An Examination of Inverse Demand Models: An Application to the U.S. Crawfish Industry

Youngjae Lee and P. Lynn Kennedy

This study analyzes quantity impacts of imported crawfish tailmeat on Louisiana crawfish tailmeat prices, and demonstrates the statistical validity and proper interpretation of cross substitution within inverse demand systems. Among five inverse demand systems, the Differential Inverse National Bureau of Research (DINBR) model shows no violation of econometric assumptions for the data analyzed. By using Allais coefficients proposed by Barten and Bettendorf (1989), we show substitutability among the five fish species.

Key Words: inverse demand systems, statistical validity, Allais coefficients

Gorman (1960) provided a theoretical basis for the price formation of perishable or semi-perishable goods like fish and vegetables. Studies thereafter, for fish and vegetables, developed similar types of inverse demand systems. In particular, four inverse demand systems—the Differential Inverse Rotterdam Demand System (DIRDS) proposed by Barten and Bettendorf (1989), the Differential Inverse Central Bureau of Statistics (DICBS) demand model introduced by Laitinen and Theil (1979), the Differential Inverse Almost Ideal Demand System (DIAIDS) presented by Barten and Bettendorf (1989), and the Differential Inverse National Bureau of Research (DINBR) demand model referred to by Brown, Lee, and Seale (1995)—were prominent achievements in inverse demand model development. Furthermore, Brown, Lee, and Seale (1995) obtained a Differential Inverse Generalized Demand System (DIGDS) from these four nested differential inverse demand systems and terminated derivation of this kind of inverse demand system. These inverse demand systems provided a useful economic framework for applied demand analyses for fish and vegetables and have contributed to a better understanding of consumer behavior (Brown, Lee, and Seale 1995, Eales, Durham, and Wessells 1997, Matsuda 2005).

However, previous demand studies in empirical demand analyses were reluctant to show the statistical validity of their empirical models. As McGuirk et al. (1995) noted, verifying the adequacy of model assumptions for the data being analyzed is not necessarily trivial. If an inverse demand study shows that regression assumptions are adequate for the empirical model, the statistical results can be viewed with some confidence on the premise of statistical validity. Statistical validity also helps to minimize erroneous conclusions often reached when, as is typical in applied econometric studies, underlying model assumptions are not tested.

In addition to issues dealing with the statistical validity of the underlying inverse demand model, former inverse demand studies did not properly document the correct methods of measuring the interaction of cross effects in these inverse demand systems. As Barten and Bettendorf (1989) showed, due to the negativity of the diagonal elements of the Antonelli matrix, price flexibilities cannot be directly used for determining substitution or complementarity between two goods. Instead, they suggested an Allais coefficient, which is able to ascertain the nature of cross effects in the off-diagonal elements of the Antonelli matrix. In particular, the Allais coefficient provides a useful, empirical way for interpreting how much

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change occurs in the price ratio depending upon one unit of change in the quantity ratio coming from a change in the quantity of one good. However, the majority of previous studies have simply utilized off-diagonal price flexibilities (rather than Allais coefficients) for substitution and complementarity calculations. With such weaknesses inherent in previous inverse demand studies, this study conducted an applied demand analysis for Louisiana crawfish tailmeat by using the above five different types of inverse demand systems to better calculate the intensity of interaction of fish species.

Overview of the Crawfish Industry

In the United States, crawfish are sold for consumption in three forms: whole live crawfish, whole boiled crawfish, and processed (peeled) tailmeat. About 12 percent of domestically harvested crawfish are processed into tailmeat, with most of the remainder sold as whole live and boiled crawfish (88 percent). Whole live and boiled crawfish is dominated mainly by U.S. producers and sold to restaurants and retail consumers in Louisiana. Crawfish tailmeat can be sold fresh (chilled) or frozen. Like whole live and boiled product, fresh tailmeat is quite perishable and does not keep for more than a few weeks, so the U.S. market for fresh crawfish tailmeat is also dominated by U.S. producers. Frozen tailmeat, however, can keep for a year or more and is the product most directly affected by Chinese imports. Crawfish tailmeat is purchased by restaurants, distributors, and retail food stores, and is sold predominantly within Louisiana or to national distributors' local outlets (U.S. International Trade Commission 2003).

Historically, whenever whole live or boiled crawfish supplies exceeded what could be moved through market channels to restaurants and retail consumers, excess product found its way to processing plants to be peeled and sold as fresh or frozen tailmeat. This marketing outlet served to moderate price swings and provided regional economic benefits in terms of adding value and creating employment. After the mid-1990s, however, these enterprises met a new challenge from low-priced imported frozen crawfish tailmeat from China, resulting in price instability not only for frozen tailmeat but also for fresh tailmeat and whole live and boiled crawfish (U.S. International Trade Commission 2003).

In light of these circumstances, we intend to identify the economic impacts of crawfish imports and the other related fishery products such as catfish, shrimp, and oysters which are expected to compete with crawfish products. Since inverse demand systems provide a useful economic model to analyze quantity impacts on price, this study will use five inverse demand systems to analyze the quantity impact of imported crawfish tailmeat and other related fish on domestic crawfish tailmeat price. In doing this work, we will provide the necessary statistical tests to show the statistical validity of the underlying models.

In order to achieve these purposes, we proceed as follows: in the next section the five inverse demand systems are briefly reviewed; this review will provide a clear understanding of the theoretical relationship among these five inverse demand systems. Thereafter, the necessary statistical tests of the underlying system models will be discussed, which will provide sensible standards for adequate model specification for the applied demand analysis of Louisiana crawfish tailmeat. Then, we discuss the empirical results, inspecting/discussing test results of the underlying system models and more stringently interpreting the cross effects in the specified inverse demand system.

Inverse Demand Systems

As Anderson (1980), Barten and Bettendorf (1989), and Brown, Lee, and Seale (1995) defined, the features of an inverse demand system are (i) that price is endogenous and quantity is exogenous, (ii) that the system equations of endogenous prices are expressed using budget shares, which then leads to the adding up of the system equations, and (iii) that the mathematical form of variables in the system equations is that of differential logarithms. The above five inverse demand systems are summarized into one nesting equation specified as

$$(1)\quad w_i d\ln\pi_i = (h_i - \theta_1 w_i)d\ln Q + \sum_j \left(h_{ij} - \theta_2 w_i(\delta_{ij} - w_j)\right) d\ln q_j,$$

where w_i is budget share of good i, π_i is the normalized price of good i,

$$d \ln Q = \sum w_j d \ln q_j$$

is the Divisia volume index, h_i is a scale coefficient, θ_1 and θ_2 are nesting parameters, h_{ij} is the Antonelli coefficient, and δ_{ij} is the Kronecker delta. We can also obtain the other nested models by restricting both nesting parameters as follows:

(2) $$w_i d \ln \pi_i = h_i d \ln Q + \sum_j h_{ij} d \ln q_j$$

DIRDS for $\theta_1 = 0$ and $\theta_2 = 0$,

(3) $$w_i d \ln \frac{p_i}{P} = c_i d \ln Q + \sum_j h_{ij} d \ln q_j$$

DICBS for $\theta_1 = 1$ and $\theta_2 = 0$,

(4) $$dw_i = c_i d \ln Q + \sum_j c_{ij} d \ln q_j$$

DIAIDS for $\theta_1 = 1$ and $\theta_2 = 1$,

(5) $$dw_i - w_i d \ln Q = h_i d \ln Q + \sum_j c_{ij} d \ln q_j$$

DINBR for $\theta_1 = 0$ and $\theta_2 = 1$.

In equation (3), p_i is the nominal price of good i,

$$d \ln P = \sum w_j d \ln p_j$$

is the Divisia price index, and $c_i = h_i + w_i$, and in equation (4), $c_{ij} = h_{ij} + w_i \delta_{ij} - w_i w_j$. The preceding models were applied to a variety of studies for fish and vegetables.

Theoretical and Statistical Validity

One inherent, crucial feature of inverse demand systems is the adding-up condition, which results in singularity problems in the estimation procedure because the covariance of disturbance in the system of equations will be zero. Due to the weakness of finite discrete numbers in the empirical model, the adding-up condition is not always satisfied. Even though the inverse demand systems presented above use the same price and quantity data, a different transformation of the discrete price and quantity data highlights the necessity of confirming whether the summations of the residuals of the five equations in each of the individual inverse demand systems are zero. Should the summations of the residuals be zero for an individual inverse demand system, the full equation estimation will be singular.

Related to the statistical assumptions of the underlying systems of equations, this study will briefly discuss those statistical tests employed to obtain a valid statistical model. Initially, an econometric regression assumes a normal distribution of the disturbances, which implies that the skewness and kurtosis coefficients of disturbances are

$$\alpha_3 = [E(u_t' \Omega^{-1} u_t)^3]^{1/2} = 0$$

and

$$\alpha_4 = E(u_t' \Omega^{-1} u_t)^2 = n(n+2),$$

respectively (where n is the number of equations in the system). If we find the disturbances to be non-normal, one has to proceed with some plausible assumptions like gamma, χ^2, lognormal, and Pareto distributions. Unfortunately, these specific distributions have not been investigated to any great extent.

Secondly, economic theory itself, while useful in helping economists identify economic variables that may be relevant in a particular problem setting, gives very little guidance as to correctly identifying the corresponding functional form. For example, economic knowledge does not elucidate as to whether or not the log-linear relationship of the underlying inverse demand system is preferable. By using an auxiliary regression system, McGuirk, Driscoll, and Alwang (1993) examined system functional form tests that can be used to determine which underlying inverse demand model is preferable for the log-linear relationship.

In order to obtain efficient estimators that are both unbiased and variance consistent, it is necessary that the random error terms of a statistical model should be specified as homoskedastic. The system's homoskedasticity can be tested through using the auxiliary regression system, which is used to investigate whether the variances of residuals from the different equations are homoskedastic or not. The consequence of heteroskedasticity is that estimators are not best linear unbiased estimators (BLUE).

As in the case of the Louisiana crawfish market, there may be a structural change between two different periods of time. For example, the average market share of crawfish tailmeat can be compared for the periods before and after 1997. The average market share of domestic crawfish tailmeat markedly decreased from 88 percent before 1997 to 35 percent after 1997, while the average market share of imported crawfish tailmeat correspondingly increased from 12 percent before 1997 to 65 percent after 1997. In this case, the stability of conditional mean parameters could be doubted. To gain insight into the relevance of the parameter-stability assumption for each equation in the system separately, Chow and variance-stability tests are conducted. Parameter stability of the crawfish equation in the system is assessed using a Chow test to determine if β differs between the first and second time periods of the sample. Because the Chow test assumes equal variances in the two periods, we also conduct an *F*-test of variance equality. The variance test statistic is

$$\left(\frac{RSS_2}{RSS_1}\right)\left(\frac{T_1-K}{T_2-K}\right) \sim F(T_2-K, T_1-K)$$

under H_0, where RSS_1 and RSS_2 are the residual sums of squares using the first T_1 and last T_2 observations, respectively.

The concept of independence in econometrics represents the idea that dependent variables are independent across time, resulting from the function, $f(y_{it} \mid X_{it} : \beta_i)$, $t = 1, 2, ..., T$. McGuirk, Driscoll, and Alwang (1993) provided the *t*-type test for independence, in which lags of the dependent variable are included in the auxiliary regression and *t*-values of the lag-dependent variable will be used to judge independence.

If the underlying inverse demand model satisfies these theoretical and statistical assumptions, then the econometric framework of these inverse demand systems yields minimum variance and unbiased estimators. Consequently, each of these assumptions should be tested and verified before conducting applied demand analyses.

Quantitative Analysis

Data

Data is a crucial part of this research because the utility of the economic framework of inverse demand systems will depend upon the availability of empirical data. In inverse demand system analysis, studies used short-term (monthly or quarterly) data rather than yearly data because of price endogeneity and quantity exogeneity. Unfortunately, this study met limitations regarding data availability in conducting inverse demand system analysis of crawfish tailmeat. In fact, there is no official institution that regularly records macroeconomic data of the relatively small local crawfish tailmeat business on a short-term basis. Therefore, this study used annual data from 1989 to 2005. However, since sample size has a profound effect on statistical results, this smaller number of observations may lead to decreased precision in estimates of various properties of the population given all else being equal. Since only a set of small observations is available, then the standard error of the sample mean will be great, given by the formula of standard error, $\sigma / \sqrt{n}$. It can be shown that as *n* becomes small, this variability becomes large. This leads to less sensitive hypothesis tests with smaller statistical power and greater confidence intervals.

Data of domestic and imported crawfish tailmeat, shrimp, and oysters were provided by the National Marine Fisheries Service (NMFS), while catfish data were collected from Catfish Processing Reports published by the National Agricultural Statistics Service. The quantity of domestic crawfish tailmeat represents the summation of quantity supplied by Louisiana crawfish tailmeat processors. Domestic crawfish tailmeat for 1991 was supplied by 64 Louisiana crawfish processors, whose numbers had decreased to 24 by 2005. Domestic crawfish tailmeat price is an average weighted price of supplied crawfish tailmeat.[1] Total quantity and value data of imported crawfish tailmeat were also provided by NMFS. Owing to the lack of data on market prices for imported crawfish tailmeat, unit values (value divided by the volume of imports) were used as a proxy for market prices. The value of imports is

[1] This study calculated the prices as follows:

$$p_i = \frac{v_d + v_m}{q_d + q_m},$$

where v_d = domestic price × domestic volume is domestic value, v_m is imports value, q_d is domestic volume, and q_m is imports volume.

generally defined as the price actually paid or payable for merchandise when sold for exportation to the United States, excluding U.S. import duties charged at the point of entry into the United States. Therefore, unit values may not be a perfect measure of market prices because trade restrictions (such as an anti-dumping tax) may not change the unit value for imported crawfish from a particular import source, but might impact the market price of imported crawfish because of supply restrictions. The quantity of catfish, shrimp, and oysters represents the summation of domestic and imported products. Domestic prices of catfish, shrimp, and oysters are used in this study.

System Estimates

To estimate scale and price flexibility parameters of the inverse demand system equations (1)–(5), the specifications must be modified to reflect the discrete nature (as to time) of the data as follows:

$$\text{(6)} \quad \bar{w}_{it}\Delta \ln \pi_{it} = g_i \Delta \ln Q_t + g_{ij}\Delta \ln q_{jt} + u_{it}\,, \quad i, j = 1,2,3,4,5\,,$$

where $g_i = (h_i - \theta_1 w_i)$ and $g_{ij} = h_{ij} - \theta_2 w_i(\delta_{ij} - w_j)$ are scale and price flexibility parameters of DIGDS, respectively, and $\bar{w}_{it} = (w_{it} + w_{it-1})/2$ is the two years' moving average in the share of good i in total expenditures. In the empirical model, this study used moving average share to avoid a simultaneity problem (Haden 1990). The scale and price flexibility parameters and dependent variables would be changed according to the value of nesting parameters, θ_1 and θ_2, as shown in equations (2)–(5). The discrete forms of parameters and dependent variables are as follows:

(7) $\Delta \ln \pi_{it} = \Delta \ln \pi_{it} - \Delta \ln \pi_{it-1}$
(differential log normalized price of good i)

$\Delta \ln q_{it} = \Delta \ln q_{it} - \Delta \ln q_{it-1}$
(differential log quantity of good i)

$\Delta \ln p_{it} = \Delta \ln p_{it} - \Delta \ln p_{it-1}$
(differential log price of good i)

$\Delta \ln Q_t = \sum_i \bar{w}_{it}\Delta \ln q_{it}$
(Divisia volume index)

$\Delta \ln P_t = \sum_i \bar{w}_{it}\Delta \ln p_{it}$
(Divisia price index)

$\bar{w}_{it}\Delta \ln \pi_{it}$
(dependent variable of DIRDS and DIGDS)

$\bar{w}_i \Delta \ln \dfrac{p_{it}}{P_t} = \bar{w}_{it}(\Delta \ln p_{it} - \Delta \ln P_t)$
(dependent variable of DICBS)

$\Delta \bar{w}_{it} = \bar{w}_{it} - \bar{w}_{it-1}$
(dependent variable of DIAIDS)

$\Delta \bar{w}_{it} - \bar{w}_{it}\Delta \ln Q$
(dependent variable of DINBR).

Zellner's (1962) Seemingly Unrelated Regression (SUR) was used as an econometric methodology because it is sensible to assume that individual fish products are contemporaneously correlated in consumption as substitutes. In the econometric estimation procedure, this study will confirm that the summation of the residuals of the five equations in the systems is equal to zero, before dropping one equation to show whether or not the adding-up condition is sufficiently satisfied. Before imposing theoretical restrictions such as homogeneity and symmetry, we test for the statistical assumptions discussed in the previous section to judge the statistical validity of those models. After obtaining a statistically appropriate model, the scale and price flexibility parameters will be estimated by imposing theoretical restrictions on the SUR model. Furthermore, as we stated previously, price endogeneity should be tested for because the yearly quantity of fish is taken to be predetermined and also because fish consumption is presumed to respond to price incentives. Accordingly, this study will investigate the predeterminedness of quantity consumed by using a Wu-Hausman test (Wu 1973, Hausman 1978, and Thurman 1986).

Misspecification Tests

The adding-up condition assumes that the summation of residuals of the equations of the system is equal to zero,

$$\hat{u}_{1t} + \hat{u}_{2t} + \hat{u}_{3t} + \hat{u}_{4t} + \hat{u}_{5t} = 0\,,$$

which causes a singular contemporaneous residual covariance matrix of systems of equations. Under adding up, the parameters should be estimated in the n–1 equations of the systems; moreover, these estimates are invariant to the equation deleted. Table 1 shows the summation of residuals of five equations in individual inverse demand systems. The adding-up assumption was not satisfied in both DIRDS and DIGDS, while the assumption was satisfied in the DIAIDS, DICBS, and DINBR models. The contemporaneous residual covariance matrix of DIRDS and DIGDS will not be singular, while it will be singular in DIAIDS, DICBS, and DINBR, so scale and price flexibility parameters of DIAIDS, DICBS, and DINBR should be estimated in the SUR models of n–1 equations.

This study performed multivariate and univariate tests of normality of the individual inverse demand system. The multivariate tests provided are Mardia's skewness and kurtosis test and the Henze-Zirkler $T_{t,\beta}$ test. The univariate test employed is the Shapiro-Wilk W test. The null hypothesis for all these tests is that the residuals are normally distributed. The results from the multivariate and univariate tests of normality are reported in Table 2. The p-values from the tests indicate a possible violation of the null hypothesis. Mardia'a skewness tests for the systems of models confirmed that both the DIRDS and DIGDS models violated the assumption of symmetrical distribution of residuals at the 5 percent significance level. For individual equation tests in the system, the assumption of normality of distribution for error variance was violated in the shrimp equation of both the DIRDS and DIGDS models at the 5 percent significance level. The test results showed that the other equations and systems did not reject the null hypothesis at the 5 percent level.

A question related to variable selection is that of choosing the functional form appropriate for a particular economic relationship. Ideal tests of functional form have high power over a wide variety of alternative hypotheses. A RESET type test, which uses powers (2nd and higher order) of the model's fitted values $(\hat{y})$, can be viewed as a general functional form test (Ramsey 1969). The RESET tests, in which the price variable was used as a dependent variable and the quantity variable was used as an independent variable, use an F-statistic to assess the significance of the squares and possibly the cubes of the predictions from the inverse demand models in an auxiliary regression in which the squares and the cubes of the model's fitted values are used as additional explanatory variables. Significance of the coefficients of the additional variables is intended to be indicative of some kind of specification error of underlying inverse demand models such as omitted variables or incorrect functional form. Test results are reported in Table 3. Except for F-statistics in RESET2 of the imported crawfish price equation in DICBS and RESET3 of the imported crawfish price equations in DIAIDS and DICBS, F-statistics were lower than the critical value at the 10 percent significance level.

One of the key assumptions of an econometric regression is that the variance of the disturbances is constant across observations. If the residuals of the underlying model have constant variance, as assumed herein, each model will provide an efficient, unbiased estimator with a consistent variance. This study used a multivariate WHITE and Godfrey Lagrange Multiplier (LM) test for checking for static homoskedasticity, and an ARCH test for checking for autoregressive conditional heteroskedasticity. The null hypothesis for the static and dynamic homoskedasticity tests is that the covariance matrix of residuals is homoskedastic. The test results are reported in Table 4. The test results showed no static heteroskedasticity except for the catfish and shrimp equations in the DIRDS, DICBS, and DIGDS models, with the Godfrey LM test at the 5 percent level. The Q statistics test and Lagrange Multiplier tests in dynamic homoskedasticity tests indicate no autoregressive conditional heteroskedasticity at the 5 percent level.

In order to examine whether or not there have been structural changes in the domestic crawfish tailmeat market between the periods before and after 1997, this study used a Chow test to determine if the conditional mean parameters, β's, differ between before and after 1997. This study also conducts an F-test of variance equality between the first and second period of the sample provided by McGuirk, Driscoll, and Alwang (1993). The null hypothesis for conditional mean and variance stability tests is that the variance-

Table 1. Summation of Residuals of Individual Inverse Demand System

t	DIRDS	DIAIDS	DICBS	DINBR	DIGDS	DINBR [a]
1	0.00007	0.00000	0.00000	0.00000	-0.00324	0.00000
2	-0.00020	0.00000	0.00000	0.00000	-0.00240	0.00000
3	-0.00011	0.00000	0.00000	0.00000	-0.00047	0.00000
4	0.00024	0.00000	0.00000	0.00000	0.00101	0.00000
5	-0.00013	0.00000	0.00000	0.00000	0.00423	0.00000
6	-0.00001	0.00000	0.00000	0.00000	0.00116	0.00000
7	0.00014	0.00000	0.00000	0.00000	-0.00518	0.00000
8	-0.00027	0.00000	0.00000	0.00000	0.00156	0.00000
9	-0.00005	0.00000	0.00000	0.00000	0.00329	0.00000
10	-0.00019	0.00000	0.00000	0.00000	-0.00033	0.00000
11	0.00006	0.00000	0.00000	0.00000	0.00304	0.00000
12	0.00045	0.00000	0.00000	0.00000	0.00258	0.00000
13	0.00007	0.00000	0.00000	0.00000	0.00149	0.00000
14	-0.00011	0.00000	0.00000	0.00000	-0.00747	0.00000
15	0.00016	0.00000	0.00000	0.00000	0.00225	0.00000
16	-0.00011	0.00000	0.00000	0.00000	-0.00153	0.00000

[a] DINBR represents an adjusted DINBR to reflect the structural change in the U.S. crawfish market.

Table 2. The *p*-values of Multivariate and Univariate Tests of Normality

	DIRDS	DIAIDS	DICBS	DINBR	DIGDS	DINBR [a]
EQUATION (SHAPIRO-WILK W)						
Domestic crawfish	0.733	0.026	0.736	0.050	0.733	0.3998
Imported crawfish	0.299	0.935	0.302	0.995	0.299	0.2774
Catfish	0.075	0.517	0.095	0.389	0.076	0.4909
Shrimp	0.050	0.498	0.092	0.414	0.050	0.9738
Oysters	0.964	0.792	0.955	0.649	0.964	0.2461
SYSTEM						
Mardia skewness	0.034	0.091	0.105	0.095	0.034	0.1804
Mardia kurtosis	0.819	0.883	0.857	0.716	0.819	0.5837
Henze-Zirkler T	0.082	0.468	0.465	0.504	0.082	0.8441

[a] DINBR represents an adjusted DINBR to reflect the structural change in the U.S. crawfish market.

covariance matrices from the two pre-specified sample periods (composed of the first T_1:1989–1997 and last T_2:1998–2005 observations, respectively) are equal. The test results are reported in Table 5. Apparent structural change has been found using Chow tests. Both mean and variance stability tests rejected the null hypothesis at the 1 percent or 5 percent significance levels except for the variance stability test in the DINBR model, which supported the hypothesis that structural change has occurred in the domestic crawfish tailmeat market after 1997. In fact, the market

Table 3. The *p*-values of Ramsey's Functional Form Tests

Reset Test	DIRDS	DIAIDS	DICBS	DINBR	DIGDS	DINBR[a]
RESET2						
Domestic crawfish	0.436	0.142	0.271	0.180	0.242	0.348
Imported crawfish	0.651	0.124	0.084	0.152	0.182	0.883
Catfish	0.887	0.562	0.439	0.202	0.126	0.175
Shrimp	0.896	0.712	0.989	0.659	0.495	0.807
Oysters	0.781	0.670	0.881	0.725	0.412	0.788
RESET3						
Domestic crawfish	0.492	0.218	0.183	0.181	0.157	0.236
Imported crawfish	0.572	0.002	0.050	0.198	0.241	0.897
Catfish	0.679	0.812	0.742	0.395	0.347	0.358
Shrimp	0.720	0.874	0.719	0.793	0.809	0.705
Oysters	0.545	0.610	0.529	0.728	0.457	0.639

[a] DINBR represents an adjusted DINBR to reflect the structural change in the U.S. crawfish market.

Table 4. The *p*-values of Static and Dynamic Homoskedasticity Tests

Homoskedasticity		DIRDS	DIAIDS	DICBS	DINBR	DIGDS	DINBR[a]
STATIC							
White (Pr > ChiSq)	System	0.382	0.378	0.382	0.378	0.382	0.378
Godfrey (Pr > LM)	Domestic crawfish	0.538	0.023	0.611	0.118	0.538	0.013
	Imported crawfish	0.268	0.376	0.337	0.564	0.268	0.057
	Catfish	0.010	0.551	0.009	0.456	0.010	0.274
	Shrimp	0.027	0.111	0.028	0.193	0.027	0.051
	Oysters	0.809	0.476	0.851	0.417	0.808	0.094
DYNAMIC							
Arch1 (Pr > Q)	Domestic crawfish	0.923	0.609	0.974	0.935	0.923	0.175
	Imported crawfish	0.356	0.139	0.448	0.154	0.356	0.970
	Catfish	0.381	0.800	0.390	0.661	0.381	0.974
	Shrimp	0.327	0.661	0.329	0.803	0.327	0.855
	Oysters	0.204	0.380	0.214	0.435	0.204	0.708
Arch1 (Pr > LM)	Domestic crawfish	0.966	0.796	0.921	0.562	0.966	0.278
	Imported crawfish	0.453	0.189	0.560	0.185	0.453	0.160
	Catfish	0.442	0.704	0.450	0.597	0.442	0.410
	Shrimp	0.381	0.585	0.383	0.702	0.381	0.936
	Oysters	0.295	0.528	0.305	0.590	0.295	0.248

[a] DINBR represents an adjusted DINBR to reflect the structural change in the U.S. crawfish market.

Table 5. The *p*-values of Parameter (conditional mean and variance) Stability Tests

	DIRDS	DIAIDS	DICBS	DINBR	DIGDS
CHOW TEST					
Mean (β)					
Domestic crawfish	0.001	0.022	0.001	0.050	0.002
Imported crawfish	0.202	0.825	0.065	0.823	0.162
Catfish	0.722	0.156	0.714	0.241	0.801
Shrimp	0.663	0.647	0.663	0.660	0.633
Oysters	0.757	0.951	0.786	0.940	0.571
Variance (σ^2)					
Domestic crawfish	0.000	0.014	0.000	0.248	0.000
Imported crawfish	0.870	0.970	0.280	0.954	0.752
Catfish	0.860	0.957	0.880	0.903	0.706
Shrimp	0.800	0.645	0.802	0.631	0.623
Oysters	0.540	0.998	0.258	0.998	0.637

share of domestic and imported crawfish tailmeat has markedly changed since 1997. Domestic fresh tailmeat is preferable from a taste standpoint because it is fresh and has the fat on, but imports of frozen tailmeat are competitive primarily because of their very low price. Furthermore, imports of frozen tailmeat are creating a new market for large national restaurant chains because frozen imported tailmeat is available year-round in massive quantities at a price that makes it attractive to national restaurant chains. These are increasing the market share of imported crawfish tailmeat while decreasing the market share of domestic crawfish tailmeat.

Independence can be examined using a *t*-type test to assess the significance of the lag-dependent variables in the five equations' auxiliary regression system. These equation-by-equation system tests differ from single-equation independence tests as each auxiliary equation includes lagged residuals from all system equations. The test results are reported in Table 6. The test results showed that domestic crawfish and shrimp equations in the DIAIDS model and oyster equations in the DIAIDS and DINBR models violated this independence assumption at the 5 percent level.

Results

To estimate scale and price flexibility coefficients,[2] this study used the DINBR model because this model showed the best statistical validity after confirming the adding-up condition and testing for the statistical assumption of normality, functional form, homoskedasticity, parameter stability, and independence. As seen in parameter stability tests, domestic crawfish tailmeat price showed a structural change in the period from before 1997 to after 1997. In order to maintain parameter stability in estimating the scale and price flexibility coefficients, the DINBR model is augmented with one dummy variable, D_{97}, representing the time period after 1997.[3] Hence, the actual DINBR model used in the estimation procedure is as follows:

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[2] Scale flexibilities in the inverse demand model are analogous to income elasticities in the direct demand model, while price flexibilities are analogous to price elasticities.

[3] This study initially used the intercept and slope dummy variables to determine the effects of structural change in the DINBR. The intercept dummy variable is shown to be significant at the 5 percent level, while the slope dummy variable is insignificant even at the 10 percent level. Given this, of the intercept and slope dummy variables, this analysis uses only the intercept dummy variable in DINBR.

Table 6. The *p*-values of t-Type Tests of System Equations for Independence

t-tests	DIRDS	DIAIDS	DICBS	DINBR	DIGDS	DINBR[a]
Domestic crawfish	0.542	0.029	0.614	0.124	0.306	0.903
Imported crawfish	0.723	0.361	0.783	0.331	0.797	0.641
Catfish	0.220	0.497	0.281	0.439	0.327	0.697
Shrimp	0.208	0.046	0.257	0.176	0.495	0.674
Oysters	0.363	0.045	0.328	0.037	0.452	0.585

[a] DINBR represents an adjusted DINBR to reflect the structural change in the U.S. crawfish market.

(8) $$\overline{w}_i(\Delta \ln p_i - \Delta \ln P) = c_i \Delta \ln Q + \sum_j h_{ij} \Delta \ln q_j + \gamma_i D_{97} + u_{it},$$

where additional adding up implies

$$\sum_i \gamma_i = 0 .$$

As this study showed in the previous section, the adding up of the DINBR model causes the contemporaneous covariance matrix of residuals to be singular. Therefore, one equation (in this case, the catfish equation) was excluded from the system for estimation purposes. The coefficients of the dropped equation were then calculated from the adding-up restriction. Then, the study added back the catfish equation and deleted the shrimp equation and re-estimated the system in order to determine the parameters and the standard errors of the catfish equation. The results are the same as calculating the parameters of the catfish equation from the adding-up condition. The restricted SUR model, by both symmetry and homogeneity, was used to estimate scale and price flexibility parameters. Table 7 gives the estimates of the c_{ij} and h_i together with their standard errors in parentheses. For ease of comprehension, the entries have been transformed into price and scale flexibilities following the formula outlined below in Table 7.

Before discussing the estimated parameters, this study will briefly review the test results of price endogeneity as stated in the previous section. The main motivation for estimating an inverse demand system is that quantities are naturally taken to be predetermined. However, fish consumption is presumed to respond to price incentives, and the actual quantity consumed in a year is likely to be influenced by random perturbations in that year's price. As a result, the assumption of quantity predeterminedness might be questioned. Accordingly, this study tested the predeterminedness of annual quantities with the Wu-Hausman test. The Wu-Hausman test, in this instance, involves a comparison between two estimators: the first being BLUE under the null hypothesis of predetermined quantities, but inconsistent under the alternate hypothesis of endogenous quantities for the restricted SUR model, and the second being consistent under both the null and alternate hypotheses for the two-stage least square estimator (2SLS). Implementation of the two-stage least squares estimator requires instrumental variables not already included in the right-hand sides of the demand equations and be at least equal in number to the number of variables in the equation (the five quantities on the right-hand sides of the equations). The χ^2 statistic of the test was 2.78 with 11 degrees of freedom, less than the 10 percent critical value in the chi-square distribution of 17.2. In sum, the test of the predeterminedness of quantities could not reject the null hypothesis, so the restricted SUR estimates reported in Table 7 are supported by this evidence. Also, the dummy variable for policy impacts of the antidumping tariff and Byrd amendment[4] since 1997 is estimated to be positive with a high degree of precision, implying that these

[4] Lee (2007) discussed policy impacts of the antidumping tariff and Byrd amendment on the Louisiana crawfish industry. Provisions of the Byrd amendment allowed antidumping tariff revenues to be distributed within the domestic crawfish industry.

Table 7. The Price and Scale Flexibilities Estimated in the DINBR Model

	Quantity Elasticity					Scale Elasticity
	Domestic Crawfish	Imported Crawfish	Catfish	Shrimp	Oysters	
Domestic crawfish	-0.809 (0.000)	-0.002 (0.000)	0.463 (0.113)	0.435 (0.164)	-0.087 (0.004)	-0.334 (0.057)
Imported crawfish	-0.002 (0.000)	-0.990 (0.000)	-0.161 (0.001)	1.411 (0.188)	-0.258 (0.054)	3.484 (0.617)
Catfish	0.016 (0.001)	-0.007 (0.001)	-0.231 (0.009)	0.270 (0.045)	-0.048 (0.003)	-1.761 (0.252)
Shrimp	0.002 (0.002)	0.009 (0.002)	0.038 (0.011)	-0.085 (0.015)	0.036 (0.004)	-0.949 (0.346)
Oysters	-0.013 (0.000)	-0.050 (0.001)	-0.210 (0.003)	1.145 (0.380)	-0.872 (0.191)	-0.228 (0.084)
Policy impact	0.0014*					

Notes: System weighted R^2: 0.97. Own price flexibility: $f_{ii} = c_{ii} / w_i - 1 + w_i$. Cross-price flexibility: $f_{ij} = c_{ij} / w_i + w_j$. Scale flexibility: $f_i = h_i / w_i$. ( ) is standard error. * indicates significance at 5 percent level.

policy instruments have a positive effect on domestic crawfish tailmeat prices.

Consider first the estimated scale flexibilities in Table 7, which have all been estimated and are negative in sign. The estimated scale flexibility of shrimp is not significantly different from -1, implying homothetic preferences, while the estimated scale flexibilities of the other four fish classifications are significantly different from -1, implying that the underlying scale curve differs significantly from linear logarithmic form.

The first five columns of Table 7 show the estimates of the compensated own/cross-price flexibilities. These estimates show how much the price of commodity *i* must change in order to induce the consumer to absorb marginally more of commodity *j* while maintaining the same utility level (Tomek and Robinson 1990).

As expected, the own price flexibilities of domestic crawfish, imported crawfish, catfish, shrimp, and oysters have all been estimated to be negative with a high degree of precision. This result implies an increase in quantity consumed, and results from a decrease in the own price. For example, the own price flexibility is estimated as -0.809 for domestic crawfish, implying that a 1 percent increase in quantity decreases the domestic price of crawfish by 0.809 percent. In terms of its definition of corresponding elasticity, we can interpret that a 1 percent decrease in domestic crawfish price leads to a 1.236 percent increase in domestic crawfish consumption. Domestic crawfish demand is shown to be elastic, which is consistent with the estimated price flexibility being less than one in absolute terms. The other own price flexibilities are estimated as -0.990 for imported crawfish, -0.231 for catfish, -0.085 for shrimp, and -0.872 for oysters. As seen, the imported crawfish price is most sensitive to a change in its quantity, while shrimp price is the least sensitive to a change in its quantity. However, all own price flexibilities are less than one in absolute terms, implying that the demand of all species used in this study is shown to be price elastic, which is similar to results found by Barten and Bettendorf (1989), Matsuda (2005), Brown, Lee, and Seale (1995), and Wong and McLaren (2005). These results are consistent with the fact that, typically, fish consumed at home is found to be price elastic (Park, Thurman, and Easley 2004).

The cross-price flexibility represents the cross effect between two different goods. The negative cross-price flexibility represents substitutability between two goods, while the positive cross-price flexibility implies complementarity between goods (Kim 1997). Among 10 cross-price flexibilities, 5 cross-price flexibilities are positive, while the other five cross-price flexibilities are negative. In particular, this study confirmed the substitutability of imported crawfish tailmeat for domestic crawfish tailmeat. However, the magnitude of the cross effect of imported crawfish tailmeat on domestic crawfish tailmeat estimated by the DINBR model was shown to be relatively small. This result might be related to the data used in the annual basis and functional form of the specific model.

For domestic crawfish tailmeat, oysters show substitutability, while catfish and shrimp show complementarity, which is not consistent with the notion that most types of fish are mutual substitutes. As Barten and Bettendorf (1989) suggest, cross-price flexibilities are not the appropriate interaction measures among the various types of fish because of the substitution effect and adding up in the Antonelli matrix, which cause cross-price flexibilities to be biased toward complementarity. In order to obtain a more adequate measure of interaction between commodities, the Allais coefficients, $A = [\alpha_{ij}]$, were calculated following the method of Barten and Bettendorf and are as follows:

$$(9)\quad a_{ij} = h_{ij} / w_i w_j - h_{rs} / w_r w_s + (c_i / w_i - c_r / w_r) + (c_j / w_j - c_s / w_s),$$

and

$$(10)\quad \alpha_{ij} = a_{ij} / \sqrt{\left(a_{ii} a_{jj}\right)^2}\ ,$$

where the subscript r and s refer to some standard pair of goods r and s to compare relative strength of complementarity or substitutability between the pair of i and j and the standard pair.

An α_{ij} greater than zero indicates that i and j are more complementary than r and s, while an α_{ij} less than zero signifies that i and j are stronger substitutes than r and s. Clearly, $\alpha_{ij} = 0$ means that i and j have the same type of interaction as r and s. This study has selected imported crawfish tailmeat and shrimp as the standard pair of goods r and s. This selection causes all other Allais interactions to become negative, implying that all the types of fish considered here are substitutes in consumption. By construction, the diagonal entries are -1, consistent with the notion that a good is its own perfect substitute. Also, by construction, the interaction intensity between imported crawfish and shrimp is zero, so all fish are shown to be substitutes for each other. The Allais coefficients of domestic crawfish are estimated as -0.0118609 for imported crawfish, -0.0000015 for catfish, -0.1372468 for shrimp, and -0.1799581 for oysters (see Table 8). The negative estimation of Allais coefficients implies stronger substitutes between two commodities i and j than the standard pair of imported crawfish, r, and shrimp, s. The results of this analysis show that the substitutability between domestic and imported crawfish tailmeat is stronger than the substitutability between domestic crawfish tailmeat and catfish, while the substitutability between domestic and imported crawfish tailmeat is weaker than the substitutability between domestic crawfish tailmeat and either shrimp or oysters.

Conclusion

Inverse demand systems have been used in applied demand analyses and have contributed to the understanding of consumer behavior. However, former studies did not give proper attention to the statistical validity of these inverse demand models. Since the data used in the underlying model often did not show statistical consistency with the assumptions of econometric regression, the estimated results could lead to an improper conclusion. In order to avoid this violation, this study conducted statistical tests for statistical validation of the inverse demand models using five inverse demand systems. In so doing, this study constructed five inverse demand systems using five fishery products comprised of domestic and imported crawfish, catfish, shrimp, and oysters. Of the five inverse demand models considered, the DINBR model showed no violation in the assumptions of adding up, normality, functional form, homoskedasticity, or independence.

Next, we used the DINBR model to estimate the parameters of scale and price flexibilities. Before doing this, we tested price endogeneity because we used annual data rather than short-term (monthly or quarterly) data due to a limitation in the availability of short-term data. This price endogeneity test showed that the assumption of pre-determinedness of quantity was not violated.

As expected, the own price flexibilities of five fish are shown to be negative and the magnitude of own price flexibilities are shown to be dependent upon the size of budget share used in this

Table 8. Allais Coefficients of Five Types of Fish

	Domestic	Imported	Catfish	Shrimp	Oysters
Domestic	-1	-0.0118609	-0.0000015	-0.1372468	-0.1799581
Imported		-1	-0.0968102	0.0000000	-0.2424426
Catfish			-1	-0.9894646	-0.6795946
Shrimp				-1	-0.6740317
Oysters					-1

study. That is, imported crawfish tailmeat, which is a relatively small percentage of budget-share, is shown to be most sensitive to a change in quantity, while shrimp, which is a relatively large percentage of budget share, was shown to be least sensitive to a change in quantity. Scale flexibilities, except for those of shrimp, were not shown to be homothetic.

One goal of this study was to identify the cross effects of imported crawfish tailmeat on the price of domestic crawfish tailmeat. Even though this study confirmed the substitutability of imported crawfish tailmeat for domestic crawfish tailmeat, the estimated cross-price flexibility was relatively small. This result might be related to the data used in the annual basis and functional form of the specific model. It is important to note the limitations of this work and potential for improvement in future research. Despite the fact that the methods employed in this analysis tested for the statistical validation of the models, the results are limited by a low number of observations. As mentioned previously, this is due primarily to limitations in data availability. Future work could seek improvements in the data set in order to verify the results of this analysis.

In order to represent cross substitution, this study calculated Allais coefficients suggested by Barten and Bettendorf (1989) because the direct use of cross-price flexibilities can lead to a violation of substitutability of fish. As this study mentioned previously, negativity of own price flexibilities and adding up in the system equation enforce the complementarity of cross-price flexibilities. In this study, five of the cross-price flexibilities calculated indicate complementarity, violating substitutability among fish products. By defining own price flexibilities as -1 and the interaction intensity between imported crawfish and shrimp as zero (the Barten and Bettendorf standard), the results of this study show that all five fish groups are substitutes for one another, as indicated by the negative cross-price flexibilities.

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