Nonlinear Vector Error Correction Models in Price Transmission Analysis: Threshold Models vs. Markov-Switching Models

Ihle, R. [1] and von Cramon-Taubadel, S. [2]

[1] Georg-August-Universität Göttingen/ Centre for Statistics (ZfS) and Department of Agricultural Economics and Rural Development, Göttingen, Germany

[2] Georg-August-Universität Göttingen/ Department of Agricultural Economics and Rural Development, Göttingen, Germany

Abstract— **This work provides a comparison of methodologies for applied research in price transmission analysis. We compare two regime-dependent econometric models, namely the threshold vector error correction model and the Markov-switching vector error correction model. We first provide a conceptual comparison in which we find that the regime-switching mechanisms of the models differ fundamentally so that each model is suitable for a certain type of nonlinear price transmission. Furthermore, we conduct a Monte Carlo experiment in order to study the performance of each of the models' estimation techniques for simulated data. Although each model possesses an immediate economic interpretation which well matches an aspect of the theory of price transmission, the simulation results indicate that the corresponding estimation techniques yield biased estimates of low precision.**

Keywords— **price transmission, threshold vector error correction model, Markov-switching vector error correction model.**

I. INTRODUCTION

In recent years, the analysis of price transmission between spatially or vertically separated markets has increasingly drawn on methods that account not only for the common nonstationarity but also for nonlinear dynamics in the cointegration relationship of price series. Among others, [1] argue that integrated markets are unlikely to follow linear adjustment to the long-run equilibrium, i.e., that price transmission is regime-dependent. Figure 1 depicts this idea, i.e., trade of a homogenous product between markets *A* and *B* takes only place if the price differential between the markets p^B - p^A exceeds the costs involved in physical trade τ (transaction costs).

We compare two model classes that incorporate non-linearities into the adjustment process: the threshold vector error correction model (TVECM) and the Markov-switching vector error correction model (MSVECM), providing a comparison of the theoretical appropriateness, strengths and weaknesses of each.[1]

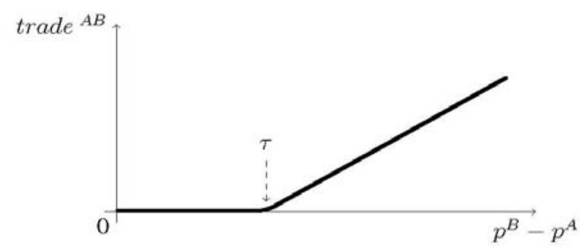

Fig. 1 Regime-dependent Price Transmission

II. CONCEPTUAL COMPARISON

A. Literature Review

The TVECM was introduced by [2]. It represents a well established methodology in price transmission analysis, see for applications, for example, [3], [4] or [5]. The MSVECM is an extension of the general Markov-switching vector autoregressive model proposed by [6]. It has been used, among others, in macroeconomic and business cycle research, see for example [7]. [8] suggest this method for analyzing vertical price transmission between agricultural products.

[1] We only consider the version of each of these models which is mainly used in applied research which are the TVECM with constant thresholds and the MSVECM with constant transition probabilities.

B. Common Properties of Both Models

Both approaches assume that the prices being studied are linked by a stable, linear long run equilibrium (cointegration) relationship. Deviations from this equilibrium of the previous period (quantified by the error correction term ect_{t-1} usually lagged by one period) are adjusted. Furthermore, both models postulate that (some of) the parameters governing price transmission are not constant but take temporarily different values, i.e., the transmission of prices between markets is regime-dependent. Hence, the deviations are not adjusted by a unique linear process but by a non-linear mechanism, i.e., the adjustment of deviations from the long-run equilibrium depends on the regime of price transmission.

The models reflect this idea by parameters which are constant only within the regimes of the data denoted by the indicator variable $J_t = j, j \in 1, 2,\ldots,M$; $t \in 1, 2,\ldots, T$. The realizations of this variable for all t form the regime-generating process $\{J_t\}$ (RGP). [9] suggests a comprehensive classification of models for nonlinear time series analysis. TVECM and MSVECM belong both to the class of piecewise linear autoregressive threshold models. The regimes might either be determined endogenously by the data itself, exogenously by some other observed or unobserved variables or by a combination of both.

C. Basic Ideas of the Models

The possibility of some kind of nonlinear behaviour of price series in international trade was first hypothesized by [10]. He supposed a band of inaction in which small deviations from the Law of One Price are not adjusted because transaction costs are higher than potential earnings due to the price differential. Moreover, such nonlinear behaviour follows from the Enke-Samuleson-Takayama-Judge spatial equilibrium model as formulated in [11]. Data characterized by such a property may in general be modelled, according to [9], with threshold autoregressive (TAR) models. The application of the TVECM to the analysis of price transmission processes possesses with Heckscher's supposition and the Takayama-Judge spatial equilibrium conditions immediate economic justification.

The MSVECM is particularly suitable in cases where "discrete shifts in regime-episodes across which the dynamic behavior of the series is markedly different" ([6]) seem to be present in the data or the time series were "occasionally disrupted by dramatic events" ([12]) or "a sudden shock" ([13]). Regimes of price transmission are likely not to be exclusively determined by the error correction term but might instead be determined by unknown and/ or immeasurable external factors or the "general state" of the trading process or the surrounding political or economic system. Hence price transmission might temporarily change due to traders' temporary insecurity resulting from government actions in form of the introduction or the elimination of legal regulations affecting trade, economic recessions, negative expectations about the near future, crop failures and further political, economic or natural factors.

D. Formulation

The two models may be written in an identical general formulation since both belong to the class of TAR models: $\Delta p_t = A^{(J_t)\prime} X_{t-1} + \varepsilon_t$.[2] Although both model regime-dependent price transmission, the philosophies regarding their underlying RGPs, i.e., the mechanisms generating the regimes J_t, differ fundamentally leading to differing appropriateness for certain types of nonlinearities and differing estimation methods and interpretation.

E. Differing Regime-Generating Processes

The regimes are, in case of the TVECM, assumed to be exclusively determined by ect_{t-1}, i.e., the first lag of

[2] p_t=(p^A_t p^B_t)\` is the vector of prices in markets A and B. ε_t are assumed to be iid random variables with $\mathrm{E}(\varepsilon_t)=0, \mathrm{Var}(\varepsilon_t)=\Sigma$. β denotes the cointegration vector of the prices p_t. $A^{(J_t)}$ denotes a (2k+2) x 2 matrix of regime-dependent coefficients containing, among others, the loading matrix α which measures the rates of error correction (speed of adjustment). The 2k+2 vector of the regressors is represented by $X_{t-1} = (1\,\beta' p_{t-1}\,\Delta p_{t-1} \ldots \Delta p_{t-k})'$ where $\beta' p_{t-1} = ect_{t-1}$. The determination of the regimes J_t at each time t is left unspecified in this formulation; it is discussed below.

some linear combination of the two price series under investigation. Hence the regimes are an explicit function of the two price series, i.e., $J_t = f(p^A_{t-1}, p^B_{t-1})$. A certain regime $J_t = j$ is taken if $\theta^{(j-1)} < ect_{t-1} \le \theta^{(j)}$ where $\theta^{(j)}$ is the j's *threshold* and M-1 equals the number of thresholds (usually M-1=2). The variable ect_{t-1} determining the relevant regime is in general called *threshold variable.* [3]

In case of the MSVECM, the RGP is assumed to follow a *homogeneous, ergodic and irreducible* Markov chain. It is characterized by the *transition probabilities* $\Pr(J_t=j|J_{t-1}=h) = \gamma_{hj}$; $h, j = 1,\ldots,M$ which are summarized in the transition matrix $\Gamma = \{\gamma\}_{h.j=1,\ldots,M}$. [4] The switching is assumed to exclusively depend on the previous regime J_{t-1} (Markov property). Conditionally on $J_{t-1}=h$, the Markov chain switches according to the transition probabilities in the h'th row of Γ to one of the M states. The RGP may be a function of one or more exogenous variables $y, z,\ldots$ which might be thought of as the "general state" of the system, i.e., $J_t = f(y_t,\ldots, y_{t-l}, z_t,\ldots, z_{t-r,\ldots})$ where $l, r \in \mathrm{N}^+$. They are furthermore allowed to be latent. Hence unlike the TVECM, no observations on the regime generating variables are required, they even may remain entirely unspecified.

The RGP of the TVECM may be reformulated in terms of transition probabilities in order to contrast its properties to those of the MSVECM which allows interesting insights in common and differing characteristics of both models. First, the probabilities of the TVECM are not constant over time, i.e., its RGP is not homogeneous. Since the probabilities depend on ect_{t-1}, the process does not satisfy the Markov property. The regime switching is endogenous because ect_{t-1} is a function of the observed prices. Its transition probabilities are furthermore restricted to equal either zero or one, whereas they can take any probability between zero and one in case of the MSVECM. In contrast, the switching mechanism of the MSVECM is exogenous since the RGP may depend on one or a range of unobserved/ unobservable exogenous factors.

F. Interpretation

The estimated thresholds of the TVECM are, based on [10] and [11], interpreted as transaction costs. Interpretation of the detected regimes is straightforward. For the MSVECM, a thorough analysis of the regimes, their duration and their timing has to be undertaken in order to make sense of the identified regimes, i.e., to identify their distinguishing characteristics since the factors causing the switching are unknown a priori. The estimated parameters and further descriptive variables have carefully to be analyzed in order to obtain insights regarding the characteristics specific to each regime.

Statements about regime incidences are always made with certainty for the TVECM, but are, in case of the MSVECM, of probabilistic nature. Such deterministic all-or-nothing statements of the former model are more restrictive than the probabilistic ones of the latter. Hence, the Markov-switching philosophy acknowledges the uncertainty concerning the unknown true data generating process, however, the TVECM approach does not. However, if the additional information contained in the error correction terms reflects the true RGP better than the general Markov-switching approach, then its estimates can be expected to be more efficient and more informative.

Both models do not model transaction costs explicitly. They are both able to detect regimes characterized by different error correction rates as well as regimes in which no adjustment behaviour takes place. Moreover, the MSVECM can be interpreted as allowing transaction costs to shift in contrast to the constant threshold (transaction costs) of the TVECM since an identified regime without adjustment may be have been caused by temporary prohibitive transaction costs.

G. Estimation

Estimation of both models faces the same challenge of interdependencies between the unknown regimes and the regime-dependent parameters. The methods of sequential conditional least squares (SCLS) and the

[3] In price transmission analysis, the error correction term of the previous period ect_{t-1} is always used as the threshold variable.

[4] The so-called ergodic probabilities can be calculated from the transition matrix. They are interpreted as the unconditional probabilities that the Markov chain is in one of its M states at arbitrary time t. Each state of the Markov chain corresponds to one regime of price transmission.

Expectation-Maximization algorithm (EMA) for the TVECM and the MSVECM respectively tackle this task in different ways.[5] Conditional on a set of candidate values, an optimization criterion is evaluated in the former case. The combination of parameters optimizing the criterion is selected as the final estimates. The EMA, in contrast, is an iterative method conditional on a set of starting values. First, inference on the unobserved regimes is obtained (expectation step) and the parameters are estimated subsequently (maximization step).

III. SIMULATION STUDY

A. Design

We are interested in assessing the performance of the estimation methods under ideal circumstances. We extend the simulation study of [14] who find “considerable uncertainty in the estimates [of the thresholds] for moderate sample sizes” for the unrestricted TVECM model. In particular, we apply the (unrestricted) SCLS estimation and the EMA to simulated data which is generated by a TVECM and a MSVECM respectively (Table 1).

Table 1 Cases of the Simulation Study

Estimation	Data generation	
	TVECM	MSVECM
SCLS (TVECM)	I	II
EMA (MSVECM)	III	IV

We simulate data sets which follow a simple non-linear VECM with three regimes $\Delta p_t = \alpha^{(J_t)} ect_{t-1} + \varepsilon_t$, M=3. The data sets differ in the speed of error correction, parameters of the transition matrix and the threshold values respectively. In each scenario we generate 1000 pairs of time series of prices in markets *A* and *B*. In each of cases I and IV, 16, 16 and 9 scenarios are simulated consisting of time series with 150, 500 and 1500 observations respectively. The mean squared error (MSE) and the percentage of correctly identified regimes are chosen as criteria to measure the method’s performance. In each of cases II and III, only one data set is generated and only the latter measure of performance is used.

B. Results

In each of the cases[6], only a very low share of regimes is correctly identified ranging from 30% in case III to 56% in case I. In case I, we furthermore find a large bias of the estimates which is strongly influenced by the true regime-dependent adjustment speed $\alpha^{(J_t)}$. It increases with decreasing $\alpha^{(J_t)}$ and increasing *T*. The variance also increases with decreasing $\alpha^{(J_t)}$ and increasing *T*. Consequently, the MSE is large and highly dependent on the true parameters, in particular it depends on $\alpha^{(J_t)}$ (Figure 2).[7] Case IV displays similar behaviour; now the bias increases with decreasing $\alpha^{(J_t)}$ and decreasing *T* and the variance increases slightly with increasing $\alpha^{(J_t)}$ and decreasing *T* (Figure 2).[8]

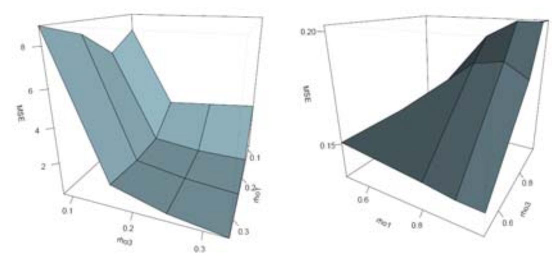

Fig. 2 MSE for Case I (left) and Case IV (right)

IV. CONCLUSIONS

This paper compares two time series models which are relevant for price transmission analysis and allow for nonlinear adjustment to the long-run equilibrium. From an economic point of view, both models are suitable to analyze price transmission processes since

[5] We only focus on these two methods since they are mainly employed in applied research.

[6] We summarize the most important results here. Further results are available from the authors.

[7] The left graph of Figure 2 shows the resulting surface of the MSE of the estimates of one threshold (vertical axis, 16 scenarios) as a function of the true adjustment speeds $\alpha^{(J_t)}$ of the outer regimes of the simulated TVECM (horizontal axes).

[8] The right graph of Figure 2 shows the resulting surface of the MSE of the estimates of one of the ergodic probabilities (vertical axis, 16 scenarios) as a function of the true adjustment speeds $\alpha^{(J_t)}$ of two regimes of the simulated MSVECM (horizontal axes).

they possess sensible interpretations. Although they seem very similar at first glance due to their common property of regime switching, their underlying statistical concepts differ fundamentally. Hence, each model is suited for a particular type of nonlinearity.

The switching is endogenous in case of the TVECM and exogenous in case of the MSVECM. The restriction in case of the former model can be reasonable and facilitates interpretation of estimation results. As noted by [15], such restrictions represent one way to ease the difficulties encountered in the interpretation of Markov-switching models.

If the price data is generated by trade alone, i.e., if it is exclusively governed by spatial arbitrage conditions and not subject to suddenly changing economic, political or natural determinants, the TVECM is the more appropriate model. However, if trade continuously takes place into one direction or is dominated by external interferences, the MSVECM can be an appropriate alternative.

Although both models can generate interesting insights into price transmission processes, their empirical estimation constitutes a drawback and is to be improved by further research.

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REFERENCES

1. McNew K P, Fackler P L (1997) Testing market equilibrium: is cointegration informative? J Agr Resour Econ 22: 191–207
2. Balke N S, Fomby T B (1997) Threshold cointegration. Int Econ Rev 38: 627–645
3. Balcombe K, Bailey A, Brooks J (2007) Threshold effects in price transmission: the case of brazilian wheat, maize, and soya prices. Am J Agr Econ 89: 308–323
4. Ben-Kaabia M, Gil J M (2007) Asymmetric price transmission in the Spanish lamb sector. Eur Rev Agric Econ 34: 53–80
5. Meyer J (2004) Measuring market integration in the presence of transaction costs — a threshold vector error correction approach. Agr Econ 31: 327–334
6. Hamilton J D (1989) A new approach to the economic analysis of nonstationary time series and the business cycle. Econometrica 57: 357–384
7. Krolzig H-M, Marcellino M, Mizon G (2002) A Markov-switching vector equilibrium correction model of the UK labour market. Empir Econ 27: 233–254
8. Brümmer B, von Cramon-Taubadel S , Zorya S (2008) A Markov-switching vector error correction model of vertical price transmission between wheat and flour in Ukraine. Under revision for the Eur Rev Agric Econ
9. Tong H (1990) Non-linear time series. A dynamical system approach. Clarendon Press, Oxford
10. Heckscher E F (1916) Växelkurens grundval vid pappersmynfot. Ekon Tidskr 18: 309–312
11. Takayama T, Judge G (1971) Spatial and temporal price allocation models. North-Holland, Amsterdam, The Netherlands
12. Hamilton J D, Raj B (2002) New directions in business cycle research and financial analysis. In Advances in Markov-switching models. Applications in business cycle research and finance (eds. Hamilton, J D, Raj, B.). Physica-Verlag, Heidelberg
13. Psaradakis Z, Sola M, Spangolo, F (2004) On Markov-switching models, with an application to stock prices and dividends. J Appl Econ 19: 69–88
14. Lo M C, Zivot E Threshold cointegration and nonlinear adjustment to the law of one price." Macroecon Dyn 5: 533–576
15. Jackman S (1995) Re-thinking equilibrium presidential approval - Markov-switching error correction. Paper presented at the 12th Annual Political Methodology Summer Conference, Indiana University, Bloomington, USA

Corresponding author:

- Author: Rico Ihle
- Institute: Georg-August-Universität Göttingen/ Department of Agricultural Economics and Rural Development
- Street: Platz der Göttinger Sieben 5
- City: 37073 Göttingen
- Country: Germany
- Email: rihle@gwdg.de