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Impact of Income on Price and Income Responses in the Differential Demand System

Mark G. Brown

An extension of the Rotterdam model is developed that makes the model's income flexibility and marginal propensities to consume varying coefficients. Frisch's duality relationships that the second partial derivatives of demand with respect to income and prices are independent of the order of differentiation are imposed with the marginal propensities to consume specified as functions of income and price, and the Slutsky coefficients specified as functions of income only. A uniform substitute specification is used to analyze the conditional demands for a group of beverages.

Key Words: demand, Rotterdam model, varying parameters

JEL Classifications: C51, D12, Q11

The differential demand system is based on the fundamental matrix equation of consumer demand derived through differentiation of the first order conditions of the utility maximization problem (Barten 1966; Theil 1975). The basic differential demand system is known as the Rotterdam model and there are two parameterizations of this model—the absolute price version and relative price version.[1] The relative price version of the Rotterdam model has been useful to impose various separability and preference-structure restrictions. To allow for increased flexibility in the income and price responses, as well as for specification of nonprice, nonincome explanatory variables, various extensions of the differential model have been suggested, including those that combine the features of the Rotterdam model and Almost Ideal Demand System (Barten 1993) and those based on the Basmann, Tintner, and Ichimura condition for the impacts of nonprice, non-income variables (e.g., Brown and Lee 1997, 2002; Duffy; Theil 1980b).

In this study, a further extension of the relative price version of the Rotterdam model is proposed to analyze the impacts of income levels on the price and income responses of the model. The impacts of prices on the income responses are also considered based on Frisch's duality relationship.[2] An example of this relationship is when a demand equation for some good, specified in levels as a linear function of its price and income, is augmented with the product term between price and income. In this case, the demand responses to price and income become dependent on income and price, respectively, and the impact of income on the demand response to price equals the impact of price on the demand response to income. Consistent interaction

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[1] The absolute price version can also be derived from the difference version of the double log model by imposing the basic properties of demand—adding up, homogeneity of degree zero in prices and income, and symmetry (e.g., Deaton and Muellbauer 1980b).

[2] The author gratefully acknowledges an anonymous referee for pointing out this relationship.

effects should similarly hold in a corresponding differential specification of demand. Such interactions between explanatory variables may be important in fitting demand equations to data and understanding changes in the demand impacts of these variables.

The relative price version of the Rotterdam model must be restricted in some fashion to be estimated and uniform substitute restrictions (Theil 1980a) are imposed in the present study. In an empirical analysis, a conditional demand system for a group of beverage products is considered. The focus is on how total expenditures on the product group (conditional income) impact the price and income coefficients of the conditional demand equations for the group.

The relationship between income and the effects of prices on demands was earlier examined by Timmer in context to food policy. Timmer's analysis suggested that as real income increases, the own-price elasticity of food tends to decline in absolute value. In contrast, previous findings based on the linear expenditure system (LES) and quadratic expenditure system (QES) supported the opposite conclusion that price elasticities increase with income level (Pollak and Wales). As suggested by Timmer, this result may be related to the restrictive nature of these demand models, exemplifying the importance of a flexible demand specification. The LES is based on an additive utility function with the result that the expenditure on a good in this demand system is a linear function of income and prices (Phlips; Theil 1975). The QES is less restrictive but, along with the LES, may not be sufficiently flexible for some applications as suggested by Theil, Chung, and Seale, and Timmer. Theil, Chung, and Seale developed and estimated a flexible cross-country demand model and found that the own-price elasticity for food did tend to decrease as real income increased. More recent analysis by Bouis supports this finding. There is, however, no reason to believe the previous findings that increases in income reduce the price responses should hold in the present study, given the conditional income variable for the beverage group examined in the study here differs from the broader definition of income used by Timmer and the other studies mentioned.

The article is organized as follows. The utility maximization problem and the basic Rotterdam model, including the absolute and relative price versions, are first reviewed. With our model extension based on the relative price version, we note that this version cannot be estimated unless restricted, followed by discussion of the restrictions imposed in the present study, those underlying the uniform substitute model in context of a conditional demand system. The uniform substitute model is then extended to make the original model's income flexibility and marginal propensities to consume functions of income. Finally, an application of the extended model to U.S. retail beverage sales data is discussed, followed by conclusions.

Model

Consider the utility maximization problem confronting consumers—how to allocate income over available goods. The solution is the affordable bundle of goods that yields the greatest utility. Formally, this problem can be written as maximization of $u = u(q)$ subject to $p'q = x$, where u is utility; $p' = (p_1, \ldots, p_n)$ and $q' = (q_1, \ldots, q_n)$ are price and quantity vectors with p_i and q_i being the price and quantity of good i, respectively; and x is total expenditures or income. The first order conditions for this problem are $\partial u/\partial q = \lambda p$ and $p'q = x$, where λ is the Lagrange multiplier, which is equal to $\partial u/\partial x$. The solution to the first-order conditions is the set of demand equations $q = q(p, x)$, and the Lagrange multiplier equation $\lambda = \lambda(p, x)$. The Rotterdam demand model is an approximation of this set of demand equations and the demand model developed in this paper is an extension of this approximation.[3]

[3] Analyses by Barnett, Byron, and Mountain show that the Rotterdam approximation is comparable to other flexible functional forms, such as the Almost Ideal Demand System (Deaton and Muellbauer 1980a).

Rotterdam Model

Following Theil (1975, 1976, 1980a,b), the absolute price version of the Rotterdam model can be written as

$$(1)\quad w_i\, d(\log q_i) = \theta_i\, d(\log Q) + \Sigma_j \pi_{ij} d(\log p_j) \qquad i = 1, \ldots, n,$$

where $w_i = p_i q_i / x$ is the budget share for good i; $\theta_i = p_i(\partial q_i/\partial x)$ is the marginal propensity to consume (MPC) for good i; $d(\log Q) = \Sigma w_i\, d(\log q_i)$ is the Divisia volume index, a measure of the change in real income or utility;[4] and $\pi_{ij} = (p_i\, p_j/x)s_{ij}$ is the Slutsky coefficient, with $s_{ij} = (\partial q_i/\partial p_j + q_j \partial q_i/\partial x)$ being the i,jth element of the substitution matrix $\mathbf{S}$. The Rotterdam model is a Hicksian or compensated demand system with the Divisia volume index indicating changes in real income and the Slutsky coefficients indicating compensated effects.

The Slutsky coefficient can be decomposed as (Theil 1975)

$$(2)\quad \pi_{ij} = \varphi(\theta_{ij} - \theta_i\theta_j),$$

where $\theta_{ij} = ((p_i p_j \lambda)/(x\varphi))u^{ij}$, with u^{ij} being the i,jth element of the inverse of the Hessian matrix, $[\mathbf{u}^{ij}] = [\partial^2 u/\partial q_i\, \partial q_j]^{-1}$. The parameter φ is referred to as the factor of proportionality or income flexibility, and is equal to the reciprocal of the elasticity of the marginal utility of income with respect to income; φ is negative based on the assumption that U is negative definite for utility maximization. The term $\varphi\theta_{ij}$ captures the specific substitution effect while the term $-\varphi\theta_i\theta_j$ captures the general substitution effect.

[4] The link between utility and the Divisia volume index can be shown by totally differentiating the direct utility function: $du = \Sigma\, \partial u/\partial q_i\, dq_i$ or, given the first order conditions ($\partial u/\partial q_i = \lambda p_i$), $du = x\, \lambda\, \Sigma\, (p_i\, q_i/x)(dq_i/q_i)$ or $du = x\, \lambda\, \Sigma\, w_i\, d(\log q_i)$. It can also be shown that the Divisia volume index is a close approximation of $d(\log x) - \Sigma w_i\, d(\log p_i)$, or the percentage change in (money) income minus the budget-share, weighted-average percentage change in all prices (Theil 1971); $d(\log Q)$ is used instead of $d(\log x) - \Sigma\, w_i\, d(\log p_i)$ to ensure adding-up.

The general restrictions on the Rotterdam model are (e.g., Theil 1975, 1976, 1980a,b)

$$(3a)\quad \text{adding up}: \quad \Sigma_i\theta_i = 1; \quad \Sigma_i\pi_{ij} = 0;$$

$$(3b)\quad \text{homogeneity}: \quad \Sigma_j\pi_{ij} = 0;$$

$$(3c)\quad \text{symmetry}: \quad \pi_{ij} = \pi_{ji}.$$

Following (3a) through (3c), the restrictions on Slutsky coefficient specification (2) are

$$(4a)\quad \text{adding up}: \quad \Sigma_i\theta_{ij} = \theta_j; \quad \Sigma_j\Sigma_i\theta_{ij} = 1;$$

$$(4b)\quad \text{homogeneity}: \quad \Sigma_j\theta_{ij} = \theta_i;$$

$$(4c)\quad \text{symmetry}: \quad \theta_{ij} = \theta_{ij}$$

The θ_{ij}*'s* are referred to as normalized price coefficients since by restriction (4a) they add up to one.

The relative price version of the Rotterdam model is obtained by replacing the Slutsky coefficient (π_{ij}) in Equation (1) by the right-hand side of Equation (2),

$$(5a)\quad w_i d(\log q_i) = \theta_i d(\log Q) + \varphi\Sigma_j(\theta_{ij} - \theta_i\theta_j) \times d(\log p_j), \quad \text{or}$$

$$(5b)\quad w_i d(\log q_i) = \theta_i d(\log Q) + \varphi\Sigma_j\big(\theta_{ij} d(\log p_j) - \Sigma_j\theta_j d(\log p_j)\big),$$

where restriction (4b) has been used to eliminate θ_i in the price term. The term $\Sigma\theta_j\, d(\log p_j)$ is known as the Frisch price index (Theil 1980a).

In this study, the relative price model (5b) is extended by making the coefficients φ and θ_i functions of real income. The relative price version of the Rotterdam model, however, cannot be estimated unless some restriction(s) is placed on the normalized price coefficients, the θ_{ij}*'s* (Theil 1971). In the absolute price version (1), the MPC can be identified from the income variable or Divisia volume index, and the Slutsky coefficients can be identified from the price variables. Defining the matrices $\theta = [\theta_i]$, $\pi = [\pi_{ij}]$, and $\Theta = [\theta_{ij}]$, Equation (2) can be written as $\pi = \varphi(\Theta - \theta\theta')$. The question is whether φ and Θ can be determined given that π and θ are known. The answer, in general, is no. Given π, θ, and φ, the solution for normalized price coefficients is $\Theta = \pi/\varphi +$

$\theta\theta'$. When π and θ are known but φ is unknown, different values of φ can be used to generate different values of Θ, but each set of estimates of ϕ and Θ would be consistent with the known π and θ. However, when one constraint is put on Θ, in addition to those for homogeneity and symmetry, the parameter ϕ can be estimated (Theil 1971). In this study, the restrictions underlying the uniform substitute model are placed on Θ. To examine alternative restrictions such as those resulting from separability, a reformation of the Rotterdam model is provided in Appendix A.

In the empirical study, we consider the demands for a group of goods and assume a two-stage budgeting process, where consumers first decide how much to spend on the group (first stage), and then how to allocate this amount to the individual goods in the group (second stage). Imposition of separability restrictions on the Rotterdam model allows specification of such a two-stage budgeting process (Theil 1976). The second-stage demand equations for individual goods in a group, called conditional demands, are functions of the amount of income allocated to the group and the prices of the goods in the group. The specifications of these conditional demand equations follow the same general structure as the unconditional demands specified above, Equations (1) and (5a) or (5b), except the real income variable or the Divisia volume index is based on income allocated to the group, the prices are those for the goods in the group, and the coefficients are conditional, being functions of the unconditional coefficients (e.g., Brown and Lee 2000; Theil 1976).

Conditional Uniform Substitute Model

Consider how the marginal utility of a dollar spent on good i changes in response to another dollar spent on good j, (i.e., $\partial^2 u/\partial(p_i q_i)\partial(p_j q_j)$). A group of goods are uniform substitutes when this change in marginal utility is the same for all pairs of goods i and j ($i \neq j$) in the group (Brown and Lee 1993, 2000; Theil 1980a). The goods in the group are assumed to be nearly identical with respect to key attributes but unique with respect to some. The nearly identical nature of goods i and j is assumed to result in generic type changes (k_0) in the marginal utilities (the more one beverage is consumed and thirst is satiated, the lower the marginal utility of all beverages), while the unique nature of the goods is assumed to result in product specific changes (k_i) in the marginal utilities. These two concepts can be expressed by $\partial^2 u/\partial(p_i q_i)\partial(p_j q_j) = k_0 + \Delta_{ij} k_i$, where Δ_{ij} is the Kronecker delta ($\Delta_{ij} = 1$ if $i = j$, otherwise $\Delta_{ij} = 0$), and both k_0 and k_i are negative. This specification of changes in marginal utilities underlies the uniform substitute model.

As shown by Theil (1980a) as well as Brown and Lee (2000), the uniform substitute model for a group of goods under block independence can be written as

$$(6)\quad w_i^* d(\log q_i) = \theta_i^* d\left(\log Q_G^*\right) + \varphi^* \theta_i^* \left(d(\log p_i) - \Sigma_{j\in G} \theta_j^* d\left(\log p_j\right)\right), \quad i, j \in G,$$

where $w_i^* = w_i/w_G$, with $w_G = \Sigma_{i\in G} w_i$; $\theta_i^* = \theta_i/\theta_G$, with $\theta_G = \Sigma_{i0G}\theta_i$; $\varphi^* = (\varphi\theta_G)/(1 - k\theta_G)/w_G$, with k being a positive parameter reflecting the commonality of the uniform substitutes in impacting utility; and $d\left(\log Q_G^*\right) = \Sigma_{i\in G}\, w_i^*\, d(\log q_i)$. All θ_i and hence θ_i^* are positive (no inferior goods) and between zero and one, given restriction (3a); and with φ being negative as previously mentioned, the coefficient φ^* is also negative (Theil 1975, 1980a). In extending model (6) below, the superscript star and group subscript G will be dropped for convenience with the understanding that the income flexibility (φ), the MPCs (θ_i), budget shares and Divisia volume index are all conditional with respect to the group in question.

It is interesting to note that the (conditional) uniform substitute model has the same general form as the Rotterdam model assuming preference independence (Theil 1980a). Thus, the uniform-substitute assumption (an additional dollar spent on a good, regardless the good, uniformly impacts the marginal utilities of the other goods) affects the demand parameterization similarly as the preference-independence assumption (the cross impacts on the marginal utilities are zero, i.e., $\partial^2 u/\partial(p_i q_i)\partial(p_j q_j) = 0$).

Extension

In the Rotterdam model extension considered in this paper, the (conditional) coefficients φ and the θ_i are made functions of income. These functions are motivated by specifications of the income flexibility suggested by Theil (1976), Theil and Brooks, and Theil, Chung, and Seale. The latter specifications are extended here to be consistent with Frisch's observation that the second derivative of demand with respect to income and one of the prices should be independent of the order that the derivative is calculated with respect to these two variables (i.e., $\partial^2 q_i/\partial p_j \partial x = \partial^2 q_i/\partial x \partial p_j$). Frisch refers to this condition as a duality relationship (see Kinnucan and Zheng for further discussion).

With the Rotterdam model being a compensated demand system in log differences, we express Frisch's duality relationship by differentiating with respect to the logs of price and real income Q (reflected by the Divisia volume index) (i.e., $w_i \partial^2 \log q_i/\partial \log p_j \, \partial \log Q = w_i \partial^2 \log q_i/\partial \log Q \, \partial \log p_j$), or

$$(7) \quad \partial \pi_{ij}/\partial \log Q = \partial \theta_i/\partial \log p_j,$$

given the Slutsky coefficients and MPCs can be written as $\pi_{ij} = w_i \partial \log q_i \big/ \partial \log p_j\, _{u=\text{constant}}$ and and $\theta_i = w_i \, \partial \log q_i/\partial \log Q$, respectively. As noted in footnote 4, $d(\log Q)$ is approximately equal to $d(\log x) - \Sigma w_i \, d(\log p_i)$, so that θ_i can also be viewed as $w_i \partial \log q_i/\partial \log x$, providing the usual interpretation of θ_i as the MPC for good i.

The specification suggested by Theil (1976), as well as by Theil, Chung, and Seale, makes the income flexibility a function of real income Q, treating the MPCs as constants, such as

$$(8) \quad \begin{aligned} w_i d(\log q_i) &= \theta_i d(\log Q) + (\varphi_0 + \alpha \log Q) \\ &\quad \times \theta_i \big(d(\log p_i) - \Sigma_j \theta_j d(\log p_j)\big), \end{aligned}$$

where φ_0 and α are coefficients.

Equation (8), however, does not satisfy Frisch's duality relationship (7) given the MPCs are constants and do not change with prices, while the Slutsky coefficients, $\pi_{ij} = (\varphi_0 + \alpha \log Q)\theta_i(\Delta_{ij} - \theta_j)$, clearly do change with real income Q. Differentiating π_{ij} with respect to the log of real income results in $\partial \pi_{ij}/\partial \log Q = \alpha \theta_i(\Delta_{ij} - \theta_j)$, which would equal $\partial \theta_i/\partial \log p_j$ if Frisch's duality relationship held. Integrating this derivative with respect to $d(\log p_j)$, across j, implies that θ_i must equal a constant plus $\alpha \theta_i$ $(\log p_i - \Sigma_j \theta_j \log p_j)$ to comply with this property. Below, the duality relationship is further developed in extending specification (8) and allowing the MPCs to vary with real income.

Although our extension is in context of the Rotterdam model in differential form, to comply with Frisch's duality relationship, consider Barten's (1989) levels version of the Rotterdam model under the uniform substitute coefficient restrictions:

$$(9) \quad \begin{aligned} w_i \log q_i &= \mu_i + \theta_i \log Q \\ &\quad + \varphi \theta_i \big(\log p_i - \Sigma_j \theta_j \log p_j\big), \end{aligned}$$

where μ_i is an intercept and $\log Q = \Sigma w_i \log q_i$, Barten's measure of real income. As in the case of the differential Rotterdam model, the coefficient θ_i is the MPC for a good and ϕ is the income flexibility, treating the budget share as a constant.

In Equation (9), real income is assumed to impact θ_i and φ, as in Equation (8), so that

$$(10) \quad \theta_{it} = \theta_{i0} + \beta_i \log Q_t,$$

$$(11) \quad \varphi_t = \varphi_0 + \alpha \log Q_t,$$

where θ_{i0} and β_i are additional coefficients, and the subscript t indicates time (the subscript t on θ_{it} indicates the value of θ_i at time t and is not to be confused with the second subscript j on the normalized price coefficients θ_{ij}'s used earlier to indicate cross price relationships). The adding up constraint (3a) requires $\Sigma \theta_{i0} = 1$ and $\Sigma \beta_i = 0$.

Substituting the right-hand sides of Equations (10) and (11) into Equation (9) results in

$$(12) \quad \begin{aligned} w_{it} \log q_{it} &= (\theta_{i0} + \beta_i \log Q_t) \log Q_t \\ &\quad + (\varphi_0 + \alpha \log Q_t) \\ &\quad (\theta_{i0} + \beta_i \log Q_t) \\ &\quad \times (\log p_{it} - \Sigma_j \\ &\quad \Big(\theta_{j0} + \beta_j \log Q_t\Big) \log p_{jt}). \end{aligned}$$

Given the income flexibility is a factor of proportionality for all price effects, a change in this term results in a general change across all goods with respect to the sensitivity of demands to prices. In contrast, changes in the θ_i's result in specific changes in the price effects.

Treating the budget share as a constant, the total differential of Equation (12) with respect to prices and real income is

$$(13)\quad \begin{aligned} w_{it}d(\log q_{it}) = \Big(&\theta_{it} + \beta_i \log Q_t + (\alpha\theta_{it} + \varphi_t\beta_i) \\ &\times (\log p_{it} - \Sigma_j\theta_{jt}\log p_{jt}) \\ &- \varphi_t\theta_{it}\Sigma_j\beta_j \log p_{jt}\Big) d(\log Q_t) \\ &+ \varphi_t\theta_{it}\big(d(\log p_{it}) \\ &\quad - \Sigma_j\theta_{jt}d(\log p_{jt})\big), \end{aligned}$$

or, expanding terms and dropping the time subscript,

$$(14)\quad \begin{aligned} &w_i d(\log q_i) \\ &= \Big(\theta_{i0} + 2\beta_i \log Q + \\ &(\alpha(\theta_{i0} + \beta_i \log Q) + (\varphi_0 + \alpha \log Q)\beta_i) \\ &\times (\log p_i - \Sigma_j(\theta_{j0} + \beta_j \log Q) \log p_j) \\ &- (\varphi_0 + a \log Q)(\theta_{i0} + \beta_i \log Q) \\ &\Sigma_j\beta_j \log p_j\Big)\, d(\log Q) + (\varphi_0 + a \log Q) \\ &(\theta_{i0} + \beta_i \log Q) \times \Big(d(\log p_i) \\ &- \Sigma_j(\theta_{j0} + \beta_j \log Q)\, d(\log p_j)\Big). \end{aligned}$$

Equation (13) or (14) is our extended model. The MPCs of this model are

$$(15)\quad \begin{aligned} \theta'_i = (&\theta_{i0} + 2\beta_i \log Q + (\alpha(\theta_{i0} + \beta_i \log Q) \\ &+ (\varphi_0 + \alpha \log Q)\beta_i) \times (\log p_i \\ &- \Sigma_j\Big(\theta_{j0} + \beta_j \log Q\Big) \log p_j\Big) - (\varphi_0 + \\ &\alpha \log Q)(\varphi_{i0} + \beta_i \log Q)\, \Sigma_j\beta_j \log p_j\Big), \end{aligned}$$

while the Slutsky coefficients are

$$(16)\quad \begin{aligned} \pi'_{ij} = &(\varphi_0 + \alpha \log Q)(\theta_{i0} + \beta_i \log Q) \\ &\times \Big(\Delta_{ij} - \Big(\theta_{j0} + \beta_j \log Q\Big)\Big). \end{aligned}$$

Equations (13) and (14) satisfy Frisch's duality condition that the second partial derivatives of demand with respect to income and prices are independent of the order of differentiation, so that

$$\begin{aligned} &w_i\partial^2(\log q_i)/\partial(\log Q)\,\partial(\log p_j) \\ &= w_i\partial^2(\log q_i)/\partial(\log p_j)\,\partial(\log Q) \\ &= (\alpha(\theta_{i0} + \beta_i \log Q) + (\varphi_0 + \alpha \log Q)\,\beta_i) \\ &\quad\times \Big(\Delta_{ij} - \Big(\theta_{j0} + \beta_j \log Q\Big)\Big) \\ &\quad - (\varphi_0 + \alpha \log Q)(\theta_{i0} + \beta_i \log Q)\,\beta_j. \end{aligned}$$

In comparison with Equation (8), if the coefficient β_i is set to zero in Equation (14), the resulting demand equation will satisfy Frisch's condition,

$$(17)\quad \begin{aligned} &w_i d(\log q_i) \\ &= (\theta_{i0} + \alpha\theta_{i0}(\log p_i - \Sigma_j\theta_{j0} \log p_i)) \\ &\quad\times d(\log Q) + (\varphi_0 + \alpha \log Q) \\ &\quad\times \theta_{i0}(d(\log p_i) - \Sigma_j\theta_{j0}\, d(\log p_j)), \end{aligned}$$

where the MPC is now $\theta_{i0} + \alpha\, \theta_{i0}\, (\log p_i - \Sigma_j\, \theta_{j0} \log p_j)$, as opposed to a constant in Equation (8), and the Slutsky coefficient is $(\varphi_0 + \alpha \log Q)\ \theta_{i0}\ (\Delta_{ij} - \theta_{j0})$, the same as in Equation (8), letting $\theta_i = \theta_{i0}$. Note that models (8) and (17) are not nested, and, in this case, imposing Frisch's duality condition does not result in an additional cost in terms of extra coefficients to be estimated.

The MPC and Slutsky coefficient of models (14) and (17) differ with respect to underlying income and price variables and interactions. The MPCs for model (14) depend on both the logarithms of real income and prices, with a number of interactions up to the cube of the log of income times the log of price, while those for model (17) depend on only the logs of prices. The Slutsky coefficients for both models are functions of the log of income but not prices, but the coefficients of model (14) depend on the log of income, its square and cube, while those in model (17) depend on the log of income only.

Application

Conditional demands for beverages were studied using Nielsen data based on retail scanner sales for grocery stores, drugstores,

Table 1. Descriptive Statistics of Beverage Sample, 06/28/03 through 06/03/06

	Gallons/Week		Price: $/Gallon		Budget Share	
Beverage	Mean	SD	Mean	SD	Mean	SD
Orange[a]	0.0428	0.0034	4.52	0.14	7.30%	0.70%
Grapefruit[a]	0.0015	0.0004	5.73	0.69	0.30%	0.00%
Apple[a]	0.0138	0.0022	3.62	0.19	1.90%	0.30%
Grape[a]	0.0039	0.0004	5.9	0.14	0.90%	0.10%
Remaining fruit juice[a]	0.0119	0.0005	6.06	0.34	2.70%	0.20%
Vegetable	0.0053	0.0006	6.78	0.39	1.40%	0.10%
Juice drinks[b]	0.0876	0.0122	3.7	0.15	12.10%	0.90%
Carbonated water	0.0099	0.0009	2.79	0.14	1.00%	0.10%
Water	0.1341	0.0247	1.68	0.04	8.40%	1.30%
Soda	0.3499	0.0428	2.62	0.15	34.20%	1.50%
Liquid tea	0.0157	0.0034	3.66	0.12	2.10%	0.40%
Milk and shakes	0.2173	0.0078	3.39	0.18	27.70%	1.70%
	Mean	SD	Minimum	Maximum		
Real income[c]	−2.5532	0.0479	−2.6717	−2.4132		

[a] 100% juice.
[b] Less than 100% juice.
[c] $\text{Log } Q = \Sigma w_i \log q_i$.

and mass merchandisers along with an estimate of Wal-Mart sales based on a consumer panel.[5] Twelve beverages were included in the model: 1) 100% orange juice, 2) 100% grapefruit juice, 3) 100% apple juice, 4) 100% grape juice, 5) remaining 100% juice, 6) vegetable juice, 7) less-than-100% juice drinks, 8) carbonated water, 9) water, 10) regular and diet soda, 11) liquid tea or tea for short, and 12) milk and shakes.

The data are weekly running from week ending June 28, 2003 through week ending June 3, 2006 (154 weekly observations). The raw data comprised gallon and dollar sales. In the study, quantity demanded was measured by per capita gallon sales, which were obtained by dividing raw gallon sales by the U.S. population; prices were obtained by dividing dollar sales by gallon sales. Sample mean per capita gallon sales, prices and budget shares are shown in Table 1.

The infinitely small changes in the logarithms of quantities and prices in the differential models were measured by discrete first differences (Theil 1975, 1976). To account for seasonality, first differences of sine and cosine variables were included—sine($2\pi t/52$) and cosine($2\pi t/52$), where $\pi = 3.14\ldots$, observation $t = 1, \ldots, 154$ and 52 is the number of weeks in a year.[6] Average budget share values underlying the differencing were used in constructing the model variables—w_{it} was replaced by $(w_{it} + w_{it-1})/2$. The levels values of Barten's (1989) real income variable and logarithms of prices were similarly constructed as $\log Q_t = \Sigma w_{it} (\log q_{it} + \log q_{it-1})/2$ and $(\log p_{it} + \log p_{it-1})/2$, respectively, following the approach taken by Theil (1976) to construct a levels value for real income.

The demand specifications studied are conditional on expenditure or income allocated to the 12 beverage categories. Income allocated to the beverage group is measured by the conditional Divisia volume index for this group, which was treated as independent of the error term added to each beverage demand

[5] Data are for U.S. grocery stores doing $2 million and greater annual sales, Wal-Mart stores (excluding Sam's Clubs), mass merchandisers, and drugstores doing $1 million and greater annual sales.

[6] See, for example, Makridakis, Wheelwright, and McGee for discussion on incorporating seasonal effects into regression equations through sine and cosine variables.

Table 2. Full Information Maximum Likelihood Estimates of Uniform Substitute Model (17) with MPCs and Slutsky Coefficients Varying with Prices and Real Income, Respectively

	MPC		Sine		Cosine	
Beverage	Estimate	SE	Estimate	SE	Estimate	SE
Orange[a]	0.0591	0.0024	0.0066	0.0014	−0.0024	0.0014
Grapefruit[a]	0.0031	0.0001	0.0001	0.0001	−0.0002	0.0001
Apple[a]	0.0230	0.0009	0.0033	0.0007	0.0007	0.0007
Grape[a]	0.0092	0.0005	0.0007	0.0003	−0.0009	0.0004
Remaining fruit juice[a]	0.0271	0.0010	0.0012	0.0005	−0.0014	0.0006
Vegetable	0.0163	0.0008	0.0006	0.0005	−0.0005	0.0005
Juice drinks[b]	0.1262	0.0053	−0.0078	0.0032	0.0010	0.0033
Carbonated water	0.0096	0.0003	−0.0006	0.0002	−0.0002	0.0002
Water	0.0775	0.0056	−0.0139	0.0041	0.0028	0.0042
Soda	0.4657	0.0079	0.0007	0.0059	−0.0038	0.0061
Liquid tea	0.0213	0.0013	−0.0039	0.0011	0.0011	0.0011
Milk and shakes	0.1620	0.0095	0.0131	0.0051	0.0038	0.0052
	Constant		Slope			
	Estimate	SE	Estimate	SE		
Income flexibility	−2.4534	0.1090	−0.2763	0.0479		

[a] 100% juice.
[b] Less than 100% juice.

equation for estimation, based on the theory of rational random behavior (Brown, Behr, and Lee; Theil 1980a). As the data add up by construction—the left-hand-side variables in the Rotterdam model sum over i to the conditional Divisia volume index—the error covariance matrix was singular and an arbitrary equation was excluded (the model estimates are invariant to the equation deleted as shown by Barten 1969). The parameters of the excluded equation can be obtained from the adding-up conditions or by re-estimating the model omitting a different equation.[7] The equation error terms were assumed to be contemporaneously correlated and the full information maximum likelihood procedure (TSP) was used to estimate the system of equations.

The estimates of general model (14) and specific model (17), which is more closely related to the model suggested by Theil (1976) and Theil, Chung, and Seale, are discussed below. Model (17) showed promise but the key slope coefficient estimate (α) for the income flexibility proved to be insignificant in context of model (14). A brief discussion of model (17) is provided first, followed by a more detailed discussion of model (14).

Estimates of model (17), where the income flexibility is dependent on income and the MPCs are dependent on prices, specified so as to be consistent with Frisch's duality condition, are shown in Table 2. The individual equation r-squares ranged from .405 (water) to .975 (soda), while the system r-square (Bewley; Buse) was .946. All MPC and income flexibility coefficient estimates were statistically significant at the 10% or smaller level; and half of the 24 seasonality coefficient estimates were statistically significant. The results that the coefficients α and θ_{i0}'s were statistically different from zero indicate that the MPCs (Equation [15] with $\beta_i = 0$) and Slutsky coefficients (Equation [16] with $\beta_i = 0$), as well as associated elasticities, vary across price and income levels, respectively. To illustrate the variation in demand responses, (condi-

[7] Re-estimating the model omitting a different equation also serves as a double check on the results. In this study, each model examined was estimated twice, once with the milk equation removed and once with the tea equation removed, and as required both set of estimates were the same.

Table 3. Conditional Income and Uncompensated Own-Price Elasticity Estimates at Selected Real Income and Price Values, Based on Estimates of Uniform Substitute Model (17)

	Income Elasticity			Uncompensated Own-Price Elasticity		
Beverage	Minimum[a]	Mean[b]	Maximum[c]	Minimum[d]	Mean[e]	Maximum[f]
Orange[g]	0.709	0.724	0.730	−1.366	−1.391	−1.420
Grapefruit[g]	0.816	0.806	0.790	−1.671	−1.703	−1.740
Apple[g]	1.175	1.170	1.163	−2.082	−2.122	−2.168
Grape[g]	0.855	0.870	0.886	−1.818	−1.853	−1.894
Remaining fruit juice[g]	0.807	0.811	0.807	−1.693	−1.725	−1.763
Vegetable	0.939	0.934	0.924	−2.038	−2.077	−2.122
Juice drinks[h]	0.978	0.987	0.985	−1.685	−1.715	−1.750
Carbonated water	0.947	0.947	0.946	−1.582	−1.612	−1.647
Water	1.058	1.072	1.082	−1.549	−1.577	−1.609
Soda	1.422	1.416	1.415	−1.732	−1.756	−1.784
Liquid tea	0.930	0.939	0.950	−1.684	−1.716	−1.753
Milk and shakes	0.571	0.566	0.563	−1.084	−1.102	−1.123

[a] Calculated at the minimum prices.
[b] Calculated at the mean prices.
[c] Calculated at the maximum prices.
[d] Calculated at the minimum value of real income, $\log Q = \Sigma w_i \log q_i$, and mean prices.
[e] Calculated at the mean of real income, $\log Q = \Sigma w_i \log q_i$, and mean prices.
[f] Calculated at the maximum value of real income, $\log Q = \Sigma w_i \log q_i$, and mean prices.
[g] 100% juice.
[h] Less than 100% juice.

tional) income elasticities ($e_i = \theta'_i/w_i$) and uncompensated own-price elasticities ($e_{ij} = \pi'_{ij}/w_i - w_j e_i$) for model (17), calculated at minimum, mean and maximum prices and income levels, and mean budget shares are shown in Table 3. (Budget shares may also change with income and price levels but are held constant at mean levels for simplification; thus, the results in Table 3 reflect changes in the MPCs and Slutsky coefficients, adjusted to elasticities for convenience in interpretation.) The variation in income elasticities is relatively small, while the variation in price elasticities is greater but still relatively moderate. Overall, these results suggest that allowing the income flexibility and MPCs to vary as such may be more promising than suggested by results obtained by Theil and Brooks, which indicated that the income flexibility was not significantly related to income, based on a study of Dutch data on aggregated goods (food, beverages, durables, and remaining goods) for the period from 1922 through 1963. A study by Paulus supported this result. Differences in data, level of aggregation of goods and model specification (the Theil and Brooks and Paulus specifications did not comply with Frisch's duality relationship as previously noted) may explain the different results. However, when model (14), the more general model that extends model (17) allowing the MPCs to depend on income as well as prices, was estimated, the foregoing results on the income flexibility did not hold up.

The estimates of uniform substitute model (14) are show in Table 4. The individual equation *r*-squares ranged from .515 (water) to .979 (soda), while the system *r*-square was .950. All the MPC constants (θ_{i0}), 7 out of 12 of the MPC slopes (β_i), the income flexibility constant (φ_0), and 13 of the 24 seasonality coefficients were statistically significant. However, the income flexibility slope (α) was not significant. This latter result is consistent with Theil and Brooks' and Paulus' findings that income does not have a significant general impact on all price responses through the income flexibility. Overall, however, the estimates of model (14) do not imply that the price responses, as well as income responses,

Table 4. Full Information Maximum Likelihood Estimates of Uniform Substitute Model (14) with MPCs Varying with Prices and Real Income and Slutsky Coefficients Varying with Real Income

	MPC Constant		MPC Slope		Sine		Cosine	
Beverage	Estimate	SE	Estimate	SE	Estimate	SE	Estimate	SE
Orange[a]	0.0239	0.0078	−0.0077	0.0025	0.0073	0.0013	−0.0024	0.0014
Grapefruit[a]	0.0022	0.0005	−0.0002	0.0002	0.0001	0.0001	−0.0002	0.0001
Apple[a]	0.0120	0.0028	−0.0023	0.0008	0.0034	0.0007	0.0007	0.0007
Grape[a]	0.0071	0.0016	−0.0003	0.0006	0.0007	0.0003	−0.0009	0.0003
Remaining fruit juice[a]	0.0176	0.0043	−0.0017	0.0015	0.0012	0.0005	−0.0014	0.0005
Vegetable	0.0117	0.0029	−0.0008	0.0010	0.0006	0.0005	−0.0005	0.0005
Juice drinks[b]	0.0388	0.0148	−0.0188	0.0042	−0.0083	0.0031	0.0011	0.0031
Carbonated water	0.0051	0.0011	−0.0011	0.0003	−0.0005	0.0002	−0.0002	0.0002
Water	0.1445	0.0381	0.0180	0.0131	−0.0143	0.0037	0.0022	0.0038
Soda	0.7032	0.0546	0.0499	0.0126	−0.0014	0.0054	−0.0027	0.0055
Liquid tea	−0.0150	0.0027	−0.0078	0.0006	−0.0038	0.0009	0.0012	0.0009
Milk and shakes	0.0489	0.0191	−0.0272	0.0052	0.0149	0.0049	0.0030	0.0050
	Constant		Slope					
	Estimate	SE	Estimate	SE				
Income flexibility	−3.0015	0.8710	−0.3418	0.3399				

[a] 100% juice.
[b] Less than 100% juice.

are independent of income as a number of the MPC slope coefficient estimates with respect to income (β_i) were significant, indicating the impact of income on the price and income responses are more product specific than general.

The negativity condition of demand requires that the Slutsky matrix is negative semidefinite, which in the present case requires the values of θ_{it} in Equation (10) and φ_t in Equation (11) be positive and negative, respectively, across all t.[8] Calculating Equations (10) and (11), based on model (14) estimates (θ_{i0}, β_i, φ_0, α) across all sample income values, revealed that the negativity condition did hold for these income values—all θ_{it} and φ_t were positive and negative, respectively.

To provide a general indication of beverage demand responses, conditional income and uncompensated price elasticity estimates for model (14), calculated at sample mean income, price and budget share values, are shown in Table 5. Corresponding standard error estimates are shown in Appendix B. Tea, soda, and apple juice have the highest income elasticities at 1.31, 1.28, and 1.20, respectively; the income elasticities for the remaining beverages range from .66 for milk and shakes to 1.09 for juice drinks. The own-price elasticities ranged from −.50 for tea to −2.33 for water. Although many are relatively small, the cross-price elasticity estimates are predominately positive, reflecting substitution.

The impacts of income on the demand elasticities are illustrated in Table 6. Income and own-price elasticities, calculated at the minimum, mean, and maximum values of the income variable and mean prices and budget shares are shown. The largest changes in the income elasticities are for water and tea, while the smallest changes are for grape juice, vegetable juice, and grapefruit juice. For water

[8] This result is based on the requirement that the sign of quadratic form $x'[\pi'_{ij}]x$, where x is an $n\times 1$vector and π'_{ij} is defined as in equation (16), be negative except when $x = k^* \iota$ where k is a scalar and ι is an $n\times 1$ vector of unit values in which case the value of this quadratic form is zero (based on the adding-up and homogeneity properties, (3a) and (3b). Hence $[\pi'_{ij}]$ is negative semi-definite with rank $n-1$.

Table 5. Uniform-Substitute Model (14) Elasticity Estimates at Sample Means[a]

		Price											
Beverage	Income	Orange	Grapefruit	Apple	Grape	Remaining Fruit Juice	Vegetable	Juice Drinks	Carbonated Water	Water	Soda	Tea	Milk and Shakes
Orange[b]	0.8404	−1.2824	0.0007	0.0072	0.0030	0.0052	0.0061	0.0092	0.0014	0.0552	0.4478	−0.0741	−0.0029
Grapefruit[b]	0.7690	0.0205	−1.7502	0.0170	0.0074	0.0175	0.0136	0.0591	0.0059	0.1081	0.7459	−0.0081	−0.0057
Apple[b]	1.1983	0.0018	0.0015	−2.0250	0.0060	0.0122	0.0117	0.0321	0.0038	0.1003	0.7642	−0.0159	−0.0908
Grape[b]	0.7954	0.0283	0.0027	0.0205	−1.9663	0.0217	0.0163	0.0752	0.0074	0.1278	0.8652	−0.0075	0.0133
Remaining fruit juice[b]	0.8014	0.0167	0.0020	0.0158	0.0068	−1.7038	0.0127	0.0523	0.0053	0.1021	0.7160	−0.0089	−0.0186
Vegetable	0.8979	0.0286	0.0028	0.0218	0.0095	0.0228	−2.1354	0.0782	0.0078	0.1367	0.9323	−0.0089	0.0059
Juice drinks[c]	1.0878	−0.0124	0.0005	0.0070	0.0028	0.0040	0.0062	−1.5269	0.0009	0.0592	0.5076	−0.0160	−0.1207
Carbonated water	0.9904	−0.0009	0.0011	0.0107	0.0045	0.0089	0.0089	0.0218	−1.6286	0.0775	0.6004	−0.0134	−0.0815
Water	1.0039	0.0358	0.0034	0.0260	0.0113	0.0274	0.0206	0.0951	0.0094	−2.3341	1.0935	−0.0095	0.0172
Soda	1.2751	0.0635	0.0054	0.0404	0.0177	0.0438	0.0319	0.1566	0.0152	0.2459	−1.9560	−0.0101	0.0707
Liquid tea	1.3072	−0.0741	−0.0029	−0.0159	−0.0074	−0.0249	−0.0112	−0.1165	−0.0097	−0.0628	−0.1720	−0.5038	−0.3059
Milk and shakes	0.6596	−0.0083	0.0003	0.0040	0.0016	0.0020	0.0035	−0.0009	0.0004	0.0342	0.2978	−0.0098	−0.9844

[a] Price elasticities are uncompensated and conditional.
[b] 100% juice.
[c] Less than 100% juice.

Table 6. Conditional Income and Uncompensated Own-Price Elasticity Estimates for Uniform Substitute Model (14) at Selected Real Income and Price Values

	Income Elasticity			Own-Price Elasticity		
Beverage	Minimum[a]	Mean[b]	Maximum[c]	Minimum[a]	Mean[b]	Maximum[c]
Orange[d]	0.862	0.840	0.815	−1.285	−1.282	−1.278
Grapefruit[d]	0.778	0.769	0.758	−1.729	−1.750	−1.775
Apple[d]	1.226	1.198	1.166	−2.017	−2.025	−2.033
Grape[d]	0.803	0.795	0.786	−1.939	−1.966	−1.999
Remaining fruit juice[d]	0.813	0.801	0.788	−1.687	−1.704	−1.723
Vegetable	0.908	0.898	0.886	−2.109	−2.135	−2.166
Juice drinks[e]	1.121	1.088	1.049	−1.536	−1.527	−1.514
Carbonated water	1.016	0.990	0.960	−1.624	−1.629	−1.632
Water	0.947	1.004	1.071	−2.244	−2.334	−2.442
Soda	1.243	1.275	1.313	−1.921	−1.956	−1.996
Liquid tea	1.383	1.307	1.217	−0.585	−0.504	−0.403
Milk and shakes	0.681	0.660	0.634	−0.994	−0.984	−0.972

[a] Calculated at the minimum value of real income, $\log Q = \Sigma w_i \log q_{i,}$ and mean prices.
[b] Calculated at the mean of real income, $\log Q = \Sigma w_i \log q_{i,}$ and mean prices.
[c] Calculated at the maximum value of real income, $\log Q = \Sigma w_i \log q_{i,}$ and mean prices.
[d] 100% juice.
[e] Less than 100% juice.

and tea, the income elasticities at the maximum income level are 13.1% greater and 12.0% less than the corresponding values at the minimum income level, respectively. All income elasticities decrease with income except those for water and soda which increase.

The largest changes in the own-prices elasticities are for tea and water, while the smallest changes are for orange juice and carbonated water. The own-price elasticities for orange juice, juice drinks, tea, and milk and shakes decrease (in absolute value) with income, while those for the other beverages increase with income. Following the impacts on the MPCs and income elasticities, the tea and water own-price elasticities at the maximum income level are 31.1% less and 8.8% greater in absolute value than the corresponding elasticities at the minimum income level, respectively. The orange juice and carbonated water own-price elasticities at the maximum income level are only 0.5% less and greater than their values at the minimum income level, respectively. The various impacts of income on the demand elasticities may be of interest to analysts, marketers, and planners in the beverage industry monitoring and seeking to understand the underlying causes for volume changes in the market.

The conditional demand findings in this study are mixed across beverages with respect to the previous unconditional findings that increases in income reduce the price responses. Our conditional demand equations differ from unconditional equations with respect to the conditional income variable for the beverage group versus the broader definition of income used by Timmer and the other studies mentioned earlier, and with respect to the definitions and restrictions imposed on the conditional versus unconditional coefficients. Conditional demands focus on the allocation of a portion of income (conditional income) across a subgroup of goods and are limited in explaining broader, unconditional changes in consumer behavior. Changes in conditional income or total beverage expenditures in the present case may be related to a number of variables, including beverage prices, prices of goods outside the beverage category, and total consumer expenditures across all goods, as well as various preference variables such as consumer demographics and advertising. The impact of the conditional income variable on the price and income coefficients may thus indirectly reflect the impacts of such other factors through their impacts on conditional income. Regardless the underlying cause for changes in conditional income, knowing how this variable impacts the beverage price and income responses may be useful to explain changes in demand.

Conclusions

This paper extends the Rotterdam model to analyze the impact of income level on the price and income responses of demand. Based on Frisch's duality relationship, the second partial derivatives of demand with respect to income and prices are independent of the order of differentiation. For the demand for some good in the Rotterdam model, this relationship means that the impact of income on a Slutsky coefficient (price response) should be the same as the impact of the associated price on that good's MPC (income response). Frisch's duality condition was imposed in the present study with the MPCs specified as functions of income and prices, and the Slutsky coefficients specified as functions of income only. These specifications were made in context of the relative price version of the Rotterdam model. The impacts of income and prices on the model coefficients were specified through the income flexibility and the original model's MPCs. The income flexibility is a component of all Slutsky coefficients and changes in this parameter thus result in a general impact across all price responses. On the other hand, changes in the MPCs result in specific changes in the Slutsky coefficients. To estimate the relative price version requires some restriction on the normalized price coefficients of the model. In the present study, uniform-substitute model restrictions were imposed.

The empirical analysis focused on the conditional demands for beverages. The results indicate that the conditional income level does impact the MPCs and Slutsky coefficients. The impacts of income on the Slutsky coefficients through the MPCs were significant, while impacts through the income flexibility were not. That is, income had specific, but not general, impacts on the beverage-demand responses to prices. The conditional income and price elasticities varied moderately, based on the income levels of the sample.

The varying-coefficient specification of the uniform-substitute model might also be useful for analyzing other product groups dominated by substitution, and when the uniform substitute assumptions are not applicable, the varying MPC and income flexibility specifications suggested here can still be used provided appropriate restrictions on the normalized price coefficients can be made for identification.

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Appendix A

Consider the price term without the income flexibility in Equation (5b), i.e., $\Sigma_j\ \theta_{ij}\ (d(\log\ p_j) - \Sigma_j\ \theta_j\ d(\log\ p_j))$. Breaking out the own-price component, this term can be written as

$$\text{(A1)}\qquad \theta_{ii}\big(d\ (\log p_i) - \Sigma_j \theta_j d\big(\log p_j\big)\big) + \Sigma_{j \neq i}\theta_{ij}\big(d\big(\log p_j\big) - \Sigma_j \theta_j d\big(\log p_j\big)\big).$$

Based on restriction (4b), $\theta_{ii} = \theta_i - \Sigma_{j\neq i}\ \theta_{ij}$. Substituting the right-hand side of this result for the first parameter θ_{ii} of Equation (A1) yields

$$\text{(A2)}\qquad \big(\theta_i - \Sigma_{j \neq i}\theta_{ij}\big)\big(d(\log p_i) - \Sigma_j \theta_j d\big(\log p_j\big)\big) + \Sigma_{j \neq i}\theta_{ij}\big(d\big(\log p_j\big) - \Sigma_j \theta_j d\big(\log p_j\big)\big),$$

or, simplifying,

$$\text{(A3)}\qquad \theta_i\big(d(\log p_i) - \Sigma_j \theta_j d\big(\log p_j\big)\big) + \Sigma_{j \neq 1}\theta_{ij}\big(d\big(\log p_j\big) - d(\log p_i)\big).$$

Substituting result (A3) for $\Sigma_j\ \theta_{ij}$ (d(log p_j) $-\ \Sigma_j\ \theta_j$ d(log p_j)) in Equation (5b) yields

$$\text{(A4)}\qquad w_i d(\log q_i) = \theta_i d(\log Q) + \varphi_0 \theta_i \times \big(d(\log p_i) - \Sigma_j \theta_j d\big(\log p_j\big)\big) + \varphi_0 \Sigma_{j \neq i}\theta_{ij} \big(d\big(\log p_j\big) - d(\log p_i)\big).$$

Equation (A4) is in a convenient form to impose separability restrictions on the cross-price parameters θ_{ij}. For example, if good i is strongly separable from the other goods, then $\theta_{ij} = 0$ for $j \neq$ i (Theil 1971, 1976). Likewise, if goods i and j belong to different weakly separable groups, say groups A and B, then $\theta_{ij} = \varphi_{AB}\ \theta_i\ \theta_j$ (Theil 1976), where φ_{AB} is another factor of proportionality.

Appendix B. Uniform-Substitute Model SEs for Elasticity Estimates at Sample Means for Table 5[a]

		Price											
Beverage	Income	Orange	Grapefruit	Apple	Grape	Remaining Fruit Juice	Vegetable	Juice Drinks	Carbonated Water	Water	Soda	Tea	Milk and Shakes
Orange[b]	0.0373	0.0632	0.0003	0.0021	0.0009	0.0024	0.0015	0.0115	0.0009	0.0136	0.0366	0.0080	0.0004
Grapefruit[b]	0.0544	0.0080	0.0847	0.0028	0.0013	0.0034	0.0021	0.0155	0.0012	0.0187	0.0542	0.0028	0.0263
Apple[b]	0.0686	0.0092	0.0005	0.0769	0.0015	0.0039	0.0024	0.0182	0.0014	0.0216	0.0498	0.0033	0.0314
Grape[b]	0.0750	0.0090	0.0005	0.0031	0.1006	0.0038	0.0023	0.0174	0.0013	0.0209	0.0622	0.0032	0.0306
Remaining fruit juice[b]	0.0388	0.0073	0.0004	0.0027	0.0012	0.0573	0.0019	0.0144	0.0011	0.0177	0.0417	0.0026	0.0243
Vegetable	0.0661	0.0094	0.0005	0.0033	0.0015	0.0039	0.0870	0.0184	0.0014	0.0226	0.0583	0.0034	0.0319
Juice drinks[c]	0.0471	0.0073	0.0004	0.0026	0.0012	0.0031	0.0019	0.0748	0.0011	0.0173	0.0474	0.0024	0.0232
Carbonated water	0.0410	0.0070	0.0004	0.0025	0.0011	0.0029	0.0018	0.0140	0.0642	0.0170	0.0456	0.0025	0.0236
Water	0.0851	0.0126	0.0007	0.0047	0.0021	0.0056	0.0036	0.0254	0.0020	0.1680	0.1037	0.0040	0.0405
Soda	0.0378	0.0104	0.0006	0.0034	0.0020	0.0043	0.0029	0.0205	0.0017	0.0295	0.0451	0.0052	0.0361
Liquid tea	0.0658	0.0080	0.0004	0.0029	0.0013	0.0036	0.0022	0.0150	0.0013	0.0155	0.0843	0.1392	0.0255
Milk and shakes	0.0382	0.0043	0.0002	0.0016	0.0007	0.0018	0.0011	0.0081	0.0007	0.0099	0.0298	0.0015	0.0575

[a] Price elasticities are uncompensated and conditional.
[b] 100% juice.
[c] Less than 100% juice.