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Firm-specific information, product differentiation, and industry equilibrium

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FIRM-SPECIFIC INFORMATION, PRODUCT
DIFFERENTIATION, AND INDUSTRY EQUILIBRIUM*

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Abstract

Where consumers have imperfect information about specific firms' prices and lack information about the market, firms have informational market power. In general, improving the consumer's information about each firm's price will not necessarily lower average market price. We show, however, that certain types of improvements will lower price. Moreover, a reduction in barriers to entry (e.g., capital costs) will lower price-holding information constant. Where a significant number (but not all) consumers have perfect information, single-price equilibria are impossible.

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I. Introduction

Research over the last three decades has shown that imperfect consumer information may enable even small firms to set their prices above marginal cost.[1] Much of the recent literature has assumed that consumers possess information about the general market but lack information about specific firms. This paper presents a new model in which consumers have imperfect information about specific firms and lack information about the market. The resulting equilibrium has very different properties than in previous models.[2]

Consumers gather information in a number of diverse ways. One method is a personal inspection or search before purchase. This prepurchase inspection may be aided by the use of screening devices and signals. Prepurchase information may also be purchased from diagnostic and testing agencies, certifiers, newspapers, and brokers. Recommendations from friends may also be used. Finally, advertising by sellers and personal experience yield information that is more or less reliable.

Most attention has been paid to the information-gathering role of search or inspection, perhaps because it contains both the result of _informational market power_ and the possibility of nonexistence of equilibrium as emphasized by Stiglitz (1979). Search or inspection has been studied by Wilde and Schwartz (1979) and a number of others since Diamond (1971).

At the same time, however, the other information-gathering institutions have been analyzed in detail. For example, Phelps (1972) analyzes screening devices. Nelson (1974) examines the role of product market signals, particularly advertising and market share. The educational signaling literature of

Spence (1973, 1974), Stiglitz (1975), Gausch and Weiss (1980), and others may be reinterpreted as product testing and certification. Leland (1979) analyzes the effect of licensing to ensure minimum quality standards. Plott and Wilde (1979) have studied diagnosticians both theoretically and experimentally. Newspaper information has been analyzed by Salop and Stiglitz (1977) and Varian (1979).

Recommendations from friends have been paid less attention, except to the extent that such information may be similar to that gained from using market share as a positive signal. The role of advertising in directly providing firm-specific information has been analyzed by Butters (1977). The behavior of brokers has been implicitly modeled in the agency literature. Moreover, the direct mailing advertising in Butters (1977) may be reinterpreted as an independent broker or salesman. The matchmaking role of brokers has been examined by Salop (1980). Personal experience has been analyzed by Phelps and Winter (1970); Grossman, Kihlstrom, and Mirman (1977); Smallwood and Conlisk (1979); and Shapiro (1980).

The model presented here might be best described as a newspaper model in that consumers are endowed with some imperfect information about each firm in the market, though the equilibrium in the market for information is not explicitly analyzed. Alternatively, it might be better described as an amalgam of all information gathering, past and present, about specific firms and brands where the number of consumers perfectly informed about every firm is initially taken to be insignificant.

On the other hand, unlike the other search, newspaper, and signaling models, the consumers here are restricted to firm-specific information. Additional general market information, such as the range or density of actual

prices in the market, is not known to the consumer. His general market in-
formation is limited to only that which may be inferred from firm-specific
data and, therefore, is redundant. This model has strikingly different pro-
perties from those of earlier models which were driven by their assumptions of
perfect general market information. Indeed, it many ways, this firm-specific
information model represents a retrenchment for it has none of the strange and
wondrous properties of search and other models. In a market restricted to
firm-specific information gathering, if only an insignificant proportion of
consumers are perfectly informed about all firms, market breakdown is far less
likely; instead, equilibrium generally exists for the model presented here.[3]

Given that firms' profit-maximizing conditions hold, a unique single-price
equilibrium does obtain; however, we have not ruled out the existence of addi-
tional multiple-price equilibria from pure or mixed strategies. Moreover, we
show that price dispersion may occur if a significant number of consumers are
perfectly informed. As the degree of information about all firms improves
from perfect ignorance to perfect information, the equilibrium price falls
continuously to the competitive price. In contrast, as Stiglitz (1979) dis-
cusses, most models have a discontinuity in that any imperfection of informa-
tion causes price to be above marginal cost.

Finally, perhaps the most striking contrast with previous models occurs
with respect to entry competition. In the search models, entry does not re-
duce price; if anything, it increases the equilibrium price by making dis-
covery of the lowest price firm more costly on average. On the other hand,
the firm-specific information model has the property that, as the number of
firms becomes sufficiently large, the equilibrium price falls to the perfectly
competitive price.

These results are discussed below. Section II sets out the basic specific-firm information framework, derives the equilibrium, and analyzes improvements in consumer information. Entry competition is examined in Section III and multiple-price equilibria in Section IV.

In Section V we show how the basic model may be reinterpreted and applied to industry equilibrium when products are differentiated. This product differentiation may be spurious, arising out of consumers' misperceptions; or it may be due to actual differences in product formulations and consumer preferences. As a model of product differentiation, the formal structure is a synthesis of the spatial approach of Hotelling (1929), Lancaster (1979), and others with the representative consumer approach of Spence (1976), Dixit and Stiglitz (1977), and Hart (1979). An analogous model of product differentiation is analyzed in detail in Perloff and Salop (1985). The welfare implications of a similar model are discussed in Sattinger (1984). Possible improvements and extensions are discussed in the conclusions.

II. Equilibrium with imperfect information

In this section we analyze a model of industry equilibrium when consumers are perfectly informed. As discussed in the introduction, this model differs somewhat from other work in its conceptualization of information imperfections and consumer decision making.

Two classes of price and quality data may be distinguished--firm-specific and general market information. By firm-specific information, we mean consumers' direct estimates of the prices and qualities of various commodities available from different firms. By general market information, we mean consumers' estimates of these parameters for the market generally. For example, in the case of price uncertainty, a consumer's firm-specific information may

be a prior probability distribution $F_i(p_i)$ over the possible prices, p_i, of
each firm, $i = 1, 2, \ldots, n$; or it may simply be a point estimate, s_i, of
each price. With respect to the market in general, the consumer may have a
probability distribution $G(\underline{p})$ of the set of all prices charged for the com-
modity in question or simply the range of prices charged.

Of course, these two classes of information are related. The general mar-
ket distribution $G(\underline{p})$ may be derived from the appropriate aggregation of the
firm-specific distributions, $F_i(p_i)$. Similarly, in the absence of any
additional firm-specific information, a consumer treats $G(\underline{p})$ as the firm-
specific distribution as well.

Models of search equilibrium, such as Diamond (1971), generally assume
that consumers' general market information is rational; that is, the prior
price distribution, $G(\underline{p})$, is self-fulfilled by the actual equilibrium dis-
tribution of prices in the market. Additional firm-specific information is
gathered from search; in particular, a consumer obtains perfect firm-specific
information by sampling a store or product. For example, Butters' (1977)
advertising model has a diffuse prior $G(\underline{p})$ and perfect firm-specific informa-
tion if an advertisement is received. The newspaper model of Salop and
Stiglitz (1977) has a rational $G(\underline{p})$ and, additionally, perfect firm-specific
information for all firms if the newspaper is purchased.

We take a different approach here. We assume that consumers have only
imperfect firm-specific information and no additional general market infor-
mation about prices beyond that implied by the firm-specific distributions.
This formalization is more in the spirit of estimation models rather than the
search literature.

Specifically, we assume each consumer j ($j = 1, 2, \ldots, L$) enters the
market armed with a point estimate s_i^j for each of the $i = 1, 2, \ldots, n$

firms in the market and purchases from the firm estimated to have the lowest price or $\min_i s_i^j$. For now, we focus on the case in which products in the industry are homogeneous and known to be homogeneous (i.e., this general market information does exist).[4]

Consumers may form their estimates, s_i^j, by gathering information in a variety of ways according to the costs and benefits of each. As discussed previously, inspection, reliable and unreliable experience, truthful and deceptive advertising, and friends and neutral third parties are among the information-gathering methods analyzed in the literature.[5] According to the exact structure of information gathering assumed, particular restrictions on the estimates are implied. For example, if a price is sampled, it will yield a perfect price estimate. For other information-gathering methods, it is difficult to determine exactly what sort of rationality restrictions to place on consumers' estimates.

In this model, we do not derive the structure of the estimates from an explicit information-gathering technology. Instead, we begin with an exogenously generated set of estimates, satisfying certain plausible conditionally. In particular, we assume that consumer j's estimates $(s_1^j, s_2^j, \ldots, s_n^j)$ are generated as follows:

$$s_i^j = p_i + \beta \theta_i^j \tag{1}$$

where $\theta_i^j \sim F_i^j(\theta)$, $\theta \in [a, b]$, $E(\theta_i^j) = 0$, $\mathrm{Var}(\theta_i^j) > 0$, and $F_i^j(\theta)$ is a continuously differentiable distribution function with density $f_i^j(\theta)$.[6]

Thus, estimates are taken to be unbiased and, if $\beta > 0$, as imperfect.[7] The scale parameter β permits a range of information states from perfect

information ($\beta = 0$) to perfect ignorance ($\beta \rightarrow \infty$). Those consumers who draw $\theta = 0$ have accurate estimates, those who draw $\theta < 0$ have an underestimate, and those with $\theta > 0$ have an overestimate of price. Estimates are related to the actual price, p_i, charged by the firm.[8] Finally, the support of θ, $\theta \in [a, b]$ may be finite or infinite. One natural restriction would be to assume price estimates must be nonnegative although, as will be demonstrated below, weaker restrictions will suffice.

Given his estimates $(s_1^j, s_2^j, \ldots, s_n^j)$, each consumer j selects the firm with the lowest estimated price, $\min_i s_i^j$, and shops there. Further comparison shopping is not permitted although the model could accommodate it; thus, we implicitly assume the cost of further search is prohibitive.[9] Instead, once at the selected store, the consumer observes the actual price, p_i, and purchases $d(p_i)$ units.

As a result of this information, a disproportionate share of each firm's sales is made to customers who underestimated its price. Comparison shopping would affect this proportion. Finally, in the static model analyzed here, no additional learning is permitted; every period is independent of the past. In contrast, a richer intertemporal model would include an analysis of the evolution of estimates over time as experienced consumers learn and eventually die, and new ignorant buyers enter the market.[10]

Given this formal structure, we may derive the form of the demand curves facing each firm in the market. It is apparent that, for $\beta > 0$, these demand curves are downward sloping even though all products are homogeneous. Since consumers are not perfectly informed of the lowest price store, higher priced stores do obtain some unlucky customers.[11] Under these circumstances, demand is elastic for two reasons: A price reduction brings forth additional customers, and each customer purchases additional units.

In the case of perfect information ($\beta = 0$), however, the lowest price store does obtain all the customers; thus, shading one's price below a common level $\bar{p}$ does yield a discontinuous demand increase (i.e., demand is perfectly elastic). In contrast, in the perfect ignorance case ($\beta \to \infty$), the flow of customers is unrelated to actual price; demand elasticity comes only from additional purchases from each customer obtained.

We now derive the exact form of the firms' demand curves from the theory of order statistics. For a representative firm i, the probability that it is selected by consumer j is the probability that s_i^j is the lowest estimate. Dropping the superscript j for convenience and substituting from equation (1), we have[12]

$$Pr_i \equiv Pr(s_i \leq s_1, \; s_i \leq s_2, \; \ldots, \; s_i \leq s_n)$$

$$= \int \prod_{k=i} \left\{ 1 - F_k\left(\frac{p_i - p_k}{\beta} + \theta\right) \right\} f_i(\theta) \, d\theta. \tag{2}$$

After selecting a firm, each consumer observes the actual price, p_i, and purchases $d(p_i)$ units there. If there are L consumers with identical demand curves, then the expected demand of firm i is given by

$$Q_i(p_1, \; p_2, \; \ldots, \; p_n) = Ld(p_i) \, Pr_i. \tag{3}$$

Given these demand curves for each firm, the industry equilibrium for an exogenous number of firms n may be derived using conventional methods. If firm i has a constant marginal cost c_i and fixed cost K_i, then its expected operating profits are given by

$$\pi_i(p_1, p_2, \ldots, p_n) = (p_i - c_i) Q_i(p_1, p_2, \ldots, p_n) - K_i. \qquad (4)$$

Each firm maximizes expected operating profits, taking the prices at other firms as given; that is, we derive a Nash-in-price equilibrium. Note that this approach assumes firms have perfect information regarding their competitors' prices in contrast to consumers.[13] Differentiating equation (4) with respect to p_i under the Nash conjectural variation and rewriting, we have[14]

$$p_i = c_i - \frac{Q_i}{\partial Q_i / \partial p_i}. \qquad (5)$$

We now derive a <u>symmetric, single price, Nash equilibrium</u>, given the structure of demand given by equation (3). By symmetry, we mean that the degree of imperfect information for all consumers and costs are identical for all firms, or

$$F_i^j(\theta) = F(\theta),$$
$$c_i = c. \qquad (6)$$

Moreover, we <u>assume</u> that equilibrium entails identical prices for all firms,[15]

$$p_i = p. \qquad (7)$$

We derive the equilibrium as follows: Assuming that all firms except firm i charge an identical price p, then after substituting into equation (3), we have

$$Q_i(p, \ldots, p_i, \ldots, p) = Ld(p_i) \int \left\{ 1 - F\left(\frac{p_i - p}{\beta} + \theta\right) \right\}^{n-1} f(\theta) \, d\theta. \qquad (8)$$

Differentiating (8) with respect to p_i under the Nash conjecture, the demand slope is given by

$$\frac{\partial Q_i}{\partial p_i} = \frac{d'(p_i)}{d(p_i)} Q_i - \left(\frac{n-1}{\beta}\right) Ld(p_i)$$

$$(9)$$

$$\int \left\{ 1 - F\left(\frac{p_i - p}{\beta} + \theta\right) \right\}^{n-2} f\left(\frac{p_i - p}{\beta} + \theta\right) f(\theta) \, d\theta.$$

Substituting the equilibrium value $p_i = p$ into (8) and (9), we have

$$Q_i = Ld(p) \int \{1 - F(\theta)\}^{n-1} f(\theta) \, d\theta = \frac{L}{n} d(p),^{16} \qquad (10)$$

$$\frac{\partial Q_i}{\partial p_i} = \frac{L}{n} d'(p) - \left(\frac{n-1}{\beta}\right) Ld(p) \int \{1 - F(\theta)\}^{n-2} \{f(\theta)\}^2 \, d\theta. \qquad (11)$$

The individual consumer's demand elasticity is

$$\eta \equiv - \frac{\partial Q_i}{\partial p_i} \frac{p_i}{Q_i} = - \frac{p_i d'(p_i)}{d(p_i)}. \qquad (12)$$

Substituting equations (10)-(12) into (5), the symmetric, single-price equilibrium price, $p(n)$, is characterized as follows when there are n firms in the market:

$$p(n) = c + \frac{1}{M(n)}, \qquad (13)$$

where

$$M(n) = \frac{\eta}{p(n)} + \frac{n(n-1)}{\beta} \int \{1 - F(\theta)\}^{n-2} \{f(\theta)\}^2 \, d\theta. \qquad (14)$$

Equations (13) and (14) define a single-price equilibrium between the competitive and monopoly prices. For example, if $\beta = 0$ (perfect information), then $M(n) \to \infty$ and $p = c$, that is, perfect competition obtains.[17] This result, of course, is analogous to the usual "Bertrand" equilibrium. At the other extreme, if $\beta \to \infty$ (perfect ignorance), then $M(n) = \eta/p$ and the monopoly price p^m obtains, where p^m satisfies the usual Lerner markup condition

$$\frac{p^m - c}{p^m} = \frac{1}{\eta}.$$

Improved information is captured by decreases in the scale parameter β. If the elasticity η is nondecreasing in price, then it is easily shown that a firm's aggregate demand becomes more elastic; thus, the equilibrium price falls. Differentiating equations (13) and (14) with respect to β, we have $\partial p/\partial \beta > 0$. That is,

Theorem 1: A reduction in consumer information (in the sense of an increase in β) raises the equilibrium price.

Moreover, as information becomes perfect, the equilibrium price approaches the perfectly competitive price continuously. This result is in contrast to Diamond's result that small but strictly positive search costs yield an equilibrium at the monopoly price. That is, in this model a small degree of imperfect information gives only a small degree of informational market power.

This difference from Diamond's result is not difficult to explain. A small search cost does not, in fact, imply a low cost to becoming perfectly informed. In fact, Diamond's result obtains because, at his monopoly price equilibrium, becoming perfectly informed entails sampling an infinite number of stores, and thus an infinite cost, if search costs are strictly positive.

It should be added that, if decreased information is formalized as a general mean-preserving spread of the density $f(\theta)$, the effect on the equilibrium price is indeterminate. This ambiguity arises because the firm's demand elasticity depends on the entire noise distribution as discussed in Appendix A. This result takes on greater importance in the analysis of product differentiation in Section V.

III. Entry competition

In this section we examine the effect of entry competition (increases in the exogenous number of firms n) on the single price equilibrium. It is a property of even traditional Cournot models of imperfect competition that entry may not lower the equilibrium price (Seade, 1980). We have not yet obtained a general entry result for small changes in the number of firms, but we have derived some asymptotic properties.

Although entry shifts each firm's demand curve inward, the elasticity of demand may not rise and, thus, equilibrium price may not fall. This ambiguity may be confirmed by differentiating the expression for M(n) in equation (14) with respect to n.

On the other hand, for the limiting case of $n \to \infty$ ($k \to 0$), a complete characterization does obtain. Of course, if each firm has strictly positive fixed costs ($k_i > 0$), the market is unable to support an infinite number of firms. Instead, ignoring the integer problem, a zero profit equilibrium is

characterized by the usual tangency of demand with average cost. Only if the level of fixed costs approches zero (perfectly free entry) may the number of competitors become infinite.

The following theorem presents a condition under which the perfectly free entry price equals the perfectly competitive price under full information. The proof is contained in Appendix B.[18]

> Theorem 2: If the support [a, b] of the noise density $f(\theta)$ is bounded from below (i.e., if a is finite), then
>
> $$\lim_{n \to \infty} p(n) = c.$$
>
> The support [a, b] must be bounded from below since all price estimates, s_i, must be positive.

Intuitively, the Nash equilibrium price approaches the competitive price if firm's Nash demand curves become perfectly elastic. If so, then even the smallest price increase causes the loss of all customers. Recall that a representative firm obtains only those customers who most underestimate its price. Indeed, for $n \to \infty$ and finite lower bound a, a firm obtains only those customers who draw the maximum underestimate $\theta = -a$ since each customer chooses a firm from an infinite sized sample from $f(\theta)$, that is, the first (lowest) order statistic equals the lower bound a. Similarly, since the sample is infinitely large, the second-order statistic also approaches the lower bound a. In other words, all of the firm's customers represent close wins, and each of these close wins is converted into a close loss if the firm raises its price even slightly. Thus, its demand is perfectly elastic and Theorem 2 holds.

Thus, perfectly free entry implies perfect competition. Setting profits equal to zero (allowing free entry) in equation (4) and substituting for p from equation (5)--the marginal revenue equals marginal cost condition--we obtain the equal number of firms.

$$n = \frac{L}{KM(n)} \cdot \qquad (15)$$

Above, we assumed $K \to 0$ so the equilibrium number of firms, n, grew without bound. Similarly, increases in the size of the market (as measured by L) increase the number of firms n which, in turn, increases M(n). In the limit, as $L \to \infty$, then $n \to \infty$, $M(n) \to \infty$, and the equilibrium price approaches the perfectly competitive level. In these cases, the firm-consumer ratio (n/L) does become zero as in Hart (1979).

Although biased estimates have not been formally analyzed here, the reader may confirm that the theorems generalize to the case of a common biased distribution $F(\theta)$. In this sense, deceptive (biased) advertising does not destroy perfect competition in the perfectly free entry case as long as the degree of bias is identical for all firms.[19]

IV. Uniqueness, mass points, and multiprice equilibria

Thus far, we have restricted our attention to single price equilibria. In this section we discuss the possible existence of multiprice equilibria as well as the uniqueness of the single-price equilibrium derived above. We turn first to the uniqueness issue.

In principle, there could be multiple single-price equilibria; however, for the conventional case where the individual consumer's demand elasticity, $\eta(p)$, is nondecreasing in price, multiple single-price equilibria cannot occur:

Theorem 3: If $\eta(p)$ is nondecreasing in price and if a single-price equilibrium exists, then it is unique.

This result may be shown be rewriting (13) as follows:

$$\frac{p - c}{p} = \frac{1}{pM(n)} .$$

The left-hand side is monotonically increasing in p, while the right-hand side is monotonically decreasing. Since the left-hand side equals zero when $p = c$ and the right-side approaches zero as p becomes infinitely large, the two sides must intersect exactly once at a positive price markup

$$\frac{p - c}{p} > 0.$$

This result does not rule out the additional possibility of multiple-price equilibria, even under the symmetric information and cost conditions set out in Section II. We do not have a general theorem on the nonexistence of multi-price equilibria; however, such equilibria can be rejected in a duopoly (n = 2) model to which we now turn.

For simplicity, suppose that consumers have perfectly inelastic demands ($\eta = 0$). Normalizing $\beta = 1$, the probability that firm 1 obtains a representative customer is

$$Pr(s_1 \leq s_2) = Pr(\theta_1 - \theta_2 \leq p_2 - p_1). \tag{16}$$

The distribution of $\mu \equiv \theta_1 - \theta_2$, $H(\mu)$ is symmetric with mean equal to zero so that $H(0) = 1/2$.[20] Substituting the definition of μ into equation (16) and normalizing L = 1 so that expected sales equal the representative probability, we have

$$Q_1(p_1, p_2) = H(p_2 - p_1), \tag{17a}$$

$$Q_2(p_1, p_2) = 1 - H(p_2 - p_1). \tag{17b}$$

Calculating expected profits and substituting into the profit-maximizing condition analogous to equation (5), we obtain

$$p_1 = c + \frac{H(p_2 - p_1)}{h(p_2 - p_1)}, \tag{18a}$$

$$p_2 = c + \frac{1 - H(p_2 - p_1)}{h(p_2 - p_1)}, \tag{18b}$$

where $h(\mu)$ is the density of $H(\mu)$. Subtracting (18a) from (18b), we have

$$p_2 - p_1 = \frac{1}{h(p_2 - p_1)} \{1 - 2H(p_2 - p_1)\}. \tag{19}$$

Since $H(0) = 1/2$, equation (19) is only satisfied for $p = p_1 = p_2$; and the unique single-price equilibrium is given by[21]

$$p = c + \frac{H(0)}{h(0)} . \tag{20}$$

Two price equilibria may be ruled out by examining (19). If $p_2 - p_1 > 0$, then $H(p_2 - p_1) > 1/2$ and, since $h(p_2 - p_1) > 0$, the right-hand side of (19) is negative while the left-hand side is positive. A similar contradiction obtains for $p_2 - p_1 < 0$.[22]

Thus, if $n = 2$ and $\eta = 0$, only a single-price equilibrium obtains. For $\eta > 0$, the result obtains if η is nondecreasing in price. However, this method of proof cannot be easily extended to the case of more than two firms. Beginning from a single price satisfying the equilibrium conditions, suppose a

deviant firm, say, firm 1, sets its price at a level other than the common price p. In this case, letting $\mu_i = \theta_1 - \theta_i$, i = 2, ..., n, the n-firm equation analogous to (19) might be derived. Unfortunately, the marginal distributions of the μ_i's are not independent, complicating the calculations.

Until now, we have ruled out mass points. Mass points are important because they lead to the possibility of ties between the lowest estimates. These ties, in turn, lead to discontinuities in demand. Mass points may occur at $\theta = 0$ if some consumers are perfectly informed.[23] The introduction of mass points greatly changes the analysis.

> Theorem 4: If the distribution function $F(\theta)$ has a mass point at $\theta = 0$, no single-price equilibrium exists.

We show this result by first ruling out a single-price equilibrium at p > c and then by ruling out a single-price equilibrium at p = c. For any p > c, one deviant firm could break all previous "ties" by shading its price slightly. Sales would jump discontinuously if there were a strict proportion of ties raising its profits.

For p = c, unless absolutely all consumers were perfectly informed about all firms, a deviant could earn positive profits by charging $p_i > c$ and relying on the occasional unlucky buyer. In contrast, nondeviants set p = c and earn zero profits.

The presence of mass points also has implications for the nature of multiprice equilibria:

> Theorem 5: If the distribution function $F(\theta)$ has a mass point at $\theta = 0$, an equilibrium price vector cannot contain two or more prices which are equal.

If two prices were equal, the previous argument would apply. One of the firms could increase its sales and profits discontinuously by shading its price slightly.

As yet, we have not been able to take the analysis much further. It appears possible for a multiprice equilibrium to exist with (given appropriate reordering of firms) $p_1 < p_2 < \ldots < p_n$. It is clear that $p_1 > c$ and $p_n \leq p^m$, the monopoly price. We have obtained no further restrictions beyond equal profitability.

Given mass points, if average costs are U-shaped, however, either single price or two or more price equilibria may obtain. Figure I illustrates possible single-price and two-price equilibria for this structure. This result is similar to Salop and Stiglitz's (1977) newspaper model. The difference is that the uninformed consumers here purchase according to their different estimates while, in the newspaper model, they purchase randomly.

These results are possible because of the demand discontinuities. Thus, common prices may only occur at the competitive price. There may still be a two-price equilibrium if there is only one high price (say, at p^h in Figure I) deviant. Three-price equilibria require only two deviants and so forth.

Although the existence of multiprice equilibria might cause an embarrassing nonuniqueness, they would enrich the model considerably. In particular, they would permit general market information to be more easily incorporated into the formal model, allowing the conventional search model to be more easily compared to this one. The existence of multiprice equilibria would remove the necessity of the restriction of only firm-specific information as follows: In the current model, where equilibrium entails only a single
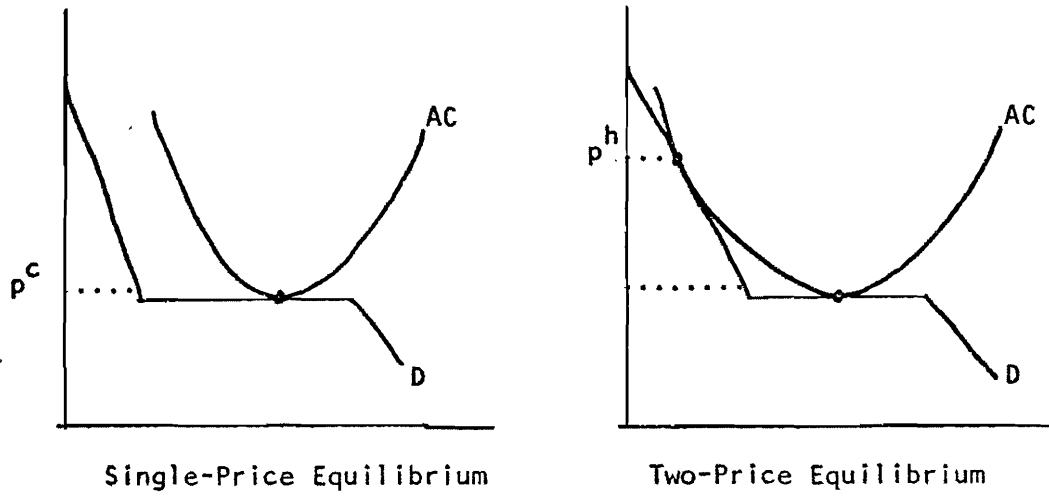
FIG. I

price, a consumer with that general market information would purchase randomly, regardless of the actual estimates drawn. Further analysis along these lines must await a sequel.

V. Spurious and actual product differentiation

As discussed earlier, the model may be reinterpreted to include both spurious and actual product differentiation. By spurious product differentiation, we mean that consumers mistakenly perceive brands to differ by more than they do actually, including the purely spurious differentiation case in which brands are actually homogeneous but are perceived to differ.[24] By actual product differentiation, we mean the case in which consumers differ in their actual valuation of different brands.

The model may easily handle spurious product differentiation by interpreting θ_i^j as quality misperceptions rather than price misperceptions. Similarly, actual product differentiation may be treated by reinterpreting θ_i^j as actual (cardinal) brand preferences. In both cases, s_i^j is redefined as the negative of consumer surplus.

All of the previous theorems hold for these variants of the basic model. Interestingly, the addition of quality misperceptions to price misperceptions may not raise the equilibrium price. As is shown in Appendix A, a mean-preserving spread in the noise density may raise or lower the equilibrium price. The actual product differentiation model is examined in more detail in Perloff and Salop (1985).

VI. Extensions and conclusions

To recapitulate the main results of the firm-specific information model, if second-order conditions are satisfied, then at least one single-price equilibrium obtains. There is a unique single-price equilibrium if individual

demand elasticities are nondecreasing in price. Multiprice equilibria appear to be possible as well although more work needs to be done to rigorously establish existence and additional properties of such equilibria.

If a mass of consumers are well informed, a single-price equilibrium cannot exist if marginal costs are constant. If average costs are U-shaped, however, then single-price equilibria at the competitive price or multiprice equilibria may obtain.

If there are an insignificant number of well-informed consumers, then the single-price equilibrium has the following properties. Improved information, in the sense of the scaling parameter defined above, lowers the equilibrium price. Entry competition lowers price for sufficiently vigorous entry; and in the case of perfectly free entry, equilibrium price falls to the competitive price.

Beyond these results, few other properties have been established. More work needs to be done here with respect to both symmetric multiprice equilibria and multiprice equilibria arising from differential costs and information endowments. The degree of information must be made endogenous. Particular distributions should be examined. The dynamics of the model must be analyzed.

Finally, and probably most important, search must be explicitly introduced into the model. This modification may be done in either of two ways. First, having arrived at a store, a consumer will often find he has underestimated the price charged so he may have a sufficient incentive to sample the firm with the second lowest estimate. Such search will probably have little or no effect on the general qualitative properties of the model.

Of course, a more sophisticated or experienced consumer may infer that his lowest estimate tends to be an underestimate. This information will not alter

his behavior significantly unless he also infers that all prices are identical if, in fact, they are.[25] In that case, if consumers ignore their firm-specific estimates and choose firms randomly, price rises to the monopoly level. Of course, in this case, if a deviant lowers his price and, hence, the firm-specific estimates of his price, will consumers rely on the information? This is the usual logical difficulty arising in search and newspaper models.[26] The problem can be avoided in the case of multiprice equilibria. At such an equilibrium, general market information corresponding to the full rational expectations hypothesis of the search and newspaper models can be well accommodated.

APPENDIX A

We rewrite the density as $f(\theta; \alpha)$ where α is a parameter represent-
ing the level of uncertainty: As α increases, uncertainty increases due to
a mean-preserving spread. Differentiating (13), it may be shown that the sign
of $\partial p/\partial \alpha$ is the same as the sign of

$$\frac{\partial}{\partial \alpha} \int_a^b \{f(\theta; \alpha)\}^2 \, d\theta.$$

Figure II shows a symmetric density to which a mean-preserving spread has been
applied. Various size regions are shown and identified by capital letters:
All regions with the same letter are of the same size.

   If $F(\theta)$ is the original density and $h(\theta)$ is the density after two
sections (labeled A, which are e by x as shown in Appendix Figure A.I) are
removed from the center and added to the tails, then the change in the in-
tegral of the squared density is given as follows:

$$\int_a^b \{h^2(\theta) - f^2(\theta)\} \, d\theta = 2 \left\{ \int_0^x [f(\theta) - e]^2 \, d\theta - \int_0^x f^2(\theta) \, d\theta \right.$$

$$\left. + \int_y^{y+x} [f(\theta) + e]^2 \, d\theta - \int_y^{y+x} f^2(\theta) \, d\theta \right\}$$

$$= 4e \left(ex + \{[F(y + x) - F(y)] - [F(x) - F(0)]\} \right).$$

This value may be either positive or negative. Graphically, it is positive if
the areas A and B are greater than C and negative if A plus B is less than C.

   Heuristically, if the density is nearly uniform, this value is positive,
so price rises as uncertainty increases. If the density is single peaked with
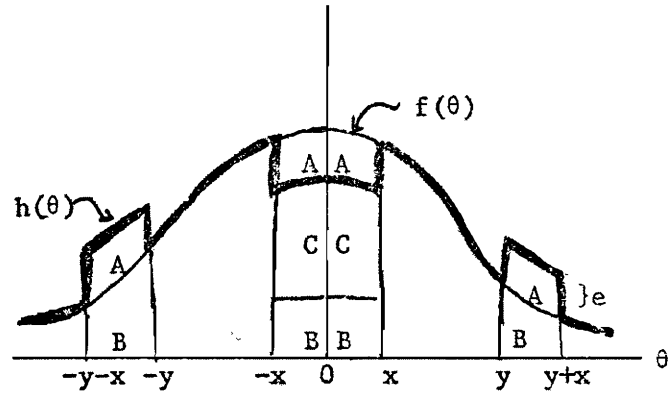
APPENDIX FIG. A.I

a large model, then the price will fall as uncertainty increases. Thus, the price effect depends on the density and the type of mean-preserving spread used.

APPENDIX B

The proof of Theorem 2 is given here. This proof assumes that the density function $f(\theta)$ has the following properties (which could be relaxed at the cost of greater complexity in the proofs):

1. $f(\theta) > 0$, $\theta \in (a, b)$.

2. $f(\theta)$ is analytic.

We wish to prove that, under the conditions given in Theorem 2, entry will drive the equilibrium price to marginal cost (even given limited consumer information). Since $p = c + 1/M(n)$, showing that $\lim_{n \to \infty} M(n) = \infty$ is sufficient to show that $\lim_{n \to \infty} p = c$. The following lemmas establish that, if a is finite, then $\lim_{n \to \infty} M(n) = \infty$.

<u>Theorem 2'</u>: The $\lim_{n \to \infty} M(n) = \infty$.

<u>Proof of Theorem 2'</u>: Since $f(\theta) > 0$ for $\theta \in (a, b)$ and $f(\theta)$ is continuous, there exists an interval $(a, a + \delta)$ subject to $\theta \in (a, a + \delta)$, $f(\theta) \geq \xi > 0$. As a result,

$$M(n) = \int_{a}^{a+\delta} n(n - 1) \{1 - F(\theta)\}^{n-2} \{f(\theta)\}^2 \, d\theta + Z$$

$$\geq \xi \int_{a}^{a+\delta} n(n - 1) \{1 - F(\theta)\}^{n-2} f(\theta) \, d\theta + Z,$$

where

$$Z = \int_{a+\delta}^{b} n(n - 1) \{1 - F(\theta)\}^{n-2} \{f(\theta)\}^2 \, d\theta.$$

Therefore,

$$\lim_{n \to \infty} M(n) \geq \lim_{n \to \infty} n\xi - \lim_{n \to \infty} \{1 - F(a + \delta)\}^{n-1} + \lim_{n \to \infty} Z = \infty.$$

We know, however, that

1. $\lim_{n \to \infty} n\xi = \infty$.

2. $\lim_{n \to \infty} n\xi \{1 - F(a + \delta)\}^{n-1} = 0$, since $1 > \{1 - F(a + \delta)\} > 0$.

3. $\lim_{n \to \infty} Z \geq 0$ since $n(n - 1) \{1 - F(\theta)\}^{n-2} \{f(\theta)\}^2 \, d\theta \geq 0$

    for all $\theta \, \epsilon(a + \delta, b)$.

Thus, $\lim_{n \to \infty} M(n) = \infty$.

FOOTNOTES

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[1]The concept that imperfect consumer information endows even small firms with <u>informational market power</u> was developed by Scitovsky (1950), Arrow (1958), and Stigler (1961) among others. The elegant modeling of this phenomenon by Diamond (1971) and the discovery of the lemons principle by Akerlof (1970) has stimulated research by economists and policy analysts on both the scope of and potential remedies for imperfect information. The policy implications are emphasized by Pitofsky (1977), Schwartz and Wilde (1979), and the Federal Trade Commission (1978, 1979).

[2]Stiglitz (1979) surveys most of the major models and discusses their properties.

[3]This assertion is true for those cases in which the usual second-order conditions for profit maximization hold for each firm (see Section II).

[4]We might note here that the estimates, s_i^j, could easily be re-interpreted as estimates of expected consumer surplus so that real or spurious product differentiation may be incorporated into this model. This extension is made below in Section V.

[5]For a nontechnical discussion of these different methods, see Federal Trade Commission (1979).

[6]Mass points to $F_i^j(\theta)$ are discussed in Section IV. The other assumed properties of these functions are presented in Appendix B.

[7]In fact, this restriction of unbiased estimates is not necessary for many of the results derived below. A weaker restriction of identical bias for all estimates would suffice.

[8]There exists some evidence on the nature of $F_i^j(\theta)$. For example, the Progressive Grocer (November, 1974, p. 39) conducted a survey of 560 shoppers in four Providence and Boston area supermarkets in July, 1974. The consumers were asked to cite the selling price of 44 popular brand-name and nationally advertised items. Only 24 percent of the shoppers tested knew the "correct" price (within 5 percent) for a specific product compared to 32 percent in a similar study in 1963. Other evidence is provided by Gabor and Granger (1961) and Uhl and Brown (1972).

[9]Further search would be induced if the actual price, p_i, exceeded the second lowest estimate, $\min_k s_k^j$, in excess of the consumer's search cost. This topic is discussed in more detail below.

[10]Cf. Phelps and Winter (1970) and Smallwood and Conlisk (1979).

[11]For example, if store 1 charges $10 and estimates are (8, 10, 12) and if store 2 charges $11 with estimates (9, 11, 13), then store 2 will obtain customers who draw the estimate pairs {(10, 9); (12, 9); (12, 11)}.

[12]If $s_i \leq s_k$, then

$$\theta_k > \frac{p_i - p_k}{\beta} + \theta_i.$$

Thus, given θ_i, the probability that $s_i \leq s_k$ is

$$1 - F\left(\frac{p_i - p_k}{\beta} + \theta_i\right).$$

Since θ_i is drawn independently, equation (2) follows.

[13]This assumption may be justified on the grounds that the gains to gathering this information are higher for firms than for individual consumers.

[14]We assume that the second-order conditions are fulfilled, an assumption that is not true in general for all $F(\theta)$ and $d(p)$. For a discussion of sufficient conditions for the second-order condition to hold, see Perloff and Salop (1985); see also footnote 21.

[15]It should be emphasized that we assume a single-price equilibrium. Although this assumption may be easily proved for the case of $n = 2$, we have not ruled out multiprice equilibria for larger n. This issue is discussed in more detail in Section IV.

[16]Since

$$\int \{1 - F(\theta)\}^{n-1} f(\theta) \, d\theta = \frac{1}{n} \ .$$

[17]Of course, if $\eta \to \infty$, then $p(n) = c$ as well.

[18]These proofs are due to Robert Willig and Janos Galambos. Any remaining errors are our own.

[19]Of course, if advertising is treated as a fixed cost, the perfectly free entry condition is not satisfied by a zero profit equilibrium.

[20]Symmetry may be shown by deriving $h(\mu)$, the density of $H(\mu)$, using a convolution with substitutions $\mu = \theta_1 - \theta_2$ and $\zeta = \theta_1 + \theta_2$. With a little manipulation, it can be shown that $h(\mu) = h(-\mu)$.

[21]When $n = 2$, in the symmetric equilibrium ($p_i = p$), if $d(p_i) = a$ constant, $\partial^2 \pi_i / \partial p_i^2 = 2 \, \partial Q_i / \partial p_i < 0$. Where $d'(p_i) < 0$, $\partial^2 \pi_i / \partial p_i^2 = 2 \, \partial Q_i / \partial p_i +$ other negative terms < 0. This result does not necessarily hold for $n > 2$.

[22]A similar analysis can be used to analyze the case of differential costs. If $c_1 < c_2$, then it can be shown that $p_1 < p_2$, that $p_1 - c_1 > p_2 - c_2$, and that the low-cost firm has a higher gross margin $(p_1 - c_1)/p_1$.

[23]By perfect information, we mean that the vectors $\underline{s}^j = \underline{p}^j$ (e.g., $\underline{\theta}^j \equiv 0$ or $\beta \equiv 0$).

[24]The classic story of spurious product differentiation concerns the consumer who forms a false belief that one aspirin brand is superior to another after it relieves a mild headache and the "inferior" brand does not relieve a more serious one. This story may not be too farfetched: Even a placebo achieves a relief rate of around 45 percent compared to a relief rate of around 80 percent for actual aspirin (Food and Drug Administration, 1977). Such spurious product differentiation has been suggested by a number of writers including Chamberlin and Galbraith with respect to a wide variety of consumer products such as beer, detergents, lemon juice, and even soft drinks. The experimental evidence is interesting on this point. Blind tests of consumers' preferences after use do not replicate market shares. In addition, they vary according to whether products are labeled with brand names. For evidence, see Tucker (1964), McConnell (1968), Morris and Bronson (1969), and Monroe (1976); for a related model, see Schmalensee (1979); and for a good discussion of some of the policy implications of this phenomenon, see Craswell (1979).

[25]The level of the expected benefits of search, of course, will be altered.

[26]Cf. the solutions of Salop and Stiglitz (1977) and Diamond and Rothschild (1978).

REFERENCES

AKERLOF, G. (1970). "The market for lemons: qualitative uncertainty and the market mechanism," Quarterly Journal of Economics, vol. 84, pp. 488-500.

ARROW, K. (1958). "Toward a theory of price adjustment," in P. Baran et al., The Allocation of Economic Resources, Stanford University Press, Stanford.

BUTTERS, G. (1977). "Equilibrium distributions of sales and advertising prices," Review of Economic Studies, vol. 44, pp. 465-491.

CRASWELL, R. (1979). "Trademarks, consumer information and barriers to competition," Federal Trade Commission, Washington, D. C.

DIAMOND, P. (1971). "A model of price adjustment," Journal of Economic Theory, vol. 3, pp. 156-168.

DIAMOND, P. and ROTHSCHILD, M. (1978). Uncertainty in Economics: Readings and Exercises, Academic Press, New York, pp. 488 and 489.

DIXIT, A. and STIGLITZ, J. (1977). "Monopolistic competition and optimum product diversity," American Economic Review, vol. 67, pp. 297-308.

FEDERAL TRADE COMMISSION (1978). Trade Regulation Rule on Advertising of Opthalmic Goods and Services, 16 CFR 456, Washington, D. C.

FEDERAL TRADE COMMISSION (1979). Consumer Information Remedies, Washington, D. C.

FOOD AND DRUG ADMINISTRATION (1977). "Proposed monograph for OTC internal analgesic, antipyretic and antirheumatic products," Federal Register, vol. 42, p. 35382.

GABOR, A. and GRANGER, C. (1961). "On the price consciousnsss of consumers," Applied Statistics, vol. 10, pp. 170-180.

GROSSMAN, S., KIHLSTROM, R., and MIRMAN, L. (1977). "A Bayesian approach to the production of information and learning by doing," Review of Economic Studies, vol. 44, pp. 533-548.

GUASCH, L. and WEISS, A. (1980). "Wages as a sorting mechanism in competitive markets with asymmetric information," Review of Economic Studies, vol. 47, forthcoming.

HART, O. (1979). "Monopolistic competition in a large economy with differentiated commodities," Review of Economic Studies, vol. 46, pp. 1-30.

HOTELLING, H. (1929). "Stability in competition," Economic Journal, vol. 39, pp. 41-57.

LANCASTER, K. (1979). Variety, Equity and Efficiency, Columbia University Press, New York.

LELAND, H. (1979). "Quacks, lemons and licensing: a theory of minimum quality standards," Journal of Political Economy, vol. 87, pp. 1328-1346.

McCONNELL, J. (1968). "The development of brand loyalty: an experimental study," Journal of Marketing Research, vol. 5, pp. 13-19.

MONROE, K. (1976). "The influence of price differences and bran familiarity on brand preferences," Journal of Consumer Research, vol. 3, pp. 42-49.

MORRIS, R. and BRONSON, C. (1969). "The chaos of competition indicated by Consumer Reports," Journal of Marketing, vol. 33, pp. 26-34.

NELSON, P. (1974). "The economic value of advertising," in Y. Brozen et al. Advertising and Society, New York University Press, New York, pp. 43-66.

PERLOFF, J. AND SALOP, S. (1985). "Equilibrium with product differentiation," Review of Economic Studies, forthcoming.

PHELPS, E. (1972). "A statistical theory of racism and sexism," American Economic Review, vol. 62, pp. 659-661.

PHELPS, E. and WINTER, S. (1970). "Optimal price policy under atomistic competition," in E. S. Phelps et al., Microeconomic Foundations of Inflation and Employment Policy, W. W. Norton and Co., Inc., New York, pp. 309-337.

PITOFSKY, R. (1977). "Beyond Nader: consumer protection and the regulation of advertising," Harvard Law Review, vol. 90, pp. 661-701.

PLOTT, C. and WILDE, L. (1979). "Seller induced demand," California Institute of Technology, Pasadena, unpublished manuscript.

SALOP, S. (1980). "The monopoly matchmaker and the theory of brokerage," Federal Trade Commission, Washington, D. C., unpublished manuscript.

SALOP, S. and STIGLITZ, J. (1977). "Bargains and ripoffs: a model of monopolistically competitive price dispersion," Review of Economic Studies, vol. 44, pp. 493-510.

SATTINGER, M. (1984). "Value of an additional firm in monopolistic competition," Review of Economic Studies, vol. 51, pp. 321-332.

SCHWARTZ, A. and WILDE, L. (1979). "Intervening in markets on the basis of imperfect information: a legal and economic analysis," University of Pennsylvania Law Review, vol. 127, pp. 630-682.

SCHMALENSEE, R. (1979). "On the use of economic models in antitrust: the ReaLemon case," University of Pennsylvania Law Review, vol. 127, pp. 994-1050.

SCITOVSKY, T. (1950). "Ignorance as a source of oligopoly power," American Economic Review, vol. 40, pp. 48-53.

SEADE, J. (1980). "On the effects of entry," Econometrica, vol. 48, pp. 479-489.

SHAPIRO, C. (1980). "Dynamic models of imperfect information and quality," Massachusetts Institute of Technology, Cambridge, unpublished manuscript.

SMALLWOOD, D. and CONLISK, J. (1979). "Product quality in markets where consumers are imperfectly informed," Quarterly Journal of Economics, vol. 93, pp. 1-23.

SPENCE, A. M. (1973). "Job market signalling," Quarterly Journal of Economics, vol. 87, pp. 355-374.

SPENCE, A. M. (1974). "Competitive and optimal responses to signals," Journal of Economic Theory, vol. 7, pp. 296-306.

SPENCE, A. M. (1976). "Product selection, fixed costs and monopolistic competition," Review of Economic Studies, vol. 43, pp. 217-235.

STIGLITZ, J. (1975). "The theory of screening, education and the distribution of income," American Economic Review, vol. 65, pp. 283-300.

STIGLITZ, J. (1979). "Equilibrium in product markets with imperfect information," American Economic Review, Papers and Proceedings, vol. 69, pp. 339-345.

STIGLER, G. (1961). "The economics of information, Journal of Political Economy, vol. 69, pp. 213-225.

TUCKER, W. (1964). "The development of brand loyalty," Journal of Marketing Research, vol. 1, pp. 32-35.

UHL, J. N. and BROWN, HAROLD L. (1971). "Consumer perception of experimental retail food price changes," Journal of Consumer Affairs, vol. 5, pp. 174-185.

VARIAN, H. (1979). "A model of sales," University of Michigan, Ann Arbor, unpublished paper.

WILDE, L. and SCHWARTZ, A. (1979). "Equilibrium comparison shopping," Review of Economic Studies, vol. 46, pp. 543-553.

WILSON, R. (1977). "A bidding model of perfect competition," Review of Economic Studies, vol. 44, pp. 511-518.