A METHOD FOR INCLUDING IN PMP MODELS ACTIVITIES NON-EXISTENT IN THE BASELINE SITUATION

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Abstract— **When working with positive mathematical programming (PMP) models it is generally admitted that it is not possible to consider in the modelled unit activities that are not present in the baseline situation of the unit. This constitutes a considerable drawback for traditional PMP techniques which cannot be applied in specific cases, in particular to the study of the impact of new agri-environmental programs that subsidise crops grown with technologies different to those applied in the baseline situation.**

This paper presents a method for dealing with these cases, which can be easily implemented as an extension of the traditional calibration techniques of PMP. The method is applied to a specific problem, using modified calibration expressions derived from the necessary Khun-Tucker conditions, assuming increasing marginal costs. The analysis of the results and their comparison with those obtained using a linear programming model permits a first evaluation of this methodological proposal.

Keywords— **Positive mathematical programming extensions, Agri-environmental measures.**

I. INTRODUCTION

Positive mathematical programming (PMP), proposed by Howitt [1], has in recent years undergone new extensions and a great number of applications in analysing the impact of new measures of the Common Agricultural Policy in Europe. Reviews of these extensions and applications can be found in Heckelei and Britz [2] and Henry de Frahan et al. [3].

When working with agri-environmental measures modellers are often obliged to consider several variants for different crops. When all the variants are present in the baseline situation- or base year- there are two possible procedures for dealing with these variants: one, proposed by Röhn and Dabbert [4] as an extension of PMP which establishes a link between the variants of each crop and the other, used traditionally with PMP, which considers each variant as an independent activity (see for instance Buisse et al., [5]). Neither of these two procedures can be applied when some variants do not exist in the base year. This is a specific case where models need to take new activities into account, thus making the application of PMP difficult.

The objective of this paper is to present a procedure that allows the inclusion in PMP models variants of crops that do not exist in the base year, using binary variables and assuming we know which existing variants for each crop are to be replaced, when less profitable, by the new ones. Although the procedure is very general it is developed in the context of solving the problem formulated below.

II. THE PROBLEM

Assuming that two hypothetical farms grow irrigated crops, the problem to solve is two-fold: on the one hand, to determine the minimum agri-environmental premium per hectare which ought to be paid to each farmer for their changeover to non irrigated crops, and on other hand to obtain the new crop distribution should this premium be applied. The crop distribution on these farms in the base year and the characteristics of the possible irrigated and non irrigated crops that can be grown are shown in Table 1.

Both variants of crops, irrigated and non irrigated, need to be taken into account to address this problem. The PMP model presented in the next section considers these variants even though the non irrigated crops are not grown in the base year.

Table 1 Characteristics and base year area of crops

	Area, in ha in base year ($\bar{x}_i$)		Yield in ton/ha (y_{ij})	Price in €/ton (p_{ij})	Variable costs in €/ha (c_{ij})	Acreage subsidy in €/ha (a_{ij})	Water consumption in m^3/ha (w_{ij})
	Farm 1	Farm 2					
Barley (i=1)	25	25	-	-	-	-	-
Irrigated (i=1,j=1)	25	25	4.00	110	200	265	850
Non-irrigated (i=1,j=2)	0	0	2.80	110	125	265	0
Wheat (i=2)	15	15	-	-	-	-	-
Irrigated (i=2,j=1)	15	15	3.00	144	190	265	1040
Non-irrigated (i=2,j=2)	0	0	1.55	144	120	265	0
Pea (i=3)	5	5	-	-	-	-	-
Irrigated (i=3,j=1)	5	5	1.58	151	130	370	580
Non-irrigated (i=3,j=2)	0	0	0.94	151	85	370	0
Irrigated maize (i=4,j=1)	0	2.5	11.00	140	800	384	6700

III. THE MODEL

Defining x_{ij} the area, in hectares, of the variant j of crop i, and Z as the cultivated area of the farm when it has made a 100% reduction of irrigation water, the following model is used to solve the problem of Section 2 :

$$\max: \sum_i \sum_j \left[p_{ij} y_{ij} + a_{ij} - (\alpha_{ij} + 1/2\beta_{ij} x_{ij})\right] x_{ij} + eZ \qquad (1)$$

$$\text{s.t.}: \sum_i \sum_j x_{ij} \leq A \qquad (2)$$

$$\sum_j I_{ij} \leq 1 \quad \forall i \qquad (3)$$

$$x_{ij} - GI_{ij} \leq 0 \quad \forall (i,j) \qquad (4)$$

$$\sum_j \sum_i w_{ij} x_{ij} - GY \leq 0 \qquad (5)$$

$$-\sum_i \sum_j x_{ij} + Y + Z \leq 0 \qquad (6)$$

$$I_Y + I_Z = 1 \qquad (7)$$

$$Y - GI_Y \leq 0 \qquad (8)$$

$$Z - GI_Z \leq 0 \qquad (9)$$

$$x_{ij},\ Y,\ Z \geq 0$$

$$I_{ij},\ I_Y,\ I_Z \text{ binary variables}$$

where A is the area of the farm (45 hectares in Farm 1 and 47.5 in Farm 2), G is an unrestricted positive real number and e is the agri-environmental premium per hectare, not existing in the base year, and received only when the whole farming area is non irrigated. The minimum premium per hectare to be paid for converting to non irrigated crops is obtained by a parametric analysis of e.

Expression (1) represents the gross margin of the farm, where $\sum_i \sum_j (\alpha_{ij} + 1/2\beta_{ij} x_{ij}) x_{ij}$ is the cost function whose parameters α_{ij} and β_{ij} have the expressions presented below.

Equation (2) is the land constraint. Constraints (3) and (4) allow each crop to be present or not in the solution with only a single variant of the crop appearing in the first case. These two constraints, together with the expressions of α_{ij} and β_{ij} allow the calibration of the model in the base year. These expressions are:

$$\alpha_{ij} = 2c_{ij} - (p_{ij} y_{ij} + a_{ij} - \lambda_L) - d_{ij}$$

$$\beta_{ij} = 2(p_{ij} y_{ij} + a_{ij} - c_{ij} - \lambda_L)/\bar{x}_i + (d_{ij}/\bar{x}_i)$$

where λ_L is the opportunity cost of the land and d_{ij} the difference between the gross margins of the variant of crop i existing in the base year ($j = 1$), and that of other variants ($j \neq 1$) of the crop.

For the variants existing in the base year d_{ij} is zero and α_{ij} and β_{ij} have the traditional expressions (obtained through the necessary conditions of Khun-Tucker and assuming increasing marginal costs) which allow the model to reproduce the level of each crop existing in the base year when no other crops are considered. The expressions of α_{ij} and β_{ij} for the other variants are adjusted to reflect the assumption that a farmer will not object to growing the variant j ($j \neq 1$) of crop i instead of the variant $j = 1$ if both variants produce the same gross margin, that is if the variant $j \neq 1$ receives an additional subsidy of d_{ij} euros.

If the value of e is large enough to make the 100% reduction of irrigation water profitable for the farm then: $Y = 0$ and $Z > 0$. In this case, constraint (5) forces all the irrigated variants to be zero. This constraint, together with (6), determines the level of the non irrigated area, Z. The possibility of having $Y = 0$ and $Z > 0$ simultaneously is given by the equations (7)-(9) which also offer as alternative result: $Y > 0$ and $Z = 0$ when e is not large enough, as happens in the base year when $e = 0$. In this case, all the farming area is irrigated (when binary variables I_{ij} are considered) or a part is irrigated, and another part non irrigated (when binary variables I_{ij} are not considered).

IV. ANALYSIS OF RESULTS

The results obtained with GAMS/SBB are presented in Table 2. The opportunity cost of the land was assumed to be € 350 in the two farms.

A. Crop distribution

The distribution of crops when farms are converted from irrigated to non irrigated does not change dramatically as happens when working with linear programming. When all crops existing in the base year can be replaced by their non irrigated variants (Farm 1) part of the crop area which loses most gross margin in the conversion (wheat) is replaced by the other crops (barley and pea).

In Farm 2 the area of maize must be zero when the farm becomes non irrigated. In this case the area of maize and a part of the area of wheat is replaced by the other crops.

Table 2 Results

	Farm 1			Farm 2		
		100% water reduction			100% water reduction	
	Base year	Value	Variation (%)	Base year	Value	Variation (%)
Barley (ha) Irrigated ((ha) Non-irrigated (ha)	25 25 0	27.074 0 27.074	8.30	25 25 0	28.438 0 28.438	13.75
Wheat (ha) Irrigated (ha) Non-irrigated (ha)	15 15 0	12.321 0 12.321	-17.86	15 15 0	13.129 0 13.129	-12.47
Pea (ha) Irrigated (ha) Non-irrigated (ha)	5 5 0	5.604 0 5.604	12.08	5 5 0	5.933 0 5.933	18.66
Irrigated maize (ha)	0	0		2.5	0	
Minimum premium (€/ha)(1)		81.23			122.47	

B. Minimum premium per hectare for converting farms from irrigated to non irrigated

Firstly, if we assume that in Farm 1 there will be no change in its crop distribution, the minimum premium required to make the conversion from an irrigated to a non irrigated farm economically interesting would be slightly greater than € 83.67 per hectare. The minimum premium obtained with the model which permits changes in the crop distribution, 81.23 €, is not far off the above mentioned amount.

In the case of Farm 2, it is not as easy to reach an approximate figure for the premium without the model because it is assumed that one crop (maize) cannot be grown without irrigation. A larger premium in this case, as shown in Table 2, is a reasonable result.

C. Effects of allowing in the results only a single variant of each crop by means of binary variables

The model does not reproduce the base year situation of the farms when binary variables are not considered. This is not the only effect produced when these variables are excluded; the minimum premium per hectare for converting farms to non irrigated also changes, amounting to €123.63 in Farm 1, instead of €81.23 and €163.87 in Farm 2, instead of € 122.47.

Although the premium increases, the crop distribution after the farms have been converted is the same as when binary variables were considered. The increase in the premium is due to the fact that without binary variables, crop distribution on the farms appearing in the base year lead to an increase in the gross margin (in Farm 1, for instance, this amounts to € 24,532.14 without binary variables and to € 22,622.9 when binary variables are included).

D. Results with a linear programming model that maximizes the gross margin of the farms

If the problem of Section 2 is solved with a linear programming model, the results, identical with and without binary variables, are obvious and also unrealistic if we consider the crop distribution established in the baseline situations of the farms. In the case of 100% reduction in irrigation water, the total area in both farms is cultivated with non irrigated barley, which is the crop having the largest gross margin among the possible non irrigated crops that can be grown on the farms.

The minimum premium per hectare required to accomplish the changeover from irrigated to non irrigated farms is slightly greater than the difference between the gross margin per hectare of the irrigated crop having the largest gross margin (wheat in Farm 1 and maize in Farm 2), and the gross margin per hectare of the non irrigated barley. This difference is € 59 in Farm 1 and € 676 in Farm 2.

V. FINAL REMARKS

The method proposed in this paper using the concepts of PMP models with increasing marginal costs can be easily implemented and extended on the basis of other PMP calibration methods such as those which consider decreasing yields or exogenous supply elasticities.

In this study only a single variant of each crop is admitted. In a more general framework, this constraint can be avoided by adding to the model variables representing combinations of the variants in different proportions.

The method has been presented in the context of solving a specific problem. Nevertheless it can be applied in dealing with other problems when crop variants non-existent in the base year need to be considered, and which, like the problem solved in this paper, cannot be addressed with traditional PMP models or with linear programming models.

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