AN ASSESSMENT OF DYNAMIC BEHAVIOR IN THE U.S. CATFISH MARKET: AN APPLICATION OF THE GENERALIZED DYNAMIC ROTTERDAM MODEL

Andrew Muhammad
Department of Agricultural Economics, Mississippi State University, PO Box 5187, Mississippi State, MS 39762; Phone: (662) 325-0200;
E-mail: muhammad@agecon.msstate.edu

Keithly G. Jones
U.S. Department of Agriculture, Economic Research Service, 1800 M Street NW, Washington DC; Phone: 202-694-5172;
Email: kjones@ers.usda.gov

Selected Paper prepared for presentation at the Southern Agricultural Economics Association Annual Meeting, Atlanta, Georgia, January 31-February 3, 2009

ABSTRACT: Dynamic demand systems have been employed in a number of studies to account for habit formation and inventory adjustments in demand. Few studies have attempted to provide a theoretical foundation for the dynamic demand structures employed. Recently, Bushehri (2003) showed how a generalized dynamic Rotterdam model could be derived from the neoclassical intertemporal utility maximization problem; however, no empirical application is provided in his study. This paper provides an empirical application of the generalized dynamic Rotterdam model to the demand for processed catfish products in the U.S. The two-period dynamic Rotterdam model explained a significant amount of the variation in U.S. catfish demand and was preferred to the one-period and static models. Estimates suggest that buyers adjust short-run inventories such that the past sales negatively affect current sales. Given inventory adjustment behavior, demand was relatively more inelastic in the long-run.

I. INTRODUCTION

Dynamic demand systems have been employed in a number of studies to account for habit formation and inventory adjustments in demand (Sexauer, 1977; Blanciforti and Green, 1983; Pollak and Wales, 1992; Arnade and Pick, 1994; Balcombe and Davis, 1996; Karagiannis, Katranidis and Velentzas, 2000). The conventional approach has been to include lag terms as demand determinants or to employ time series methods (e.g. an error correction model). Few studies have attempted to provide a theoretical foundation for the dynamic demand structures employed. An exception is Brown and Lee (1992); however, they employed a method (translation) that is typically used to incorporate any non-price/income variable (i.e. advertising) into a demand system. More recently, Bushehri (2003) showed how a generalized dynamic Rotterdam model could be derived from the neoclassical intertemporal utility maximization problem; however, no empirical application is provided in his study. This paper provides an empirical application of the generalized dynamic Rotterdam model developed by Bushehri (2003) to the demand for processed catfish products in the U.S.

II. MODEL DERIVATION

Bushehri (2003) shows how a generalized dynamic Rotterdam model can be derived from the consumer's intertemporal utility maximization problem. In this section we provide the mathematical derivations; however, readers are referred to Bushehri (2003) for the complete theory.

Given the intertemporal utility maximization problem we can define the optimal demand for the i^{th} good at time t as follows:

$$q_i(t) = g_i(x(t), \mathbf{p}(t), \mathbf{h}(t)). \tag{1}$$

$q_i(t)$ is the quantity of good i; g_i denotes the demand function; $x(t)$ is consumer expenditures; $\mathbf{p}(t)$ is an n-vector of prices where n denotes the total number of goods within the consumer's choice set; and $\mathbf{h}(t)$ is an n-vector of stock of habits. All are specified at time t.

The above specification requires an additional stage in the consumer budgeting process. The conventional utility tree approach assumes that consumers first allocate total expenditures across product groups and then allocate group expenditures on goods within groups (Theil, 1980; Deaton and Muellbauer, 1980). To arrive at equation (1), it must also be assumed that at the initial stage of the budgeting process, consumers allocate lifetime wealth to specific time periods and that expenditures are allocated across goods (or groups) without reconsidering the intertemporal optimization problem. Otherwise, demand at time t would be a function of lifetime wealth and the static conditions implied by consumer demand theory would not hold (Bushehri, 2003).

Differentiating Equation 1 with respect to time yields:

$$\dot{q}_i = \frac{\partial g_i}{\partial x(t)}\dot{x} + \sum_{j=1}^{n}\frac{\partial g_i}{\partial p_j(t)}\dot{p}_j + \sum_{j=1}^{n}\frac{\partial g_i}{\partial h_j(t)}\dot{h}_j\ . \tag{2}$$

Note that for any variable y, $\dot{y} = \mathrm{d}y(t)/\mathrm{d}t$. If we divide both sides of equation (2) by $q_i(t)$, and if we multiply the first, second and third terms on the right hand side by $x(t)/x(t)$, $p(t)/p(t)$ and $h(t)/h(t)$, respectively, with some manipulation we get the following growth equation:

$$\frac{\dot{q}_i}{q_i(t)} = \eta_i\frac{\dot{x}}{x(t)} + \sum_{j=1}^{n}\eta_{ij}\frac{\dot{p}_j}{p(t)} + \sum_{j=1}^{n}\phi_{ij}\frac{\dot{h}_j}{h_j(t)}\ . \tag{3}$$

Note that η_i is the expenditure elasticity and η_{ij} is the uncompensated price elasticity. $\phi_{ij} = (\partial g_i/\partial h_j)(h_j/g_i)$ is the responsiveness of the quantity demanded for good *i* to changes in the stock of habit for good *j*.

The last step is to substitute the Slutsky equation for the uncompensated price elasticity and to multiply both sides of Equation 3 by the i^{th} budget share $w_i = p_i q_i / \sum_i p_i q_i$.[1] This yields the following demand equation:

$$w_i\frac{\dot{q}_i}{q_i(t)} = \sum_{j=1}^{n} w_i\phi_{ij}\frac{\dot{h}_j}{h_j(t)} + w_i\eta_i\left[\frac{\dot{x}}{x(t)} - \sum_{j=1}^{n} w_j\frac{\dot{p}_j}{p(t)}\right] + \sum_{j=1}^{n} w_i\eta_{ij}^{*}\frac{\dot{p}_j}{p(t)}\ . \tag{4}$$

Without the stock of habits term $\sum_j w_i\phi_{ij}(\dot{h}_j/h_j(t))$ equation (4) is similar to the absolute price version of the Rotterdam model in Theil (1980) and Theil and Clements (1987) where the term in brackets is the change in real expenditures and the last term is the impact of prices on quantity demanded.

The dynamic Rotterdam model is used in estimating the demand for U.S. catfish products. Theil (1980) and Theil and Clements (1987) show that the static Rotterdam model is a theoretically separable functional form, that is if product groups are separable (weak or strong), the Rotterdam model is sufficient for representing the demand for goods within a single product group. We assume the same holds true for the dynamic Rotterdam model which should be the case if lifetime wealth is pre-allocated (Bushehri, 2003).

To put equation (4) in empirical form, we replace continuous changes with discrete time changes. Theil (1987) and Bushehri (2003) suggest the one-period log difference which is used in most demand studies. Therefore, we approximate the changes in quantities and prices as follows:

$$\Delta q_t = \log q_t - \log q_{t-1} \approx \dot{q} / q(t) \text{ and } \Delta p_t = \log p_t - \log p_{t-1} \approx \dot{p} / p(t).$$

The term in brackets in equation (4) is equal to the Divisia volume index (Theil, 1980). We replace this term with a discrete measure of the Divisia volume index ΔQ_t where

$$\Delta Q_t = \sum_{i=1}^{n} w_i \Delta q_{it} = \Delta x_t - \sum_{j=1}^{n} w_j \Delta p_j \approx \dot{x} / x(t) - \sum_{j=1}^{n} w_j (\dot{p}_j / p(t)). \tag{5}$$

Bushehri (2003) also suggests the following habit specification for discrete time periods:

$$\sum_{j=1}^{n} \phi_{ij} \frac{\dot{h}_j}{h_j(t)} = \alpha_i^* + \sum_{k=1}^{p} \sum_{j=1}^{n} \alpha_{ijk} \Delta q_{jt-k} \tag{6}$$

where $\sum_k \sum_j \alpha_{ijk} \Delta q_{jt-k}$ is a distributed lag of the quantities consumed in log-difference form.

Given equations (5) and (6), the empirical version of the dynamic Rotterdam model is expressed as follows:

$$\bar{w}_{it} \Delta q_{it} = \gamma_i^* + \sum_{k=1}^{p} \sum_{j=1}^{n} \gamma_{ijk} \Delta q_{jt-k} + \theta_i \Delta Q_t + \sum_{j=1}^{n} \pi_{ij} \Delta p_{jt} + \varepsilon_{it} \tag{7}$$

where $\bar{w}_{it} = 0.5(w_{it} + w_{it-1})$; $w_{it} = p_{it} q_{it} / \sum_i p_{it} q_{it}$; $\gamma_i^* = \bar{w}_{it} \alpha_i^*$; $\gamma_{ijk} = \bar{w}_{it} \alpha_{ijk}$; $\theta_i = \bar{w}_{it} \eta_i$; and $\pi_{ij} = \bar{w}_{it} \eta_{ij}^*$. γ_i^* , γ_{ijk} , θ_i and π_{ij} are parameters to be estimated and ε_{it} is a random disturbance term. Equation (7) suggests that the effects of habit on consumption is captured by past consumption where consumption of a particular good depends not only on present expenditures and prices but also on the past consumption of that good and all other related goods.

Demand theory requires the following restrictions on parameters: $\sum_i \gamma_i^* = 0$, $\sum_i \gamma_{ijk} = 0$ for all j and k, $\sum_i \theta_i = 1$, $\sum_i \pi_{ij} = 0$ (adding up); $\sum_j \pi_{ij} = 0$ (homogeneity); $\pi_{ij} = \pi_{ji}$ (symmetry); and $\mathbf{\Pi}_{n \times n} = \left[\pi_{ij}\right]$ is negative semidefinite (negativity).

The short-run conditional expenditure and compensated price elasticities (Hicksian) are defined as θ_i / w_i and π_{ij} / w_j respectively. The short-run uncompensated price elasticity (Marshallian) is defined as $\pi_{ij} / w_j - \theta_i w_j / w_i$ (Seale, Sparks and Buxton, 1992). The long-run expenditure elasticity, compensated price elasticity, and uncompensated price elasticity are respectively defined as (Bushehri, 2003)

$$\eta_i^L = \frac{\theta_i}{w_i - \left(\sum_k \gamma_{ijk}\right)} \tag{8}$$

$$\eta_{ij}^{*L} = \frac{\pi_{ij}}{w_j - \left(\sum_k \gamma_{ijk}\right)} \tag{9}$$

$$\eta_{ij}^L = \frac{\pi_{ij}}{w_j - \left(\sum_k \gamma_{ijk}\right)} - \frac{\theta_i}{w_i - \left(\sum_k \gamma_{ijk}\right)} w_j \,. \tag{10}$$

III. EMPRICAL RESULTS

Monthly disaggregated catfish quantities at the processor level measured in 1,000 pounds and U.S. processor prices measured in dollars per pound were provided by the USDA, National Agricultural Statistical Service (NASS). The time period for the data was January 1996 to January 2007. Processed catfish was disaggregated into six products: fresh whole fish, fillets and other; frozen whole fish, fillets and other. The other category included steaks, nuggets and other products not elsewhere specified. Variable statistics are presented in Table 1.

Given that a dynamic model of lag-length k is nested within a model of lag-length k + 1, a likelihood ratio (LR) tests can be used to test for the appropriate lag (Brown and Lee, 1992). Tests results are presented in Table 2. LR tests indicated that the static Rotterdam model was rejected in favor of the one-period lag model, and that the one-period lag model was rejected in favor of the two-period lag model. However, there was little difference between the log-likelihood values for the three-period and two-period lag models. Therefore, we assume a maximum lag length of two months. Additionally, homogeneity and symmetry failed to be rejected in the 2-period lag model. All results that follow assume a two-month lag, homogeneity and symmetry.

Estimation of the dynamic Rotterdam model was accomplished using the LSQ procedures in TSP (version 5.0) (Hall and Cummins, 2005). Preliminary results indicated that the constant term should be excluded from the model. Thus, equation (7) was estimated without a constant term which suggests that there were no trending variables (in levels) that determined catfish demand (Seale, Marchant and Basso, 2003). Overall, the dynamic Rotterdam model performed well. All expenditure effects θ_i (marginal shares) were positive and significant at the 1% level. These estimates reflect how a dollar increase in real expenditures is allocated across the six products. Given that fillets (fresh and frozen) are the more popular products, their marginal share estimates were relatively larger. All own-price effects π_{ii} were negative and significant at the 1% level (except frozen whole fish). A number of cross-price effects reflected a competitive relationship between products, particularly between fresh fillets and frozen fillets (0.385).

The lag effects are presented in Table 4. Positive own-lag effects reflect habit formation and negative effects reflect short-run inventory adjustments (Sexauer, 1977). Note that all own-lag effects are significant and negative for all products suggesting inventory adjustment behavior on the part of buyers. Given the relative durability of frozen products, there own-lag effects were negative in both periods whereas the own-lag effects for the fresh products were mostly significant in the first period only. The signs and magnitudes of the cross-lag effects depend on the relationship between products (substitutes versus complements) and the adjustment behavior of buyers (habits versus inventories). For example, if any two products are substitutes (complements) and unrelated to all other goods, we would expect their cross-lag effect to be positive (negative) if buyers adjust consumption behavior based on short-run inventories.

Lastly, the short-run and long-run expenditure and price elasticities are presented in Table 5. Given inventory adjustments, demand in the short-run was relatively more elastic. The short-run expenditure elasticities were close to unity for all products. In the long-run, the effect of expenditures on demand was significantly smaller. The short-run own-price elasticities (Hicksian and Marshallian) indicated that the demand for the fresh products was elastic and the demand for frozen fillets was also elastic. In the log-run, the demand for all products except fresh fillets was inelastic. Given that fresh fillets are relatively more perishable the impact of short-run inventory adjustments was relative small resulting in similar short-run and long-run own-price elasticities.

IV. SUMMARY & CONCLUSION

This paper provides an empirical application of the generalized dynamic Rotterdam model presented by Bushehri (2003) which was used in estimating disaggregated catfish demand. Test results showed significant information when the static model or 1-period lag model was assumed. The 2-period dynamic model explained a significant amount of the variation in U.S. catfish demand. The lag estimates suggest that buyers adjust short-run inventories such that sales in the previous two periods have a negative effect on current period sales. Given inventory adjustment behavior, the demand was relatively more inelastic in the long-run.

NOTES

[1] The Slutsky equation is defined as $\eta_{ij} = \eta_{ij}^{*} - \eta_i w_j$, where η_{ij}^{*} is the compensated price elasticity and $w_j = p_j q_j / \sum_i p_i q_i$ is the budget share for good j.

REFERENCES

Arnade, C., Pick, D. and Vasavada, U. (1994) Testing dynamic specification for import demand models: the case of cotton, *Applied Economics*, 26, 375-80.

Balcombe, K. G. and Davis, J. R. (1996) An application of cointegration theory in the estimation of the almost ideal demand system for food consumption in Bulgaria, *Journal of Agricultural Economics*, 15, 47-60.

Blanciforti, L. and Green, R. (1983) An almost ideal demand system incorporating habits: an analysis of expenditures on food and aggregate commodity groups, *The Review of Economics and Statistics*, 65, 511-15

Brown, M. G. and Lee, J. Y. (1992) A dynamic differential demand system: an application of translation, *Southern Journal of Agricultural Economics*, 24(2), 1-10.

Bushehri, M. A. M. (2003) Dynamic generalization of the Rotterdam model, *Applied Economics Letters*, 10, 303-06.

Deaton, A. and Muellbauer, J. (1980) *Economics and Consumer Behavior*, Cambridge University Press, Cambridge.

Hall, B. H. and Cummins, C. (2005) *TSP International Reference Manual Version 5.0*, TSP International, Palo Alto, California.

Karagiannis, G., Katranidis, S. and Velentzas, K. (2000) An error correction almost ideal demand system for Greece, *Agricultural Economics*, 22, 29-35.

Pollak, R. A. and Wales, T. J. (1992) *Demand System Specification & Estimation*, Oxford University Press, New York.

Seale, J. L., Marchant, M. A. and Basso, A. (2003) Imports versus domestic production: a demand system analysis of the U.S. red wine market, *Review of Agricultural Economics*, 25, 187-202.

Seale, J. L., Sparks, A. L., and Buxton, B. M. (1992) A Rotterdam application to international trade in fresh apples: a differential approach, *Journal of Agricultural and Resource Economics*, 17, 138-49.

Sexauer, B. (1977) The role of habits and stocks in consumer expenditure, *Quarterly Journal of Economics*, 91, 127-42.

Theil, H. (1980) *The System-Wide Approach to Microeconomics*, The University of Chicago Press, Chicago.

Theil, H. and Clements, K. W. (1986) *Applied Demand Analysis: Results from System-Wide Approaches*, Ballinger Publishing Company, Cambridge.

Table 1. Descriptive Statistics for U.S. Processed Catfish: January 1996 – January 2007

	Fresh			Frozen		
	Whole	Fillet	Other	Whole	Fillet	Other
Price ($/lb)						
Mean	1.56	2.77	1.67	1.97	2.67	1.62
Standard Deviation	0.13	0.18	0.10	0.10	0.17	0.15
Minimum	1.24	2.44	1.40	1.80	2.36	1.32
Maximum	1.81	3.28	1.90	2.27	3.09	1.96
Monthly Quantity (1000 lbs)						
Mean	3,297	4,836	1,300	1,138	9,705	3,917
Standard Deviation	428	789	228	158	1,163	640
Minimum	2,426	3,222	768	793	6,296	2,384
Maximum	4,467	6,815	2,156	1,500	12,362	5,364
Expenditure Share						
Mean	0.094	0.242	0.039	0.041	0.469	0.115
Standard Deviation	0.013	0.017	0.006	0.005	0.020	0.009
Minimum	0.071	0.209	0.026	0.031	0.410	0.091
Maximum	0.131	0.282	0.050	0.056	0.510	0.139

Table 2. Likelihood Ratio Tests Results

Models	Log-likelihood Value	LR Statistic	P-value
Lag structure†			
3-period	2,610.81		
2-period	2,606.47	8.68	0.999 (30)‡
1-period	2,575.32	62.31	0.000 (30)
Static	2,535.58	79.46	0.000 (30)
Economic constraints (2-period model)			
Unrestricted	2,614.16		0.000 (18)
Homogeneity	2,613.54	1.26	0.939 (5)
Symmetry	2,606.47	14.12	0.168 (10)

† All models have homogeneity and symmetry model.
‡ The number of restrictions is in parenthesis.

Table 3. Conditional Demand Estimates for Processed Catfish

Products	Price Coefficients π_{ij}						Marginal Share θ_i
	Fresh			*Frozen*			
Fresh	Whole	Fillet	Other	Whole	Fillet	Other	
Whole	-0.146 (.017)***	0.054 (.028)*	-0.023 (.009)***	-0.017 (.012)	0.124 (.036)***	0.009 (.008)	0.093 (.004)***
Fillet		-0.475 (.090)***	0.029 (.019)	-0.009 (.027)	0.385 (.095)***	0.016 (.018)	0.258 (.009)***
Other			-0.056 (.008)***	0.011 (.007)	0.037 (.023)	0.002 (.005)	0.044 (.003)***
Frozen							
Whole				-0.035 (.018)*	0.047 (.029)*	0.003 (.006)	0.041 (.003)***
Fillet					-0.619 (.125)***	0.025 (.025)	0.456 (.012)***
Other						-0.055 (.016)***	0.108 (.008)***
Equation R^2	.84	.89	.78	.72	.93	.71	

System R^2 = .965
[a] Asymptotic standard errors are in parentheses. Homogeneity and symmetry are imposed. *** Significance level = .01; ** Significance level = .05; * Significance level = .10

Table 4. Dynamic Adjustment Estimates

Products	Lag Coefficients γ_{ijk}					
	Fresh			*Frozen*		
	Whole	Fillet	Other	Whole	Fillet	Other
Fresh	One-period Lag (Δq_{jt-1}) Effects					
Whole	-0.287	0.033	0.203	0.183	-0.018	0.082
	(.074)***	(.028)	(.119)*	(.109)*	(.015)	(.038)**
Fillet	-0.373	-0.116	0.322	-0.103	0.139	-0.021
	(.164)**	(.061)*	(.264)	(.241)	(.034)***	(.084)
Other	0.048	0.048	-0.334	0.021	-0.008	0.004
	(.049)	(.018)***	(.080)***	(.072)	(.010)	(.025)
Frozen						
Whole	0.174	0.010	-0.139	-0.404	0.037	-0.004
	(.052)***	(.019)	(.084)*	(.076)***	(.011)***	(.027)
Fillet	0.581	0.012	-0.092	-0.041	-0.225	0.327
	(.216)***	(.081)	(.352)	(.321)	(.045)***	(.112)***
Other	-0.143	0.014	0.040	0.343	0.075	-0.388
	(.139)	(.052)	(.224)	(.207)*	(.029)***	(.073)***
	Two-period Lag (Δq_{jt-2}) Effects					
Fresh	Whole	Fillet	Other	Whole	Fillet	Other
Whole	-0.196	-0.021	0.162	0.099	0.026	0.111
	(.073)***	(.028)	(.118)	(.102)	(.016)*	(.037)***
Fillet	-0.209	0.058	0.189	-0.006	0.087	0.035
	(.162)	(.062)	(.261)	(.226)	(.034)**	(.082)
Other	0.017	-0.008	-0.099	0.003	0.012	0.010
	(.048)	(.018)	(.078)	(.067)	(.010)	(.025)
Frozen						
Whole	0.180	-0.011	-0.237	-0.287	-0.022	0.004
	(.051)***	(.020)	(.083)***	(.072)***	(.011)**	(.026)
Fillet	0.432	-0.091	-0.049	0.574	-0.194	0.073
	(.215)**	(.082)	(.346)	(.301)*	(.046)***	(.109)
Other	-0.222	0.072	0.034	-0.382	0.047	-0.233
	(.139)	(.053)	(.223)	(.195)**	(.030)	(.070)***

Asymptotic standard errors are in parentheses.
*** Significance level = .01; ** Significance level = .05; * Significance level = .10

Table 5. Short-run and Long-run Demand Elasticities

	Short-run Elasticities			Long-run Elasticities		
Fresh	Expenditure	Hicksian own-price	Marshallian own-price	Expenditure	Hicksian own-price	Marshallian own-price
Whole	0.991	-1.559	-1.652	0.161	-0.254	-0.269
Fillet	1.067	-1.960	-2.218	0.861	-1.582	-1.790
Other	1.113	-1.421	-1.465	0.093	-0.118	-0.122
Frozen						
Whole	1.011	-0.850[a]	-0.891[b]	0.056	-0.047[a]	-0.050[a]
Fillet	0.971	-1.318	-1.774	0.513	-0.696	-0.937
Other	0.941	-0.480	-0.588	0.147	-0.075	-0.092

[a] Significance level =.10; [b] Significance level =.05.
All others are significant at the .01 level or lower.