# Farm Level Nonparametric Analysis of Profit Maximization Behavior with Measurement Error

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# Farm level Non parametric Analysis of Profit Maximization Behavior with Measurement Error

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*This paper tests the farm level profit maximization hypothesis using a nonparametric production analysis approach allowing for measurement error in the input and output variables. All farms violated Varian's deterministic Weak Axiom of Profit Maximization (WAPM). The magnitude of minimum critical standard errors required for consistency with profit maximization, convex technology production was smaller after allowing technological change during the sample period. Results indicate strong support for the presence of technological change during the sample period.*

Keywords: nonparametric analysis, profit maximization, measurement error, technological change

Conventional analysis of neoclassical theory of production proceeds by first postulating a parametric form for the production function and then using standard statistical techniques to estimate the unknown parameters from the observed data. This procedure suffers from a possible defect that the maintained hypothesis of parametric form can never be directly tested (Varian, 1984). However, nonparametric production analyses do not require a functional form of the production function to be defined.

Nonparametric approaches are of two types. One type compares a firm with another firm for a given year (Fare et al., 1985). The second type, which is used in this analysis, compares current input/output choices to decisions made previously (Varian,1984). These two approaches were developed and popularized by researchers such as Afriat (1967, 1972), Hanoch and Rothschild (1972), Varian (1983, 1984, 1985), Diewert and Parkan (1983, 1985), Swofford and Whitney (1987), Chavas and Cox (1988), Fawson and Shumway (1988), and Chalfant and Alston (1988), Fare, Grosskopf,

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and Lovell (1985). Lim and Shumway (1992a) applied nonparametric techniques to statewide aggregate production data for the United States from 1956 through 1982. Estimated measurement errors of about 3% from the stochastic test results were consistent with the profit-maximization hypothesis in nearly all states. Lim and Shumway (1992b) also used nonparametric analysis to investigate separability in state-level agricultural technology. Featherstone et al. (1995) applied nonparametric techniques to analyze agricultural technology and production behavior for a sample of 289 Kansas farms, using annual farm level data for an 18 –year period, 1973 to 1990. Their results rejected strict adherence of the observed data to the hypotheses of cost minimization and profit maximization.

A limitation of many previous applied studies addressing optimizing behavior and the structure of technology is that they typically used national or state level rather than individual farm data. Microeconomic theory is based upon optimization by individual agents. Featherstone et al. (1995) argue that the use of statewide data to characterize individual agents' optimization behavior can cause problems by possibly introducing aggregation bias because of summing across farms.

Empirical evidence suggests that when firm/farm level data are used, the cost minimization or profit maximization hypothesis is rejected in most cases, whereas the optimization hypothesis is not rejected when aggregate data are used. That is why Love (1998) suggested that stochastic nonparametric test procedures be used when testing firm-level data for cost-minimizing or profit maximizing behavior. The objective of this paper is to test the farm level profit maximization hypothesis using Varian's nonparametric production approach allowing for measurement error in variables. In particular, a nonparametric production analysis approach on a sample of Kansas farms tests the profit maximization hypothesis consistent with the existence of a closed, convex, negative monotonic production set using data from 1988-2007. The analysis proceeds by using quadratic programming to determine the minimum perturbation of the input and output set to calculate measurement error necessary to be consistent with profit maximization for each farm. Unlike Lim and Shumway (1992a), we employ an additive error with transformed data to accommodate multiproduct analysis with some outputs equal to zero. Output data are aggregated into two commodities: crops and livestock. Three categories of inputs are also used: labor, capital and purchased inputs.

The rest of the paper is organized as follows: first, description of the nonparametric production analysis approach, equations used to test deterministic and stochastic profit maximization tests with and without accounting for technical change is presented. Then data and methodology is briefly discussed. Finally, we present the main findings of the empirical results followed by discussion.

## Nonparametric Production Analysis

As developed by Varian (1984, 1985), two procedures are used. One is a deterministic test and the other is a stochastic test of the magnitude of measurement error required for consistency with the profit maximization hypothesis when some observations violate the deterministic test. Varian derived a test statistic that permits the latter procedure to be interpreted in terms of the classical statistical frame-work of hypothesis testing.

The deterministic test is an all-or-none test, in that the entire test fails if the optimizing hypothesis is violated once. The stochastic test allows for measurement error in the data when considering consistency with the optimizing behavior. Varian (1985, 1990) derived test statistics that permit the results of the stochastic procedure to be interpreted using classical statistical hypothesis testing.

**Deterministic Tests**

Following Varian (1985), let T be the production possibility set of all input-output bundles (-x, y) compatible with available technology. The production possibility set T is nonempty, closed, bounded from above, convex, and allows for free disposal. A specific production set at time t is represented by a netput vector **Y** = ($Y_1$, . . . $Y_m$) in T, where positive $Y_i$ s represent outputs and negative $Y_i$ s represent inputs. The set of all feasible production plans, **Y**, a subset of T, is closed, convex, and negative monotonic (Varian 1984). The boundary of the convex set reflects an efficient production frontier, because no other way exists to produce the given output with fewer inputs or to produce more output with given inputs. This implies that profit ($\Pi_t$) at any time, t, is the product of the netput vector, Y', and its price vector, $P_t$, where t=l . . . m. Varian (1985) showed that the following conditions are equivalent: (1) There exists a production set that p-rationalizes the data and (2) $P_tY_t \geq P_tY_s$ for all t, s = 1, 2... n, and (3) there exists a closed, convex, negative monotonic production set that p-rationalizes the data.

Under constant technology over the sample period, consistency of the observed data with profit maximization requires:

$$P_tY_t \geq P_tY_s \quad \text{for all t, s = 1, 2, ..., n,} \tag{1}$$

where $Y_t$ is in Y. Varian (1984) calls this the Weak Axiom of Profit Maximization (WAPM). This axiom implies that if profit is maximized given $P_t$, then that profit should be greater than or equal to any other profit generated by any other set of outputs and inputs evaluated at $P_t$. In this case, one needs to use straight forward exhaustive checking of the above inequalities to test for adherence of the data set with profit maximization hypothesis.

**Stochastic Tests**

The deterministic test is an all-or-none test, in that the entire test fails if the optimizing hypothesis is violated once. However, data could fail the test because producers make decision errors, don't always operate on the efficient boundary, and/or because observations aren't perfect measurements (Hanoch and Rothschild, 1972; Varian, 1985) in addition to output risk. Focusing on the measurement error, Lim and Shumway (1992a) studied state-level analysis for the US agriculture.

Varian (1985) proposed a general nonparametric method of statistical hypothesis testing when data are subject to measurement error. Consider the null hypothesis, $H_O$, that the data ($Y^i$, $p^i$) satisfy the joint hypothesis of profit maximization, convex technology, and monotonic nonregressive technical change. Assume that the true netput k quantity for observation i is related to the observed netput quantity in the following manner:[2]

$$Q_{ik} = Y_{ik} + \varepsilon_{ik} \tag{2}$$

where $Q_{ik}$ is the true netput quantity, $Y_{ik}$ is the observed netput quantity, and $e_{ik}$ is a random error term that is independently and identically distributed N(0, $\sigma^2$). Since netputs are measured in different units (e.g., tons, bushels, pounds), we chose to work

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[2] Varian (1985) used this equation to relate true and observed factor demands in a cost minimization problem.

with mean scaled data since some outputs are equal to zero so that we can use the additive error relationship. If we could observe the true data, ($Q_{ik}$), we could calculate the test statistic:[3]

$$T = \sum_{i=1}^{n} \sum_{k=1}^{m} (Q_{ik} - Y_{ik})^2 / \sigma^2 \sim X^2 \qquad (3)$$

Under $H_O$

Since $Q_{ik}$ is not observed and the true variance is unknown, the chi-squared test in (3) cannot be carried out in the usual manner. It can be used, however, to obtain a critical lower bound estimate of the variance when the null hypothesis is true. The following quadratic programming problem is formulated:

$$Min_z R = \sum_{i=1}^{n} \sum_{k=1}^{m} (Z_{ik} - Y_{ik})^2$$

Subject to

$$\sum_{k=1}^{m} p_{ik} z_{ik} \geq \sum_{k=1}^{m} p_{ik} z_{jk} \text{ for all i= 1,2,…,n} \qquad (4)$$

where $Z_{jk}$ are solutions to the quadratic programming problem that minimize the sum of squared additive residuals, R. Under $H_O$, the true data ($Q_i$, $p_i$) satisfy the constraints. Hence, the minimum sum of squares, R, must be no larger than the test statistic, $\sigma^2$ T. This means that whenever R is greater than $\sigma^2 C_\alpha$, where $\sigma^2 C_\alpha$ is the critical value for a given significance level, $\alpha$, we reject $H_O$. Since $\sigma^2$ is unknown, a critical lower bound estimate of the standard error is computed at $\alpha$ as $\bar{\alpha}$ = $(R / C_\alpha)^{0.5}$. If the investigator

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[3] Varian (1985), and Lim and Shumway (1992a) used these equations to relate optimization problems for cost minimization and profit maximization with measurement errors.

believes the true standard error of measurement is less than this lower bound, then the null hypothesis is false.

So far the setup has been considering constant technology along the years in consideration. Technological progress increases the efficiency of inputs used in the production of output. In this kind of set up, it is possible to modify the above equations to allow for technological change as:

$$Min_z R = \sum_{i=1}^{n} \sum_{k=1}^{m} (Z_{ik} - Y_{ik})^2$$

Subject to

$$\sum_{k=1}^{m} p_{ik} z_{ik} \geq \sum_{k=1}^{m} p_{ik} z_{jk} \text{ for all i= 1,2,…,n , and } j \leq i, \qquad (5)$$

**Data and Methods**

The nonparametric approach was used to evaluate the profit maximization behavior of 377 Kansas farms observed from 1988 to 2007. Specifically, consistency with deterministic profit-maximization behavior was tested for each farm. Adherence to the stochastic profit-maximization hypothesis under monotonic non-regressive, technical change was also examined for each of the farms.

Income and balance sheet data for the 377 farms were obtained from the Kansas Farm Management Association databank (Langemeier, 2003). The farms were defined to have three inputs:  labor, purchased input and capital input. The farms were also assumed to have two outputs: crops and livestock. Our results will be conditioned upon this aggregation across commodities. Price indexes for inputs and outputs were obtained from USDA’s Kansas Agricultural Statistics (USDA, 1988-2007a) and Agricultural Prices (USDA, 1988-2007b). Physical input indices for quantities were obtained by dividing the farms' cash operating expenses in each of the three input categories by the price for each input. Similarly, physical output indices for quantities were calculated by dividing the farms' gross income in each of the two output categories by the price of each output. The price and quantity measurements were then scaled so that the mean of each price and quantity is equal to one.

## Results

### Deterministic tests

Given 20 years data, checking for the deterministic nonparametric tests involved 380 price-output comparisons. The number of profit maximization violations for the individual farms ranged from 184 to 207, with a mean of 191.5. The standard deviation of violations was 2.784. All farms violated Varian's deterministic WAPM. The number of violations of profit maximization under non-regressive technical change ranged from 8 to 167, with a mean of 72.9, and with standard deviation of 44.8 (Table 1).

**Table 1. Summary statistics for farm-level nonparametric analysis for 377 Kansas farms**

| Hypothesis | Mean | Standard deviation | Minimum | Maximum |
|---|---|---|---|---|
| Deterministic profit maximization violations | 191.5 | 2.9 | 184 | 207 |
| Deterministic profit maximization under non-regressive technical change | 72.9 | 44.8 | 8 | 167 |

### Stochastic tests

Using equation 4, we estimate the quadratic program for each farm. The minimized values follow a chi square distribution with 380 degrees of freedom. These minimized values were used to calculate the standard error $\bar{\alpha}$. Assuming no technological change, the minimum standard error required to maintain the hypothesis of profit maximization ranged from 0.095 to 1.55 with a mean of 0.227 and a standard deviation of

0.160. The farms with excessively high standard error were those which were inconsistent in their production patterns. More than half of the farms (56%) had less than 0.20 minimized standard error and 87% of the farms had standard errors required for consistency with the profit maximization of less than 0.30.

Using equation (5), consistency of the profit maximization hypothesis under technological change was tested. The minimized values follow a chi square distribution with 190 degrees of freedom and with alpha level of .05 used to compute the critical values. After accounting for technological change, the critical minimum standard errors required for profit maximization hypothesis decreased substantially. The critical standard errors ranged from 0.018 to 1.02 and with a mean value of 0.109 and with a standard deviation of 0.117. The distribution of the standard errors required for consistency with the stochastic profit maximization in the case with technological change is quite different when compared with the without technological change case. A large majority of the farms (92%) had standard errors less than 0.20 and 95% of the farms had standard errors of less than 0.30.

The standard errors without technological change were much larger than with technological change. The distribution of standard errors of measurement for the stochastic profit maximization with and without technological changes is shown in Figure 1. The minimized standard errors are skewed more to the lower range of standard errors under the analysis conducted with technological change and concentrated towards the larger range of standard error in the analysis conducted without technological change.

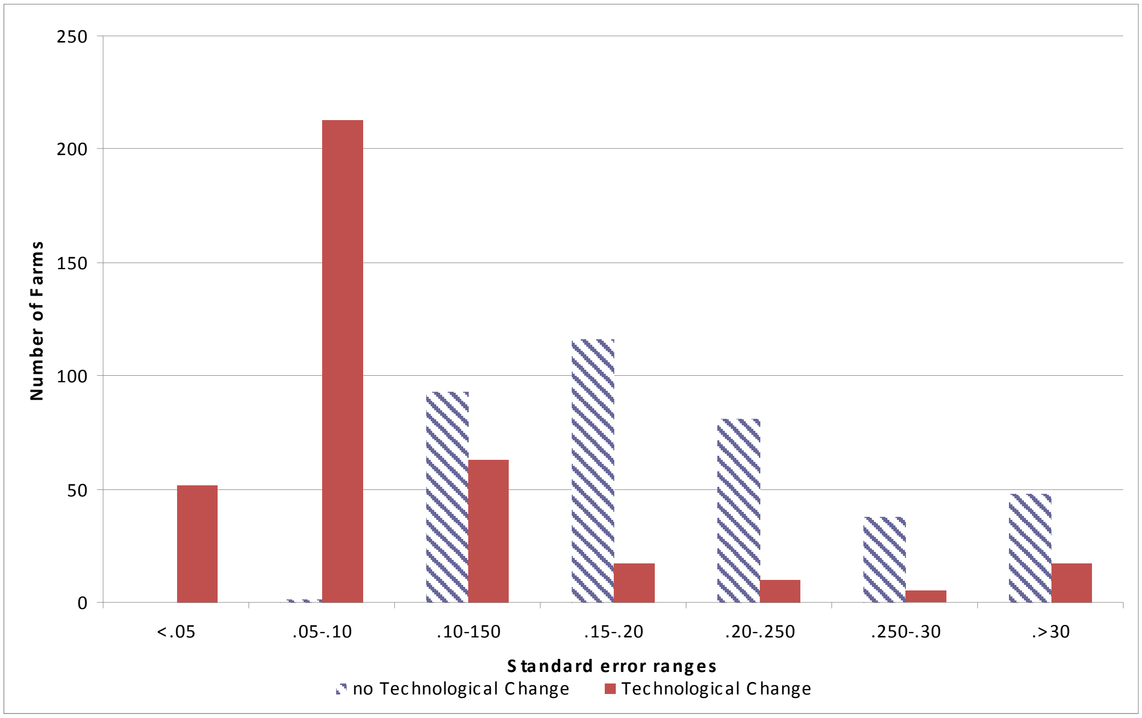

**Figure 1. Distribution of minimal standard error of measurement for profit maximization hypothesis with and without technological change at $\alpha$ =5%**

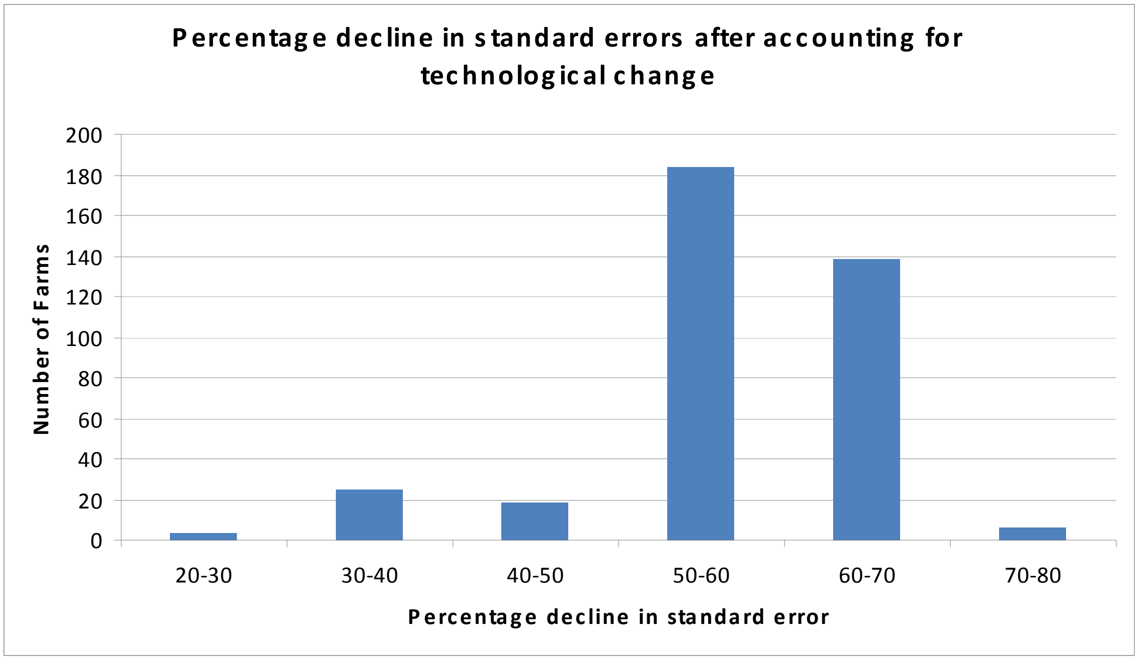

**Figure 2. Distribution of percentage decline in profit maximization hypothesis violations after accounting for technological change at $\alpha$ =5%**

This implies that there might be more violation of the hypothesis of profit maximization with constant technology than with technological change. Out of 377 farms, 60% of the farms showed a difference of between .10-.20 standard error when comparing with and without technological change and a total of 4% a difference of standard error more than .20 when comparing with and without technological change. In absolute terms, the change in the standard error by relaxing the constant technology constraint ranged from a minimum of 0.074 to a maximum of 0.525.

One can also look at the percentage change in the magnitude of standard error before and after accounting for a technological change as shown in Figure 2. This ranged from 21.7% to 80.3 % with a mean 56.9%. This decline in the measurement error for the profit maximization hypothesis can be interpreted as strong support in the presence of technological change using farm level data.

## Discussion

This study applied deterministic and stochastic nonparametric tests to individual farm data for 377 Kansas farms to test adherence of these data with profit-maximization hypotheses before and after considering technical change. All farms violated the hypothesis of deterministic profit maximization. Due to the nature of agriculture, errors in measurement of variables are inevitable. The stochastic analysis used in this paper provides a way of testing the adherence of farms to profit maximization hypothesis that would otherwise been rejected without accounting for measurement error, even though farmers might have been in reality profit maximizers. The magnitude of the critical minimum standard error required to maintain profit maximization was on average 0.227 assuming constant technology. After considering technical change, this figure was reduced to a mean value of 0.109. Results provide strong support for the presence of technological change during the sample period.

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