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Equilibrium with product differentiation

Jeffrey M. Perloff [*] Steven Salop [†]

[*]University of California, Berkeley and Giannini Foundation
[†]Federal Trade Commission

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EQUILIBRIUM WITH PRODUCT DIFFERENTIATION

by

Jeffrey M. Perloff and Steven C. Salop

California Agricultural Experiment Station
Giannini Foundation of Agricultural Economics
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1. INTRODUCTION

Product differentiation has been a topic of interest to economists at least since the 1920s and 1930s. Two basic formalizations of product differentiation have been explored in detail.

One is the Hotelling (1929) spatial competition model. In this formalization, heterogeneous consumers have diverse preferences among the available brands. As analyzed originally by Hotelling and as generalized and extended more recently by Lancaster (1979) and Salop (1979), competition is treated as a localized phenomenon. Each consumer purchases a limited number of brands (often one) from a small subset that are most preferred. As such, this model might better be classified as a location model rather than as monopolistic competition. In this model, brands may or may not be reformulated (relocated) in the event that entry occurs.

In contrast to this formalization is the representative consumer model often associated with Chamberlin (1933). As recently analyzed by Dixit and Stiglitz (1976) and Spence (1976), a representative consumer purchases many brands, varying the proportions of each according to their prices and exogenously given utility weights. These models entail competition by all brands for each representative consumer in contrast to the localized competition of the linked oligopoly model.

Neither formalization is clearly superior to the other for all industry settings. The representative consumer model has the desirable property of permitting multibrand competition. Yet, the spatial model has the desirable property of focusing clearly on brand attributes and brand reformulation (or

the inability to reformulate) the face of entry competition. In addition, for many product classes, consumers purchase only one brand.

In this paper we begin to synthesize these two diverse approaches. By using consumer valuations of all (possible) brands as a starting point, our model focuses on brand attributes. At the same time, however, competition is not localized. In principle, every brand competes with every other brand.

The next section of the paper develops the basic model of consumer preferences for differentiated products and examines the properties of a single-price equilibrium. In particular, we show that, as the intensity of consumer preferences increases, the equilibrium price rises.

In the third section, entry competition is analyzed. As the number of firms increases (say, due to a decrease in fixed costs), the equilibrium price-cost markup asymptotically disappears if the utility a consumer receives from a particular brand is bounded. If this condition is not met, entry may not eliminate the markup. Rather, an equilibrium may obtain in which each of a large number of small firms has a significant degree of market power. These conditions are compared to Hart's (1979) conditions.

In the fourth section, we return to the fixed number of firms case and examine the uniqueness of single-price equilibria and the possibility of multiprice equilibria. We show that, if a single-price equilibrium exists, it is unique. For the case of duopoly, we show that only a single-price equilibrium is possible. In contrast, if a significant number of consumers have identical tastes (i.e., if the distribution of consumer tastes has a mass point), a single-price equilibrium may not exist.

The fifth section discusses localized competition in more detail. If consumers only consider a finite subset of brands or stores (say, due to location), entry will never fully eliminate market power. The sixth section

examines the concept of spurious product differentiation and its effect on market power. The last section contains conclusions and returns to the issue of alternative approaches to product differentiation.

2. A MODEL OF CONSUMER PREFERENCES FOR DIFFERENTIATED PRODUCTS

In this section, we analyze a model of industry equilibrium when there are differentiated brands in a product class. Suppose there are an unlimited possible number of distinct brands indexed by the subscript $i = 1, 2, \ldots$. Each consumer attaches relative values to these brands according to his preference vector, $\underline{\hat{a}} = (\hat{a}_1, \hat{a}_2, \ldots)$. Initially, we assume that there are n brands ($i = 1, 2, \ldots, n$) available and a finite number of consumers L, each of whom have no monopsony power.

Each consumer purchases the brand among those available that maximizes his net surplus, or

$$s_i = \hat{a}_i - p_i, \qquad i = 1, 2, \ldots, n \tag{1}$$

where p_i is the price of the ith brand, s_i is its surplus, and $\hat{a}_i$ is an element of the consumer's preference vector $\underline{\hat{a}} = (\hat{a}_i, \ldots, \hat{a}_n)$.[1] The term best buy refers to the brand with the highest net surplus ($\max_i s_i$) for a particular consumer.

Of course, for some prices $(p_1, p_2, \ldots, p_n)$ even the best buy may give negative surplus ($s_i < 0$), or surplus less than some threshold opportunity value, $\bar{v}$, if "outside goods" are included in the analysis as in Salop (1979). In these cases, demand for even the best buy would be zero. However, permitting zero purchases adds unnecessary complication to the current model. Therefore, we assume consumers purchase the best buy regardless of its cardinal level of net surplus.[2]

For simplicity, we assume that preferences are symmetric in the sense that aggregate preferences for each particular brand i are independent and identically distributed as summarized by the density functions[3]

$$g(\theta) = g_i(\theta_i). \tag{2}$$

Given prices $(p_1, p_2, \ldots, p_n)$ for the n available brands, a consumer will choose that brand for which his surplus, s_i, is maximized—his best buy. If $s_i \leq s_j$ for a given consumer, then $\theta_j \leq p_j - p_i + \theta_i$. Thus, given θ_i, the probability that $s_i \geq s_j$ is $Pr(s_i \geq s_j) = G(p_j - p_i + \theta_i)$ where $G(\cdot)$ is the cumulative distribution function corresponding to $g(\cdot)$.

Since the θ_j are distributed independently, the proportion of consumers who purchase brand i is given by

$$Pr(s_i \geq \max_{j \neq i} s_j) = \int \prod_{j \neq i} [G(p_j - p_i + \theta_i)] \; g(\theta_i) \, d\theta_i. \tag{3}$$

We examine the special case where each consumer purchases exactly one unit of his best buy.[4] In this case, the expected demand for brand i, $Q_i(p_1, p_2, \ldots, p_n)$, equals the proportion of consumers who buy that brand [given by equation (3)] times the number of consumers (total number of units of all brands sold) L:

$$Q_i(p_1, p_2, \ldots, p_n) = Pr(s_i \geq \max_{j \neq i} s_j) \; L. \tag{4}$$

Assuming that each firm has a constant marginal cost c, then its expected profits π_i are given by

$$\pi_i(p_1, p_2, \ldots, p_n) = (p_i - c)\, Q_i(p_1, p_2, \ldots, p_n) - K \tag{5}$$

where K is the common level of fixed (sunk) costs of each firm.

We assume each risk-neutral firm maximizes expected profits. We also assume firms take the prices of the other firms as given; that is, we analyze the "Bertrand Nash in price" equilibrium. Differentiating equation (5) with respect to p_i under the "Nash" conjectural variation ($\partial p_j/\partial p_i = 0$, for all $j \neq i$) and rewriting, we have the marginal revenue equals marginal cost condition,

$$p_i = c_i - \frac{Q_i(p_1, \ldots, p_n)}{\partial Q_i/\partial p_i}. \tag{6}$$

We now consider the existence of a single-price equilibrium. While a single-price equilibrium may seem plausible given our symmetry assumption, it may not be necessary.[5] Thus, we assume for now that equilibrium entails identical prices for all firms, or

$$p_i = p, \qquad i = 1, 2, \ldots, n. \tag{7}$$

This symmetric industry equilibrium is easy to characterize. Assuming that all firms except firm i charge an identical price p, then, after substituting into equation (7), we have

$$Q_i(p, \ldots, p_i, \ldots, p) = L \int [G(p - p_i + \theta)]^{n-1} g(\theta)\, d\theta. \tag{8}$$

Differentiating equation (8) with respect to p_i (under the Nash conjecture), the slope of the demand curve of firm i is given by

$$\frac{\partial Q_i}{\partial p_i} = -(n-1)\, L \int [G(p - p_i + \theta)]^{n-2}\, g(p - p_i + \theta)\, g(\theta)\, d\theta. \tag{9}$$

Substituting the equilibrium value $p_i = p$ into equations (8) and (9), it is easy to show that each brand has an equal expected market share,

$$Q_i = L \int [G(\theta)]^{n-1}\, g(\theta)\, d\theta = \frac{L}{n}, \tag{10}$$

and its demand curve slope equals

$$\frac{\partial Q_i}{\partial p_i} = -(n-1)\, L \int [G(\theta)]^{n-2}\, [g(\theta)]^2\, d\theta. \tag{11}$$

Substituting equations (10) and (11) into (6) and denoting this symmetric, single-price equilibrium price by $p(n)$, we have

$$p(n) = c + \frac{1}{M(n)} \tag{12}$$

where

$$M(n) = n(n-1) \int [G(\theta)]^{n-2}\, [g(\theta)]^2\, d\theta. \tag{13}$$

Equations (12) and (13) determine a single-price equilibrium which lies strictly above the competitive price (i.e., price equal to marginal cost) as long as $M(n) > 0$.[6] This condition obtains for all finite number of firms n so long as the preference density is differentiable.

It is sometimes argued that advertising or experience information increases the intensity of consumers' preferences leading to higher prices in the market. We can explore the relationship between preference intensity and equilibrium prices by examining the following formalization of preference intensity. We denote the typical preference vector $\underline{\theta}$ as a scaled form of a standard intensity vector $\underline{\theta}^0$ or

$$\underline{\theta} = \beta\underline{\theta}^{o}, \tag{14}$$

where $\beta \geq 0$. Thus, a larger β represents more intense preferences. Perfect substitutes are captured by $\beta = 0$ in the sense that the consumer likes all brands equally well.[7]

Substituting equation (14) into (1) and repeating the above reasoning, it may be shown that increased preference intensity (β) decreases price elasticity, thereby raising the equilibrium price.[8] In particular, equation (13) may be rewritten as follows:[9]

$$p(n) = c + \frac{\beta}{M(n)}. \tag{15}$$

Thus, it is easy to see that $dp(n)/d\beta = 1/M(n) > 0$.

This result may be summarized as follows:

Proposition 1. An increase in preference intensity (in the sense of an increase in β) raises the equilibrium price.[10]

If brands are perfect substitutes (in the sense of $\beta \to 0$), the equilibrium price approaches the competitive price c. Where brands are perfect substitutes, we have the usual "Bertrand" price competition model for which $p = c$ for all $n \geq 2$. Of course, this formulation of preference intensity is not the only one possible. A preference density $h(\underline{\theta})$ might be called more intense than another density $g(\underline{\theta})$ if $h(\underline{\theta})$ differs from $g(\underline{\theta})$ by a mean-preserving spread. Under this more general formalization, the equilibrium price need not rise as intensity increases [(as shown in Perloff and Salop (1980)].

We analyze the limiting case of $n \to \infty$ ($K \to 0$) in the next section and then return to a finite number of firms to examine mass points in the following section.

3. ENTRY COMPETITION

Up to now, we have assumed the number of firms is fixed. Entry competition, in the sense of an exogenous increase in the number of firms n will generally affect the single-price equilibrium. Even in traditional Cournot models of imperfect competition, entry may not lower the equilibrium price (Seade, 1980). Similarly, we have not obtained a general result for entry effects. Although entry shifts each firm's demand curve inward, the elasticity of demand may not rise and, thus, the equilibrium price may not fall. This ambiguity may be confirmed by differentiating the expression for M(n) in equation (13) with respect to n.

On the other hand, a complete characterization obtains for the limiting case of unbounded entry ($n \to \infty$). Of course, if each firm has strictly positive fixed costs K, the market is unable to support an infinite number of firms. Instead, ignoring the "integer" problem, a zero-profit equilibrium is characterized by the usual Chamberlinian tangency of demand with average cost.[11]

The single price, free entry equilibrium is determined by equations (12) and (13) and a zero profits condition (price equals average cost):

$$p = c + \frac{nK}{L} \tag{16}$$

where L/n is the quantity each firm sells in a symmetric equilibrium. Only if the level of fixed costs approaches zero (perfectly free entry) may the number

of competitors become infinite. The following two propositions present conditions for the perfectly free entry price to equal the perfectly competitive price even when consumers have distinct brand preferences and brands are not perfect substitutes. The proofs are presented in the Appendix.

Proposition 2. If the support [a, b] of the preference density $g(\theta)$ is bounded from above (i.e., if b is finite), then (as $K \to 0$, so that $n \to \infty$),

$$\lim_{n\to\infty} p(n) = c.$$

Proposition 3. If the domain [a, b) is unbounded from above (i.e., if $b = \infty$) and if

$$\lim_{\theta\to\infty} \frac{g'(\theta)}{g(\theta)} = -\infty,$$

then (as $K \to 0$, so that $n \to \infty$),

$$\lim_{n\to\infty} p(n) = c.$$

Intuitively, the Nash equilibrium price approaches the competitive price if all firms' Nash demand curves become perfectly elastic, for then even a small price increase causes a firm to lose all its customers. There are two cases to consider. First, if preferences are bounded from above (Proposition 2), a brand only obtains those customers for whom the brand is the best buy (largest s_i); then, as the number of firms grows infinitely large, a brand only obtains those customers who value the brand at the level equal to the upper bound of the consumer preference density, b. Similarly, since the number of brands is infinitely large, any consumer's preference for his next-highest-valued brand also approaches the upper bound b. There are many "near ties" among consumer preferences for the available brands.

In other words, all of the firms have close substitutes for the firm's brand. As a result, if the firm raises its price even slightly, each of these consumers will choose another brand instead. Since small price increases cause a loss of all customers, demand is perfectly elastic, and price is driven down to the competitive level.

Next, consider the case of an unbounded preference density. If the preference density is unbounded,[12] then a consumer's valuations of his most preferred brand and the next-best substitute may not cluster together. As a result, demand may not be perfectly elastic. Instead, the elasticity of demand depends on the rate at which the preference density approaches zero as measured by the condition given in Proposition 3. Proposition 2 and Proposition 3 are related, of course. It is easy to confirm that, in the case of a finite support, the ratio $g'(\theta)/g(\theta)$ becomes unbounded as $\theta \to b$ (see Lemma 3 in the Appendix).

The preference density conditions underlying Propositions 2 and 3 have intuitive interpretations. That all consumers place a finite maximum valuation on all brands (as given in Proposition 2) implies that each brand's demand curve cuts the price axis at some finite price. That is, there surely exists some sufficiently high but finite price premium at which any brand is not purchased by any consumer.

Proposition 3 covers those cases in which the demand curve does not cut the price axis. Even at an infinitely high price premium, some consumers prefer a brand. The condition given in Proposition 3 thus concerns the elasticity of demand at high prices.

Since the 1930s, there has been a lively debate concerning the necessary conditions for firms to possess monopoly power. Robinson (1934) and Kaldor

(1935) took issue with Chamberlin's (1933) contention that the theory of the monopolistically competitive firm and the perfectly competitive firm are distinct. They maintained that monopoly power would disappear in both the homogeneous and differentiated product cases if there were a sufficiently large number of very small firms in the industry. In a recent paper, Hart (1979) points out that if marginal costs are initially falling, then a (limiting) output level of zero is not consistent with competitive behavior. He argues that the joint assumptions that the number of firms in an industry is large and that each firm is very small are misleading.

If each firm's profit-maximizing output is small relative to the economy as a whole, then Hart shows that perfect competition is obtained even if products are differentiated. "Thus, contrary to the Chamberlinian point of view, what ensures that a firm behaves like a perfect competitor is not the presence of other firms producing close substitutes, but rather the fact that the firm is a negligible part of the aggregate economy . . ." (Hart, 1979, p. 2). In Hart's model, the ratio n/L of the number of firms n to the number of consumers L is the crucial determinant of monopoly power.[13]

We have previously discussed the effect of reductions in the level of fixed costs K. As $K \to 0$, we showed that the number of firms the market can support becomes unbounded. However, only if the conditions of Propositions 2 or 3 hold does perfect competition obtain. In either event, the firm-consumer ratio becomes zero. If the conditions of neither of the propositions hold, in the perfectly free entry case, an unbounded number of firms is consistent with price in excess of marginal cost.

Solving equations (12)-(14) for the equilibrium number of firms, we have

$$\frac{n}{L} = \frac{1}{KM(n)}. \tag{17}$$

In the cases in which Propositions 2 or 3 obtain, increases in the size of the market (as measured by L) increase the number of firms n which in turn increases M(n). Thus, in the limit, as $L \to \infty$, then $n \to \infty$, $M(n) \to \infty$, and the equilibrium price approaches the perfectly competitive level. In these cases, the firm-consumer ratio (n/L) does become zero, as in Hart.

If the assumptions of Propositions 2 or 3 do not obtain, the economy may not approach perfect competition as the size of the market increases and the number of firms becomes unbounded. In this case, the firm-consumer ratio is nonzero. Consider the following example.

Suppose that $\theta = -u$ and the density of u is exponential, i.e., $h(u) = \lambda e^{\lambda u}$ (where $u = -\theta < 0$).[14] In this case, $\lim_{\theta \to \infty} g'(\theta)/g(\theta) = -\lambda$ $(> -\infty)$. Thus, equations (12) and (13) imply a constant price above marginal cost, regardless of the number of firms, or[15]

$$p = \bar{p} = c + \lambda, \quad \text{for all } n \geq 2.^{16}$$

Substituting this exponential distribution into equation (20), the number of firms is given by $n = \lambda L/K$. Thus, $n/L = \lambda/K > 0$ even as $L \to \infty$.

In summary, entry competition (due to reductions in fixed costs or increases in the number of consumers) does not guarantee that the price falls to marginal cost. Price approaches marginal cost if (i) the demand curve cuts the price axis (Proposition 2) or (ii) the speed of convergence toward the axis is fast enough (Proposition 3). However, cases do exist in which the market is very large, there are an unbounded number of firms, yet each firm has market power.

For a firm to maintain market power under these conditions though, preferences must be unbounded. If consumers' willingness to pay are bounded (since

their assets are), we may reject these possibilities and concentrate on the bounded preference case. We now return to the case of a fixed number of firms, n.

4. UNIQUENESS, MASS POINTS, AND MULTIPRICE EQUILIBRIA

So far, we have restricted our attention to single-price equilibria. In this section, the uniqueness of single-price equilibria and the possible existence of multiprice equilibria are examined. We begin with the uniqueness issue by proving that a multiplicity of single-price equilibria is impossible given a fixed number of firms:

Proposition 4. If a single-price equilibrium exists, then it is unique.

To show this result, we start by rewriting equation (12) as follows:

$$\frac{p - c}{p} = \frac{1}{pM(n)} \cdot \qquad (12')$$

The left-hand side of (12') is monotonically increasing in p, while the right-hand side is monotonically decreasing. Since the left-hand side equals zero at $p = c$ and the right-hand side is positive for all positive finite p, a single equality must obtain at a price above c.

Of course, this result does not rule out the additional possibility of multiprice equilibria, even under the symmetric information and cost conditions set out in Section I. While we have not ruled out the existence of multiprice equilibria in general, the possibility of such equilibria can be rejected in the case of a duopoly ($n = 2$).

In a duopoly, the probability that firm 1 obtains a representative customer with preferences $\underline{\theta} = (\theta_1, \theta_2)$ is given by

$$\Pr(s_1 \geq s_2) = \Pr(\theta_1 - \theta_2 \geq p_2 - p_1). \tag{18}$$

The distribution $H(\mu)$, where $\mu \equiv \theta_1 - \theta_2$, is symmetric with mean equal to zero, so that $H(0) = 1/2$.[17] Substituting the definition of μ into equation (21) and normalizing the number of consumers to one ($L = 1$) so that expected sales equal the representative probability, we have

$$Q_1(p_1, p_2) = H(p_2 - p_1), \tag{19a}$$

$$Q_2(p_1, p_2) = 1 - H(p_2 - p_1). \tag{19b}$$

Calculating expected profits and substituting into the profit-maximizing condition analogous to equation (6), we have

$$p_1 = c + \frac{H(p_2 - p_1)}{h(p_2 - p_1)}, \tag{20a}$$

$$p_2 = c + \frac{1 - H(p_2 - p_1)}{h(p_2 - p_1)}, \tag{20b}$$

where $h(\mu)$ is the density of $H(\mu)$. Subtracting (20a) from (20b), we obtain

$$p_2 - p_1 = \frac{1}{h(p_2 - p_1)} \; [1 - 2H(p_2 - p_1)]. \tag{21}$$

Since $H(0) = 1/2$, equation (21) is only satisfied at the single price $p = p_1 = p_2$. This single-price equilibrium is unique and given by[18]

$$p = c + \frac{H(0)}{h(0)}. \tag{22}$$

Two-price equilibria can be easily ruled out in this case. If $p_2 - p_1 > 0$, then $H(p_2 - p_1) > 1/2$. Since $h(p_2 - p_1) > 0$, the right-hand side of equation (21) is negative while the left-hand side is positive. Thus, such an equilibrium is impossible. A similar contradiction occurs for $p_2 - p_1 < 0$.[19] Thus, if $n = 2$, only a single-price equilibrium obtains. Unfortunately, this method of proof cannot easily be extended to cases of more than two firms.[20]

So far, we have assumed that the preference density is continuous. Mass points could occur if a significant number of consumers have identical tastes. If there are mass points at the equilibrium, then there can be "draws" among brands as best buys leading to discontinuities in demand curves so that a single-price equilibrium may not exist.[21] Given mass points, if average costs are U-shaped, either single-price or more than single-price equilibria may obtain.[22]

5. IMPERFECT INFORMATION AND LOCALIZED COMPETITION

The model can be reinterpreted to examine the interaction of imperfect information and product differentiation as follows. Suppose that consumers are imperfectly informed about the availability of competing brands in the market.[23]

For example, consider a market in which there are three brands but each consumer is aware of only two of them. If each of the three possible pairs of brands is equally likely to be known by any given consumer, then on average one-third of all consumers know and thus choose between brands one and two, a third choose between brands one and three, and a third choose between brands two and three. These subsets thus define three duopoly submarkets.

Calculating demands as earlier, the slope of the demand curve faced by the first brand for each of its two submarkets is given by equation (12) for $n = 2$. Since it has two submarkets, the slope of brand one's demand curve is twice the slope given by equation (12). However, its price elasticity is unchanged because the quantity sold in the two duopoly submarkets is also twice that of one submarket. Thus, the equilibrium price for the aggregate market consisting of two duopoly submarkets equals the duopoly equilibrium price.

Generalizing this argument to an n brand industry, suppose each consumer is aware of only $k < n$ brands. Then there are $m = \binom{n}{k}$ equal-sized submarkets, each consisting of k brands competing for L/m consumers. Given the strong symmetry assumptions made earlier, each submarket is identical and equilibrium is achieved at the k-firm equilibrium price p(k).

A similar analysis could be made for the type of localized competition that characterizes Hotelling-style models of spatial competition.[24] In such a model, consumers strictly prefer the k stores located nearby to the other $n - k$ more distant stores.

6. SPURIOUS AND ACTUAL PRODUCT DIFFERENTIATION

The model may also be reinterpreted to include "spurious" as well as actual product differentiation (which was discussed above, where consumers accurately perceived true variations across brands). By spurious product differentiation, we mean the case in which consumers mistakenly perceive brands to differ by more than they actually do.[25] In the model above, it does not matter why θ_i is high--only that it is high; so spurious differentiation may be treated the same as actual differentiation. If, however, there

is spurious differentiation in addition to actual differentiation, then the model must be expanded accordingly.

Let $\underline{\varepsilon} = (\varepsilon_1, \ldots, \varepsilon_n)$ be a set of "spurious valuations" a consumer places on the n available brands in addition to the "actual valuations" given by $\underline{\theta}$. Thus, the consumer perceives surplus to be:

$$s_i = \theta_i + \varepsilon_i - p_i, \qquad i = 1, 2, \ldots, n. \tag{1'}$$

If each ε_i were drawn independently from an identical distribution $F(\varepsilon)$ with zero mean which is uncorrelated with $G(\theta)$, then spurious product differentiation is equivalent to a mean-preserving spread of the preference density. The effect of general mean-preserving spreads on equilibrium prices is ambiguous as shown in Perloff and Salop (1980). If, however, the spurious element "intensifies" preferences in the multiplicative sense of increases in β as in Proposition 1, the equilibrium price increases, of course. Similarly, if the spurious element reduces the intensity of preferences in that sense, the equilibrium price falls.

7. CONCLUSIONS

This paper has formulated a general model of product differentiation and explored its properties. Although the model has a number of significant limitations, in particular inelastic individual demand and the absence of outside goods, the results suggest the type of extensions necessary to analyze those more general cases.

The key results of the basic model may be summarized as follows. First, as preferences become more intense (in a multiplicative fashion), equilibrium price increases. Second, if the value consumers obtain from every possible

brand is bounded, perfectly free entry eliminates monopoly markups. Third, only one single-price equilibrium exists. Fourth, if a significant number of consumers have identical tastes (so that there are mass points in the distribution function of consumer tastes), then a single-price equilibrium may not exist.

Another appeal of this analysis is that it suggests a reasonably general framework that synthesizes the alternative models of monopolistic competition. In this context, the model analyzed in detail here is Chamberlinian in nature; every brand competes equally with every other available brand. Although the model explicitly considers differences in the preferences of individual consumers, a "representative consumer" model of the sort analyzed by Spence (1976) and Dixit and Stiglitz (1977) could be obtained by treating the joint preference density as the "aggregate" preferences of a representative consumer. A special type of localized (spatial) competition was analyzed in Section IV by restricting consumers' preferences (or attention) appropriately.

The underlying product space of brand attributes is not analyzed explicitly here. However, the particular form of preferences and the manner in which entry affects demand suggests an implicit set of special assumptions on brand formulations and competition in product space. In particular, additional brands "crowd" product space so that, on average, consumers get additional utility when more brands are available. That the density of brand preferences, $g_i(\theta)$, is not altered upon entry represents an assumption that brands are not reformulated (relocated) after entry. In a similar model, Sattinger (1983) has examined the welfare implications of additional firms.

Although every consumer has some most-preferred brand in this approach, the concept of "localized" (spatial) competition is only imperfectly captured

by treating consumers as considering only a subset of the available brands. In this case, entry does not eliminate market power. The alternative approach, which is closer to the conventional model of spatial competition, would be for every consumer to have exactly one brand valuation equal to some θ_{max} and all other brands lower valued according to some compensation (transport cost) function. That formulation, which places a somewhat different structure on the preference density, might be captured with a more complex analysis. That extension, however, is beyond the scope of this paper.

APPENDIX

The proofs of Propositions 2 and 3 are given here.[26] These proofs assume that the density function $g(\cdot)$ has the following properties (which could be relaxed at the cost of greater complexity in the proofs):

1. $g(\theta) > 0$, $\theta \in (a, b)$.
2. $g(\theta)$ is analytic.
3. $\lim_{\theta \to b} g'(\theta) \neq 0$.

We wish to prove that, under the conditions given in Propositions 2 and 3, entry will drive the equilibrium price to marginal cost (even given a diversity of consumer tastes). Since $p = c + 1/M(n)$ as given in equation (15), then showing that

$$\lim_{n \to \infty} M(n) = \infty$$

is sufficient to show that

$$\lim_{n \to \infty} p = c.$$

The following lemmas establish that, if either b is finite or if

$$\lim_{\theta \to b} \frac{g'(\theta)}{g(\theta)} = -\infty,$$

then

$$\lim_{n \to \infty} M(n) = \infty.$$

Lemma 1: If $g(b) > 0$, then $\lim_{n \to \infty} M(n) = \infty$.

Proof of Lemma 1: By the continuity of $g(\theta)$, if $g(b) > 0$, then there exists an interval $(b - \delta, b]$ such that for $\theta \in (b - \delta, b]$, $g(\theta) \geq \xi > 0$. As a result,

$$M(n) = \int_{b-\delta}^{b} n(n-1)\,[G(\theta)]^{n-2}\,[g(\theta)]^2\,d\theta + K$$

$$\geq \xi \int_{b-\delta}^{b} n(n-1)\,[G(\theta)]^{n-2}\,[g(\theta)]\,d\theta + K$$

where

$$K = \int_{a}^{b-\delta} n(n-1)\,[G(\theta)]^{n-2}\,[g(\theta)]^2\,d\theta.$$

Therefore,

$$\lim_{n\to\infty} M(n) \geq \lim_{n\to\infty} n\,\xi - \lim_{n\to\infty} n\,\xi\,[G(\theta)]^{n-1} + \lim_{n\to\infty} K.$$

However, we can show that

1. $\lim_{n\to\infty} n\,\xi = \infty$.
2. $\lim_{n\to\infty} n\,\xi[G(b-\delta)]^{n-1} = 0$, since $1 > G(b-\delta) > 0$.
3. $\lim_{n\to\infty} K \geq 0$, since $n(n-1)\,[G(\theta)]^{n-2}\,[g(\theta)]^2 \geq 0$ for all $\delta \in (b-\delta, b]$. Indeed, it can be shown then that $\lim_{n\to\infty} K = 0$.

Thus, $\lim_{n\to\infty} M(n) = \infty$.

Lemma 2: If $g(b) = 0$ and $\lim_{\theta\to b} \frac{g'(\theta)}{g(\theta)} = -\infty$, then $\lim_{n\to\infty} M(n) = \infty$.

Proof of Lemma 2: Since $g(b) = 0$, then by integrating by parts,

$$M(n) = -\int_{a}^{b} n[G(\theta)]^{n-1}\,g(\theta)\left[\frac{g'(\theta)}{g(\theta)}\right]d\theta$$

since $g(\theta) > 0$, $\theta \in (a, b)$. Further, since $g'(\theta)$ is continuous near b, $g'(\theta) \leq 0$ near b so that $-g'(\theta)/g(\theta) \geq 0$ near b. If $\lim_{\theta\to b} g'(\theta)/[g(\theta)] = -\infty$, then for $\xi > 0$, there exists a δ such that, if $\theta \in (b-\delta, b]$, $-g'(\theta)/g(\theta) \geq \xi$. Then,

$$M(n) = \bar{K} + \int_{b-\delta}^{b} n[G(\theta)]^{n-1}\,g(\theta)\left[\frac{-g'(\theta)}{g(\theta)}\right]d\theta$$

where

$$\bar{K} = -\int_{a}^{b-\delta} n[G(\theta)]^{n-1}\,g(\theta)\left[\frac{g'(\theta)}{g(\theta)}\right]d\theta.$$

Therefore,

$$M(n) \geq \bar{K} + \xi \int_{b-\delta}^{b} n[G(\theta)]^{n-1} g(\theta)\, d\theta = \bar{K} + \xi\{1 - [G(b - \delta)]^n\}.$$

Then,

$$\lim_{n\to\infty} M(n) \geq \lim_{n\to\infty} \{\xi - \xi[G(b - \delta)]^n + \bar{K}\} = \xi$$

since

1. $1 > G(b - \delta) > 0$, $\lim_{n\to\infty} G(b - \delta)^n = 0$.
2. It can be shown that $\lim_{n\to\infty} \bar{K} = 0$.

Since ξ is arbitrary, we can make it arbitrarily large. Therefore,

$$\lim_{n\to\infty} M(n) = \infty.$$

Lemma 3: If b is finite and $g(b) = 0$, then

$$\lim_{\theta\to b} \frac{g'(\theta)}{g(\theta)} = -\infty.$$

Proof of Lemma 3: Since, by assumptions 1 and 3, $g(\theta) > 0$ for $\theta \in (a, b)$ and $g(b) = 0$, $\lim_{\theta\to b} g'(\theta) < 0$. As a result, $\lim_{\theta\to b} g'(\theta)/g(\theta)$ is a negative number divided by zero; that is, the limit is negative infinity.

Combined with our earlier discussion, Lemmas 1 through 3 establish Proposition 2. Lemma 1 shows the proposition true if $g(b) > 0$, and Lemmas 2 and 3 show it is true if $g(b) = 0$. Proposition 3 follows from our earlier discussion and Lemma 2.

FOOTNOTES

*University of California, Berkeley.

**Georgetown University Law Center.

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[1]Sattinger (1984) presents a similar model in which consumers maximize θ_i/p_i rather than $\theta_i - p_i$.

[2]As an alternative, one could assume that $\underline{\theta}$ is sufficiently large so that the inclusion of outside goods leaves the equilibrium unaffected. Formally, letting $\bar{v}$ denote the surplus that can be obtained from an outside good if $\theta_{min} > p_{max} + \bar{v}$ (where θ_{min} is the lowest possible value for each θ_i in $\bar{g}(\underline{\theta})$ and p_{max} exceeds the highest price charged), then $\underline{\theta}$ is "sufficiently large" to allow us to ignore outside goods.

[3]Nonsymmetric preference densities could be used in our analysis, but some of the stronger results we obtain below would not hold.

[4]Variable purchases could be included in two ways. If the number of units purchased depends on the price of the best buy, $d(p_i)$, variable purchases are easily incorporated into the model as shown in Perloff and Salop (1980). Alternatively, if the number of units purchased depends on net surplus, $d(s_i)$, then the mathematical complexity increases substantially.

[5]While we assume the existence of a single-price equilibrium here, this assumption will be proved for the two-brand case below. In Section III, we also examine multiple-price equilibria for $n > 2$.

[6]The second-order condition for profit maximization (where $p_j = p$ for $j \neq i$) is:

$$\frac{\partial^2 \pi_i}{\partial p_i^2} = 2 \frac{\partial Q_i\ (p, \ldots, p, p_i, p, \ldots, p)}{\partial p_i} + (p_i - c) \frac{\partial Q_i^2\ (p, \ldots, p, p_i, p, \ldots, p)}{\partial p_i^2}.$$

Since $\partial Q_i / \partial p_i < 0$ (as can be shown by differentiating), a sufficient condition for $\partial^2 \pi_i / \partial p_i^2 < 0$ is that $\partial^2 Q_i / \partial p_i^2 < 0$.

Substituting in the first-order condition to eliminate $(p_i - c)$, the second-order condition may be expressed as

$$\frac{\partial^2 \pi_i}{\partial p_i^2} = \frac{1}{\partial Q_i / \partial p_i} \left[2 \left(\frac{\partial Q_i^2}{\partial p_i} \right) - Q_i \frac{\partial^2 Q_i}{\partial p_i^2} \right].$$

Thus, another sufficient condition for the second-order condition to hold is that the term in brackets be positive. For example, if $g(\theta)$ is uniform on $[-a, a]$, then, in the relevant range, the term in the brackets is positive so the second-order condition holds. The second-order condition can be shown to hold for other distributions. See, for example, the "negative exponential" discussed below.

In the symmetric equilibrium, where $p_i = p$, also, the second-order condition becomes:

$$\frac{\partial^2 \pi_i}{\partial p_i^2} = - \frac{2}{n} M(n)\ L + \frac{n - 2}{n} L.$$

Thus, the second-order condition holds at the symmetric equilibrium when $n = 2$ [since $M(n) > 0$], or, more generally, $(n - 2)/n < M(n)$.

[7]If outside goods were explicitly included in the analysis, then the alternative form $\underline{\theta} = \beta\underline{\theta}^{o} + v^{o}$ (where $v^{o} > 0$) would maintain a positive cardinal valuation even as $\beta \to 0$; see, also, footnote 2.

[8]The elasticity ε facing each firm is given by

$$\varepsilon = -\frac{\partial Q_i}{\partial p_i}\frac{p_i}{Q_i} = p(n)\, M(n).$$

[9]Substituting equation (14) into (2), then equation (9) becomes

$$\frac{\partial Q_i}{\partial p_i} = -(n-1)\, L \frac{1}{\beta} \int \left[G\left(\frac{p - p_i}{\beta} + \theta\right)\right]^{n-2} g\left(\frac{p - p_i}{\beta} + \theta\right) g(\theta)\, d\theta,$$

setting $p_i = p$, and following the reasoning in the text, equation (15) obtains.

[10]In this formulation, because the monopoly price is inelastic (consumers buy one unit--at least up to some "choke" price), as β becomes unbounded, the monopoly price becomes unbounded. If individuals have an elastic demand with elasticity η for each brand, as discussed in footnote 4, then the monopoly price, p^m, obtains for sufficiently large β, where p^m is determined by the usual Lerner mark-up condition:

$$\frac{p^m - c}{p^m} = \frac{1}{\eta}.$$

For a similar model with elastic consumer demands, see Perloff and Salop (1980).

[11]For a discussion of the integer issue, see, e.g., Seade (1980).

[12]An unbounded preference density (or, as stated below, a brand demand curve that does not cut the price axis) may represent an unreasonable assumption when the number of brands is finite. With an infinite number of brands, however, perhaps each brand would give unlimited value to at least one consumer.

[13]Interestingly, Chamberlin (1956) recognized the same condition as Hart, although he rejected the possibility that it would occur in monopolistic competition: The assumption that buyers are infinitely divisible "would remove completely any reasons for a flattening out of the demand curve with infinite divisibility (or products or firms), since sellers would not become more numerous and closer together relative to buyers" (p. 199).

[14]More generally, Weibull densities on the same support have this property.

[15]Cf., Wilson (1977) for a similar result with this density in his competitive bidding model.

[16]A sufficient condition for the second-order condition to hold is that $\lambda < \sqrt{2}$; see, also, footnote 6.

[17]Symmetry may be proved by deriving $h(\mu)$, the density of $H(\mu)$, using a convolution with substitutions $\mu = \theta_1 - \theta_2$ and $\zeta = \theta_1 + \theta_2$. After a little manipulation, it can be shown that $h(\mu) = h(-\mu)$.

[18]As shown in footnote 6, the second-order condition holds for $n = 2$ at the symmetric equilibrium, which is sufficient, since we show below that a two-price equilibrium is impossible.

[19]If $c_1 < c_2$, then it can be shown that $p_1 < p_2$, that $p_1 - c_1 > p_2 - c_2$, and that the low-cost firm has a higher gross margin or $(p_1 - c_1)/p_1 > (p_2 - c_2)/p_2$.

[20]Beginning from a single price satisfying the equilibrium conditions, suppose a deviant firm, say, firm 1, sets its price at a level other than the common price p. In this case, letting $\mu_i = \theta_1 - \theta_i$, $i = 2, \ldots, n$, the n-firm equation analogous to equation (21) might be derived. Unfortunately, the marginal distributions of the μ_i's are not independent, complicating the calculations.

[21]As an example, suppose that the number of firms is arbitrarily set to n and firms have no fixed cost. Many customers consider the brands identical so that $G(\theta)$ has a mass point at zero. Consider a single-price equilibrium at $p > c$; for any $p > c$, one "deviant" firm could shade its price slightly, thereby winning many previous "ties." If there had been a proportional sharing of ties, the deviant's sales would have jumped discontinuously--thereby raising profits.

For $p = c$, unless absolutely all consumers were indifferent between all brands, a deviant firm i could earn positive operating profits in excess of variable cost by charging $p_i > c$ and relying on those few buyers who prefer its product. By contrast, as nondeviant firms set $p = c$ and earn zero profits, the deviant firm finds its behavior more profitable.

[22]This model is similar to Salop and Stiglitz's (1977) newspaper model. However, the consumers here purchase according to their different tastes while, in the newspaper model, they purchase randomly. Because of this difference, equilibria with more than two prices may obtain. [(See Perloff and Salop (1980).]

[23]This section follows Porter (1979).

[24]For example, see Salop (1979) and Lancaster (1979).

[25]Spurious product differentiation has been suggested by a number of writers including Chamberlin and Galbraith, with respect to a wide variety of consumer products such as beer, detergents, lemon juice, and even soft drinks. One class of spurious product differentiation involves drug placebo effects. For example, suppose that a consumer forms a false belief that one aspirin brand is superior to another after it relieves a mild headache and the other so-called "inferior" brand does not relieve a more serious one. This story may not be too farfetched: Even a placebo achieves a headache relief rate of about 45 percent compared to a relief rate of around 80 percent for actual aspirin (Food and Drug Administration 1977). These high relief rates are often interpreted as illustrative of the self-limiting nature of headaches--they often just go away by themselves. In contrast, if the placebo induces a classic "placebo effect" of speeding this relief, then one may be incorrect in calling this "spurious" product differentiation. If the headache is relieved, then the remedy is successful by definition. Other experimental evidence is also interesting on this point. Blind tests of consumers' preferences after use do not replicate market shares. In addition, they vary according to whether products are labeled with brand names. For evidence, see Tucker (1964), McConnell (1968), Morris and Bronson (1969), and Monroe (1976); for a related model, see Schmalensee (1979); and for a good discussion of some of the policy implications of this phenomenon, see Craswell (1979).

[26]Several of these proofs are independently due to Robert Willig and Janos Galambos in personal communications. Any remaining errors are our own. Somewhat similar theorems are proved in Miyao and Shapiro (1979).

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