Competitive Equilibrium of an Industry with Labor Managed Firms and Price Risk

by
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Abstract

This paper studies the effect of output-price uncertainty in an industry comprised of labor-managed firms (LMFs) in which the number of LMFs and their membership are determined endogenously. The exit condition for a risk-averse LMF member is formulated and the effect of various economic variables on the equilibrium quantities and prices are examined. We find that the equilibrium in our setting is similar to the one that emerges in a 'capitalistic' economy where firms are owned by profit-maximizing agents. However, the effects of increases in risk and risk aversion differ from those found in a short-run analysis of a single LMF.

Keywords: Labor Managed Firms, Cooperatives, Price Risk, Risk Aversion, Long-Run.

Introduction

In the last two decades, traditionally socialist, centrally planned, Eastern European economies have implemented economic liberalization, tending towards a free-market economy. In these economies, however, cooperatives and labor-managed firms (LMFs) still occupy a significant position in the production sector, especially in agriculture. The development of the LMF sector in the former Soviet Union

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following the acts of 1987, which legalized private enterprises in the form of either individual or cooperative production, is examined by Plokker (1990). The estimated number of cooperatives in the former Soviet Union in 1989 was over 100,000.

It is well known that firms operating in such economies face an unstable environment, and are exposed to risks of all kinds. In the long run, LMFs can only survive in a competitive environment if members, within the cooperative, enjoy their respective reservation utility level, available to them elsewhere. The principal motivation for this paper is derived from an interest in studying the survival prospects of such LMFs in the presence of opportunities to their members in other sectors of the economy. To accomplish this, the paper develops a positive theory, describing the equilibrium of an industry that consists of LMFs which face risk. By terminating their membership, LMF members have the opportunity to be hired by capitalistic firms in other industries.

The LMF phenomenon is not unique to eastern Economies: these firms comprise a significant sector in western economies as well. For example, the Italian LMF sector, the largest in Western Europe, consists of 11,000 firms and half a million laborers (Estrin, 1985). Other examples include the UK and France, where there are about 1,400 and 1,000 firms, respectively. In Israel, the LMF sector is comprised of 270 “kibbutzim", populated by 125,000 members. The findings of this paper are directly relevant to the study of these LMF sectors in western economies. In particular, government market interventions by means of minimum wage, subsidies/taxes and stabilization schemes are common in most western economies. The framework developed in this paper can be applied to a study of the consequences of such interventions in the LMF sector.

Moreover, as observed by Sexton (1984), eastern LMFs “are closely analogous to agricultural marketing cooperatives. Cooperatively processing and marketing the raw labor input is conceptually very similar to processing and marketing a raw agricultural commodity such as milk or grain.” (p. 429). Specifically, he noted that the LMF theorists explore the same range of solutions as those developed by the agricultural cooperative theorists. Therefore, the theory developed in this paper is directly applicable to an analysis of agricultural marketing and processing cooperatives.

Literature Review

The initial motivation for the analysis of LMFs was derived mostly from an interest in the Yugoslavian experiment of self-managed firms which started in the early forties. To explain the behavior of those firms, a theory of a Labor-Managed

Firm (LMF) under certainty was developed by Ward (1958). The theory has been extended by many others, most notably Domar (1966).

Although this literature is diverse, there are three principles which are generally accepted. First, the objective of the members (owners-operators) of the LMF is the maximization of profit per laborer. In the case of marketing/processing cooperatives, this would mean maximization of profits per unit of raw product, supplied by members. The second principle is that capital and all other non-labor inputs are purchased from free markets at given prices. Third, instead of paying wages, the LMF divides its net revenue of non-labor costs equally among its members.

In later years, the literature evolved to account for risk and uncertainty, e.g., Muzondo (1979), Bonin (1980). The analysis was extended to include the long-run effects, when both labor and capital are optimally chosen, by Horowitz (1982), Kahana and Paroush (1984) and Choi and Feinerman (1991). Dynamic considerations and the possibility of learning about the stochastic demand process over time were introduced by Horowitz (1992). More recently, the literature has been expanded to account for other dimensions of economic behavior and for an empirical examination. Hwang, Lin and Mai (2001) introduced space into the LMF theory. Neary and Ulph (1997) investigated the possibility of coexistence of a LMF and a capitalistic firm in a duopoly. In a recent paper, Podivinsky and Geo (2003) employed a panel of UK data to perform an empirical analysis of the determinants of entry and exit decisions of LMFs.

However, with the exception of Haruna (1988), the literature on LMF decisions under price risk, thus far, has focused on the individual firm, and ignored the industry. Moreover, it presumes an interior solution, ignoring the possibility of quitting business, the very point which is the focus of the discussion on industry equilibrium. This paper attempts to fill this gap.

Modeling Cooperative Decisions under Price Risk

Consider a LMF consisting of many homogeneous members, each of which supplies the cooperative with one unit of labor. The LMF's total profit is given by Π,

$$\Pi(l,k,p,r,T) = pQ(l,k) - rk - T,$$

where T denotes fixed cost, l is the total labor input measured in laborers, k is a capital input, r is a capital-input price and p denotes the random output price. The random output price is given by $p \equiv \mu + \gamma\xi$, where $\gamma \geq 0$, $E(\xi) = 0$,

$V(\xi)=1$, and $\mu = h(S,\beta)$. The function h decreases with S - the total industry output, and increases with β – a demand shifter (*e.g.* an immigration wave or taste change). The production function, $Q(l,k)$, is assumed to be concave and non-decreasing in both l and k.

The cooperative members choose l and k, *ex-ante*, so as to maximize the expected value of the von Neumann – Morgenstern utility function, U, defined over profits per cooperative member:

$$\max_{l\geq 0, k\geq 0} \int U[\pi(l,k,p,r,T),\rho]dF(p),$$

where $\pi = \frac{\Pi}{l}$ is the profit per laborer[1]. The utility function $U(\pi,\rho)$ is assumed to be differentiable in the index of risk aversion, ρ, and twice differentiable, non-decreasing, and concave in π. The subjective cumulative distribution function of p, F, is assumed to have finite moments.

Employing the notion of a cost function, the cooperative's choice problem can be described in an equivalent form, involving a two-stage solution. In the first stage, the optimal mix of l and k is chosen so as to minimize the cost per cooperative member for a given level of output per member, $q \equiv Q/l$. The minimal cost per cooperative member, to produce a given amount of output per member, is denoted $C(q)$ and defined through the following minimization problem[2]:

$$C(q) \equiv \min_{l\geq 0, k\geq 0} \frac{rk+T}{l}$$
$$s.t. \frac{Q(l,k)}{l} = q.$$

Focusing on an industry equilibrium, we assume a long-run planning horizon, namely the LMF can optimally choose both inputs, k and l, in which case $T=0$. However, for the sake of completeness and comparison with previous studies, a short-run analysis with fixed capital and positive fixed cost will also be considered in the last section of the paper.

The necessary conditions for an interior solution of the above cost minimization problem are given by

1 The labor input l is treated as a continuous variable. This poses no problem for the meaning of l as the number of members as long as the labor unit is small relative to the total labor input. When there is only a small number of members, l cannot be interpreted as an undetermined number of laborers.

2 Obviously, C is a function of r and T, as well. For the sake of clarity these parameters are subsumed.

$$(i)\; r - \lambda Q_k = 0$$

$$(ii) - rk - \lambda (Q_l l - Q) = 0$$

where subscripts of a function stand for partial derivatives with respect to the subscript and λ is the Lagrange multiplier[3].

Further insight may be gained by combining these two conditions, yielding

$$Q_l l + Q_k k = Q.$$

This last condition is referred to as a local constant return to scale (CRS) condition (*e.g.* Estrin 1985). It states that at the optimum, the production function exhibits a CRS. As in the case of a capitalistic firm, if the production function of a LMF is linearly homogeneous (exhibits global CRS), then the profit-maximizing second-order conditions are not satisfied and equilibrium output is indeterminate. However, under CRS, the cost-minimization problem of a capitalistic firm yields, in general, a unique solution, whereas the solution for the analogous problem of a LMF is not unique (all input bundles that produce q cost the same).

The second stage of solving the cooperative problem is to utilize $C(q)$ for the maximization of expected utility, defined, as before, over profits per cooperative member. However, the choice variable in the second stage is the cooperative output per member, rather than the levels of the two inputs l and k. Formally, the second stage of the LMF's optimization problem is given by[4]

$$\max_{q \geq 0} \int U[pq - C(q)]dF(p).$$

The necessary condition for an interior solution of the above problem is given by

$$\int U_\pi \cdot [p - C_q(q)]dF(p) = 0.$$

The two-stage solution must be shown to be equivalent to the direct expected utility maximization with respect to the two inputs. As is formally shown in the appendix, this assertion is valid since manipulation of the first-order conditions for the latter problem yields the necessary conditions for the former one. On intuitive grounds, the two-stage solution is justified by the following reasoning: the choice of input mixture does not affect the risk faced by the cooperative member. The latter is affected only by each member's level of output. Since the cooperative

3 The existence and efficiency of a general equilibrium of an economy with LMFs, uncertainty and incomplete markets was investigated by Kihlstrom and Lafonnt (2002).

4 Haruna (1988) did not investigate the optimal inputs allocation and its relationships to the LMF's total output. Accordingly he started his formal analysis with an optimization problem identical to our second stage decision problem.

member's utility increases with π, the input mixture is chosen so as to minimize the cost per member for any given level of each member's output, independent of both price risk and risk attitudes. In the second stage, the level of output per member is chosen according to the member's risk preferences.

Properties of the cost and conditional demand functions

Under the assumption that the production function Q is concave, the cost function C is well defined and one can show several properties of C that will be useful below. These are formally stated in lemma 1.

Lemma 1: *Assuming that* Q *is concave,*

$$(i)\ C_q \geq 0,$$
$$(ii)\ C_{qq} \geq 0,$$
$$(iii)\ C_r \geq 0,$$
$$(iv)\ C_T \geq 0.$$

The usefulness of lemma 2 in the equilibrium analysis which follows lies in the fact that it describes the relationships between output per member to inputs utilization and total output of the LMF.

Lemma 2: *Let* $\hat{l}(p)$, $\hat{k}(p)$ *and* $\hat{Q}(p)$ *be the levels of input and output which maximize profit per member when* p *is deterministic; and let* $\widehat{l}(q)$, $\widehat{k}(q)$ *and* $\widehat{Q}(q)$ *be the levels of input and output which minimize cost per member for a given level of output per member under uncertainty. Then*

$$(i)\ sgn\left(\frac{\partial \widehat{l}}{\partial q}\right) = sgn\left(\frac{\partial \hat{l}}{\partial p}\right),$$
$$(ii)\ sgn\left(\frac{\partial \widehat{k}}{\partial q}\right) = sgn\left(\frac{\partial \hat{k}}{\partial p}\right),$$
$$(iii)\ sgn\left(\frac{\partial \widehat{Q}}{\partial q}\right) = sgn\left(\frac{\partial \hat{Q}}{\partial p}\right).$$

Lemma 2 states that the effects of a change in output per member on inputs and output are qualitatively identical to the effect of a change in output price on them. Its power stems from the fact that the behavior of input demand and output supply with respect to output price under certainty determines the relationships between output per member and output and input use under uncertainty. The outline of the proof is given in the appendix. In addition, we note that when l and k are substitutes ($F_{lk} < 0$) / complements ($F_{lk} > 0$), then (i) is negative/indeterminate and (ii) is positive/indeterminate.

Entry and Exit Decisions of a Risk-Averse LMF

About three decades ago, in his seminal paper, Sandmo (1971) showed that a risk-averse competitive firm facing output price risk requires a positive expected profit to remain in business. Following his pioneering study, extensive research into decisions regarding entry or exit of capitalistic firms in the long run evolved in the literature (e.g., Flacco, 1983). Flacco generalized Sandmo's results, showing that the competitive firm prefers operating (quitting business) when expected profits exceed (fall short of) the risk premium. Later, Finkelshtain and Chalfant (1991) generalized these results to a case in which the producer's preferences are state-dependent.

This section considers the entry and exit decisions of a risk-averse, expected-utility-maximizer LMF. To date, the entry or exit decisions of a LMF have only been analyzed by Haruna (1988). He showed that if the LMF is risk-averse, then output per worker is determined when the expected output price equals marginal cost plus marginal risk premium. Moreover, assuming that in industry with free entry LMFs would continue to enter as long the expected utility is positive, he found that a risk-averse LMF will produce less output per worker than that for which average cost is at a minimum.

Our analysis extends Haruna's results in a few directions. Firstly, we assume that a representative laborer will join the LMF as long as his expected utility exceeds the utility from his opportunity wage, which need not equal zero as, implicitly, assumed by Haruna. Secondly, this assumption is utilized to characterize an expected price level threshold in terms of the sum of cost per member, opportunity wage rate, and the member's risk premium, all averaged by the output level per laborer; under which the LMF is indifferent between remaining and quitting the industry. Finally, we examine the impact of an increase in price risk and degree of risk aversion on the expected price threshold and discuss its economic implications.

In the absence of uncertainty, laborers prefer being members in the cooperative as long as the dividend per cooperative member, π, exceeds the alternative wage, w. If this condition is not satisfied, the cooperative is dismantled and its members are hired by other firms. Proposition 1 establishes the analogous condition under price risk and demonstrates that the larger the risk or risk aversion, the larger the expected price required for the cooperative to survive in the industry.

Proposition 1: *(i) A positive level of output is produced by the LMF in the long run if and only if*

$$\mu \geq \frac{C(q)+\phi+w}{q},$$

where ϕ *is the individual member's risk premium and* w *is the ongoing wage rate available for hired laborers in other industries.*

(ii) Call μ^* *the level of expected price for which the cooperative members are just indifferent between staying in the cooperative or dismantling it. Then*

$$(i)\ \frac{\partial \mu^*}{\partial \rho} \geq 0,$$

$$(ii)\ \frac{\partial \mu^*}{\partial \gamma} \geq 0.$$

The first part of the proposition states that a necessary and sufficient condition for the cooperative to remain in (quit) the industry is that the expected price exceed (fall short of) the sum of the cost per member, opportunity wage, and the risk premium, all averaged by the output per member. The second part of the proposition states that the expected output-price threshold, for which the cooperative members are indifferent between staying in business and quitting it, is increasing in both the degree of risk aversion and the level of price risk. The proof is provided in the appendix.

Turning to the implications for industry equilibrium, it is again useful to begin with the absence of risk. Similar to a capitalistic industry, a competitive LMF industry is at a long-run equilibrium when the price equals average cost per member, where the latter includes the opportunity wage, w, of the members as hired employees elsewhere. Proposition 1 facilitates an analogous characterization of the cooperative industry's long-run equilibrium under risk. It implies that if laborers are risk-averse, long-run equilibrium is characterized by expected dividend above the normal, thus exceeding the opportunity wage.

The second part of the proposition implies that the evolution of LMF industries involves natural self-selection: the LMFs' members would tend to consist of the less risk-averse laborers, while the more risk-averse individuals would prefer being hired employees. In addition, an increase in the uncertainty of the economic environment would cause more laborers to prefer a risk-free wage as hired employees over membership in a LMF.

An additional insight requires the assumptions that all the LMFs possess identical technologies, and have the same beliefs and risk attitudes. Under these assumptions, the industry is in long-run equilibrium when the level of output per member of each individual LMF and the equilibrium price distribution both satisfy

$$\mu = \frac{C(q^*)}{q^*} + \frac{w}{q^*} + \frac{\phi(q^*)}{q^*}.$$

This condition replaces the traditional “price equals average cost per laborer” or zero economic dividend condition governing the long-run equilibrium of the industry under certainty. Thus, risk and risk aversion drive a wedge between expected price and average cost per member, equal to the member’s average risk premium. This latter condition (or its equivalent in expected utility terms), coupled with the first-order condition, $Q = Q_l l + Q_k k$, create the framework for the comparative static analysis of the industry to which we now turn.

Comparative Statics of LMF Industry under Risk

We shall begin this section with a general framework. In the sequel we will study the effects of various parameter changes on equilibrium variables. Propositions 2-5 are derived under the assumption of a long-run planning horizon, where the LMF enjoys full flexibility with regard to its choice of inputs. At the very end, proposition 6 is derived under the assumption of a short-run horizon, where labor is the only variable input, while capital is fixed.

For the sake of completeness, the effects of mean-preserving spread (MPS) in the output price distribution, which were analyzed by Haruna (1988), are rederived in Proposition 3. In addition to Haruna’s results, we present new findings concerning the effect of MPS on input use as well as the effects of other parameters.

A General Framework

Utilizing the notion of cost per member to model the LMF’s behavior, we closely follow the methodology introduced by Appelbaum and Katz (1986) for the analysis of a capitalistic industry. The two conditions that characterize the long-run equilibrium of the LMF industry are

$$a(q, \mu) = E[U_\pi \cdot (p - C_q(q))] = 0,$$

and

$$b(q, \mu) = E[U(\pi) - U(w)] = 0,$$

where, as before, $p = h(S, \lambda) + \gamma\varepsilon$. It is useful to illustrate the conditions $a = 0$ and $b = 0$ with diagrams in the μ and q space. Figures 1a and 1b describe the possible locus of pairs of μ and q that satisfy either condition $a = 0$ or $b = 0$.

Appelbaum and Katz presented analogous figures in the plane of μ and Q. We begin with the slope of each locus.

Totally differentiating the condition $a = 0$ and rearranging terms yields

$$\frac{\partial \mu}{\partial q}\Big|_{a=0} = -\frac{S.O.C.}{E[U_{\pi\pi} \cdot (p - C_q(q)) + EU_\pi]},$$

where $S.O.C.$ denotes the second-order (sufficient) condition for maximization of expected utility by a single LMF, which is negative by assumption. Assuming decreasing absolute risk aversion (DARA), Sandmo's technique can be used to show that the denominator is positive, implying that the locus $a = 0$ is positively sloped, as in Figure 1a. If the absolute risk aversion is increasing, the locus $a = 0$ may (but not necessarily) have a negative slope, as in Figure 1b.

Totally differentiating the condition $b = 0$ and rearranging terms gives

$$\frac{\partial \mu}{\partial q}\Big|_{b=0} = -\frac{a(q,\mu)}{qEU_\pi},$$

Thus, the locus of μ and q satisfying the condition $b = 0$ is a U-shaped curve with a minimum where the first-order (necessary) condition $a = 0$ is satisfied.

The equilibrium values of μ and q, denoted by μ^* and q^*, respectively, are achieved at the intersection of the curves $a = 0$ and $b = 0$. The effect of any parameter change on μ is determined only by the shift of the curve $b = 0$, an upward shift implying an increase in μ and a downward shift implying the opposite. The same effect on q, however, is determined by the relative shifts of both $a = 0$ and $b = 0$ along the line $q = q^*$. For example, inspection of Figure 1a shows that if $b = 0$ shifts upward more than $a = 0$, then q^* is increased.

Comparative Statics

We start with a demand perturbation, such that the mean price increases while preserving constant variance for every level of the industry supply, S.

Proposition 2: *A spread-preserving increase in demand raises industry output, and the number of LMFs in the industry. The equilibrium price, output per member, use of inputs and total output per firm remain unchanged. These results are valid, regardless of assumptions about the measures of risk aversion.*

Proof: A spread-preserving increase in demand is modeled by an increase in λ. Such a change only affects μ and leaves the locus that satisfies the condition $a = 0$, or the one that satisfies the condition $b = 0$ unchanged, *i.e.* there is movement on the curves rather than of the curves. Hence, the equilibrium levels of expected price and output per member do not change. Since l and k are functions of only q and r, which do not change, these variables are left unchanged as well, implying that Q, output per firm, also does not change. Therefore, the spread-preserving increase in demand can only be accommodated by a corresponding increase in the number of LMFs in the industry.

The above results concerning the industry equilibrium differ substantially from the results obtained by others, such as Kahana and Paroush ,(1984); and Choi and Feinerman (1991), who focused on a single LMF. Kahana and Paroush showed that in the short run, when labor is the only variable production factor, DARA implies that the LMF's membership and output levels both decrease with a spread-preserving increase in demand. Choi and Feinerman showed that in the long run, when both labor and capital are variable inputs, then, in general, the effects on the firm's output and its use of inputs are indeterminate.

Moreover, the effects of a spread-preserving increase in demand on a single capitalistic firm are quite different from its effects on a single LMF. Specifically, while the capitalistic firm responds to such a spread by increasing its output level (Sandmo 1971), the LMF decreases it. However, when the industry equilibrium is considered, the qualitative results for both types of firms coincide and a spread-preserving increase in demand does not affect output on input choices.

A second type of demand perturbation which is worth studying is a mean-preserving spread in the output-price distribution. This is because, for example, the recent trade liberalization in the Eastern European economies may increase the variance of price while keeping its mean constant. This issue is studied in Proposition 3, which is proved in the appendix.

Proposition 3: *A mean-preserving spread of the price distribution leads to:*

(i) an increase in the equilibrium expected price and a decrease in industry supply;

(ii) a reduction in output per member;

(iii) a decrease (increase) in both l and k, if they are positively (negatively) affected by an increase in output price under certainty;

(iv) a decrease (increase) in the LMF's output and an indeterminate change (decrease) in the number of LMFs, if output supply under certainty is positively (negatively) sloped.

Corollary: *If labor is an inferior (normal) input and capital is a normal (inferior) one, then an increase in risk decreases (increases) the LMF's output, increases (decreases) the number of laborers and decreases (increases) capital use.*

The proof of this corollary is immediate when noting that, as shown by Choi and Feinerman (1991), the LMF's supply curve under certainty is positively (negatively) sloped if labor is an inferior (normal) input and capital is a normal (inferior) one. Our qualitative results regarding the equilibrium values of output price and industry output are identical to those derived by Appelbaum and Katz (1986) for a capitalistic industry. Turning to the effect of mean-preserving spread on the equilibrium output level of a single firm, Ishii (1989) showed that, for a capitalistic industry, an increase in γ decreases Q and ambiguously affects the number of firms, even if DARA is assumed. If the LMF's supply curve under certainty is upward sloping, then the same result is obtained with regard to the LMF's output level (proposition 3). However, if the supply curve under certainty is downward sloping the result is reversed, namely, mean-preserving spread increases the LMF's output level. Moreover, in the latter case, proposition 3 also shows that in equilibrium the number of LMFs, n, decreases.

It can be inferred from the analysis for a single LMF in the short run that DARA implies a reduction in equilibrium output in response to mean-preserving spread. Extending the analysis of a single LMF to the long run, Choi and Feinerman (1991) found that the effect of mean-preserving spread on the LMF's output level is ambiguous. The current study further extends the analysis by considering the long-run industry equilibrium and obtains unambiguous results with regard to the effect of mean-preserving spread on Q, depending on the slope of the LMF's supply curve under certainty.

We turn now to an analysis of marginal variation in the risk attitudes of the industry participants. This question was never fully addressed in previous studies of LMFs under price risk. However, a partial study of the subject is provided by Choi and Feinerman (1991) who compared risk-neutral and risk-averse LMFs. Since in real life various degrees of risk aversion are observed rather than only risk neutrality or risk aversion, a marginal analysis is of interest and is conducted in Proposition 4, which is proved in the appendix.

Proposition 4: *An increase in risk aversion leads to:*

(i) an increase in the equilibrium expected price and as a result, a decrease in industry supply;

(ii) indeterminate changes in output per member, input levels, LMF's output and number of LMFs in the industry.

It is somewhat surprising that while the effect of an increase in price risk can be inferred once the slope of the supply curve under certainty is known, the effect of an increase in risk aversion is ambiguous. These results are consistent with those of Choi and Feinerman (1991) who compared the behavior of risk-neutral and risk-averse LMFs and found that the impact of risk aversion is, in general, ambiguous[5].

The long-run viability of a cooperative firm depends on the alternative wage available to the laborers in other firms. It is therefore important to examine the effect of this reservation wage on the equilibrium of the LMF industry. This issue is investigated in proposition 5.

Proposition 5: *An increase in the reservation wage, w, leads to:*

(i) an increase in the equilibrium expected price and a decrease in industry Supply, and assuming DARA,

(ii) a rise in output per member;

(iii) a decrease (increase) in both l and k, if they both decrease (increase) in output price under certainty;

(iv) an increase (decrease) in the LMF's output and a decrease (indeterminate change) in the number of LMFs, if the LMF's output supply is positively (negatively) sloped.

Proof: (i) Totally differentiating the condition $b = 0$ and rearranging gives

$$\frac{\partial \mu}{\partial w}\Big|_{b=0,q=q^*} = \frac{U_\pi(w)}{qEU_\pi(\pi)} \geq 0.$$

Thus, regardless of any specific assumption about the measures of risk aversion, an increase in the reservation wage must increase the mean price, resulting in a reduction of industry output. Since a change in the reservation wage does not affect the condition $a = 0$, the increase in expected price implies an increase in output per member if the condition $a = 0$ is upward sloping. The latter is ensured by DARA. The proof is completed by considering the effects of a change in output per member on l, k and Q established by lemma 2 above.

Corollary: *If labor is an inferior (normal) input and capital is a normal (inferior) one, then an increase in reservation wage increases (decreases) the LMF's output, decreases (increases) the number of laborers and increases (decreases) capital use.*

The proof is similar to that of the corollary following proposition 3, and is available upon request. The results regarding the equilibrium values of expected

5 It is worth mentioning, however, that Choi and Feinerman showed that if labor and capital are substitutes and labor is inferior, then a risk-averse LMF produces more than a risk-neutral LMF.

price and industry output are similar to those derived by Appelbaum and Katz (1986). If inferior labor and normal capital are assumed, our findings with regard to output per firm also coincide with their result, which does not depend on factor characteristics. However, assuming inferior capital and normal labor, the result derived here, with regard to firm output, is opposite to the one derived by Appelbaum and Katz.

The analysis conducted so far presumes that all inputs can be optimally chosen by the LMF, namely a long-run planning horizon is assumed. For the sake of completeness and comparison with previous studies, in proposition 6 we assume that capital is fixed and perform a comparative static analysis with respect to fixed cost, T. The proposition is proved in the appendix.

Proposition 6: *Regardless of any specific assumption about the measures of risk aversion, an increase in fixed cost leads to:*

(i) a rise in expected price and a reduction in total industry output;

(ii) a decrease in output per member and an increase in the number of laborers;

(iii) an increase of the LMF's output supply and a decrease in the equilibrium number of LMFs.

These results are the same as those derived by Appelbaum and Katz (except the one with regard to l which is not discussed by them). This similarity is not trivial, however, since in the current case the fixed cost per member decreases with number of employees. It is also worth noting that without the industry equilibrium condition, the capitalistic firm would reduce its output as the fixed costs rise (Sandmo, 1971) while the LMF would increase it (Kahana and Paroush 1984).

Summary and Concluding Remarks

A recent theme in the economic literature is the rigorous study of alternative organizational forms of firms and markets and their implication for industry performance. This paper extends the literature on LMFs by allowing their endogenous entry and exit and free adjustment of their membership.

In contrast to previous studies which analyzed the effect of output price uncertainty in a market with a single LMF, in our setting the reaction of LMFs to changes in price distribution is similar in direction to the effect in a 'capitalistic' economy, but not in its magnitude. The important distinction between an industry comprised of LMFs and a capitalistic industry is in the ability of the firm owners to diversify their portfolios. In a capitalistic economy, most of the shareholders regard idiosyncratic shocks as if they were risk-neutral.

Moreover, short of a catastrophic low-probability event, capitalistic firms usually possess enough resources to overcome temporary troughs. In contrast, moral hazard issues inhibit cooperative members from insuring against idiosyncratic shocks, where they often invest most of their wealth. Since, LMFs do not distinguish between wages and profits, they are more susceptible to bad states and therefore require a risk premium in addition to the market wage. These considerations entail that cooperatives react to risks as if they had a greater risk aversion then capitalistic firm owners.

By Proposition 4, this implies that an economy with LMFs will result in higher output prices and lower output than in a capitalistic economy. A way of overcoming this problem is for the government to offer insurance either to the cooperatives directly or to their members in case the cooperative dissolves.

There are several directions in which the current analysis can be profitably extended. First, the assumption of homogeneous LMFs can be relaxed and the analysis may proceed with either inter- or intra- firm differences, *e.g.* in technology, expectations, risk attitudes, or reservation wages due to heterogeneity in skills.

Second, since many LMFs face technological risk, it is important to extend the framework to accommodate this additional risk source. Third, a real challenge for future research is the formulation of an equilibrium model allowing coexistence of both capitalistic firms and LMFs. Such a study may facilitate the investigation of three important issues: i) conditions under which both types of firms can (or cannot) coexist; ii) the effects of the share of each organization form on industry performance; and iii) the effects of exogenous parameters, such as governmental policies, on the long-run industry share of each organization type.

Appendix

Proof of the Equivalence between the Direct and Two-Stage Maximizations

The expected utility maximization with respect to the two inputs and the first of the two-stage maximizations is expressed formally as:

$$(A) \qquad \max_{l,k} EU\left(\frac{pf(k,l)}{l} - \frac{rk}{l}\right) \qquad \text{and}$$

$$(B) \qquad \min_{l,k} \frac{rk}{l} \qquad \text{subject to:} \qquad \frac{f(k,l)}{l} \geq q,$$

respectively. The first-order-conditions for (B) are

$$k: \quad \frac{r}{l} = \lambda \frac{f_k}{l}$$

$$l: \quad -\frac{rk}{l^2} = \lambda \frac{f_l l - f}{l^2}$$

where λ is a lagrange multiplier. Multiplying the conditions by kl and $-l^2$, respectively, equating the two right-hand sides, and dividing by λ gives: $f_k k + f_l l = f$. The first-order conditions for (A) are (see Choi and Feinerman 1991)

$$k: \quad \frac{1}{l} E\{U' \cdot pf_k\} = \frac{1}{l} E\{U' \cdot r\}$$

$$l: \quad \frac{1}{l} E\{U' \cdot p(f_l - f/l)\} = -\frac{1}{l} E\{U' \cdot rk/l\}$$

Multiplying the conditions by kl and $-l^2$, respectively, equating the two left-hand sides, and dividing by $E\{U' \cdot p\}$ gives: $f_k k + f_l l = f$. Thus, if a choice of l and k solves the maximization in (A), it also solves the minimization in (B).

Corollary: Problem (A) is equivalent to $\max_q E(pq - c(q))$ where $c(q)$ is the minimum cost per worker function defined as (B).

Outline of the Proof of Lemma 2

The proofs of parts (i) and (ii) of lemma 2 are immediate when comparing the consequences of comparative static results from the following two systems.

1. Total differentiation of the first-order conditions resulting from profit maximization $pQ_k - r = 0$ and $pQ_l l - pQ + rk = 0$ with respect to l, k and p.

2. Total differentiation of the local CRS condition resulting from expected-utility maximization and the definition of output per member $Q_l + Q_k \frac{k}{l} - q = 0$ and $\frac{Q}{l} - q = 0$ with respect to l, k and q.

The proof of part (iii) of Lemma 2 follows by differentiating the production function with respect to p:

$$\frac{dQ}{dp} = Q_l \frac{\partial l}{\partial p} + Q_k \frac{\partial k}{\partial p},$$

with respect to q:

$$\frac{dQ}{dq} = Q_l \frac{\partial l}{\partial q} + Q_k \frac{\partial k}{\partial q},$$

and using parts (i) and (ii) of Lemma 2.

Proof of Proposition 1: (i) A representative laborer prefers to be a cooperative member as long as his expected utility exceeds the utility from the opportunity wage, *i.e.*

$$E[U(\pi)] = U(\bar{\pi} - \phi) \geq U(w),$$

where $\bar{\pi} \equiv E(\pi)$. Since U is monotonically increasing, the above condition can be rewritten as

$$\bar{\pi} - \phi \geq w \Longleftrightarrow \mu \geq \frac{C(q) + \phi + w}{q};$$

(ii) note that μ^* is defined by

$$\mu^* \equiv \frac{C(q^*) + \phi(q^*) + w}{q^*},$$

where q^* is the level of output when the right-hand side is at a minimum. Since ϕ is increasing in both γ and ρ (*e.g.* Diamond and Stiglitz 1974) for every given level of q, it follows that the right-hand side is increasing in these parameters and its minimum is increasing as well, thereby completing the proof.

Proof of Proposition 3: (i) Formally, mean-preserving spread is modeled by an increase in γ. Now, recall that the effect on the equilibrium expected price is solely determined by the shift of the locus $b=0$ (see Figures 1a and 1b). Following Sandmo (1971), it can be shown that for every risk-averse firm, $EU_{\pi}\varepsilon<0$. Hence, $\frac{\partial\mu}{\partial\gamma}|_{b=0,q=q^*}=-\frac{E[U_{\pi}\varepsilon]}{EU_{\pi}}\geq 0$ implying that, regardless of specific assumptions about the measures of risk aversion, a mean-preserving increase in the price variance necessarily increases the equilibrium mean price leading to a reduction in industry output.

(ii) Now we have to consider the relative shifts of the curves $a=0$ and $b=0$ along the line $q=q^*$. The difference between the shifts is given by

$$\frac{\partial\mu}{\partial\gamma}|_{b=0,q=q^*}-\frac{\partial\mu}{\partial\gamma}|_{a=0,q=q^*}=-\frac{E[U_{\pi}\varepsilon]}{EU_{\pi}}+\frac{qE[U_{\pi\pi}(p-C_q)\varepsilon]+E[U_{\pi}\varepsilon]}{qE[U_{\pi\pi}(p-C_q)]+EU_{\pi}}$$

$$=-q\frac{E[U_{\pi}\varepsilon]\cdot E[U_{\pi\pi}(p-C_q)-EU_{\pi}\cdot E[U_{\pi\pi}(p-C_q)\varepsilon]}{EU_{\pi}[qE[U_{\pi\pi}(p-C_q)]+EU_{\pi}]}$$

The numerator of the last expression is positive, independent of assumptions regarding the measures of risk aversion. If the LMF exhibits DARA, then the denominator is positive (Sandmo 1971), and the curve $b=0$ shifts upward less than the curve $a=0$. In this case, q decreases (see Figure 1a). If the measure of absolute risk aversion is non-decreasing, then the denominator might be negative implying that the curve $b=0$ shifts upward more than the curve $a=0$. However, inspection of Figure 1b shows that this case also results in a reduction in q. Therefore, an increase in the price risk reduces the output per member regardless of assumptions on the measures of risk aversion.

(iii) The proof of this part is an immediate consequence of lemma 2.

(iv) By lemma 2, a positively sloped supply curve under certainty implies that a decrease in output per member will decrease the LMF's output. Since both the industry output and the LMF's output decrease, the change in the number of firms is indeterminate. If, however, the supply curve is negatively sloped, the reduction in q leads to an increase in Q. The latter, coupled with the reduction in industry output, yield a decrease in the number of LMFs in the industry.

Proof of Proposition 4: (i) An increase in risk aversion is modeled by an increase in ρ, a parameter that increases the concavity of the utility function. Totally differentiating the condition $b=0$ with respect to μ and ρ, yields

$$\frac{\partial\mu}{\partial\rho}|_{b=0,q=q^*}=-\frac{EU_{\rho}(\pi)-U_{\rho}(w)}{qEU_{\pi}}.$$

Thus, the sign of the above derivative is opposite that of $EU_\rho(\pi) - U_\rho(w)$. But

$$E[U_\rho(\pi) - U_\rho(w)] = \int U_\rho[dF^1(\pi) - dF^2(\pi)],$$

where $F^1(\pi)$ is the probability distribution function of π induced by the distribution of p, and $F^2(\pi)$ is the degenerate distribution of w. Integrating the right-hand side of the last equation by parts twice yields

$$-\int U_{\rho\pi}[F^1(\pi) - F^2(\pi)]d\pi = -\int \frac{U_{\rho\pi}}{U_\pi} U_\pi[F^1(\pi) - F^2(\pi)]d\pi$$

$$= \int \frac{\partial^2 U_\pi}{\partial\pi\partial\rho} \int^{\pi} U_\pi(\theta)[F^1(\pi) - F^2(\pi)]d\theta d\pi$$

where θ is an integration variable. The first term, $\frac{\partial^2 logU_\pi}{\partial\pi\partial\rho}$, is negative by the definition of ρ. Since the distribution F^2 is less risky, the second term, $\int U_\pi(\theta)[F^1(\pi) - F^2(\pi)]d\theta d\pi$, is positive (Diamond and Stiglitz 1974). Thus

$$\frac{\partial\mu}{\partial\rho}|_{b=0,q=q^*} \geq 0.$$

(ii) The effect of an increase in risk aversion on q^* depends on the relative shifts of the curves $a = 0, b = 0$ along the line $q = q^*$. Unfortunately, the difference

$$\frac{\partial\mu}{\partial\rho}|_{b=0,q=q^*} - \frac{\partial\mu}{\partial\rho}|_{a=0,q=q^*}$$

cannot be signed, therefore the effect on q^* is indeterminate. Since l, k, Q and n are all determined by the level of q^* the effects of an increase in ρ on these variables are also indeterminate.

Proof of Proposition 6: (i) The effect of an increase in fixed cost on the expected price is found through the condition $b = 0$, evaluated at a predetermined level of k

$$\frac{\partial\mu}{\partial T}|_{b=0,q=q^*} = \frac{EU_\pi C_T}{qEU_\pi} = \frac{C_T}{q} \geq 0.$$

Thus, an increase in the fixed cost must increase the mean price and reduce industry output.

(ii) To further explore this case, we have to consider the relative shifts of the curves $a=0$ and $b=0$ along the line $q=q^*$ for a fixed k. The difference between the shifts is given by

$$\frac{\partial\mu}{\partial T}\Big|_{b=0,q=q^*} - \frac{\partial\mu}{\partial T}\Big|_{a=0,q=q^*} = \frac{C_T}{q} - \frac{E[U_{\pi\pi}(p-C_q)]C_T + EU_\pi C_{Tq}}{qE[U_{\pi\pi}(p-C_q)] + EU_\pi}$$
$$= \frac{EU_\pi C_T - qEU_\pi C_{Tq})}{q[qE[U_{\pi\pi}(p-C_q)] + EU_\pi]}.$$

The sign of the numerator is the same as $sgn(C_T - C_{Tq})$. In this case $C = \frac{T}{l(q)}$, giving

$$sgn(C_T - C_{Tq}) = sgn(\frac{1}{l} + \frac{q}{l^2}\frac{\partial l}{\partial q}).$$

Substituting $(\frac{\partial Q}{\partial q} - l)\frac{1}{q}$ for $\frac{\partial l}{\partial q}$ shows that

$$sgn(C_T - C_{Tq}) = sgn(\frac{\partial Q}{\partial q}),$$

which is clearly negative in this case. Hence, the numerator is negative, regardless of the assumption on the measures of risk aversion. If the denominator is positive (absolute risk aversion is decreasing), then the curve $b=0$ shifts upward less than the curve $a=0$ and, as implied by Figure 1a, q decreases. If the denominator is negative, the curve $b=0$ shifts upward more than $a=0$. However, as implied by Figure 1b, q also decreases. Thus, an increase in fixed costs reduces the output per member, regardless of assumptions on the measures of risk aversion. The increase in number of laborers, $\frac{\partial l}{\partial T} > 0$, follows since in this case $\frac{\partial l}{\partial q}$ is negative.

(iii) There are two ways of seeing why the LMF's output increases: first, it is implied by the increase in l; second, in this case, Q moves in the opposite direction to q (Kahana and Paroush, 1984). It follows immediately that the number of LMFs decreases.

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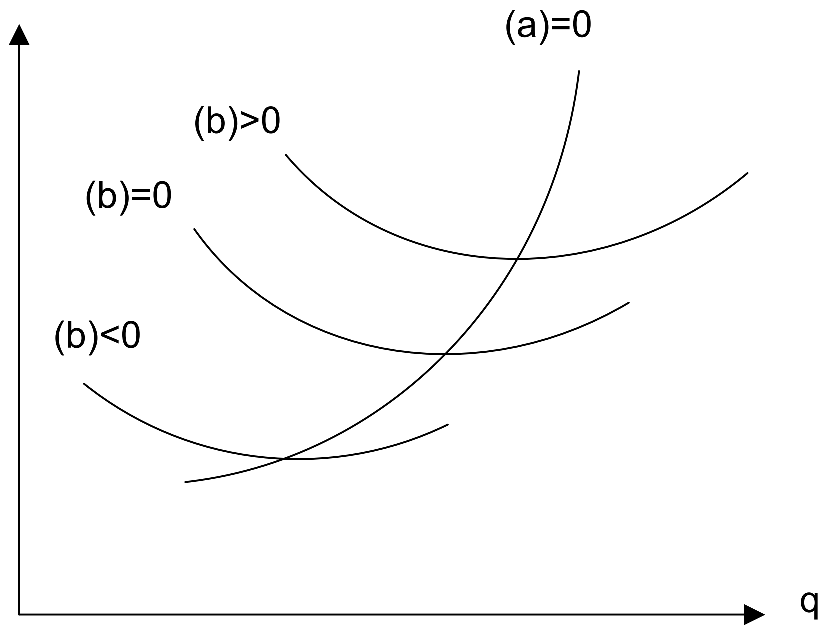

Figure 1a: Equilibrium with positively sloped supply

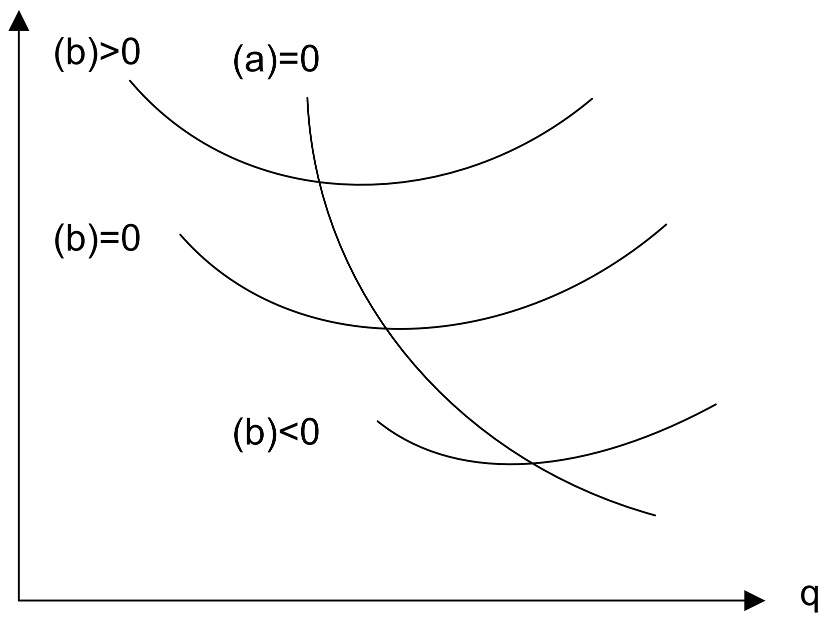

Figure 1b: Equilibrium with negatively sloped supply