Duality theorems summarizing some unambiguous qualitative comparative static results for processing cooperatives – The restricted profit function applied

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Duality theorems summarising some unambiguous qualitative comparative static results for processing cooperatives – The restricted profit function applied

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Abstract— **Cooperatives can broadly be divided into processing and marketing cooperatives on the one hand, and purchasing and supplying cooperatives on the other. In the early economic cooperative literature it is suggested that a processing cooperative may pursue either one of the following four objectives: *a)* Maximisation of the net average revenue product, *b)* Maximisation of the dividend, *c)* Maximisation of the processing cooperative's profit, and *d)* Maximisation of members' producer surplus plus the cooperative's profit. The corresponding objectives for purchasing and supplying cooperatives are: *e)* Minimisation of the purchasing price, *f)* Maximisation of the dividend, *g)* Maximisation of the purchasing cooperative's profit, and *h)* Maximisation of members' consumer surplus plus the cooperative's profit. This article analyses processing cooperative behaviour within a duality framework based on the restricted profit function. The five derived duality theorems concisely summarise the unambiguous qualitative comparative static results for a cooperative's ordinary or relative choice functions, depending on which objective the daily manager is assumed to pursue.**

Keywords— **Processing cooperative behaviour, Duality theory, Unambiguous comparative static results.**

I. INTRODUCTION

This paper illustrates how the behaviour of processing cooperatives operating in a perfectly competitive market environment without uncertainty can be analysed within a duality framework. A processing cooperative generally convert members' supplies of essential production factors, such as life animals, raw milk, vegetables, natural materials (in the case of agribusiness cooperatives), and manpower (in the case of labour-managed firms), into final products by means of other production factors such as energy, materials, and productive capital. In the early economic cooperative literature composed in the 1950's and 1960's and reviewed by Clare LeVay (1983) and Richard J. Sexton (1984), the following four different objectives have been suggested for processing cooperatives: *a)* Maximisation of the net average revenue product (*NARP*), *b)* Maximisation of the dividend, *c)* Profit maximisation, and *d)* Maximisation of members' producer surplus plus profit. The first objective involves two different adjustments according to whether the daily management of the processing cooperative is able to restrict member behaviour, or not. Maximisation of the net average revenue product under the assertion that the daily manager is unable to restrict member supplies corresponds with another common objective: «membership- or output maximisation subject to a no loss constraint».

The latter founding fathers[1] of today's economic cooperative theory such as Paavo Kaarlehto (1954, 1956), Oddvar Aresvik (1952, 1955), Peter Helmberger and Sidney Hoos (1962), and last, but not least Richard Carson (1977), adapted the «neoclassical» theory of the profit-maximising firm in order to investigate the adjustment of processing cooperatives. Richard J. Sexton, Brooks Wilson and Joyce Wann (1989) and Gerry Boyle (1998), independently of each other, extended the «neoclassical» theory of processing cooperatives further by developing a duality methodology based on Daniel McFadden's (1978) restricted profit function. Arvid Senhaji (2008a) proves that the econometric analyses in these two articles as well as the unambiguous qualitative comparative static results derived in Boyle (1998) are only correct when members' inverse aggregated supply of the raw agricultural product is completely vertical. The «neoclassical» analysis in this paper takes the standard processing cooperative model as its point of departure with an upward sloping inverse aggregated farm level supply schedule. The adjustment of a *NARP*-maximising processing cooperative operating in a perfectly competitive environment without uncertainty is described exhaustively by Senhaji (2008b). However, unambiguous qualitative comparative static results are still lacking for processing cooperatives operating in a similar short-run marketing setting, and

[1] The former founding fathers of today's economic cooperative theory include Edwin G. Nourse (1922), Ivan V. Emelianoff (1942), and Frank Robotka (1947).

at the same time pursuing any of the three objectives *b)–d)* introduced above. The contained analyses fill this gap. The subsequent duality theorems compactly summarise different sets of unambiguous qualitative comparative static results for a processing cooperative's ordinary or relative choice functions, depending on which objective the daily management is assumed to pursue. Corresponding theorems for purchasing cooperatives are forthcoming on the basis of the restricted cost function as illustrated in Senhaji (2008c).[2]

II. THE CONCEPTUAL FRAMEWORK AND THE MAIN ASSUMPTION

Processing cooperatives are typically established by farmers in order to restore and secure a regional processing facility, or in order to mitigate the economic damage due to oligopsony behaviour carried out by profit maximising investor-owned firms (IOFs) as outlined in Richard T. Rogers and Richard J. Sexton (1994). In this article it is presumed that the processing cooperative only collects and processes the raw agricultural produce supplied by its n^P members. In the short term membership is fixed, and all member farmers achieve a uniform raw agricultural price, maximise profits, and operate as price takers in all markets. Thus, the processing cooperative faces an aggregated raw agricultural supply schedule comprising the n^P members' profit-maximising supply functions. Whenever the cooperative's financial structure is patronage based, the aggregated raw agricultural supply schedule will be a function of the total payment that members of the cooperative society receive per unit of the raw agricultural product equal to *NARP*, together with *D* strictly positive farming production factor prices in the vector $\mathbf{w^d}$, and a policy support index denoted by T_a:

$$x_a^S = \sum_{i=1}^{n^P} \mathrm{S}_\mathrm{i}^\mathrm{a}(NARP, \mathbf{w^d}, T_a) = \mathrm{S}_\mathrm{a}^\mathrm{C}\left(NARP, \mathbf{w^d}, T_a\right). \qquad (1)$$

The n^P members collectively constitute a «multi-principal» and are presumed to be able to reach an agreement saying that the daily manager must run the processing business so as to maximise one of the three objectives whose comparative static results are still lacking in the economic literature on processing cooperatives. All net return is rebated to members according to patronage defined as a member's share of the aggregated supply of the raw agricultural product x_a^C. In the subsequent analyses retaining of funds is not considered.

The applied modelling setting resembles Senhaji (2008a, 2008b) where the production process of the cooperative is described by the production function

$$y = \mathrm{f}\left(x_1^b, \ldots, x_B^b, K, x_a^C\right) = \mathrm{f}\left(\mathbf{x^b}, K, x_a^C\right) = \mathrm{f}(\mathbf{x}, x_a^C). \qquad (2)$$

In expression (2) *y* denotes the quantity produced of the final output, *K* is the amount of productive capital that is fixed in the short run, and x_j^b is the j^th^ processing production factor in the processing input vector $\mathbf{x^b}$ with the corresponding price vector $\mathbf{w^b}$ equal to ($w_1^b \ldots\ w_B^b$) and containing *B* strictly positive processing input prices. The vector $\mathbf{x}$ includes all processing production factors given by the *B* processing factors together with productive capital *K*. The production function is assumed to be continuous from above, and to exhibit weak monotonicity in the processing production factor vector $\mathbf{x}$. The raw agricultural quantity x_a^C is a strictly essential production factor. The strictly positive final output price equals *P*, and the processing cooperative is a price taker in the output market as well as in all processing production factor markets. Let *F* denote fixed cost. The relationship between the gross revenue product (*GRP*) defined in expression (3) as total revenue minus variable processing cost and the *NARP* is as follows:

$$GRP = \left(Py - \left(\sum_{j=1}^{B} w_j^b x_j^b\right)\right), \text{ and} \qquad (3)$$

[2] This article is part of an ongoing project that also involves a «neoclassical» analysis of the behaviour of purchasing and supplying cooperatives. The theorems that concisely summarise the behaviour of purchasing cooperatives are presented in Senhaji (2008c) which constitutes my forth PhD-paper. Finally, my above-mentioned project also involves NEIO mixed – oligopoly analyses initiated just recently in cooperation with, among others, Ådne Cappelen at Bureau of Statistics, Norway and Constantine Iliopoulos at National Agricultural Research Foundation.

$$NARP=\left(\frac{Py-\left(\sum_{j=1}^{B} w_j^b x_j^b\right)-F}{x_a^{\mathrm{C}}}\right)=\left(\frac{GRP-F}{x_a^{\mathrm{C}}}\right). \quad (4)$$

With these definitions made, I am now ready to derive duality theorems summarising short-run unambiguous qualitative comparative static results describing the behaviour of a processing cooperative maximising either of the three objectives *b)* Maximisation of the dividend, *c)* Profit maximisation, or *d)* Maximisation of members' producer surplus plus profit.

III. DUALITY THEOREMS FOR A PROCESSING COOPERAATIVE IN SHORT-RUN EQUILBRIUM

All the results derived in this chapter for an agribusiness processing cooperative are also valid for labour-managed firms and other types of «production societies» where the members and owners of the business provide an essential production factor. Without loss of generality, I divide the daily manager's maximisation problem into three different stages. On stage one the daily manager treats the raw agricultural quantity x_a^C and final output y as two exogenous variables. Maximisation of any of the three objectives *b)–d)* introduced above, is then equivalent to minimising variable processing cost. Let the symbol «T» denote the transpose operator in the following. The short-run restricted variable processing cost function $\mathrm{V}^{\mathrm{R}}(\mathbf{w^b}, y, x_0^a, K)$ defined by the following minimisation problem

$$\mathrm{V}^{\mathrm{R}}\left(\mathbf{w}^{\mathrm{b}},y,x_0^a,K\right)=\min_{\mathbf{x}^{\mathrm{b}}}\left\{\left(\mathbf{w}^{\mathbf{b}\mathrm{T}}\mathbf{x}^{\mathbf{b}}\right)\mid \mathrm{f}\left(\mathbf{x},x_0^a\right)\geq y\right\}, \quad (5)$$

is the support function of the implicit processing input requirement set $L^*(y, x_0^a, K)$:

$$L^*\left(y,x_0^a,K\right)=\left\{\begin{matrix}\mathbf{x^b}:\left(\mathbf{w}^{\mathbf{b}\mathrm{T}}\mathbf{x}^{\mathbf{b}}\right)\geq \mathrm{V}^{\mathrm{R}}\left(\mathbf{w^b},y,x_0^a,K\right)\\ \text{for all } \mathbf{w^b}>\mathbf{0_B}, y\geq 0, x_0^a>0, K>0.\end{matrix}\right\} \quad (6)$$

On stage two the daily manager maximises the *GRP* with regards to final output y. The short-run restricted GRP function $\mathrm{GRP}^{\mathrm{R}}(P, \mathbf{w^b}, x_0^a, K)$, defined by the following maximisation problem

$$\mathrm{GRP}^{\mathrm{R}}\left(P,\mathbf{w^b},x_0^a,K\right)=\max_{y}\left\{Py-\mathrm{V}^{\mathrm{R}}\left(\mathbf{w^b},y,x_0^a,K\right)\right\}, \quad (7)$$

is the support function of the implicit production possibilities set $T^*(x_0^a, K)$ identical to:

$$T^*\left(x_0^a,K\right)=\left\{\begin{matrix}\left(y\ \mathbf{x^b}\right):\left(Py-\left(\mathbf{w}^{\mathbf{b}\mathrm{T}}\mathbf{x^b}\right)\right)\leq\\ \mathrm{GRP}^{\mathrm{R}}\left(P,\mathbf{w^b},x_0^a,K\right)\ \text{for all}\\ P>0,\mathbf{w^b}>\mathbf{0_B}, x_0^a>0,\ K>0.\end{matrix}\right\} \quad (8)$$

On stage three the daily manager determines the amount of the raw agricultural product to be delivered by the cooperative members. The restricted NARP function in expression (9) specifies the maximum price the daily manager can pay per unit of the raw agricultural product after processing cost V^R and fixed cost F are paid:

$$\mathrm{NARP}^{\mathrm{R}}\left(P,\mathbf{w^b},x_0^a,F\right)=\left(\frac{\mathrm{GRP}^{\mathrm{R}}\left(P,\mathbf{w^b},x_0^a,K\right)-F}{x_0^a}\right). \quad (9)$$

Senhaji (2008a: 9-11) derives the following six properties for the restricted NARP function:

1. $\mathrm{NARP}^{\mathrm{R}}(P, \mathbf{w^b}, x_0^a, F) \geq (-F / x_0^a)$;
2. If $P_2 \geq P_1$, then $\mathrm{NARP}^{\mathrm{R}}(P_2, \mathbf{w^b}, x_0^a, F) \geq \mathrm{NARP}^{\mathrm{R}}(P_1, \mathbf{w^b}, x_0^a, F)$;
3. If $\mathbf{w_2^b} \geq \mathbf{w_1^b}$,[3] then $\mathrm{NARP}^{\mathrm{R}}(P, \mathbf{w_2^b}, x_0^a, F) \leq \mathrm{NARP}^{\mathrm{R}}(P, \mathbf{w_1^b}, x_0^a, F)$;
4. $\mathrm{NARP}^{\mathrm{R}}(P, \mathbf{w^b}, x_0^a, F)$ is positively linearly homogeneous in the vector $(P\ \mathbf{w^b}\ F)$;
5. $\mathrm{NARP}^{\mathrm{R}}(P, \mathbf{w^b}, x_0^a, F)$ is convex and continuous in the vector $(P\ \mathbf{w^b}\ F)$; and finally
6. If the short-run $\mathrm{GRP}^{\mathrm{R}}(P, \mathbf{w^b}, x_0^a, K)$ is differentiable in the vector $(P\ \mathbf{w^b})$, the short-run $\mathrm{NARP}^{\mathrm{R}}(P, \mathbf{w^b}, x_0^a, F)$ is also differentiable

[3] Vector inequalities follow the subsequent convention throughout the paper: $\mathbf{w_2^b} > \mathbf{w_1^b}$ means that every element of $\mathbf{w_2^b}$ is strictly greater than the corresponding element of $\mathbf{w_1^b}$; $\mathbf{w_2^b} \geq \mathbf{w_1^b}$ means that every element of $\mathbf{w_2^b}$ is at least as large as the corresponding element of $\mathbf{w_1^b}$ and that at least one element of $\mathbf{w_2^b}$ is strictly greater than the corresponding element in $\mathbf{w_1^b}$.

in these (B+1) strictly positive prices since the latter function is a positive linear transformation of the former. The gradient of the short-run restricted NARP function in the price vector (P $\mathbf{w^b}$) is equal to (*The Viner-Wong envelope theorem*):

$$\nabla_{(P\ \mathbf{w^b})}\mathrm{NARP}^{\mathrm{R}}\left(P,\mathbf{w^b},x_0^a,F\right)= \left(\left(\frac{\mathrm{S}^{\mathrm{R}}\left(P,\mathbf{w^b},x_0^a,K\right)}{x_0^a}\right)\ \cdots\ -\left(\frac{\mathrm{D}_{\mathrm{B}}^{\mathrm{R}}\left(P,\mathbf{w^b},x_0^a,K\right)}{x_0^a}\right)\right)\Leftrightarrow \tag{10}$$

$$\left(\begin{array}{c}\dfrac{\nabla_{(P,\mathbf{w^b})}\mathrm{NARP}^{\mathrm{R}}(\cdot)}{\partial\,\mathrm{NARP}^{\mathrm{R}}(\cdot)} \\ -\dfrac{\partial\,\mathrm{NARP}^{\mathrm{R}}(\cdot)}{\partial F}\end{array}\right)=\nabla_{(P,\mathbf{w^b})}\mathrm{GRP}^{\mathrm{R}}\left(P,\mathbf{w^b},x_0^a,K\right) =\left(\mathrm{S}^{\mathrm{R}}\left(P,\mathbf{w^b},x_0^a,K\right)\ldots-\mathrm{D}_{\mathrm{B}}^{\mathrm{R}}\left(P,\mathbf{w^b},x_0^a,K\right)\right). \tag{11}$$

Let the members' aggregated marginal cost function be denoted by $\mathrm{MC_a}(\mathbf{w^d}, x_a^C, K_a, T_a)$. The dividend per raw agricultural unit D then equals:

$$D=\mathrm{NARP}^{\mathrm{R}}\left(P,\mathbf{w^b},x_a^C,F\right)-\mathrm{MC_a}\left(\mathbf{w^d},x_a^C,K_a,T_a\right). \tag{12}$$

The dividend function $\mathrm{D}(P, \mathbf{w^b}, \mathbf{w^d}, F, K_a, T_a)$ is defined by the maximisation problem[4]:

$$\mathrm{D}\left(P,\mathbf{w^b},\mathbf{w^d},F,K_a,T_a\right)= \max_{x_a^C}\left\{\mathrm{NARP}^{\mathrm{R}}\left(P,\mathbf{w^b},x_a^C,F\right)-\mathrm{MC_a}\left(\mathbf{w^d},x_a^C,K_a,T_a\right)\right\}. \tag{13}$$

Let us assume that an inner solution exists to the maximisation problem defined in expression (13). Since the dividend function $\mathrm{D}(P, \mathbf{w^b}, \mathbf{w^d}, F, K_a, T_a)$ is larger than or equal to ($-F/x_0^a$), it cannot be a support function of a corresponding implicit production possibilities set. This comes from the obvious fact that the production function in expression (2) is only defined for nonnegative output quantities. The Viner-Wong envelope theorem enables us to recapture the ordinary as well as the relative dividend-maximising choice functions through the gradient of $\mathrm{D}(P, \mathbf{w^b}, \mathbf{w^d}, F, K_a, T_a)$ in the price vector (P $\mathbf{w^b}$ F). Theorem 1 concisely summarises the short-run behaviour of a restricting processing cooperative maximising the dividend per raw agricultural unit denoted by D:

THEOREM 1: *The short-run unambiguous qualitative comparative static results of a restricting processing cooperative whose goal is to maximise the dividend, can be summarised in the statement that the matrix of cross-partial derivatives of the type* $\left(\dfrac{\partial^2\,\mathrm{D}\left(P,\mathbf{w^b},\mathbf{w^d},F,K_a,T\right)}{\partial Z_i\partial Z_j}\right)$*, where Z_i, Z_j are any of the prices in the extended price vector* (P $\mathbf{w^b}$ F)*, is symmetric and positive semidefinite.*

Whenever an inner solution exists to the maximisation problem defined in expression (13), the short-run dividend function $\mathrm{D}(P, \mathbf{w^b}, \mathbf{w^d}, F, K_a, T_a)$ exhibits the following five properties[5]:

1. If $P_2 \geq P_1$, then $\mathrm{D}(P_2, \mathbf{w^b}, \mathbf{w^d}, F, K_a, T_a) \geq \mathrm{D}(P_1, \mathbf{w^b}, \mathbf{w^d}, F, K_a, T_a)$;

2. If $\mathbf{w_2^b} \geq \mathbf{w_1^b}$, then $\mathrm{D}(P, \mathbf{w_2^b}, \mathbf{w^d}, F, K_a, T_a) \leq \mathrm{D}(P, \mathbf{w_1^b}, \mathbf{w^d}, F, K_a, T_a)$;

3. $\mathrm{D}(P, \mathbf{w^b}, \mathbf{w^d}, F, K_a, T_a)$ is positively linearly homogeneous in the vector (P $\mathbf{w^b}$ $\mathbf{w^d}$ F);

4. $\mathrm{D}(P, \mathbf{w^b}, \mathbf{w^d}, F, K_a, T_a)$ is convex and continuous in (P $\mathbf{w^b}$ F); and finally

5. If $\mathrm{NARP^R}(P, \mathbf{w^b}, x_0^a, F)$ is differentiable in the extended vector (P $\mathbf{w^b}$ F), then $\mathrm{D}(P, \mathbf{w^b}, \mathbf{w^d}, F, K_a, T_a)$ is also differentiable in the extended vector (P $\mathbf{w^b}$ F). The gradient of the short-run dividend function in the price vector (P $\mathbf{w^b}$) is equal to (*The Viner-Wong envelope theorem*):

4 The first- and second order conditions to the maximisation problem in expression (13) read:

$$\left(\frac{\partial\,\mathrm{NARP}^{\mathrm{R}}\left(P,\mathbf{w^b},x_a^C,F\right)}{\partial x_a^C}\right)=\left(\frac{\partial\mathbf{MC_a}\left(\mathbf{w^d},x_a^C,K_a,T_a\right)}{\partial x_a^C}\right), \text{ and}$$
$$\left(\frac{\partial^2\,\mathrm{NARP}^{\mathrm{R}}\left(P,\mathbf{w^b},x_a^C,F\right)}{\partial x_a^{C2}}\right)<\left(\frac{\partial\mathbf{MC_a}^2\left(\mathbf{w^d},x_a^C,K_a,T_a\right)}{\partial x_a^{C2}}\right).$$

5 The derivation of proofs for the five properties related to the dividend function defined in expression (13) is left as an exercise to the reader that may confer with Robert G. Chambers (1988: 124) or Eugene Silberberg and Wing Suen (2001: 124) for excellent guidance.

$$\nabla_{\left(P\ \mathbf{w^b} F\right)} \mathrm{D}\left(P,\mathbf{w^b},\mathbf{w^d},F,K_a,T_a\right) = \nabla_{\left(P\ \mathbf{w^b} F\right)} \mathrm{NARP^R}\left(\cdot\right) = \left(\frac{y^{D*}}{x_a^{D*}} \frac{-x_1^{D*}}{x_a^{D*}} \ldots \frac{-x_B^{D*}}{x_a^{D*}} \frac{-1}{x_a^{D*}}\right), \quad (14)$$

$$\left(\frac{\nabla_{(P,\mathbf{w^b})} \mathrm{D}\left(P,\mathbf{w^b},\mathbf{w^d},F,K_a,T_a\right)}{\left(-\frac{\partial \mathrm{D}\left(P,\mathbf{w^b},\mathbf{w^d},F,K_a,T_a\right)}{\partial F}\right)}\right) = \left(y^{D*},-x_1^{D*},\ldots,-x_B^{D*}\right). \quad (15)$$

A processing cooperative pursuing objective *c)* exerts monopsony power towards the cooperative members in order to maximise its own profit denoted by π_P^c:

$$\pi_P^C = \mathrm{GRP^R}\left(P,\mathbf{w^b},x_a^C,K\right) - \mathrm{MC_a}\left(\mathbf{w^d},x_a^C,K_a,T_a\right)x_a^C. \quad (16)$$

The short-run profit function $\pi_P^c(P,\ \mathbf{w^b},\ MFP_a,\ K)$ defined by the following maximisation problem[6]

$$\pi_{\mathrm{P}}^{\mathrm{C}}\left(P,\ \mathbf{w^b},\ MFP_a,K\right) = \max_{x_a^C}\left\{\begin{array}{l}\mathrm{GRP^R}\left(P,\mathbf{w^b},x_a^C,K\right) - \\ \mathrm{MC_a}\left(\mathbf{w^d},x_a^C,K_a,T_a\right)x_a^C,\end{array}\right\} \quad (17)$$

is the support function of the implicit production possibilities set $T_P^*(K)$:

$$T_P^*\left(K\right) = \left\{\begin{array}{l}\left(y\ \mathbf{x^b} x_a^C\right) : \left(Py - \left(\mathbf{w^{bT}x^b}\right) - MC_a x_a^C\right) \le \\ \pi_{\mathrm{P}}^{\mathrm{C}}\left(P,\ \mathbf{w^b},\ MFP_a,K\right) \text{for all } P>0, \\ \mathbf{w^b} > \mathbf{0_B}, \text{and } K>0.\end{array}\right\} \quad (18)$$

Hotelling-McFadden's lemma enables us to retrieve the restricting processing cooperative's ordinary profit maximising choice functions through the gradient of $\pi_P^c(P,\ \mathbf{w^b},\ MFP_a,\ K)$ in the price vector $(P\ \mathbf{w^b}\ MC_a)$:

$$\nabla_{\left(P\ \mathbf{w^b}\ MC_a\right)} \pi_{\mathrm{P}}^{\mathrm{C}}\left(P,\ \mathbf{w^b},\ MFP_a,K\right) = \left(y^{P*} \ldots - x_B^{P*} - x_a^{P*}\right). \quad (19)$$

Theorem 2 concisely summarises the behaviour of a restricting processing cooperative maximising profit denoted by π_P^c:

THEOREM 2: *The short-run unambiguous qualitative comparative static results of a restricting processing cooperative whose goal is to maximise its own profit, can be summarised in the statement that the matrix of cross-partial derivatives of the type* $\left(\frac{\partial^2 \pi_a^C\left(P,\mathbf{w^b},\mathrm{MFP_a}\left(MC_a\right),K\right)}{\partial Z_i \partial Z_j}\right)$*, where Z_i, Z_j are any of the prices in the price vector* $(P\ \mathbf{w^b}\ MC_a)$*, is symmetric and positive semidefinite.*

A processing cooperative pursuing objective *d)* maximises members' producer surplus plus the processing cooperative's profit denoted by π_{JP}^c. The daily manager is now presumed to pay members a cash price equal to the average raw agricultural cost denoted by AC_a. The objective function now reads:

$$\pi_{JP}^C = \mathrm{GRP^R}\left(P,\mathbf{w^b},x_a^C,K\right) - AC_a x_a^C - F. \quad (20)$$

The short-run joint profit function $\pi_{JP}^c(P,\ \mathbf{w^b},\ \mathbf{w^d},\ K,\ K_a,\ T_a)$ defined by the following maximisation problem[7]

$$\pi_{\mathrm{JP}}^{\mathrm{C}}\left(P,\mathbf{w^b},\mathbf{w^d},K,K_a,T_a\right) = \max_{x_a^C}\left\{\mathrm{GRP^R}\left(P,\mathbf{w^b},x_a^C,K\right) - \mathrm{AC}_a\left(\mathbf{w^d},x_a{}^C,K_a,T_a\right)x_a^C\right\}, \quad (21)$$

is the support function of the implicit production possibilities set $T_{JP}^*(K)$:

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6 The first- and second order conditions to the maximisation problem in expression (17) read:

$$\left(\frac{\partial \mathrm{GRP^R}\left(P,\mathbf{w^b},x_a^C,K\right)}{\partial x_a^C}\right) = \mathrm{NMRP}\left(P,\mathbf{w^b},x_a^C,K\right) = MC_a\left[1 + EL_{x_a^C} MC_a\right] \Leftrightarrow$$

$$\mathrm{NMRP}\left(P,\mathbf{w^b},x_a^C,K\right) = \mathrm{MFP_a}\left(MC_a\right) = MFP_a, \text{ and}$$

$$\left(\frac{\partial\,\mathrm{NMRP}\left(P,\mathbf{w^b},x_a^C,K\right)}{\partial x_a^C}\right) < \left(\frac{\partial\,\mathrm{MFP}\left(MC_a\right)}{\partial x_a^C}\right).$$

7 The first- and second order conditions to the maximisation problem in expression (17) read:

$$\left(\frac{\partial \mathrm{GRP^R}\left(P,\mathbf{w^b},x_a^C,K\right)}{\partial x_a^C}\right) = \mathrm{NMRP}\left(P,\mathbf{w^b},x_a^C,K\right) = \mathbf{MC_a}\left(\mathbf{w^d},x_a^C,K_a,T_a\right),$$

$$\text{and } \left(\frac{\partial\,\mathrm{NMRP}\left(P,\mathbf{w^b},x_a^C,K\right)}{\partial x_a^C}\right) < \left(\frac{\partial \mathbf{MC_a}\left(\mathbf{w^d},x_a^C,K_a,T_a\right)}{\partial x_a^C}\right).$$

$$T_{JP}^{*}\left(K\right)=\left\{\begin{array}{l}\left(y\ \mathbf{x^{b}}x_{a}^{C}\right):\left(Py-\left(\mathbf{w^{bT}x^{b}}\right)-AC_{a}x_{a}^{C}\right)\leq \\ \pi_{\mathrm{JP}}^{\mathrm{C}}\left(P,\mathbf{w^{b}},\mathbf{w^{d}},K,K_{a},T_{a}\right)\ \text{for all}\ P>0, \\ \mathbf{w^{b}}>\mathbf{0_{B}},\mathbf{w^{d}}>\mathbf{0_{D}},K>0,K_{a}>0.\end{array}\right\} \quad (22)$$

When the input prices in the two vectors $\mathbf{w^b}$ and $\mathbf{w^d}$ are distinct, Hotelling-McFadden's lemma enables us to retrieve the restricting processing cooperative's ordinary profit maximising supply schedule and processing input demand functions through the gradient of $\pi_{JP}^{C}(P, \mathbf{w^b}, \mathbf{w^d}, K, K_a, T_a)$ in the price vector $(P\ \mathbf{w^b})$:

$$\nabla_{\left(P\ \mathbf{w^{b}}\right)}\pi_{\mathrm{JP}}^{\mathrm{C}}\left(P,\mathbf{w^{b}},\mathbf{w^{d}},K,K_{a},T_{a}\right)=\left(y^{P*}-x_{1}^{P*}\ldots-x_{B}^{P*}\right). \quad (23)$$

However, if the cooperative farmers and the processing cooperative pay similar input prices for energy, fuels, or other processing production factors, the partial derivate of the joint profit function $\pi_{JP}^{c}(P, \mathbf{w^b}, \mathbf{w^d}, K, K_a, T_a)$ with respect to these common input prices will provide us with the members' aggregated demand plus the processing cooperative's demand of the corresponding production factors. Theorem 3 concisely summarises the behaviour of a restricting processing cooperative maximising members' producer surplus plus its own profit denoted by π_{JP}^{C}:

THEOREM 3: *The short-run unambiguous qualitative comparative static results of a restricting processing cooperative whose goal is to maximise members' producer surplus plus its own profit, can be summarised in the statement that the matrix of cross-partial derivatives of the type* $\left(\frac{\partial^{2}\pi_{\mathrm{JP}}^{\mathrm{C}}\left(P,\mathbf{w^{b}},\mathbf{w^{d}},K,K_{a},T_{a}\right)}{\partial Z_{i}\partial Z_{j}}\right)$, *where Z_i, Z_j are any of the prices in the price vector* $(P\ \mathbf{w^b})$, *is symmetric and positive semidefinite.*

After having derived theorems for three different short-run equilibrium adjustments whose unambiguous static results are still lacking in the economic cooperative literature, I now turn my focus towards the equilibrium behaviour of processing cooperatives in the long-run.

IV. DUALITY THEOREMS FOR PROCESSING COOPERAATIVES IN LONG-RUN EQUILBRIUM

In the long run even productive capital is an endogenous variable. Let the unit price of capital in processing be equal to w_k^b, while the unit price of productive capital in agriculture is given by w_k^a. Two different scenarios is analysed below depending upon whether the aggregated inverse raw agricultural supply schedule is upward sloping, or completely horizontal.

In the first scenario it is assumed that members' aggregated inverse raw agricultural supply schedule exhibits a general positive slope. Even in the long run it is legitimate to separate between restricting, closed-membership processing cooperatives in control over members' supplies of the raw agricultural product, and unrestricting, open-membership processing cooperatives with peak-coordinators unable to control membership, or members' aggregated supply of the farm level product. In the latter scenario, the forthcoming long-run unambiguous comparative static results for the long-run utilization of the farm level product x_0^a are derived on the basis of the following equilibrium condition:

$$\mathrm{ARP}^{\mathrm{R}}\left(P,\mathbf{w},x_{0}^{a}\right)=\mathrm{MC}_{\mathrm{a}}\left(\mathbf{w^{d}},w_{K}^{d},x_{0}^{a},T_{a}\right). \quad (24)$$

Stability requires that the inverse of the long-run aggregated raw agricultural supply function cuts the long-run ARP function from below, implying that:

$$\left(\frac{\partial MC_{a}}{\partial x_{a}^{C}}-\frac{\partial ARP^{R}}{\partial x_{a}^{C}}\right)=\beta>0. \quad (26)$$

The long-run equilibrium raw agricultural quantity x_0^a is defined implicitly by the equality in expression (24) as a function of the output price P, the processing production factor prices in the vector $\mathbf{w}$, the farming input prices in the vector $\mathbf{w^d}$, the price of productive capital w_K^d, and the agricultural policy variable T_a:

$$x_{0}^{a}=\mathrm{x}_{0}^{\mathrm{a}}\left(P,\mathbf{w},\mathbf{w^{d}},w_{K}^{d},T_{a}\right). \quad (27)$$

Based on the equilibrium condition in expression (24) and the stability condition in expression (26), four theorems related to the equilibrium raw agricultural schedule defined in expression (27) are forthcoming:

THEOREM 4: *The equilibrium long-run raw agricultural quantity x_0^a is upward sloping in the final output price P.*

Proof:
A positive change in the output price P will shift the long-run ARP function upwards, and the equilibrium long-run raw agricultural quantity x_0^a increases:

$$\left(\frac{\partial \mathrm{x}_0^{\mathrm{a}}\left(P,\mathbf{w},\mathbf{w^d},w_K^d,T_a\right)}{\partial P}\right)=\left(\frac{\left(\frac{\partial \mathrm{ARP^R}(\cdot)}{\partial P}\right)}{\beta}\right)=\left(\frac{\left(\frac{\mathrm{S^R}\left(P,\mathbf{w},\mathrm{x}_0^{\mathrm{a}}(\cdot)\right)}{\mathrm{x}_0^{\mathrm{a}}(\cdot)}\right)}{\beta}\right)\geq 0. \qquad (28)$$

THEOREM 5: *The equilibrium long-run raw agricultural quantity x_0^a is downward sloping in the processing production factor price w_j^b, j=1,…, B+1.*

Proof:
A positive change in a processing production factor price w_j^b will shift the long-run ARP function downwards, leading to a reduction in the equilibrium raw agricultural quantity x_0^a:

$$\left(\frac{\partial \mathrm{x}_0^{\mathrm{a}}\left(P,\mathbf{w},\mathbf{w^d},w_K^D,T_a\right)}{\partial w_j^b}\right)=\left(\frac{\left(\frac{\partial \mathrm{ARP^R}(\cdot)}{\partial w_j^b}\right)}{\beta}\right)=-\left(\frac{\left(\frac{\mathrm{D_j^R}\left(P,\mathbf{w},\mathrm{x}_0^{\mathrm{a}}(\cdot)\right)}{\mathrm{x}_0^{\mathrm{a}}(\cdot)}\right)}{\beta}\right)\leq 0,\ \mathrm{j}=1,\ldots,B. \qquad (29)$$

THEOREM 6: *If the production factor x_j^d, j=1,…, D, is a normal production factor*[8] *in agriculture, the equilibrium raw agricultural quantity x_0^a is downward sloping in the input price w_j^d, j=1,…, D as well as in w_K^d.*

Proof:
Let $\mathrm{C_a}(\mathbf{w^d}, w_K^d, x_a^C; T_a)$ denote the aggregated cost function of the cooperative farmers. If the production factor x_j^d, j = 1,…, D, is a normal input factor in agriculture, we have that:

$$\left(\frac{\partial \mathrm{S_a^{-1}}\left(\mathbf{w^d},x_a^C,T_a\right)}{\partial w_j^d}\right)=\left(\frac{\partial^2 \mathrm{C_a}\left(\mathbf{w^d},w_K^d,x_a^C,T_a\right)}{\partial x_a^C \partial w_j^d}\right)=\left(\frac{\partial \mathrm{MC_a}\left(\mathbf{w^d},w_K^d,x_a^C,T_a\right)}{\partial w_j^d}\right)>0,\ \mathrm{j}=1,\ldots,D. \qquad (30)$$

Thus, an increase in the price of a normal production factor x_j^d, j = 1,…, D, in agriculture, will shift the inverse aggregated long-run raw agricultural supply schedule upwards, leading to a reduction in the equilibrium long-run raw agricultural quantity x_0^a:

$$\left(\frac{\partial \mathrm{x}_0^{\mathrm{a}}(\cdot)}{\partial w_j^d}\right)=-\left(\frac{\left(\frac{\partial^2 \mathrm{C_a}\left(\mathbf{w^d},w_K^d,x_a^C,T_a\right)}{\partial w_j^d \partial x_a^C}\right)}{\beta}\right)=-\left(\frac{\left(\frac{\partial \mathrm{MC_a}\left(\mathbf{w^d},w_K^d,x_a^C,T_a\right)}{\partial w_j^d}\right)}{\beta}\right)<0,\ \mathrm{j}=1,\ldots,D. \qquad (31)$$

$$\mathrm{ARP^E}\left(P,\mathbf{w},\mathbf{w^d},w_K^d,T_a\right)=\mathrm{ARP^R}\left(P,\mathbf{w},\mathrm{x}_0^a(\cdot)\right). \qquad (32)$$

Unfortunately, no long-run unambiguous comparative static results for the long-run stable-equilibrium choice functions can be derived on the basis of the long-run stable-equilibrium ARP function defined in expression (32) for the processing cooperative's stable-equilibrium supply- and processing input demand functions.

In the case of closed-membership and restricting processing cooperatives with the ability to control members' supplies of the raw agricultural product, the number of cooperative farmers n^P is yet another endogenous variable. Normally, it is presumed that

[8] The farming production factor x_j^d is a normal input factor when the aggregated marginal cost function of the cooperative farmers denoted by $\mathrm{MC_a}(\mathbf{w^d}, w_a^d, x_a^C, T_a)$, is increasing in the input price w_j^d.

cooperative farmers are identical in the long term sharing the same production technology. Once again, and without loss of generality, the daily manager's maximisation problem is divided into three subsequent stages. On stage one long-run processing cost is minimised giving rise to the long-run restricted processing cost function $V^R(\mathbf{w}, y, x_0^a)$ where the price vector $\mathbf{w}$ is identical to $(w_1^b \ldots w_B^b\ w_k^b)$. The long-run restricted GRP function is defined directly by the maximisation problem solved on stage two:

$$\mathrm{GRP}^R\left(P,\mathbf{w},x_0^a\right)=\max_y\left\{Py-\mathrm{V}^R\left(\mathbf{w},y,x_0^a\right)\right\}. \tag{33}$$

On the third and final stage, the optimal raw agricultural quantity is determined. A processing cooperative pursuing objective *a)* maximises the long-run restricted average revenue product (*ARP*) defined as

$$\mathrm{ARP}^R\left(P,\mathbf{w},x_0^a\right)=\left(\frac{\mathrm{GRP}^R\left(P,\mathbf{w},x_0^a\right)}{x_0^a}\right), \tag{34}$$

with respect to the raw agricultural quantity x_0^a. The reader should notice that the long-run restricted ARP function defined in expression (34) exhibits the following five properties that are closely related to those pertaining to the short-run restricted NARP function defined in expression (9) above:

1. $\mathrm{ARP}^R(P, \mathbf{w}, x_0^a) \geq 0$;
2. If $P_2 \geq P_1$, then $\mathrm{ARP}^R(P_2, \mathbf{w}) \geq \mathrm{ARP}^R(P_1, \mathbf{w})$;
3. If $\mathbf{w_2^b} \geq \mathbf{w_1^b}$, then $\mathrm{ARP}^R(P, \mathbf{w_2^b}, x_0^a) \leq \mathrm{ARP}^R(P, \mathbf{w_1^b}, x_0^a)$;
7. $\mathrm{ARP}^R(P, \mathbf{w^b}, x_0^a)$ is positively linearly homogeneous in the vector $(P\ \mathbf{w})$;
8. $\mathrm{ARP}^R(P, \mathbf{w}, x_0^a)$ is convex and continuous in the vector $(P\ \mathbf{w})$; and finally
9. If the long-run $\mathrm{GRP}^R(P, \mathbf{w}, x_0^a)$ is differentiable in the vector $(P\ \mathbf{w})$, the long-run $\mathrm{ARP}^R(P, \mathbf{w}, x_0^a)$ is also differentiable in these $(B+1)$ strictly positive prices since the latter function is a positive linear transformation of the former. The gradient of the long-run restricted ARP function in the price vector $(P\ \mathbf{w^b})$ is equal to (*The Hotelling-McFadden's lemma*):

$$\nabla_{(P\ \mathbf{w})}\mathrm{ARP}^R\left(P,\mathbf{w^b},x_0^a\right)=\left(\left(\frac{\mathrm{S}^R\left(P,\mathbf{w},x_0^a\right)}{x_0^a}\right)\ \cdots\ -\left(\frac{\mathrm{D}^R_{B+1}\left(P,\mathbf{w},x_0^a\right)}{x_0^a}\right)\right). \tag{35}$$

Differentiating the long-run restricted ARP function defined in expression (34) with respect to the raw agricultural quantity x_0^a, and inserting the first-order conditions from the two previous optimising problems solved on stage one and stage two[9] respectively, yields:

$$EL_{x_a^C}\,\mathrm{ARP}\left(P,\mathbf{w},x_a^C\right)=\left(\frac{P\,y}{x_a^C\,ARP}\right)(\varepsilon-1)=\left[EL_P\,\mathrm{ARP}\left(P,\mathbf{w},x_a^C\right)\right](\varepsilon-1). \tag{36}$$

Expression (36) clearly states that the production process of a processing cooperative maximising *ARP* will exhibit constant returns to scale in long-run equilibrium. Furthermore, the restricted long-run ARP function defined in expression (34) is increasing (decreasing) in the raw agricultural quantity x_a^C whenever the production process exhibits increasing (decreasing) returns to scale. Since a similar argument can also be made for the short-run ARP function, it follows that the short-run production process of a restricting *NARP*-maximising marketing cooperative exhibits decreasing returns to scale in short-run equilibrium whenever the inverse aggregated farm level supply schedule cuts the NARP function to the right of the latter function's apex.

[9] Minimising long-run processing cost provides us with the following $(B+1)$ first-order conditions: $w_i^b=\lambda\left(\frac{\partial \mathrm{f}\left(\mathbf{x},x_a^C\right)}{\partial x_j^b}\right)$, $\mathrm{i}\in[1,\ldots,B]$, and $\mathrm{w}_K^b=\lambda\left(\frac{\partial \mathrm{f}\left(\mathbf{x},x_a^C\right)}{\partial K}\right)$. The vector $\mathbf{x}$ is equal to $(\mathbf{x^b}, K)$. The first-order condition on stage two where long-run *GRP* is maximised, reads: $P=\mathrm{MPC}\left(\mathbf{w},y,x_a^C\right)=\lambda$. The raw agricultural shadow price equals: $\mathrm{NMRP}\left(P,\mathbf{w},x_a^C\right)=\left(\frac{\partial \mathrm{GRP}^R\left(P,\mathbf{w},x_a^C\right)}{\partial x_a^C}\right)=P\left(\frac{\partial \mathrm{f}\left(\mathbf{x},x_a^C\right)}{\partial x_a^C}\right)$. Finally, the scale parameter in expression (36) is identical to $\varepsilon=\left[\sum_{j=1}^{B+1}\left(\frac{\partial \mathrm{f}\left(\mathbf{x},x_a^C\right)}{\partial x_j}\right)\left(\frac{x_j}{y}\right)+\left(\frac{\partial \mathrm{f}\left(\mathbf{x},x_a^C\right)}{\partial x_a^C}\right)\left(\frac{x_a^C}{y}\right)\right]$.

Let the long-run unrestricted ARP function be defined by the following maximisation problem

$$\mathrm{ARP}^{\mathrm{U}}\left(P,\mathbf{w}\right)=\max_{x_0^a}\left\{\mathrm{ARP}^{\mathrm{R}}\left(P,\mathbf{w},x_0^a\right)\right\}= \max_{x_0^a}\left\{\frac{\mathrm{GRP}^{\mathrm{R}}\left(P,\mathbf{w},x_0^a\right)}{x_0^a}\right\}. \tag{37}$$

The long-run equilibrium utilisation of the farm level product for a restricting processing cooperative maximising the *ARP* while facing a long-run upward sloping inverse farm level supply schedule is denoted by x_a^{U*}, and defined implicitly by the first-order condition to the maximisation problem in expression (37):

$$x_a^{U*}=\mathrm{x}_{\mathrm{a}}^{\mathrm{U}}\left(P,\mathbf{w}\right). \tag{38}$$

The unrestricted ARP function in expression (37) exhibits the following five properties:

1. $\mathrm{ARP}^{\mathrm{U}}(P, \mathbf{w}) \geq 0$;
2. If $P_2 \geq P_1$, then $\mathrm{ARP}^{\mathrm{U}}(P_2, \mathbf{w}) \geq \mathrm{ARP}^{\mathrm{U}}(P_1, \mathbf{w})$;
3. If $\mathbf{w_2} \geq \mathbf{w_1}$, then $\mathrm{ARP}^{\mathrm{U}}(P, \mathbf{w_2}) \leq \mathrm{ARP}^{\mathrm{U}}(P, \mathbf{w_1})$;
4. $\mathrm{ARP}^{\mathrm{U}}(P, \mathbf{w})$ is positively linearly homogeneous in the price vector (P $\mathbf{w}$);
5. $\mathrm{ARP}^{\mathrm{U}}(P, \mathbf{w})$ is convex and continuous in the vector (P $\mathbf{w}$); and finally
6. If $\mathrm{ARP}^{\mathrm{U}}(P, \mathbf{w})$ is differentiable in the vector (P $\mathbf{w}$), the gradient of this function is equal to (*Hotelling-McFadden's lemma*):

$$\nabla_{(P\ \mathbf{w})}\mathrm{ARP}^{\mathrm{U}}\left(P,\mathbf{w}\right)=\left(\left(\frac{\mathrm{S}^{\mathrm{U}}\left(P,\mathbf{w}\right)}{x_a^{U*}}\right)\ \cdots\ -\left(\frac{\mathrm{D}^{\mathrm{U}}_{\mathrm{B+1}}\left(P,\mathbf{w}\right)}{x_a^{U*}}\right)\right). \tag{39}$$

The long-run unrestricted ARP function is a support function of the relative implicit production possibilities set T_A^*:

$$T_A^*=\left\{\begin{array}{l}\left(\dfrac{y}{x_a^C}\ \ \dfrac{\mathbf{x}}{x_a^C}\right):\left(\dfrac{Py-\left(\mathbf{w}^{\mathbf{b}\mathrm{T}}\mathbf{x}^{\mathbf{b}}\right)-w_k^bK}{x_a^C}\right)\leq \\ \mathrm{ARP}^{\mathrm{U}}\left(P,\mathbf{w}^{\mathbf{b}},w_k^b\right)\text{for all } P>0,\ \mathbf{w}^{\mathbf{b}}>\mathbf{0}_{\mathbf{B}}, \\ \text{and } w_k^b>0.\end{array}\right\} \tag{40}$$

Theorem 7 concisely summarises the long-run behaviour of a restricting processing cooperative maximising the *ARP* while facing a long-run upward sloping inverse aggregated raw agricultural supply schedule.

THEOREM 7: *The long-run unambiguous qualitative comparative static results of a restricting processing cooperative whose goal is to maximise ARP, can be summarised in the statement that the matrix of cross-partial derivatives of the type* $\left(\dfrac{\partial^2\,\mathrm{ARP}^{\mathrm{U}}\left(P,\mathbf{w}\right)}{\partial Z_i \partial Z_j}\right)$*, where Z_i, Z_j are any of the prices in the price vector* (P $\mathbf{w}$), *is symmetric and positive semidefinite.*

The analyses of the long-run adjustments of a restricting processing cooperatives pursuing either one of the remaining objectives *b)–d)* while facing an upward-sloping inverse aggregated farm level supply schedule, resembles the corresponding short-run analyses undertaken in the third chapter. These derivations are therefore not repeated here. It should be noticed though, that the long-run dividend function $\mathrm{D}(P, \mathbf{w^b}, \mathbf{w^d}, w_k^b, w_k^a, T_a)$ which is now a nonnegative function, is a support function of a corresponding relative implicit production possibilities set. Theorem 8 concisely summarises the long-run behaviour of a processing cooperative pursuing either one of the objectives *b)–d)*.

THEOREM 8: *The long-run unambiguous qualitative comparative static results of a restricting processing cooperative whose goal is to maximise either the dividend, the profit, or the joint profit while facing a long-run upward sloping inverse raw agricultural supply schedule, can be summarised in the statement that the matrixes of cross-partial derivatives of the type* $\left(\frac{\partial^2 \mathrm{D}\left(P,\mathbf{w},\mathbf{w}^{\mathbf{d}},w_k^a,T_a\right)}{\partial Z_i \partial Z_j}\right)$, $\left(\frac{\partial^2 \pi_p^C\left(P,\mathbf{w},MFP_a\right)}{\partial Z_k \partial Z_l}\right)$ *and* $\left(\frac{\partial^2 \pi_{\mathrm{JP}}^{\mathrm{C}}\left(P,\mathbf{w},\mathbf{w}^{\mathbf{d}},w_k^d,T_a\right)}{\partial Z_m \partial Z_n}\right)$, *where* Z_i, Z_j *are any of the prices in the price vector* (P $\mathbf{w}$), Z_k, Z_l *are any of the prices in the price vector* (P $\mathbf{w}$ MC_a), *and* Z_m, Z_n *are any of the prices in the price vector* (P $\mathbf{w}$), *are both symmetric and positive semidefinite.*

In scenario two where the aggregated inverse raw agricultural supply schedule is completely horizontal and identical to the long-run agricultural unit cost $c_a(\mathbf{w^d}, w_k^a, T_a)$, all four objectives *a)-d)* analysed previously coincide and lead to the same equilibrium adjustment. The first-order equilibrium condition to the current maximisation problem reads:

$$\mathrm{ARP}^{\mathrm{R}}\left(P,\mathbf{w},x_a^C\right)=\mathrm{NMRP}=\mathrm{c_a}\left(\mathbf{w}^{\mathbf{d}},w_k{}^a,\ T_a\right). \tag{41}$$

$$x_a^{U*}=\mathrm{x}_\mathrm{a}^\mathrm{U}\left(P,\mathbf{w}\right). \tag{42}$$

Theorem 4 above concisely summarises the long-run behaviour of an *ARP*-maximising processing cooperative confronting a completely vertical inverse long-run aggregated raw agricultural supply schedule. More important, theorem 9 summarises the long-run unambiguous comparative static results of a mixed processing food industry containing processing cooperatives maximising *ARP* and investor-owned firms (*IOFs*) maximising profits, in a perfectly competitive market setting without uncertainty.

THEOREM 8 (Eugene Silberberg, 1974): *The long-run unambiguous qualitative comparative statics of processing cooperatives maximising ARP and IOFs maximising profits due, for example to zero entry and adjustment cost conditions in the processing industry, can be summarised in the statement that the matrix of cross-partial derivatives of the type* $\left(\partial\left(\frac{x_i^{b*}}{y^*}\right)/\,\partial w_j^b\right)$ *where i. j = 1,..., B+1, is symmetric and negative semidefinite.*

It is of outmost importance that the reader notices that the output price P is now assumed to be an endogenous variable in line with Silberberg (1974), due to the following underlying long-run equilibrium conditions:

$$P=\mathrm{AC}\left(\mathbf{w},c_a\right)=\left[\frac{\mathrm{V}^{\mathrm{R}}\left(\mathbf{w},y,x_a^{U*}\right)+\mathrm{c_a}\left(\mathbf{w}^{\mathbf{d}},w_K^d,T_a\right)x_a^{U*}}{y^*}\right], \tag{43}$$

$$\nabla_{\mathbf{w},c_a}P=\left[\left(\frac{x_1^{b*}}{y^*}\right)\ldots\left(\frac{x_{B+1}^{b\ \ *}}{y^*}\right)\left(\frac{x_a^{U*}}{y^*}\right)\right], \tag{44}$$

$$y^*=\mathrm{f}\left(\mathbf{x}\left(\mathbf{w},c_a\right),\mathrm{x}_\mathrm{a}^\mathrm{U}\left(\mathbf{w},c_a\right)\right). \tag{45}$$

From the first-order condition in expression (41) it is obvious that the long-run equilibrium behaviour of identical *ARP*-maximising processing cooperatives confronting identical and completely vertical inverse aggregated farm level supply schedules, can be modelled «as if» they where average cost minimisers, just like the processing industry's identical IOFs. This terribly important result has so far been overlooked in the economic literature on cooperatives and mixed industries with groups of firms pursuing different objectives.

In order to determine the long-run equilibrium number of farmers for a processing cooperative pursuing either one of the four objectives analysed in this chapter, we first derive the representative farmer's optimal supply of the raw agricultural product denoted by x_a^*:

$$\begin{aligned}&x_a^*\left(\mathrm{ARP}^{\mathrm{R}}\left(P,\mathbf{w},x_S^{C*}\right),\mathbf{w}^{\mathbf{d}},w_k^a,T_a\right)=\\&\underset{x^a}{\operatorname{argmax}}\left\{\mathrm{ARP}^{\mathrm{R}}\left(P,\mathbf{w},x_S^{C*}\right)x^a-\mathrm{v^a}\left(\mathbf{w}^{\mathbf{d}},w_k^a,T_a\right)\right\}.\end{aligned} \tag{46}$$

The index «s» in expression (46) runs over the four different objectives represented by their symbols in the set [ARP, D, $\pi_P{}^C$, $\pi_{JP}{}^C$]. Let the optimal input of the raw agricultural product in the four different equilibriums be denoted by $x_s{}^{C*}$, where the index «s» again runs over the four different objectives in the set [ARP, D, $\pi_P{}^C$, $\pi_{JP}{}^C$]. The equilibrium number of farmers for a processing cooperative pursuing either one of the four objectives introduced above is determined by the following expression:

$$n_s^{P*} = \left(\frac{x_s^{C*}}{x_a^*\left(\mathrm{ARP}\left(P, \mathbf{w}, x_s^{C*}\right), \mathbf{w^d}, w_k^a, T_a\right)} \right), \tag{47}$$

$\mathrm{s} \in \left[ARP,\ D,\ \pi_P{}^C,\ \pi_{JP}{}^C \right].$

V. CONCLUSION

The article analyses the short-run and long-run behaviour of processing cooperatives within a duality framework. Unambiguous qualitative comparative static results are derived for a processing cooperative pursuing any of the four objectives: *a)* Maximisation of the *NARP*, *b)* Maximisation of the dividend, *c)* Profit maximisation, and *d)* Maximisation of members' producer surplus plus profit. The first objective involves two different adjustments according to whether the daily management of the cooperative business is able to restrict member behaviour, or not. Maximisation of the net average revenue product under the assertion that the daily manager is unable to restrict member supplies corresponds with another common objective: «membership- or output maximisation subject to a no loss constraint». The short-run analysis in the third chapter only focuses on objectives *b)–d)*, since the short-run unambiguous qualitative comparative statics results for a *NARP*-maximising processing cooperative has already been derived in Senhaji (2008b). However, all the four different objectives are addressed in the fourth chapter focusing on the long-run adjustment where membership is an endogenous variable together with productive capital. The optimal number of farmers is determined in the four different equilibriums under the assumption that all farmers share the same technology and are identical. Finally, a terribly important theorem is stated that sheds light on the comparative statics of processing cooperatives confronting a long-run completely vertical farm level schedule $c_a(\mathbf{w^d}, w_k{}^a, T_a)$.

The duality theorems presented in this article illustrates how the restricted profit function should be applied in order to derive unambiguous qualitative comparative static results for processing cooperatives facing an aggregated inverse raw agricultural schedule with a general upward slope. Corresponding duality theorems that concisely summarise the behaviour of purchasing and supplying cooperatives can be derived on the basis of the cost function.

Future research should shed light on what happens to the derived unambiguous comparative static results when we introduce into the model setting principal-agent conflicts that are embedded in the relationship between the collective multi-principal and the peak coordinator being responsible for the daily management of the cooperative business. In order to be even more useful to empirical analysis, the contained cooperative «neoclassical» theory must also be extended to cover firms operating with multi-output production technologies in an imperfect competition environment where different coalitions of cooperative farmers supply heterogeneous farm level products. A natural point of departure is McFadden's multi-output restricted profit function in the case of processing cooperatives, and multi-output restricted cost function in the case of purchasing and supplying cooperatives. Finally, uncertainty must also be introduced into the contained «neoclassical» cooperative theory. Robert G. Chambers and John Quiggen (2000) constitute an obvious point of departure in this regard, but their state-contingent approach should be brought one step further to a continuous approach.

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