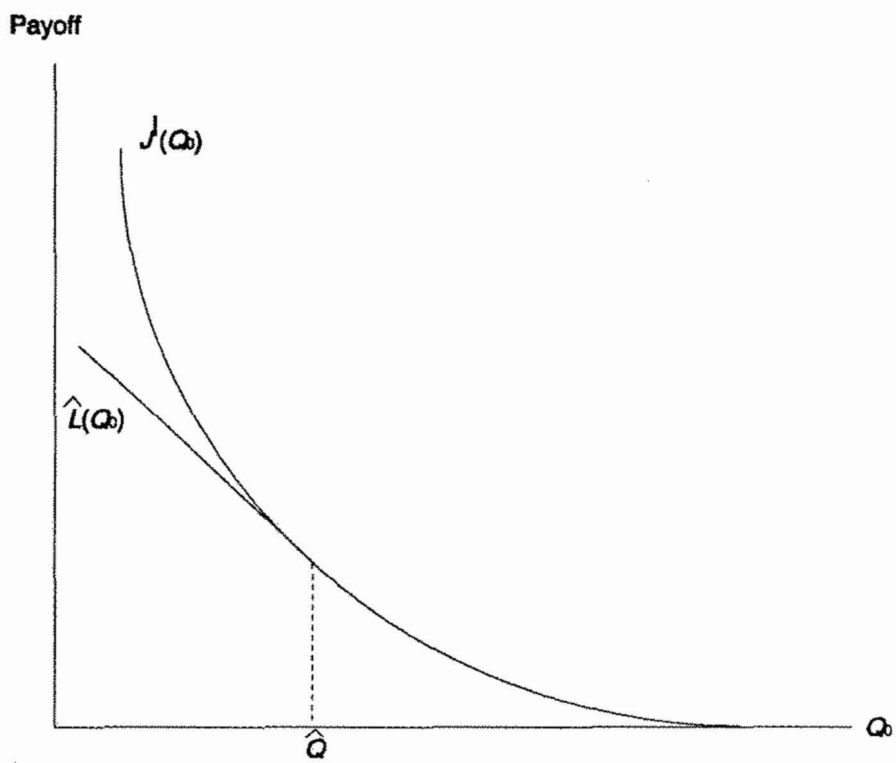

Figure 3

Optimal Switching Stock with Asymmetric Information

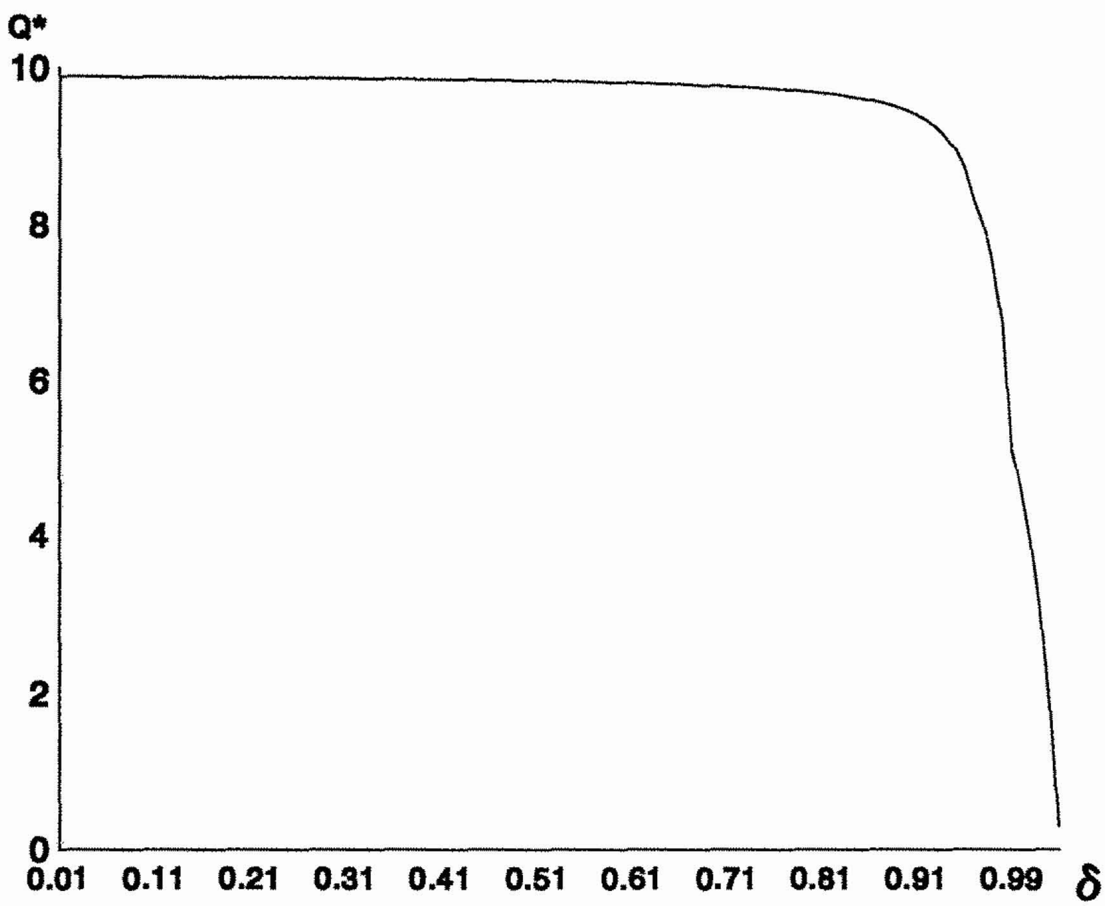

Figure B1

Solution for $\tilde{\theta}$