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## **Market Structure and the Penetration of Alternative Energy Technologies**

by

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# Market structure and the penetration of alternative energy technologies

Yacov Tsur\*      Amos Zemel<sup>◇</sup>

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## Abstract

Energy market prices ignore external effects, hence miss-allocate energy generation between (polluting) fossil fuels and (clean) solar technologies. Correcting the failure requires understanding the market allocation forces at hand. An important feature of solar energy is that its cost of supply is predominantly due to upfront investments in capital infrastructure (rather than to the actual supply rate) and this feature has far reaching implications for the market allocation outcome. Studying the market allocation process, we specify the conditions under which solar technologies penetrate the energy sector. The framework is then used to analyze policy regulation in the form of taxing fossil energy and subsidizing investments in solar energy. The first policy measure addresses undesirable environmental effects associated with the use of fossil fuels and the second internalizes the benefits of learning by doing in the solar industry. Under certain conditions, a temporary subsidy on solar energy investments gives rise to a flourishing, self-sustained solar industry that will (eventually) drive fossil energy out of production.

**Keywords:** energy, solar technologies, fossil fuels, price thresholds, regulation, environmental damage, learning by doing.

**JEL classification:** C61, Q42, Q58

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# 1 Introduction

Fossil fuels are often mentioned as the main culprit for an impressive list of undesirable consequences, including acid rain, smog, increased atmospheric concentration of greenhouse gases and financing of failed states and terror organizations. Yet, they continue to be the primary source of energy generation worldwide, fueling over 80% of total energy production and this share is not expected to decline anytime soon (International Energy Agency 2008). The obvious reason is that the market price of fossil energy is in most places cheaper than any of the alternative energy sources available. Market prices, however, ignore externalities and the adverse consequences listed above are all external effects par excellence. Our aim in this work is to understand how market forces determine the allocation of energy generation between fossil fuels and alternative sources and to offer regulation schemes that internalize some of the external effects.

We consider a stylized economy in which energy is a primary input of production along with the traditional labor and capital inputs. Energy can be derived from fossil fuels or from alternative sources, e.g. solar, wind or hydro, referred to generically as solar energy. Solar energy entails none of the external effects listed above, but it also differs in an important respect: while fossil energy generation depends on supply of fuels that give rise to a substantial variable cost component, solar energy generation is based on capital designated especially for that purpose. Once the solar infrastructure (wind turbines, solar collectors, photovoltaic panels) has been installed, the generation of solar energy entails very little additional cost. This distinguishing feature is important for understanding the economic forces underlying the energy sector

and the ensuing market allocation of fossil and solar energy. They are also instrumental in designing regulation schemes to internalize the external effects associated with fossil energy.

The economy, described in Section 2, consists of a final good sector, an energy sector, and households who own labor and capital. The energy sector consists of fossil energy firms and solar energy firms. This structure extends that of Tsur and Zemel (2008b) by treating solar energy as an endogenous sector of the economy.<sup>1</sup> In characterizing the market allocation, we first derive the optimal allocation without the external effects associated with fossil energy (Section 3), and then show (Section 4) that it constitutes a competitive allocation. This last step is not straightforward because the intertemporal nature of solar investment decisions and the irreversibility of solar capital could distort the conditions under which social and market allocations concur.

External effects are addressed in Section 5, which discusses two regulation policies: taxing fossil energy and subsidizing investment in solar energy. The rationale for taxing fossil energy is well understood and such a policy is commonplace (see e.g. Stern 2007, and myriad other references and newspaper reports by hitting "carbon tax" in google.com). The rationale for subsidizing solar energy stems from the potential improvement in the efficiency of energy generation via learning by doing in this relatively young industry (Arrow 1962, Sheshinski 1967). The second policy may well complement the first, as we show that a temporary subsidy early on could eventually lead to a flourishing, self-sustained solar energy industry driving fossil energy out of production.

The literature on energy economics and the competition among various technologies is vast and no attempt is made to review it here. Early concerns

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<sup>1</sup>In Tsur and Zemel (2008b) solar energy can be purchased at a given (exogenous) price.

revolved around scarcity of fossil resources and the limit it imposes on economic growth (Barnett and Morse 1963). Technological progress and discoveries of new coal, oil and gas reserves on the one hand, together with rapidly deteriorating environmental quality on the other, have swung the pendulum towards environmental concerns. R&D efforts to develop a backstop substitute for fossil fuels have been suggested as an answer to both the scarcity and environmental concerns (Chakravorty et al. 1997, Tsur and Zemel 2003, 2005). The recent Stern report (Stern 2007) and the debate following it (Nordhaus 2007, Weitzman 2007, Dasgupta 2007) added urgency to the environmental concerns and renewed interest in threats associated with advancing occurrence of catastrophes of global scale (Clarke and Reed 1994, Tsur and Zemel 1996, Alley et al. 2003, Nævdal 2006, Weitzman 2009). The regulation literature deals primarily with tradeoffs between prices (carbon tax) and quantity (cap-and-trade) measures (see Stern 2007, and reference cited there). The present effort studies the market forces underlying the penetration of solar energy technologies and provides the analytic framework to investigate regulation in a dynamic context.

## 2 The economy

The economy consists of a final good sector, an energy sector and households. We discuss each in turn.

### 2.1 Final good

Firm  $i$  uses capital  $K_i$ , energy  $X_i = X_i^f + X_i^a$  and labor  $L_i$  to produce output  $Y_i$  according to the constant-returns-to-scale technology  $Y(K_i, X_i, L_i)$ , where  $X_i^f$  is fossil energy and  $X_i^a$  is energy derived from alternative sources,

such as solar and wind, which serve as perfect substitutes. We refer to these alternative sources generically as ‘solar energy’. Thus,

$$Y_i = L_i y(k, x) \quad (2.1)$$

where  $y(k, x) \equiv Y(k, x, 1)$ ,  $k \equiv K_i/L_i$  and  $x \equiv X_i/L_i$  are the same across firms that use the same technology, hence the firm subscript  $i$  can be dropped. The production function  $y(\cdot, \cdot)$  satisfies the standard properties

$$\begin{aligned} y(0, x) = 0; \quad y(k, 0) = 0; \quad y_k(k, x) > 0; \quad y_x(k, x) > 0; \quad y_k(0, x) = \infty; \\ y_{kk}(k, x) < 0; \quad y_{kx}(k, x) > 0; \quad y_{xx}(k, x) < 0; \\ y_{kk}(k, x)y_{xx}(k, x) - y_{kx}^2(k, x) > 0, \end{aligned} \quad (2.2)$$

where  $k$  and  $x$  subscripts signify partial derivatives with respect to  $k$  and  $x$ .

Firms take as given the capital rental rate  $r$ , the prices of fossil and solar energy,  $p^f$  and  $p^a$ , and the wage rate  $w$  and plan production in order to maximize instantaneous profit

$$L_i[y(k, x) - (r + \delta)k - p^f x^f - p^a x^a - w] \quad (2.3)$$

where  $x^f$  and  $x^a$  are, respectively, the per worker fossil and solar energy inputs and  $\delta$  is the capital depreciation rate. Necessary conditions for profit maximization include

$$y_k(k, x) = r + \delta \quad (2.4)$$

and

$$y_x(k, x) = p \equiv \min(p^f, p^a). \quad (2.5)$$

As fossil and solar energy are perfect substitutes, firms will use only the cheaper source if  $p^f \neq p^a$  and will be indifferent between the two sources if  $p^f = p^a$ .

Thus, the per capita energy cost can be expressed as

$$p^f x^f + p^a x^a = px. \quad (2.6)$$

## 2.2 Energy

Aside from the external effects listed in the introduction (addressed in Section 5), fossil and solar energy differ in one main respect: while fossil energy depends on the supply of fuels that give rise to a substantial variable cost component, solar energy supply is based on capital designated especially for that purpose (wind turbines, solar collectors, photovoltaic panels). Once the solar infrastructure has been installed, the generation of solar energy entails hardly any further cost. When solar capital is irreversible (cannot be rented in and out), this feature implies that the decisions of managers of solar energy firms are of an intertemporal investment type. These considerations are explicitly addressed below.

### 2.2.1 Fossil energy

Let  $\zeta$  represent the unit cost of fossil energy, assumed constant. The supply curve of fossil energy is therefore horizontal and the competitive price of fossil energy is

$$p^f = \zeta. \quad (2.7)$$

When the price of energy is  $\zeta$ , final-good firms will demand the energy input  $x$  such that (cf. (2.5))

$$y_x(k, x) = \zeta. \quad (2.8)$$

For any capital stock  $k$ , we denote by  $x^\zeta(k)$  the energy input  $x$  that satisfies (2.8).



### 2.2.2 Solar energy

Production of solar energy uses capital designated solely for that purpose such that the energy output of solar energy firm  $j$  is

$$X_j^a = bA_j, \quad (2.9)$$

where  $A_j$  is the firm's stock of solar capital and  $b$  is a technological parameter indicating the rate of energy output per unit of capital. Solar capital depreciates at the same rate  $\delta$  as capital  $k$ , but is irreversible in that it cannot be rented in and out. Thus, the firm is locked with its existing capital and will supply the solar energy it produces at the going market price  $p(t)$ , obtaining the revenue flow  $p(t)bA_j(t)$ .

Based on (expectation regarding) the energy price  $p(t)$  and the capital rental rate  $r(t)$  processes, the solar firm decides on its investment policy  $I_j(t)$  according to:

$$\max_{\{0 \leq I_j(t) \leq \bar{I}_j\}} \int_0^\infty [p(t)bA_j(t) - I_j(t)] e^{-\int_0^t r(\tau) d\tau} dt \quad (2.10)$$

subject to

$$\dot{A}_j(t) = I_j(t) - \delta A_j(t) \quad (2.11)$$

and  $A_j(0) = 0$ . The upper bound  $\bar{I}_j$  on the investment rate is due to physical and financial constraints.<sup>2</sup> The optimal solar investment policy is discussed in Section 4.

We let  $I(t) = \sum_j I_j(t)$  represent aggregate investment in solar energy capital,  $A(t) = \sum_j A_j(t)$  denotes the aggregate stock of solar capital,  $X^a(t) = bA(t)$  is the aggregate solar energy supply rate and  $\iota(t)$ ,  $a(t)$  and  $x^a(t) = ba(t)$

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<sup>2</sup>The exact value of the upper bound is insignificant so long as it is large enough to avoid feasibility restrictions on the optimal processes.

denote their per-capita counterparts. The per-capita solar capital evolves in time according to

$$\dot{a}(t) = \iota(t) - \delta a(t) \quad (2.12)$$

where  $\iota(t)$  is constrained by the upper bound  $\bar{\iota}$ , and the per-capita total energy supply rate is

$$x(t) = x^f(t) + x^a(t) = x^f(t) + ba(t). \quad (2.13)$$

### 2.3 Households

The household income at time  $t$  consists of wage income  $w(t)$  plus interest on savings  $r(t)k(t)$  plus revenues from solar energy firms (owned by the households)  $p^a(t)x^a(t)$  minus the investment costs of solar firms  $\iota(t)$ . In equilibrium, the wage rate that clears the labor market gives vanishing profits to the final good producers, implying, noting (2.3) and (2.7),

$$w(t) = y(k(t), x(t)) - \zeta x^f(t) - p^a(t)x^a(t) - [r(t) + \delta]k(t). \quad (2.14)$$

The household income, thus, equals  $y(k(t), x(t)) - \zeta x^f(t) - \iota(t) - \delta k(t)$ , which the household allocates between consumption  $c(t)$  and saving, giving rise to the intertemporal budget constraint

$$\dot{k}(t) = y(k(t), x(t)) - \zeta x^f(t) - \iota(t) - \delta k(t) - c(t). \quad (2.15)$$

The utility from consuming at the rate  $c$  is  $u(c)$ , assumed increasing, strictly concave and satisfying  $u(0) = -\infty$ , so that some positive consumption is essential. A consumption stream  $c(t)$ ,  $t \geq 0$ , generates the payoff

$$\int_0^\infty u(c(t))e^{-\rho t} dt, \quad (2.16)$$

where  $\rho$  is the pure (utility) rate of discount. The household seeks the consumption stream  $c(t)$  that maximizes (2.16) subject to (2.15), given  $k(0) = k_0$ .

In doing so households take firms (energy and final good) decisions exogenously.

## 2.4 Equilibrium

The economy is in equilibrium when all actors (households, managers of final good firms, managers of fossil energy firms and managers of solar energy firms) act rationally and none has an incentive to modify decisions. In equilibrium, the energy and capital price processes,  $\{p(t), r(t), t \geq 0\}$ , clear the energy and capital markets, i.e., at each point of time energy demand by the final good firms just equals the energy supply of fossil and solar firms and households savings just equal the capital demand of the final good firms.

Absent market failures, it is well known that the competitive equilibrium (at least one if multiple equilibria exist) is optimal. We verify that this property is retained here in spite of the irreversibility of solar capital which gives rise to the solar investment problem (2.10). We characterize the optimal allocation policy (ignoring the external effects associated with fossil fuels) in the next section and verify that it constitutes a competitive equilibrium in Section 4. Section 5 addresses the external effects.

## 3 Optimal allocation without external effects

The optimal allocation is the outcome of

$$\max_{\{c(t), x^f(t), \iota(t)\}} \int_0^{\infty} u(c(t)) e^{-\rho t} dt \quad (3.1)$$

subject to (2.12)-(2.15),  $c(t) \geq 0$ ,  $x^f(t) \geq 0$ ,  $\iota(t) \in [0, \bar{\iota}]$ , given  $k(0) = k_0 > 0$  and  $a(0) = 0$ . This is an intertemporal optimization problem with two states (the capital stocks  $k$  and  $a$ ) and three controls ( $c$ ,  $\iota$  and  $x^f$ ). We characterize

the optimal allocation by means of three characteristic curves, defined in the  $(k, x)$  plane.

### 3.1 Characteristic curves

We introduce three curves that divide the  $(k, x)$  plane into distinct regions, in each of which the optimal policy is restricted to a particular behavior. The first curve corresponds to the “singular” policy under which

$$y_k(k(t), x(t)) = by_x(k(t), x(t)), \quad (3.2)$$

i.e., the marginal products of  $k$  and of  $a$  are equal during a non-vanishing time interval. Condition (3.2), called the singular condition, defines a curve in the  $(k, x)$  plane, denoted  $x^s(k)$  and referred to as the singular curve. The term “singular” comes from the property that problem (3.1) is linear in the solar investment rate  $\iota(t)$ . This implies that the optimal  $\iota(t)$  process can either assume the corner values  $\iota = \bar{\iota}$  or  $\iota = 0$  or a singular, intermediate value (see Appendix B). The latter policy  $\iota = \iota^s$  is optimal when the  $(k, x)$  process proceeds along the singular curve with  $x^f = 0$ . Invoking (2.12)-(2.15) this requires

$$\iota^s = \frac{x^{s'}(k)[y(k, x^s(k)) - c - \delta k] + \delta x^s(k)}{b + x^{s'}(k)}, \quad (3.3)$$

where  $x^{s'}(k) \equiv dx^s/dk$ .<sup>3</sup>

A second curve in the  $(k, x)$  plane, denoted  $x^e(k)$ , is defined by the condition

$$y_k(k, x) = \rho + \delta. \quad (3.4)$$

Points residing on this curve satisfy the Ramsey condition (Ramsey 1928) for a steady state, hence we refer to it as the steady state curve.

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<sup>3</sup>The singular policy  $\iota^s$  is feasible when the optimal consumption rate  $c$  yields  $\iota^s \in [0, \bar{\iota}]$ .

The third curve, denoted  $x^\zeta(k)$ , corresponds to energy demand when the unit price of energy is  $\zeta$ . It is defined by condition (2.8), which relates the marginal product of energy to the unit cost of fossil fuel. This curve depicts the demand for energy as a function of capital  $k$  when some fossil energy is used.

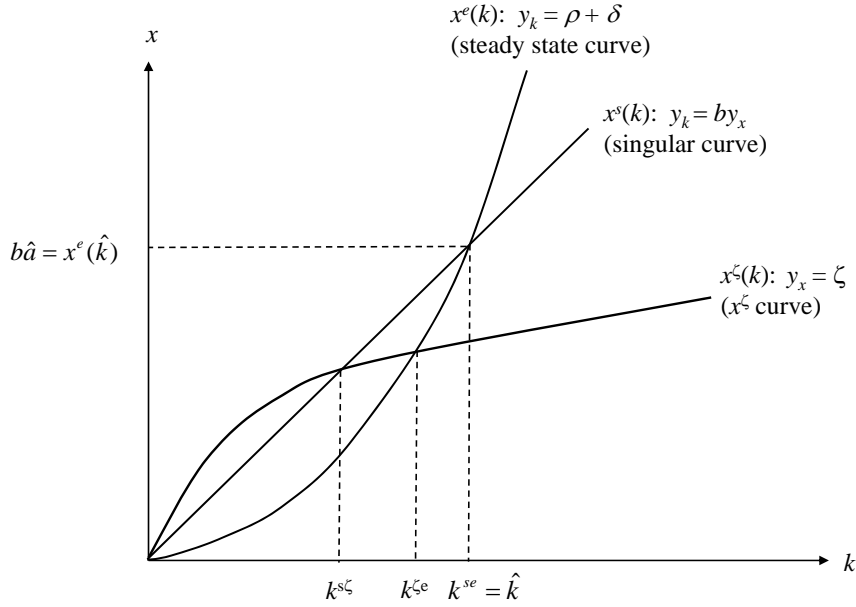


Figure 1: Characteristic curves for the case  $\rho + \delta < b\zeta$

Figures 1 and 2 display the three curves for a Cobb-Douglas technology  $y = y_0 k^\alpha x^\beta$  with  $\alpha > 0$ ,  $\beta > 0$  and  $\alpha + \beta < 1$ . As a matter of notation, we say that  $(k, x)$  is above or below  $x^j(\cdot)$ ,  $j = e, s, \zeta$ , if  $x > x^j(k)$  or  $x < x^j(k)$ , respectively. The geometrical relations among the three curves underlie the characterization of the optimal policy in Appendix B. For example, a point above the singular curve represents a surplus of solar capital (relative to physical capital  $k$ ) and implies that  $\iota = 0$  (no solar investment) is optimal. Using assumption (2.2), we verify in Appendix A the following properties:

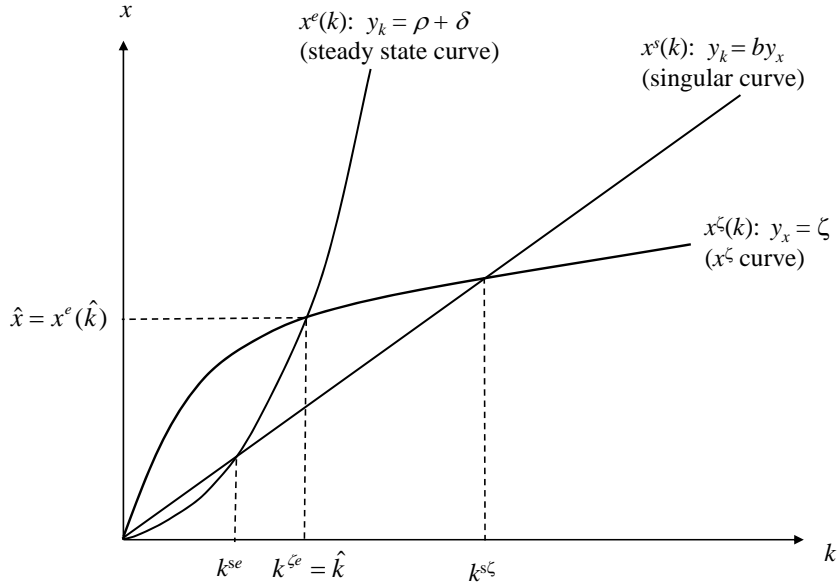


Figure 2: Characteristic curves for the case  $\rho + \delta > b\zeta$

**Property 3.1.** *The three characteristic curves (i) converge at the origin  $(0, 0)$ , and (ii) are increasing (i.e.  $dx^j(k)/dk > 0$ ,  $j = e, s, \zeta$ ).*

Assuming that each pair of curves cross at least once away from the origin (i.e. with  $k > 0$ ,  $x > 0$ ), their relative geometry is completely determined:

**Property 3.2.** *(i)  $x^\zeta(\cdot)$  crosses  $x^s(\cdot)$  once from above. (ii)  $x^e(\cdot)$  crosses  $x^s(\cdot)$  once from below. (iii)  $x^\zeta(\cdot)$  crosses  $x^e(\cdot)$  once from above.*

Let  $k^{se}$  denote the  $k$  level at which  $x^s(\cdot)$  and  $x^e(\cdot)$  intersect,  $k^{s\zeta}$  be the  $k$  level at which  $x^s(\cdot)$  and  $x^\zeta(\cdot)$  intersect, and  $k^{\zeta e}$  be the  $k$  level at which  $x^\zeta(\cdot)$  and  $x^e(\cdot)$  intersect. Note that  $k^{\zeta e}$  must always fall between the two other intersection points (Figures 1-2). In general, the three intersection points differ and the long term evolution of the economy depends on their relative positions. We investigate the long run performance of the economy in the next subsection and study the transitional path in subsection 3.3.

## 3.2 Long run performance

Define

$$\hat{k} = \max(k^{se}, k^{\zeta e}) \quad (3.5a)$$

and let

$$\hat{a} = \begin{cases} x^e(\hat{k})/b & \text{if } k^{se} > k^{\zeta e} \\ 0 & \text{otherwise} \end{cases}, \quad (3.5b)$$

$$\hat{x}^f = \begin{cases} 0 & \text{if } k^{se} > k^{\zeta e} \\ x^e(\hat{k}) & \text{otherwise} \end{cases}, \quad (3.5c)$$

$$\hat{y} = y(\hat{k}, \hat{x}^f + b\hat{a}), \quad (3.5d)$$

$$\hat{i} = \delta\hat{a} \quad (3.5e)$$

and

$$\hat{c} = \hat{y} - \zeta\hat{x}^f - \delta(\hat{k} + \hat{a}). \quad (3.5f)$$

Then (see proof in Appendix B):

**Proposition 3.1.** *The optimal policy converges to the steady state specified by equations (3.5) from any endowment  $k_0 > 0$  and  $a_0 = 0$ .*

From (3.5b)-(3.5c) we see that solar energy prevails in the long run if  $k^{se} > k^{\zeta e}$ . To see why, notice that  $k^{se} > k^{\zeta e}$  implies  $y_x(k^{se}, x^e(k^{se})) < \zeta$  (see Figure 1 and note that  $y_x < \zeta$  above  $x^\zeta$ ). The singular and steady state curves intersect at  $(k^{se}, x^e(k^{se}))$  where  $\rho + \delta = y_k(k^{se}, x^e(k^{se})) = by_x(k^{se}, x^e(k^{se}))$ . Thus, solar energy prevails in the long run (i.e.,  $k^{se} > k^{\zeta e}$ ) if and only if  $\rho + \delta < b\zeta$  or

$$\frac{1}{b}(\rho + \delta) < \zeta. \quad (3.6)$$

The threshold condition (3.6) bears a simple economic interpretation. The solar capital stock  $1/b$  generates a constant energy flow of a unit rate and inflicts the instantaneous cost of  $1/b$  times the effective discount rate (the rate of interest plus the depreciation rate); the effective rate equals  $\rho + \delta$  in the long run.<sup>4</sup> Alternatively, the same unit-rate energy flow can be derived from fossil sources at the instantaneous cost  $\zeta$ . Thus, (3.6) merely states that solar energy is more cost effective. When the reverse of condition (3.6) holds, fossil energy is cheaper in the long run, hence will (eventually) prevail. These considerations are summarized in:

**Proposition 3.2.** *(i) When condition (3.6) is satisfied, the use of fossil energy gradually diminishes and long run production is based exclusively on solar energy. (ii) When the reverse of (3.6) holds, energy is supplied in the long run exclusively from fossil sources.*

In view of the proposition we refer to economies satisfying condition (3.6) (depicted in Figure 1) as solar-based economies while economies for which the reverse condition holds are classified as fossil-based economies (Figure 2).

### 3.3 Transition to the steady state

Proposition 3.2 specifies where the economy is heading to in the long run. Here we characterize the entire transitional path. Consider an economy with a capital endowment  $k_0 < \min(k^{s\zeta}, k^{se})$  and a vanishing solar capital stock. Regardless of whether the economy is fossil-based or solar-based, initially the optimal policy is to avoid investments in solar capital ( $\iota = 0$ ) and to grow along the  $x^\zeta$  curve using fossil energy exclusively. For fossil-based economies

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<sup>4</sup>When  $k^{se} > k^{\zeta e}$ , Proposition 3.1 and (3.4) imply  $y_k(\hat{k}, \hat{x}) = y_k(k^{se}, x^e(k^{se})) = \rho + \delta$ .



(depicted in Figure 2), investment in solar capital never takes place (i.e., the  $\iota = 0$  regime prevails indefinitely) and the economy approaches a steady state at the point  $(k^{\zeta e}, x^{\zeta}(k^{\zeta e}))$ , where the steady state curve and the  $x^{\zeta}$  curve intersect and conditions (3.4) and (2.8) are satisfied.

A solar-based economy (depicted in Figure 1) evolves along the  $x^{\zeta}$  curve until it reaches  $(k^{s\zeta}, x^s(k^{s\zeta}))$ , where the  $x^{\zeta}$  curve intersects the singular curve. Upon reaching this point, the solar investment policy switches to the singular regime, building up solar capital  $a(t)$  at the singular rate

$$\iota^s(t) = y(k^{s\zeta}, x^s(k^{s\zeta})) - c(t) - \delta k^{s\zeta} - \zeta[x^s(k^{s\zeta}) - ba(t)], \quad (3.7)$$

leaving  $k$  constant at  $k^{s\zeta}$  and reducing the use of fossil energy such that the total energy use remains fixed at  $x^s(k^{s\zeta})$ . As soon as  $a(t)$  is large enough to supply the entire energy demand, i.e., when  $ba(t) = x^s(k^{s\zeta})$ , both types of capital,  $k(t)$  and  $a(t)$ , grow simultaneously along the singular curve (with solar investment given by (3.3)) towards the steady state  $(k^{se}, x^s(k^{se}))$ , where the singular curve intersects the steady state curve (see Figure 1) and conditions (3.4) and (3.2) are satisfied.

We summarize this behavior in:

**Proposition 3.3.** *The optimal policy for a small economy (endowed with  $k(0) = k_0 < \min(k^{s\zeta}, k^{se})$  and  $a(0) = 0$ ) is characterized as follows:*

(i) *When condition (3.6) holds (solar based economies), the optimal processes evolve along the following three phases: (a) An initial fossil phase (with  $\iota(t) = a(t) = 0$ ), in which the economy grows along the  $x^{\zeta}$  curve until it reaches the intersection point  $(k^{s\zeta}, x^s(k^{s\zeta}))$  with the singular curve  $x^s$ . (b) A coexistence phase, in which  $k(t)$  and  $x(t)$  are held fixed at  $k^{s\zeta}$  and  $x^s(k^{s\zeta})$ , respectively, while fossil energy input  $x^f(t)$  shrinks and solar energy input  $ba(t)$  increases*

until the use of fossil energy vanishes. (c) A solar phase ( $x^f(t) = 0$ ), where  $k(t)$  and  $a(t)$  grow together along the singular curve towards a steady state at the intersection point  $(k^{se}, x^s(k^{se}))$  of the singular and steady state curves.

(ii) When the reverse of condition (3.6) holds (fossil based economies), no investment in solar energy ever takes place ( $\iota(t) = a(t) = 0$ ) and the economy evolves along  $x^\zeta$  towards a steady state at the intersection point  $(k^{\zeta e}, x^e(k^{\zeta e}))$  of  $x^\zeta$  and the steady state curve.

The proof is provided in Appendix B.

**Remark 1.** It follows from the explicit specification in Proposition 3.3 that the optimal policy is unique.

Each of the phases described in Proposition 3.3 can be recast as a standard dynamic optimization problem with a single state variable. Solving for the optimal processes during each phase, and determining the durations of the phases by the transversality conditions associated with the transition from one phase to the other, the complete time dependence of the socially optimal processes is derived. We denote the optimal state processes by  $k^*(t)$  and  $a^*(t)$ , and the associated consumption, investment and fossil energy processes by  $c^*(t)$ ,  $\iota^*(t)$  and  $x^{f*}(t)$ , respectively. The corresponding capital and energy prices, defined by (2.4) and (2.5) evaluated at the optimal processes, are denoted  $r^*(t)$  and  $p^*(t)$ .

## 4 Competitive allocation

The dependence of solar energy generation on irreversible solar capital and the upper bound imposed on the rate of solar investments require managers of solar firms (unlike managers of final good and fossil energy firms) to make

forward-looking investment decisions.<sup>5</sup> This raises the question regarding whether the optimal allocation characterized above is also a competitive equilibrium, i.e., whether (i) households anticipating the processes  $\iota^*(t)$ ,  $x^{f*}(t)$  and  $a^*(t)$  will choose to consume  $c^*(t)$  and save  $k^*(t)$ , (ii) final good firms facing the energy price  $p^*(t)$  and the capital rental rate  $r^*(t)$  will demand the inputs  $k^*(t)$  and  $x^*(t)$  to produce  $y(k^*(t), x^*(t))$ , (iii) fossil energy firms facing the energy price  $p^*(t)$  and solar capital  $a^*(t)$  will supply  $x^{f*}(t)$ , and (iv) managers of solar firms, anticipating the energy and capital price processes  $p^*(t)$  and  $r^*(t)$ , will adopt the investment policy  $\iota^*(t)$  which gives rise to the solar capital process  $a^*(t)$ . Parts (i)-(iii) are straightforward and will be discussed briefly for the sake of completeness. We verify (iv) for a solar-based economy.

We begin by relating the optimal prices  $p^*(t)$  and  $r^*(t)$  along the path to the steady state. Let the times  $0 < t_1 < t_2 < \infty$  denote the boundaries of the three subperiods  $[0, t_1]$ ,  $[t_1, t_2]$  and  $[t_2, \infty)$  corresponding to the three phases of Proposition 3.3(i), along which the following relations hold:

$$p^*(t) = \begin{cases} y_x(k^*(t), x^*(t)) = \zeta, & 0 \leq t \leq t_2 \\ y_x(k^*(t), x^*(t)) < \zeta, & t > t_2 \end{cases} \quad (4.1)$$

$$\begin{cases} y_k(k^*(t), x^*(t)) > by_x(k^*(t), x^*(t)), & 0 \leq t < t_1 \\ y_k(k^*(t), x^*(t)) = by_x(k^*(t), x^*(t)), & t \geq t_1 \end{cases} \quad (4.2)$$

and

$$r^*(t) + \delta = y_k(k^*(t), x^*(t)) \quad \forall t. \quad (4.3)$$

Thus,

$$\begin{cases} p^*(t) < (r^*(t) + \delta)/b, & 0 \leq t < t_1 \\ p^*(t) = (r^*(t) + \delta)/b, & t \geq t_1 \end{cases}. \quad (4.4)$$

---

<sup>5</sup>Since households are also forward looking, the interaction between the various actors in the economy is more involved than the standard situation in which all firms maximize instantaneous profits.

## 4.1 Solar firms

The solar firm investment problem is recast in a per-capita form as:

$$\max_{\{0 \leq \iota(t) \leq \bar{\iota}\}} \int_0^\infty [p^*(t)ba(t) - \iota(t)]e^{-\int_0^t r^*(\tau)d\tau} dt \quad (4.5)$$

subject to (2.12) and  $a(0) = 0$ . Define the discount factor  $\Theta(t) \equiv e^{-\int_0^t r^*(\tau)d\tau}$  and note that  $\dot{\Theta}(t) = -r^*(t)\Theta(t)$ . Then,  $\lim_{t \rightarrow \infty} r^*(t) = \rho > 0$  (cf. Proposition 3.1) implies  $\lim_{t \rightarrow \infty} \Theta(t) = 0$ .

In view of (4.4), the contribution of the coexistence and solar phases to the objective in (4.5) is

$$v^s = \max_{\{0 \leq \iota(t) \leq \bar{\iota}\}} \int_{t_1}^\infty \{(r^*(t) + \delta)a(t) - \iota(t)\}\Theta(t)dt \quad (4.6)$$

subject to (2.12) and given  $a(t_1)$ . Using (2.12) we write

$$(r^*(t) + \delta)a(t) - \iota(t) = r^*(t)a(t) - \dot{a}(t). \quad (4.7)$$

However, since  $\Theta(\infty) = 0$  and  $\dot{\Theta}(t) = -r^*(t)\Theta(t)$ ,

$$\begin{aligned} \int_{t_1}^\infty \dot{a}(t)\Theta(t)dt &= -a(t_1)\Theta(t_1) - \int_{t_1}^\infty a(t)\dot{\Theta}(t)dt \\ &= -a(t_1)\Theta(t_1) + \int_{t_1}^\infty a(t)r^*(t)\Theta(t)dt, \end{aligned} \quad (4.8)$$

implying that  $v^s = a(t_1)\Theta(t_1)$  for every feasible trajectory.

Using (4.4) and  $a(t) \geq 0$  we can bound the contribution to the objective of the fossil phase by

$$v^s \leq \int_0^{t_1} \{[r^*(t) + \delta]a(t) - \iota(t)\}\Theta(t)dt = -a(t_1)\Theta(t_1) \quad (4.9)$$

where the latter equality is derived in the same way as in the singular phase, with  $a(0) = 0$ . The inequality in (4.9) is strict unless  $a(t) = 0$  for all  $t < t_1$ . Thus, the objective of (4.5) is strictly negative unless  $\iota(t) \equiv 0$  during the

fossil phase. Therefore, the solar firm can avoid a loss only if it refrains from investing during this phase, in agreement with the optimal policy of Section 3.

For  $t > t_1$ , the energy and capital prices,  $p^*(t)$  and  $r^*(t)$ , are such that individual managers of solar firms are indifferent regarding their own investment policy, in that any feasible investment policy will give rise to the same value, provided each firm is too small to affect the market prices (which continue to satisfy (4.3)-(4.4)). This is in contrast to the optimal plan of Section 3, in which the optimal solar investment policy must proceed along the  $\iota^*(t)$  trajectory. Nevertheless, market forces ensure that solar firms will invest at the optimal rate  $\iota^*(t)$ , which for  $t > t_1$  is the singular policy, and the ensuing solar capital will just suffice to maintain (4.3)-(4.4).

To see this, suppose that enough firms overinvest so that the  $(k, ba)$  process falls above the singular curve and  $y_k > by_x$  (Figure 1). This implies that  $p(t) < (r(t) + \delta)/b$  and the considerations that deter the solar firms from investing during the fossil phase will apply again, reducing solar capital  $a$  until equilibrium is restored. Similarly, if enough solar firms under-invest, the  $(k, ba)$  process is driven below the singular curve, leading to  $p(t) > (r(t) + \delta)/b$ . Repeating the evaluation of the objective integral, we see that the solar firms can now make a positive profit during this phase, and this profit increases with  $a$ . Thus, more solar firms will be induced to invest at the maximal rate, returning solar capital to the equilibrium level. We conclude that solar firms facing the price trajectories  $p^*(t)$  and  $r^*(t)$  will invest at the rate  $\iota^*(t)$  and maintain the solar capital stock  $a^*(t)$ .

## 4.2 Households, final good firms and fossil energy firms

Households and the social planner maximize the same objective ((2.16) and (3.1)) subject to the same budget constraint (2.15). When  $x^f(\cdot)$ ,  $\iota(\cdot)$  and  $a(\cdot)$  evolve along their optimal trajectories, the household optimal consumption-saving policy must coincide with  $c^*(\cdot)$  and  $k^*(\cdot)$ , for otherwise the latter processes could not have been socially optimal (cf. Remark 1).

Regarding final good firms, equations (2.4) and (2.5), together with assumption (2.2) imply that the prices  $p$  and  $r$  of energy and capital uniquely determine  $k$  and  $x$ . It follows from (4.1) and (4.3) that observing the energy price  $p^*(t)$  and capital rental rate  $r^*(t)$ , final good firms will demand the capital and energy inputs  $k^*(t)$  and  $x^*(t)$  at each point of time.

Given energy price  $p^*(t)$ , energy demand  $x^*(t)$  and solar capital  $a^*(t)$ , fossil firms will supply the entire energy demand during the fossil phase  $t \leq t_1$ , while  $p^*(t) = \zeta$ , because  $a^*(t) = 0$  and no solar energy can be supplied. During the coexistence phase  $t \in [t_1, t_2]$ , while the energy price remains fixed at  $\zeta$ , they will supply the residual demand  $x^*(t) - ba^*(t)$ . This is so because the solar firms will supply their energy output  $ba^*(t)$  at whatever the going market rate turns out to be while the fossil firms will supply nothing at any price below  $\zeta$ . This gives solar firms a “deterrence” wedge over fossil firms and implies that the latter will supply only the residual demand. During the solar phase ( $t \geq t_2$ ),  $p^*(t) < \zeta$  and fossil firms will be driven out of the market.

We summarize the above discussion in:

**Proposition 4.1.** *The optimal policy characterized in Proposition 3.3 constitutes a competitive equilibrium.*

## 5 Regulation

We use the allocation model developed above as a framework for analyzing possible regulation of the adverse external effects associated with fossil fuels and the presence of learning by doing in the (relatively young) solar industry.

### 5.1 Environmental cost of fossil energy

Fossil energy entails a variety of polluting emissions that seriously degrade environmental quality. Since the latter is a public bad, firms will tend to overlook their own contribution to the cumulative pollution and will fail to account for the ensuing damage in their resource allocation decisions. The economy as a whole, however, may be bound by exogenous constraints (such as international agreements) to reduce its overall emission rates.

Suppose that the economy is of a fossil-based type (Figure 2), and the equilibrium emission rate associated with  $\hat{x} = \hat{x}^f$  of (3.5c) exceeds the permitted rate. A common policy to control emission is to levy a tax  $\beta$  on the use of fossil energy, so that  $\zeta + \beta$  replaces  $\zeta$  as the effective unit cost. This implies a lowering of the  $x^\zeta$  curve with a corresponding reduction of  $\hat{x}^f$ . In particular, if the tax rate  $\beta$  is sufficiently high so that

$$\zeta < \frac{\rho + \delta}{b} < \zeta + \beta, \quad (5.1)$$

then the added cost of fossil energy, i.e. the tax rate  $\beta$ , converts the economy from fossil-based to solar-based. (Condition (3.6) fails when the cost of fossil energy is  $\zeta$  but holds when it is  $\zeta + \beta$ ). Thus, the use of fossil energy will gradually be reduced until vanishing completely, as the economy evolves along the trajectory characterized in Proposition 3.3.

The same characterization provides further insights on the implications of

this policy. First, the lowering of the  $x^\zeta$  curve implies a corresponding decrease in the steady state capital stock  $\hat{k}$  and consumption rate  $\hat{c}$ . Thus, the benefits of the decreased emission can be weighted against the reduced value of the objective for every value of  $\beta$ , helping to determine the optimal tax rate for the economy. Second, the lowering of the  $x^\zeta$  curve entails immediate reduction in emissions, since  $x^\zeta(k)$  is lower at any  $k$  level along the curve and not only at the eventual steady state. This raises interesting possibilities regarding the optimal time profile of the tax rate  $\beta(t)$ . Early on, when  $x^\zeta(k)$  falls short of the exogenous bound, a lower tax rate may be imposed, to be gradually increased as the economy expands and the demand for energy increases. We leave a more detailed study of these questions to future work.

## 5.2 Learning by doing in the solar energy industry

Since solar technologies are relatively young, there is plenty of room to improve efficiency and lower their costs. Fossil power generation, on the other hand, is a mature technology for which most of the important innovations have already been implemented. Of particular interest are technical changes associated with learning by doing, as the experience gained due to installation of solar capital increases its efficiency parameter  $b$ . Such technological progress mechanisms are modeled in our framework via  $b = b(A)$ , where  $b(\cdot)$  is an increasing function of the aggregate solar capital  $A$ . The latter, then, turns into a public good because it serves not only to provide solar energy but also to enhance the efficiency at which this energy is produced. This externality gives rise to a market failure, because the solar firms tend to ignore the latter contribution hence will under invest in solar capital relative to the socially optimal rate.



From equation (A.1) in Appendix A, we see that the slope of the singular curve increases with  $b$ . Consider an economy where the initial efficiency  $b(0)$  (with no solar energy experience) fails to satisfy (3.6), so that without investment in solar capital, the economy is fossil based. Assume that any solar capital above  $\bar{A}$  reverses condition (3.6), i.e.

$$\frac{\rho + \delta}{b(0)} > \zeta = \frac{\rho + \delta}{b(\bar{A})}. \quad (5.2)$$

Thus, as solar capital accumulates, the singular curve increases its slope and the geometry of the characteristic curves changes from that of Figure 2 to that of Figure 1.

Under the (unregulated) market allocation, however, no investment in solar capital will ever take place, hence the economy will not be able to realize the potential benefits of learning-by-doing. A possible remedy for this market failure comes in the form of a subsidy  $\sigma(t)$  on investment in solar capital, such that solar firms face the reduced interest rate  $r(t) - \sigma(t)$  when making investment decisions. As the use of solar energy increases, the subsidy rate will diminish, but by then the efficiency parameter  $b(\cdot)$  will be large enough to justify investment by the solar firms and further intervention on the part of the regulator will no longer be required.

In actual practice, the proceeds from the emission tax  $\beta$  can be used to finance (some or all of) the solar investment subsidy, resembling a double-dividend policy.

## 6 Conclusions

We study prospects for the penetration of solar energy technologies in a competitive economy, where energy (an essential factor of production) can be

generated from fossil fuels or by solar technologies. We characterize the entire evolution path of the competitive market allocation and provide a condition for solar energy to prevail in the long run. This condition is specified in terms of the efficiency of solar energy generation ( $b$ ), the price of fossil fuel ( $\zeta$ ) and the long term price of capital ( $\rho + \delta$ ) and shows the effects of these parameters on the economic viability of solar energy in a competitive environment. We then discuss regulation policies in the form of taxing fossil fuels and subsidizing investment in solar energy, addressing external effects of fossil fuels and learning-by-doing in the (relatively infant) solar industry. Under certain conditions, a temporary solar investment subsidy gives rise to a flourishing, self-sustained solar energy industry that will eventually drive fossil energy out of production. The temporary subsidy, thus, is capable of addressing both types of external effects. Moreover, carbon tax proceeds can be used to finance some or all of the subsidy cost.

Our analysis simplifies in a number of ways. First, the price of fossil energy is assumed constant. Relaxing this assumption can give rise to coexistence of fossil and solar energy also in the long run (under constant fossil price, such coexistence can only be temporary). Secondly, no account is taken here of fossil fuels scarcity. With scarcity included, the price of fossil energy will increase over time as the fossil reserves shrink, increasing the desirability of solar technologies. In this respect our analysis is somewhat overpessimistic about the prospects of solar energy. Thirdly, innovations in the solar industry need not be restricted to learning-by-doing. Dedicated R&D efforts that consume resources can contribute to increase efficiency and reduce costs (as in Tsur and Zemel 2003, 2005). Finally, externalities under conditions of sustained growth can be considered (as in Tsur and Zemel 2008a). Any of

these changes will constitute a valuable extension.

## Appendix

### A Properties of the characteristic curves

#### Property 3.1

*Proof.* Suppose  $x^e(0) = x_0 > 0$ , then  $y_k(0, x_0) = \rho + \delta$  violating (2.2) which states that  $y_k(0, x) = \infty$  for all  $x > 0$ . Suppose that the  $x^e(\cdot)$  curve crosses the  $k$  axis at some state  $k_0 > 0$ , then  $y_k(k_0, 0) = \rho + \delta$  violating (2.2) which implies  $y_k(k, 0) = 0$  for all  $k > 0$ . Therefore the  $x^e(\cdot)$  curve must approach the origin. Similar considerations apply for the other two curves, establishing (i).

Taking the derivatives of (3.2), (3.4) and (2.8) we obtain

$$\frac{dx^s(k)}{dk} = \frac{by_{kx} - y_{kk}}{y_{kx} - by_{xx}} > 0, \quad (\text{A.1})$$

$$\frac{dx^e(k)}{dk} = -\frac{y_{kk}}{y_{kx}} > 0 \quad (\text{A.2})$$

and

$$\frac{dx^\zeta(k)}{dk} = -\frac{y_{kx}}{y_{xx}} > 0, \quad (\text{A.3})$$

establishing (ii). □

#### Property 3.2:

*Proof.* Evaluated at a crossing point, (A.1) and (A.3) give

$$\frac{dx^s(k)}{dk} - \frac{dx^\zeta(k)}{dk} = \frac{y_{kk}y_{xx} - y_{kx}^2}{-y_{xx}(y_{kx} - by_{xx})} > 0.$$

Since multiple crossings imply alternating signs for the slope difference, this entails (i). A similar comparison of (A.1) and (A.2) at a crossing point gives

$$\frac{dx^s(k)}{dk} - \frac{dx^e(k)}{dk} = \frac{-b(y_{kk}y_{xx} - y_{kx}^2)}{y_{kx}(y_{kx} - by_{xx})} < 0,$$

establishing (ii). Finally

$$\frac{dx^s(k)}{dk} - \frac{dx^e(k)}{dk} = \frac{y_{kk}y_{xx} - y_{kx}^2}{y_{kx}y_{xx}} < 0,$$

which verifies (iii). □

## B Characterization of the optimal allocation

The optimal allocation problem is formulated as

$$v(k_0) = \max_{\{c(t) \geq 0, x^f(t) \geq 0, \iota(t) \in [0, \bar{\iota}]\}} \int_0^\infty u(c(t))e^{-\rho t} dt \quad (\text{B.1})$$

subject to (2.12)-(2.15), given  $k(0) = k_0$  and  $a(0) = 0$ . The bound  $\bar{\iota}$  is assumed to be large enough so that the singular policy is feasible, and the  $k(\cdot)$  process decreases under the  $\iota = \bar{\iota}$  regime for the relevant  $k$  domain.

When no risk of confusion arises, we suppress the time argument  $t$ . The current value Hamiltonian corresponding to (B.1) is

$$\mathcal{H} = u(c) + \lambda[y(k, x^f + ba) - \zeta x^f - \iota - c - \delta k] + \gamma[i - \delta a] \quad (\text{B.2})$$

where  $\lambda$  and  $\gamma$  are the current value costates of  $k$  and  $a$ , respectively. Defining

$$\phi \equiv \gamma - \lambda, \quad (\text{B.3})$$

the necessary conditions for optimum include:

$$u'(c) = \lambda, \quad (\text{B.4})$$

and

$$y_x(k, x) \leq \zeta, \quad \text{equality holding if } x^f > 0. \quad (\text{B.5})$$

The necessary condition for  $\iota$  gives

$$\iota = \begin{cases} \bar{\iota} & \text{if } \phi > 0 \\ 0 & \text{if } \phi < 0 \\ \iota^s & \text{if } \phi = 0 \end{cases} \quad (\text{B.6})$$

where  $\iota^s$  is the singular policy, defined by (3.3) or (3.7). The costate variables evolve according to

$$\dot{\lambda} = -\lambda[y_k(k, x) - (\rho + \delta)], \quad (\text{B.7})$$

and

$$\dot{\gamma} = -\lambda b y_x(k, x) + \gamma(\rho + \delta), \quad (\text{B.8})$$

The transversality condition is

$$\lim_{t \rightarrow \infty} \mathcal{H}(t) = 0. \quad (\text{B.9})$$

Define

$$\Lambda(k, x) \equiv y_k(k, x) - b y_x(k, x) \quad (\text{B.10})$$

and combine (B.3), (B.7) and (B.8) to obtain

$$\dot{\phi} \equiv \dot{\gamma} - \dot{\lambda} = \Lambda \lambda + \phi(\rho + \delta), \quad (\text{B.11})$$

which can be integrated to give (for any arbitrary time  $t_0 \geq 0$ )

$$\phi(t) e^{-(\rho+\delta)t} = \phi(t_0) e^{-(\rho+\delta)t_0} + \int_{t_0}^t \Lambda(k(\tau), x(\tau)) \lambda(\tau) e^{-(\rho+\delta)\tau} d\tau. \quad (\text{B.12})$$

The analysis is carried out in terms of the geometry of the optimal process vis-a-vis the three characteristic curves. Each of these curves divides the  $(k, x)$  plane to regions above and below it (i.e. with  $x$  exceeding or falling short of

$x^j(k)$ ,  $j = e, s, \zeta$ , respectively). We say that the  $(k, x)$  process crosses a curve *from below* when it moves from the region below the curve to the region above it, even if the  $x(\cdot)$  process decreases at the crossing time (in which case the crossing might be more appropriately described as *from the right*). We also say that some policy is maintained *indefinitely* if it is followed from some time onwards to  $t \rightarrow \infty$ . Unless specified otherwise, all statements referring to "the processes" describe optimal processes.

**Property B.1.** *Under the optimal policy: (i)  $\dot{c} > 0$  if  $(k, x)$  is above  $x^e(\cdot)$ ; (ii)  $\dot{c} < 0$  if  $(k, x)$  is below  $x^e(\cdot)$ ; (iii) a steady state must reside on the  $x^e(\cdot)$  curve.*

*Proof.* Taking the time derivative of (B.4) and using (B.7), we find

$$-u''(c)\dot{c}/u'(c) = y_k(k, x) - (\rho + \delta). \quad (\text{B.13})$$

Condition (3.4) which defines the steady state curve and assumption (2.2) imply that  $y_k(k, x) > \rho + \delta$  above  $x^e(\cdot)$  and the reverse relation holds below  $x^e(\cdot)$ . Noting that  $-u''/u' > 0$ , we conclude that  $\dot{c} > 0$  above  $x^e(\cdot)$  and  $\dot{c} < 0$  below  $x^e(\cdot)$ . A steady state entails  $\dot{c} = 0$ , hence it must reside on the steady state curve.  $\square$

**Property B.2.** *The optimal  $(k, x)$  process proceeds along or above  $x^\zeta(\cdot)$ . This is achieved by adjusting  $x^f$  such that*

$$x^f = \begin{cases} 0 & \text{if } x^\zeta(k) - ba \leq 0 \\ x^\zeta(k) - ba & \text{if } x^\zeta(k) - ba > 0 \end{cases},$$

*Proof.* (i) According to (2.8) and (2.2),  $y_x(k, ba) > \zeta$  when  $x^\zeta(k) - ba > 0$ . This situation violates (B.5) and  $x^f = x^\zeta(k) - ba > 0$  must be invoked to augment  $ba$  and satisfy (B.5), shifting  $(k, x)$  to reside along the  $x^\zeta(\cdot)$  curve.

If  $x^\zeta(k) - ba < 0$  then  $y_x(k, ba) < \zeta$ . However when  $x^f > 0$ , (2.2) implies  $y_x(k, ba) > y_x(k, x^f + ba) = \zeta$ , where the latter equality follows from (B.5). The contradiction implies that  $x^f = 0$  holds above  $x^\zeta(\cdot)$ .  $\square$

The following corollary holds:

**Property B.3.** *Maintaining the  $\iota = \bar{\iota}$  regime indefinitely cannot be optimal.*

*Proof.* According to Property B.2, the  $(k, x)$  process proceeds on or above  $x^\zeta(\cdot)$ . If the  $\iota = \bar{\iota}$  regime is followed indefinitely, the decreasing  $k$  process will fall below  $k^{\zeta e}$  at some finite time, following which  $x^\zeta(k) > x^e(k)$  holds. Thus,  $\dot{c} > 0$  (Property B.1) and, with  $\iota = \bar{\iota}$ , the capital stock  $k$  will be depleted in finite time, which (with  $\eta > 1$ ) reduces utility to  $-\infty$  and cannot be optimal.  $\square$

**Property B.4.** (i) *Under the singular regime the  $(k, x)$  process proceeds along the singular curve. (ii) If  $\iota = 0$  at some time when  $(k, x)$  is below  $x^s(k)$  then: (a) the process cannot switch to another  $\iota$ -regime as long as  $(k, x)$  remains below  $x^s(k)$  and (b) the  $(k, x)$  process must eventually cross  $x^s(k)$ . (iii) If  $\iota = \bar{\iota}$  at some time when  $(k, x)$  lies above  $x^s(k)$  then: (a) the process cannot switch to another  $\iota$ -regime as long as  $(k, x)$  remains above  $x^s(k)$  and (b) the  $(k, x)$  process must eventually cross  $x^s(k)$ . (iv) Except for the intersection point  $(k^{s\zeta}, x^\zeta(k^{s\zeta}))$ , a singular process must proceed with  $x^f = 0$ .*

*Proof.* (i) According to (B.6), the singular regime entails  $\phi = \dot{\phi} = 0$ , hence (B.11) implies  $\Lambda(k, x) = 0$  which defines  $x^s(\cdot)$ .

(ii) The properties of  $y(\cdot, \cdot)$  (see (2.2)) imply that  $\Lambda(k, x)$  is negative or positive for  $(k, x)$  below or above  $x^s(\cdot)$ , respectively. Suppose that a  $(k, x)$

process is initiated at some time  $t_0$  below  $x^s(\cdot)$  with  $\iota(t_0) = 0$ , so that according to (B.6)  $\phi(t_0) < 0$ . With  $\lambda > 0$  and  $\Lambda(k, x) < 0$ , (B.12) ensures that  $\phi(t)e^{-(\delta+\rho)t}$  is bounded from above by the negative constant  $\phi(t_0)e^{-(\delta+\rho)t_0}$  so long as the  $(k, x)$  process remains below the singular curve, establishing (a). To verify (b), notice that if  $(k, x)$  never crosses  $x^s(\cdot)$ , the policy  $\iota = 0$  will be retained indefinitely (since  $\phi$  remains negative), which cannot be optimal for the following reason. With  $\gamma > 0$ , we see that  $\lambda(t)e^{-(\delta+\rho)t}$  is also bounded away from zero by a positive constant. Integrating (B.7) we find

$$\lambda(t)e^{-(\delta+\rho)t} = \lambda(t_0)e^{-(\delta+\rho)t_0} \exp \left[ - \int_{t_0}^t y_k(k(\tau), x(\tau)) d\tau \right],$$

which is bounded away from zero only if  $k \rightarrow \infty$  at large  $t$ . Under the  $\iota = 0$  regime, the state  $a$  decreases, hence eventually  $x^\zeta(k) - ba > 0$  must hold and from that time on both  $k$  and  $x$  increase along  $x^\zeta(\cdot)$  (Property B.2(i)). For large enough  $k$ , Property 3.2(iii) ensures that  $x^\zeta(k) < x^e(k)$ , where  $\dot{c} < 0$  (Property B.1). However, the policy of keeping  $k$  and  $x$  constant, diverting the resources required to increase them to enhance consumption, is feasible and yields a higher value.

(iii) Suppose that a  $(k, x)$  process is initiated at some time  $t_0$  above  $x^s(\cdot)$  with  $\iota(t_0) = \bar{\iota}$ , so that  $\phi(t_0) > 0$ . Repeating the above argument, we show that  $\phi(t)e^{-(\delta+\rho)t}$  is bounded away from zero by a positive constant as long as  $(k, x)$  is above  $x^s(\cdot)$ , hence the  $\iota$  regime will be maintained. According to property B.3, this regime cannot hold indefinitely, hence the singular curve must be crossed.

(iv) The singular process proceeds along the singular curve  $x^s(\cdot)$ , which lies below or above  $x^\zeta(\cdot)$  for  $k < k^{s\zeta}$  or  $k > k^{s\zeta}$ , respectively. According to Property B.2, no process is optimal below  $x^\zeta(\cdot)$ , while  $x^f = 0$  holds above



$x^\zeta(\cdot)$ . □

**Property B.5.** (i) If a singular  $(k, x)$  process leaves the singular curve to the region above it, the corresponding  $\iota$  regime changes from singular to  $\iota = \bar{\iota}$  at the departure time. (ii) If a singular  $(k, x)$  process leaves the singular curve to the region below it, the corresponding  $\iota$  regime changes from singular to  $\iota = 0$  at the departure time.

*Proof.* The singular  $(k, x)$  process proceeds with  $\dot{\phi} = \phi = 0$ . Suppose that by mistuning  $\iota$  the  $(k, x)$  process is driven above the singular curve at some time  $t_0$ . With  $\phi(t_0) = 0$  and  $\Lambda(k(t), x(t)) > 0$ , (B.12) implies that  $\phi(t) > 0$  at  $t$  just after  $t_0$ , hence  $\iota = \bar{\iota}$  is adopted above  $x^s(\cdot)$ . The same considerations show that leaving to the region below  $x^s(\cdot)$  (where  $\Lambda(\cdot, \cdot) < 0$ ) implies  $\iota = 0$ . □

**Property B.6.** The optimal  $(k, x)$  process does not cross  $x^s(\cdot)$  from above with  $\iota = \bar{\iota}$ .

*Proof.* Under the  $\iota = \bar{\iota}$  regime,  $k(\cdot)$  decreases and  $a(\cdot)$  increases. For the  $(k, x)$  process to cross  $x^s(k)$  from above, its slope must exceed that of the singular curve, i.e.  $\dot{x}/\dot{k} > x^s(k) > 0$  must hold at the crossing time. Suppose  $x^f = 0$  then  $\dot{x}^f \geq 0$  hence  $\dot{x} \geq b\dot{a} > 0$  while  $\dot{k} < 0$ , so crossing from above cannot occur.

Crossing with  $x^f > 0$  can take place only along the  $x^\zeta$  curve. The latter crosses the singular curve at  $k^{s\zeta}$  from above, hence the crossing requires that  $k(\cdot)$  increases, which cannot occur under this  $\iota$  regime. □

**Property B.7.** Above  $x^s(\cdot)$ , the optimal policy is to set  $\iota = 0$ .

*Proof.* Suppose that  $\iota = \bar{\iota}$  when  $(k, x)$  is above  $x^s(\cdot)$ . According to Property B.4, the  $(k, x)$  process must cross  $x^s(\cdot)$  from above before changing the  $\iota$  regime,

violating property B.6. This rules out this regime. The singular regime can only be applied along  $x^s(\cdot)$ , so the only possibility left above the singular curve is  $\iota = 0$ .  $\square$

We now show that maximal solar investment can be optimal only at the initial phase:

**Property B.8.** *A switch to  $\iota = \bar{\iota}$  from any of the other two  $\iota$  regimes cannot be optimal.*

*Proof.* Above the singular curve, the  $\iota = 0$  regime is optimal (Property B.7) hence a switch to  $\iota = \bar{\iota}$  will not take place in this region. Proceeding along the singular curve cannot change the sign of  $\phi(\cdot)$  (see (B.12)) while leaving it (with  $\phi = 0$ ) to the region below implies  $\iota = 0$  (property B.5). Below the singular curve, the singular regime never holds and the  $\iota = 0$  regime cannot be switched (Property B.4).  $\square$

**Property B.9.** *If  $k < k^{s\zeta}$  then the optimal policy is to set  $\iota = 0$ .*

*Proof.* When  $k < k^{s\zeta}$  the  $x^\zeta(\cdot)$  curve lies above the singular curve, hence the  $(k, x)$  process (which must proceed on or above  $x^\zeta$  – see Property B.2) evolves above the singular curve. The optimal policy, then, is to set  $\iota = 0$  (Property B.7).  $\square$

An immediate corollary of properties B.8 and B.9 is

**Property B.10.** *A small economy, with  $k_0 < k^{s\zeta}$ , will never adopt the  $\iota = \bar{\iota}$  regime.*

**Property B.11.** *If  $k < \min(k^{s\zeta}, k^{\zeta e})$  then the optimal  $k(\cdot)$  process increases.*

*Proof.* When  $k < \min(k^{s\zeta}, k^{\zeta e})$  the  $x^\zeta(\cdot)$  curve lies above the other two curves, hence the  $(k, x)$  process (which must proceed on or above  $x^\zeta$  – see Property B.2) evolves above the other two curves. This implies an increasing  $c(\cdot)$  process (Property B.1) and  $\iota = 0$  as the optimal choice (property B.9). Under this  $\iota$ -regime,  $a(\cdot)$  does not increase. We show that  $k(\cdot)$  increases. Consider the function

$$D(t) \equiv y(k(t), x(t)) - c(t) - \zeta x^f(t) - \delta k(t) \quad (\text{B.14})$$

With  $\iota = 0$ ,  $\dot{k} = D$ . Taking the time derivative, we find

$$\ddot{k} = \dot{D} = [y_k - \delta]\dot{k} + [y_x - \zeta]\dot{x}^f + y_x b \dot{a} - \dot{c} \quad (\text{B.15})$$

Now, the second term of (B.15) vanishes because  $x^f = \dot{x}^f = 0$  above  $x^\zeta(\cdot)$  and  $y_x - \zeta = 0$  on  $x^\zeta(\cdot)$ . If  $k$  decreases, the first term is negative above  $x^e(\cdot)$  because  $y_k > \rho + \delta > \delta$ . The third term is not positive when  $\iota = 0$  while  $\dot{c} > 0$  above the steady state curve. Thus, both  $\dot{k}$  and  $\ddot{k}$  are negative, implying that if  $k$  decreases it must vanish at a finite time, which cannot be optimal.  $\square$

**Property B.12.** *The optimal state trajectory does not cross the steady state curve  $x^e(\cdot)$  from below with  $\iota = 0$  or  $\iota = \iota^s$ .*

*Proof.* Crossing  $x^e(\cdot)$  from below must occur at  $k \geq k^{\zeta e}$ , i.e. above or along the  $x^\zeta(\cdot)$  curve. In the former case,  $x^f = 0$  and  $\iota = 0$  implies that  $a$  decreases, hence  $k$  must also decrease. (Otherwise, the  $(k, x)$  process moves away from  $x^e(\cdot)$ .) It follows that all the terms of (B.15) are negative or vanishing at and after the crossing time, hence  $\ddot{k} < 0$ . Thus,  $k$  will continue to decrease at an increasing rate and will inevitably fall below  $\min(k^{s\zeta}, k^{\zeta e})$ , violating property B.11. Crossing  $x^e(\cdot)$  from below along  $x^\zeta(\cdot)$  at  $k^{\zeta e}$  also involves a decreasing  $k$  process, hence is ruled out using the same argument, which can also be used to rule out the crossing under the singular regime.  $\square$

## B.1 Solar based economies

The economy is solar based when  $k^{se} > k^{\zeta e}$  or equivalently, when  $\rho + \delta < b\zeta$  (see the derivation of 3.6). In this case  $\hat{k} = k^{se}$  and  $\hat{a} = x^e(\hat{k})/b$ , as depicted in Figure 1.

**Property B.13.** *The optimal state trajectory does not cross the singular curve  $x^s(\cdot)$  at  $k < \hat{k}$  from below with  $\iota = 0$ .*

*Proof.* Crossing the singular curve from below must occur at  $k \geq k^{s\zeta}$ , i.e. above or along the  $x^s(\cdot)$  curve. A crossing with  $\iota = 0$  implies for both cases that both  $k$  and  $a$  do not increase. For  $k < \hat{k}$ , the crossing occurs above the steady state curve. It follows that all the terms of (B.15) are negative or vanishing at and after the crossing time, hence  $\ddot{k} < 0$ . Thus,  $k$  decreases at an increasing rate and will inevitably fall below  $k^{s\zeta}$ , violating property B.11.  $\square$

**Property B.14.** *When  $(k, x)$  is below  $x^s(\cdot)$  and  $k < \hat{k}$ , then the optimal policy is to set  $\iota = \bar{\iota}$ .*

*Proof.* If  $\iota = 0$  the  $(k, x)$  process must cross  $x^s(\cdot)$  from below before the  $\iota$  regime is switched (Property B.4). Increasing  $k$  only moves the  $(k, x)$  process further away (below) from  $x^s(\cdot)$ , hence the crossing must take place with  $k < \hat{k}$ , violating property B.13 and ruling out the  $\iota = 0$  policy. The singular policy is also ruled out away from the singular curve, and the only possibility left is  $\iota = \bar{\iota}$ .  $\square$

### B.1.1 More on singular processes

**Property B.15.** *A singular process cannot leave the singular curve while  $k < \hat{k}$ .*

*Proof.* In view of property B.5, driving a singular  $(k, x)$  process above the singular curve entails  $\iota = \bar{\iota}$  above this curve, violating Property B.7. Driving a singular  $(k, x)$  process below  $x^s(\cdot)$  entails  $\iota = 0$  at  $k < \hat{k}$ , violating Property B.14. □

**Property B.16.** *A singular process with  $k^{s\zeta} < k < \hat{k}$  must increase (i.e. both  $k$  and  $a$  increase).*

*Proof.* Above the  $\zeta$  curve,  $x^f = 0$  hence  $x = ba$  and  $a$  and  $k$  vary in the same direction along the increasing singular curve. With  $k < \hat{k}$ , the singular process proceeds above the steady state curve, where  $c(\cdot)$  increases. Thus, the process cannot settle at a steady state in this region. Suppose that it decreases, then it cannot reverse its direction, nor can it leave the singular curve. The decreasing process, then, must proceed towards  $k^{s\zeta}$  where it is forced to leave the singular curve, violating property B.15. This leaves an increasing process as the optimal option. □

The crossing point  $(k^{s\zeta}, x^\zeta(k^{s\zeta}))$  marks an exception to this rule because a time period during which both  $k(t) = k^{s\zeta}$  and  $x(t) = x^f(t) + ba(t) = x^\zeta(k^{s\zeta})$  remain fixed while the solar-fossil mix varies cannot be ruled out. Indeed, the solar investment rate (3.7) is adopted during this period. Once  $x^f$  vanishes and the solar component takes over, however, the process must leave the crossing point and increase along the singular curve with  $\iota^s$  given by (3.3) in accordance with property B.16.

**Property B.17.** *A singular process with  $k^{s\zeta} < k < \hat{k}$  must approach the intersection point  $(\hat{k}, x^s(\hat{k}))$ .*

*Proof.* While  $k < \hat{k}$  the singular process cannot leave  $x^s$  (property B.15)

or settle at a steady state hence it must increase (property B.16) towards  $(\hat{k}, x^s(\hat{k}))$ .  $\square$

**Property B.18.** *A singular process with  $k > \hat{k}$  must decrease.*

*Proof.* Consider a singular process proceeding along the singular curve segment with  $k > \hat{k}$ , i.e. below the steady state curve. According to property B.1,  $c(\cdot)$  must decrease along this process. Since the steady state curve will never be crossed (property B.8) this decrease in consumption will never reverse. If  $k(\cdot)$  increases,  $a(\cdot)$  must increase as well along the increasing singular curve. This behavior, however, is inconsistent with decreasing consumption, since the alternative policy of maintaining both capital stocks fixed, diverting the resources required to increase them to enhanced consumption is also feasible and yields a higher utility. A steady state cannot be optimal away from the steady state curve, hence the process must decrease.  $\square$

### B.1.2 Convergence

The  $\iota = \bar{\iota}$  regime can hold only during the initial phase of the optimal policy, and only for a final duration (properties B.3 and B.8). Thus there exists some finite time  $t_0$  following which only the other two  $\iota$  regimes can be optimal. (For small economies,  $t_0 = 0$ , see property B.10.) To study long term behavior, we restrict attention to  $t > t_0$  hence consider only these other two regimes.

**Property B.19.** *An optimal process converges to  $(\hat{k}, \hat{b}) = (k^{se}, x^e(k^{se}))$ .*

*Proof.* Proceeding below the steady state curve, the optimal process can never cross to the region above it (property B.12), hence the consumption process must decrease indefinitely. To avoid vanishing consumption at a finite time,

the rate of decrease must approach zero, hence the process must approach the steady state curve (property B.1). The point of approach cannot have  $k < \hat{k}$ , because this region implies the excluded regime  $\iota = \hat{\iota}$  (property B.14). At  $k > \hat{k}$ , the  $\iota = 0$  regime holds and with  $x^f = 0$ ,  $x(\cdot) = ba(\cdot)$  decreases exponentially, hence the  $(k(\cdot), x(\cdot))$  process, restricted to the vicinity of the steady state curve, must converge to the intersection point with the singular curve, where the singular  $\iota = \iota^s$  policy allows to maintain  $a(\cdot)$  fixed at its steady state value.

Suppose that the  $(k(\cdot), x(\cdot))$  process proceeds above the steady state curve. If it crosses this curve, it can never return back to the region above it, hence it will converge to the steady state as shown above. We can, therefore consider processes restricted to the region above the steady state curve indefinitely. In this region, the process must also proceed on or above the singular curve (outside the interval  $[k^{s\zeta}, \hat{k}]$  the singular curve lies below one of the other curves so the process must proceed above it, while within this interval the region below the singular curve implies the excluded  $\iota = \bar{\iota}$  policy, see property B.14). The process, then can proceed either above all three curves, or along the singular curve with  $k(\cdot) \in [k^{s\zeta}, \hat{k}]$  or along  $x^\zeta$  with  $k(\cdot) < k^{s\zeta}$ . In the former case,  $x^f = 0$  and the  $\iota = 0$  regime implies that  $x(\cdot) = ba(\cdot)$  decreases exponentially, and since  $k(\cdot)$  is bounded from below by  $\min(k_0, k^{s\zeta})$  (see property B.11) the process must reach the singular or the  $\zeta$  curve (whichever lies higher at the point of arrival) in finite time. Neither curve can be crossed, nor can the process return to the region above them. It can, however, either increase along  $x^\zeta$  with  $\iota = 0$ , using fossil energy at the required rate to make up for the shrinking solar capital, or switch to the singular regime and increase along  $x^s$  towards the intersection point with the steady state curve (property B.16).

If the ride along  $x^\zeta$  takes place first, it must end up at  $(k^{s\zeta}, x^s(k^{s\zeta}))$ , where the singular regime takes over, eventually bringing the process along  $x^s$  to the steady state.  $\square$

**Property B.20.** *Characterization of the optimal process for a small solar-based economy.*

Consider a small economy endowed with  $k_0 < k^{s\zeta}$  and  $a_0 = 0$ . The initial policy for this economy must proceed with  $\iota = 0$  (property B.9) hence the  $\iota = \bar{\iota}$  policy will never be adopted (property B.10). With  $a(\cdot) \equiv 0$  the optimal process must increase (property B.11) along the  $\zeta$  curve because the region above this curve requires that  $x^f = 0$  and  $ba(\cdot) = x(\cdot) > x^\zeta(k(\cdot))$  (property B.2). This ride along  $x^\zeta$  must proceed until the singular curve is reached at  $k^{s\zeta}$ . The latter curve cannot be crossed, because the region below it implies the excluded  $\iota = \bar{\iota}$  policy (property B.14). Neither can the process leave the  $\zeta$  curve with  $a = 0$  as stated above. Moreover, the process cannot stay at the crossing point under the  $\iota = 0$  regime because a steady state is not allowed away from  $x^e$ . Thus a switch to the singular regime must occur upon reaching  $k^{s\zeta}$ . The process, however cannot leave the crossing point and increase along the singular curve (i.e. above  $x^\zeta$ ) so long as the solar stock  $a(\cdot)$  falls short of  $x^\zeta(k^{s\zeta})$ , because otherwise a positive rate of fossil energy would be required above  $x^\zeta$ , violating property B.2. Thus, a quasi-stationary and singular coexistence phase must take place to allow solar capital to build up, leaving  $k(\cdot)$  and  $x(\cdot)$  fixed but shrinking  $x^f(\cdot)$  gradually to make room for the increasing use of solar energy. Once the use of fossil energy ceases, staying at  $(k^{s\zeta}, x^s(k^{s\zeta}))$  is no longer possible, because this point does not qualify as a steady state. Leaving the singular curve is not possible below  $\hat{k}$  (property



B.15) hence the singular process must increase along this curve (property B.16) reaching towards the intersection point  $(k^{se}, x^s(k^{se}))$  with the steady state curve. A further increase along the singular curve implies crossing the steady state and an indefinitely decreasing consumption process hence the process must settle at a steady state in the intersection point.

## B.2 Fossil based economies

We consider now the case  $\rho + \delta > b\zeta$  which implies  $k^{se} < k^{\zeta e}$ , so that  $\hat{k} = k^{\zeta e}$ ,  $\hat{x} = x^e(\hat{k})$  and  $\hat{a} = 0$ , as depicted in Figure 2.

Observe first that the crossing point  $(k^{se}, x^e(k^{se}))$  of the steady state and singular curves that served as a steady state in the previous subsection lies here below the  $\zeta$ -curve hence cannot belong to an optimal process (Property B.2). What other point might serve as a steady state? Obviously it must lie on the steady state curve, hence above the singular curve so the singular policy cannot be optimal in this state. The  $\iota = \bar{\iota}$  regime cannot hold indefinitely, hence does not correspond to a steady state. With  $\iota = 0$ , a steady state implies also  $a = 0$  so  $x = x^f$ . Thus, the state must lie on the  $\zeta$ -curve, hence the only possibility is the crossing point  $(k^{\zeta e}, x^e(k^{\zeta e}))$ . Indeed, for this type of economies the following property holds

**Property B.21.** *If  $k < k^{\zeta e}$  then  $\iota = 0$  and  $k(\cdot)$  increases.*

*Proof.* When  $k < k^{\zeta e}$  the  $x^\zeta$  curve lies above the other two curves, hence the  $(k, x)$  process (which must proceed on or above  $x^\zeta$ , see Property B.2) evolves above the other two curves. The proof, then, follows that of properties B.9 and B.11. □

Next, we verify that the corresponding proofs are not affected by the change

in geometry and all the properties established for the solar based economies (except those that deal with the interval  $k^{s\zeta} < k < k^{se}$  which is not relevant for the present geometry) hold also in the the present case. Put together, they entail the following characterization

**Property B.22.** *Under the optimal policy, the  $(k(\cdot), ba(\cdot))$  process converges to  $(\hat{k}, b\hat{a}) = (k^{\zeta e}, 0)$  with  $\hat{x} = \hat{x}^f = x^e(\hat{k})$ .*

The proof follows closely the reasoning for the case of solar based economies, hence will not be reproduced here.

**Property B.23.** *Characterization of the optimal process for a small fossil-based economy.*

Consider a small economy endowed with  $k_0 < k^{\zeta e}$  and  $a_0 = 0$ . The initial policy for this economy must increase with  $\iota = 0$  (property B.21) along  $x^\zeta$  (because  $a(\cdot) = 0$  excludes the region above this curve). The process cannot increase beyond the intersection at  $k^{\zeta e}$  because crossing the steady state line implies that the consumption process will decrease indefinitely, while settling at a steady state at the intersection point (diverting the resources required to increase  $k(\cdot)$  and  $x^f(\cdot)$  to enhanced consumption) is feasible and yields a higher value. A transition to the  $\iota = \bar{\iota}$  regime is not allowed, nor can a switch to the singular regime take place away from  $x^s$ . The optimal process, then, is restricted to a fossil-based increase along  $x^\zeta$  towards the steady state  $(\hat{k}, b\hat{a}) = (k^{\zeta e}, 0)$  with  $\hat{x} = \hat{x}^f = x^e(\hat{k})$ , as asserted in property B.22.

Put together, properties B.19 and B.22 establish Proposition 3.1, and properties B.20 and B.23 establish Proposition 3.3.

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