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A Micro Economic Approach based on Mathematical Programming

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Numerical Estimation of Agricultural Supply Functions
- A Micro Economic Approach based on Mathematical Programming

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Abstract
This paper describes a cost function approach to modelling production and resource use at the level of the individual farm firm. The procedure for deriving the supply function using mathematical programming under fairly general condition of a convex input set is shown, and the model is demonstrated. It is suggested that the model is used as the basic building block in agricultural sector models, the advantage being that the technology is modelled as a production frontier providing better opportunities for modelling input and output substitution.

Keywords: Cost function, linear programming, economies of scale, agricultural sector models, production frontier.

1. Introduction and background

There is a long tradition for using agricultural sector models as a tool for estimating consequences of changes in economic and technological variables and for policy analysis in relation to the agricultural sector. Such models may cover one or more regions, countries or even larger areas like the EU, and are typically based on one of three major modelling approaches; econometric, general/partial equilibrium, or mathematical programming. CAPRI (Heckelei & Britz, 2001), DRAM (Helming, 2005), DREMFIA (Lehtonen, 2001), ESMERALDA (Jensen, 1996), and others (Balmann (1997), (Lansink, 1997), (Ekman, 2002)) are examples of such models. The models are often micro (farm) based, and data describing production and resources are typically historical data from agricultural accounts (such as FADN-data). The models are normally calibrated so that they are able to replicate historical observations.

The production technology is often described in the form of *average productivity* and fixed coefficients of production, i.e. no or limited possibility for changing *input mix* or *output mix*. Even in cases where the sector models are based on differentiated micro units (representative farm types) the empirical basis (the production parameters/coefficients) is often *average* productivity being the only parameters describing the production technology.

This is a problem because adjustment to changing economic and technological conditions is based on *marginal productivity* and substitution possibilities. Models that do not describe the

production frontier but only some point on the production frontier are basically unable to project the consequences of changes in economic and other conditions. It is difficult in these models to take into account *economies of scale*, and the potential use of *new technologies*. Further, *model calibration* is often done "mechanically" without really considering the cause of the model not providing the "correct" result.

In this paper we present and demonstrate a farm model which can be used as the basic building block of agricultural sector models without facing the problems mentioned above. The building blocks are farm models in the form of a Mathematical Programming models constructed in a way so that they can be used to generate (numerically) the ex ante supply function for the main product of the individual farm types taking into account input and output substitution, varying production elasticity, and the possibility to include new technologies. We also describe how these basic building blocks in the form of individual farm type models may be combined into a partial equilibrium agricultural sector model.

2. The general procedure

The approach is based on the *ex ante cost function*[1)] of the individual farm firm. If one knows the ex ante cost function it is possible to derive the supply function of the firm under the assumption of profit maximization. Aggregating the supply functions of all the individual firms in the region generates the market output supply function, and determines aggregate input demand for the region in question. If the corresponding equilibrium prices on the input market deviate from the original input prices used in the first estimation step, input prices are adjusted, and ex ante cost functions are estimated once again. This iterative procedure continues until equilibrium is established on the input markets where equilibrium is required (partial equilibrium model). In the case of agent-based modelling, this last procedure may be carried out in one step using cellular automata (Balmann (1997); Berger (2001)).

3. The farm model

3.1. The economic model

The (short run) supply function of the individual firm can be derived from the (ex ante) cost function C which is:

(1) $$C = C(\mathbf{w}, \mathbf{y}, \mathbf{b})$$

[1] An *ex ante* cost function is the relationship between the (planned) amount of output and the minimum cost of producing this output given the specified technology and input prices.

where **w** is an *N*'-vector of variable input prices, **y** is an *M*'-vector of outputs, and **b** is a *K*-vector of quasi-fixed inputs. Assuming profit maximization under perfect competition, the supply function is the marginal cost function determined from profit maximization:

(2) $$MC_m(\mathbf{w},\mathbf{y},\mathbf{b}) = p_m \qquad (m=1,\ldots,M')$$

where p_m is the price of output *m* and

(3) $$MC_m = \frac{\partial C}{\partial y_m} \qquad (m=1,\ldots,M')$$

For operational reasons it is easier to consider the production and supply of just one (main) output, *y*. Therefore, instead of considering explicitly a vector of (variable) inputs with prices **w**, and the vector of output **y**, we consider a *netput vector* **x** of input and output, consisting of the *N*' elements of the original variable input vector, and the negatively signed *M*'-1 elements of the original output vector **y**. At the same time, the interpretation of the price vector **w** is changed, so that **w** is now (and in the following) an *N*'+*M*'-1-vector of *netput prices*.

Equations (1), (2), and (3) change to:

(4) $$C = C(\mathbf{w}, y, \mathbf{b})$$

where **w** is an *N*-vector (*N*=*N*'+*M*'-1) of *netput prices*, *y* is (main) output, and **b** is a *K*-vector of quasi-fixed inputs.

The condition for profit maximization changes to:

(5) $$MC(\mathbf{w}, y, \mathbf{b}) = p$$

where *p* is the price of the main output and the marginal cost is:

(6) $$MC = \frac{\partial C}{\partial y}$$

The supply function is derived from (5) by solving for *y*. Thus:

(7) $$y_j = s_j(\mathbf{w}, p, \mathbf{b})$$

where $s_j(\bullet)$ is the supply function for firm *j*.

If we consider the region in question represented by J firms and their aggregate demand for netput $\mathbf{X}$, then the netput prices may depend on the production and therefore the output price p. Thus, the effective supply function of firm j is

(8) $$y_j = \psi_j(p, \mathbf{b}) \equiv s_j(\mathbf{w}(\mathbf{X}(y(p))), p, \mathbf{b})$$

where $\mathbf{w}(\mathbf{X}(y(p))) = (w_1(X_1(y(p))), w_2(X_2(y(p))), \ldots, w_N(X_N(y(p))))$.[2] Therefore, the aggregate (region) supply $S(p)$ is:

(9) $$y = \sum_j y_j = \sum_j \psi_j(p) = S(p)$$

To estimate $\psi_j(p, \mathbf{b})$ (numerically) it is necessary to consider the relation between the output price p, production y, and the netput price elasticities. The simple case is when farmers trade only on the world market and not with each other. In this case the netput price elasticity is zero (exogenous prices) and no quantity restriction is needed. The other extreme is when farmers trade only with each other (for instance piglets). In this case we need the market clearing restriction ($\sum_{j=1}^{J} x_{ij} = 0$). If the netput is sold on the world market *or* to other farmers, we need the restriction $\sum_{j=1}^{J} x_{ij} \leq 0$. Finally, if the netput is bought on the world market *or* from other farmers, we need the restriction $\sum_{j=1}^{J} x_{ij} \geq 0$.

Using these restrictions it is possible through an iterative procedure to estimate the aggregate output supply function for the region in question. At the same time, shadow prices on the market clearing restrictions determine implicitly netput prices (opportunity cost).

3.2. Estimation of the cost function of individual farm firms

The ex ante cost function (4) for an individual firm j can be numerically estimated by solving the following Mathematical Programming problem

$$\min_{\mathbf{x}} \mathbf{wx}$$

subject to

(10) $$y_j = f_j(x_1, \ldots, x_N, z_1, \ldots, z_K) \geq Y_j$$

[2] This is the simple case, with no cross product terms (the price of netput i only depends on the amount of netput i (and not the amount of other netput)). If necessary cross product terms may be included. However, this will make the following expressions more complicated.

$$\mathbf{z} \leq \mathbf{b}_j$$

where **x** is a vector of input (positive values) and output[3)] (negative values), **w** is a vector of corresponding netput prices, **z** ($z_1, \ldots, z_K$) is a vector of inputs available in the fixed amounts $\mathbf{b}_j$ ($b_{1j}, \ldots, b_{Kj}$), $f_j(\bullet)$ is the production function for the main product y, and Y_j is a parameter measuring the "target production" of output y.

The problem in (10) may be solved for different values of the parameter Y_j, thus generating the cost function (4) and the marginal cost function (6) in numeric terms.[4)] The number of discrete values of parameter Y_j that should be applied depends on how closely one wants to approximate the numeric representation of the cost function to a smooth cost curve.

Assuming profit maximising behaviour, farmers produce an amount so that marginal cost equates to marginal revenue[5)], which in a competitive market is the (expected) output price p.

3.3. The production model

The production technology in the model (10) is specified in the form of the production function $y = f(x_1, \ldots, x_N, z_1, \ldots, z_K)$ and the restriction $\mathbf{z} \leq \mathbf{b}$.[6)]

Mathematical forms of production functions are rarely available. Therefore, in empirical work it is often necessary to use other approaches to describe the production technology.

The production technology in (10) may be described in more general terms in the form of the technology set (production set):

(11) $$T = \{(\mathbf{x}, \mathbf{z}, y, \mathbf{b}) : y \leq f(\mathbf{x}, \mathbf{z}) \ \textit{and} \ \mathbf{z} \leq \mathbf{b}\}$$

The typical assumption concerning T is that it is non-empty, closed, has free disposability of inputs and outputs, and that isoquants are convex, i.e. diminishing marginal returns applies.

In the context of cost minimization it is more convenient to consider the input requirement set (or just the input set) $V(y, \mathbf{b})$:

(12) $$V(y, \mathbf{b}) = \{(\mathbf{x}, \mathbf{z}) : (\mathbf{x}, \mathbf{z}, y, \mathbf{b}) \in T\}$$

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[3] Output other than the main product y.

[4] Marginal cost is the shadow price of the restriction $y_j = f_j(x_1, \ldots, x_N, z_1, \ldots, z_K) \geq Y_j$ in (10), which is provided as output from most mathematical programming software.

[5] Or rather, *expected* marginal revenue

[6] For convenience the index for farms j is dropped.

where T is defined in (11) and where V is the set of all feasible input-output (netput) combinations for which $\mathbf{z} \leq \mathbf{b}$.

Based on the assumption that $V(y, \mathbf{b})$ is convex, it is possible to approximate the input set V by choosing a set Φ of discrete production plans such that the frontier of $V(y, \mathbf{b})$ is the convex hull of Φ and $V(y, \mathbf{b})$ is the convex hull of Φ combined with the assumption of free disposability of input. For empirical application the set Φ must consist of a finite number of points, for example the set of S production plans $(\mathbf{x}^1, \mathbf{z}^1, y, \mathbf{b})$, $(\mathbf{x}^2, \mathbf{z}^2, y, \mathbf{b})$, …, $(\mathbf{x}^S, \mathbf{z}^S, y, \mathbf{b})$, where $(\mathbf{x}^1, \mathbf{z}^1)$,…, $(\mathbf{x}^S, \mathbf{z}^S)$ are discrete points on the production frontier corresponding to a given amount y of output, i.e. $y=g_1(\mathbf{x}^1, \mathbf{z}^1) = g_2(\mathbf{x}^2, \mathbf{z}^2) = \ldots = g_S(\mathbf{x}^S, \mathbf{z}^S)$. If the term $r(y)$ is used to denote the set of production plans that are chosen to approximate (the relevant part of) $V(y, \mathbf{b})$,[7] then the complete description of the technology set for all the different levels of output Y chosen when solving (10), is $\{r(Y_1) \cup r(Y_2) \cup \ldots \cup r(Y_P)\} = R$. Thus R ($R \subseteq T$) is a set of discrete production plans that can be used to approximate T (for farm j)

This way of modelling the technology set is based on the process analysis approach to the neoclassical theory of production first discussed by (Georgescu-Roegen, 1972).

To identify the convex hull that approximates the frontier of V, an appropriate method to use is Linear Programming, in which case the technology will be piecewise linear (a polytope). To illustrate, consider the functional form of $g_s(\bullet)$ (see (12) below)) (s= 1,…, S) as Leontief production functions, i.e.:

$$(13) \qquad g_{sj}(\mathbf{x}, \mathbf{z}) = \min\{\alpha_1^{sj} x_1, \alpha_2^{sj} x_2, \ldots, \alpha_N^{sj} x_N, \alpha_{N+1}^{sj} z_1, \alpha_{N+2}^{sj} z_2, \ldots, \alpha_{N+K}^{sj} z_K\} \qquad (s \in r(y))$$

where $g_{sj}(\bullet)$ is the production of output y using production plan s on farm j, and α_i^{sj} is the amount of output y pr. unit of input i (i=1,…, N+K) using production process s on farm j. (In the next section where the mathematical programming model is described in more detail, the vector $(1/\alpha_1^s, \ldots, 1/\alpha_N^s, 1/\alpha_{N+1}^s, \ldots, 1/\alpha_{N+K}^s) = (a_1^s, \ldots, a_N^s, a_{N+1}^s, \ldots, a_{N+K}^s)$ is called a *production vector* A^s or a *production process*).

Assuming constant returns to scale, the production on farm j may be described as the convex combination of Leontief production functions:

$$(14) \qquad y_j = f_j(\mathbf{x}, \mathbf{z}) = \sum_{s \in R} \lambda_s g_{sj}(\mathbf{x}, \mathbf{z})$$

where λ_s are variables such that:

[7] In the present example, $r(y)=\{(\mathbf{x}^1, \mathbf{z}^1), \ldots, (\mathbf{x}^S, \mathbf{z}^S)\}$

(15) $$\sum_{s \in R} \lambda_s = 1, \quad 0 \le \lambda_s \le 1, \quad (s \in R)$$

A potential problem with the Leontief production function *g* in (13) is that it has constant returns to scale, and therefore it is apparently not possible to model *economies of scale*. For instance in milk production, the amount of labour necessary to produce one unit of output typically declines with the scale of production, so it cannot be modelled using (13), (14) and (15).

One way to avoid this problem is to include *scale specific production plans* (production vectors). Consider for instance the following example: The production of milk per unit of labour depends on the scale of production. With a scale of y = 400,000 kg of milk, the production of milk is 250 kg pr. hour of labour input. With a scale of y = 600,000 kg milk, the production of milk is 300 kg pr. hour. And with a scale of y = 800,000 kg milk, the production of milk is 350 kg pr. hour.

In this example one could include three scale specific production processes A^{s_1}, A^{s_2}, and A^{s_3} that differ in the sense that A^{s_1} has a parameter $\alpha_i^{s_1}$=250, A^{s_2} has a parameter $\alpha_i^{s_2}$=300, and A^{s_3} has a parameter $\alpha_i^{s_1}$=350.

If such scale specific production vectors are included in the set *R* that describes the production plans, then it is also possible to model *economies of scale*. However, to make sure that only allowable production processes are in use, further restrictions are needed. For instance, in the example above, to make sure that the production process A^{s_2} is not in use unless the scale of production is 600,000 kg or more, it is necessary to restrict the parameters λ_{s_2} and λ_{s_3} in (14) and (15) to be zero when production y is less than 600,000 kg. Similarly it is necessary to restrict λ_{s_3} to be zero when production y is less than 800,000 kg.

The complete production model that allows for modelling economies of scale in the sense that all scale specific production processes can produce more than the specific scale but not less, is accomplished by including the scale specific production processes in the set *R*, and by adding the following restrictions to the model (14) and (15):

(16) $\lambda_s = 0$ if A^s is a scale specific production process and $y_i < \bar{Y}^s$

where $\bar{Y}^s$ is the specific "scale" limit for production process A^s.

The problem (14), (15), and (16) is a so-called mixed integer programming problem, and may be solved using GAMS.

3.4. The Linear Programming model

The linear programming model to be used for estimating the cost function (10) for an individual farm firm has a general form.

The starting point is a set $\{A^1 \ldots A^S\}$ of production vectors capable of producing output y (for instance milk). A production vector A^s has $1+N+K$ elements and is of the form:

$$A^s = \begin{bmatrix} -1 \\ a_1^s \\ a_2^s \\ \vdots \\ a_N^s \\ a_{N+1}^s \\ a_{N+2}^s \\ \vdots \\ a_{N+K}^s \end{bmatrix} \tag{17}$$

where a_i^s (s=1,…, S) is the use of netput i (i=1,…, $N+K$) pr. unit of output y applying production process s. If a_i^s is positive then it is an input. If a_i^s is negative then it is an output.

Further, define a set $\{D^1 \ldots D^N\}$ of production vectors potentially capable of producing netput n (n=1,…, N). A production vector D^n has $1+N+K$ elements, and is of the form:

$$D^n = \begin{bmatrix} 0 \\ d_1^n \\ \vdots \\ d_{n-1}^n \\ -1 \\ d_{n+1}^n \\ \vdots \\ d_N^n \\ d_{N+1}^n \\ \vdots \\ d_{N+K}^n \end{bmatrix} \tag{18}$$

where d_i^n (n=1… N) is the amount of netput i (i=1…N+K) used to produce one unit of netput n. If d_i^n is positive then it is an input. If d_i^n is negative then it is an output.[8)]

Define the [(N+K+1)×S] matrix **A** as:

(19) $$\mathbf{A} \equiv \left[A^1 \cdots A^S \right],$$

Define the [(N+K+1)×N] matrix **I** as:

(20) $$\mathbf{I} \equiv \begin{bmatrix} 0_N \\ \mathbf{I}_N \\ \mathbf{0}_{KN} \end{bmatrix}$$

where 0_N is a row vector of N zeros, $\mathbf{I}_N$ is a (N×N) matrix with ones in the diagonal and zeros elsewhere (an Nth order identity matrix), and $\mathbf{0}_{KN}$ is an(K×N) matrix of zeros,

Define the (N+K+1) vector:

(21) $$\mathbf{B} \equiv \begin{bmatrix} -\bar{Y} \\ 0_N^{'} \\ \mathbf{b} \end{bmatrix}$$

where $\bar{Y}$ is the required production level of product y (milk), $0_N^{'}$ is a N element column vector of zeros, and **b** is a K-element column vector of quasi fixed input.

Define the [(1+N+K)×N] matrix **D** as:

(22) $$\mathbf{D} \equiv \left[D^1 \cdots D^N \right],$$

Then the problem (10) can be formulated as a linear programming problem as:

(23) $$\begin{aligned} &\min_{\mathbf{u},\mathbf{v},\mathbf{x}} \{\mathbf{wx}\} \\ &st \\ &\mathbf{Av} + \mathbf{Du} - \mathbf{Ix} \le \mathbf{B} \\ &\mathbf{v} \ge 0; \quad \mathbf{u} \ge 0; \end{aligned}$$

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[8] As defined here, there is only one production process for each netput x_n. As with the output y, one could include the possibility of having additional scale specific production plans for production of each netput. To keep things simple this has not been done here. If needed, the extension is straight forward.

where **w** is a N-vector of netput prices, **x** is an N-vector of the amount of netput that the firm purchases (negative) or sells (positive), **u** is an N-vector of the amount of netput that the firm produces, and **v** is an S-vector of output y that the firm produces applying production process s (s=1,…, S).

Problem (23) can be solved using the software GAMS. To ensure that scale specific production plans are not applied below the required scale (as specified in (16)), it is necessary to use binary variables. GAMS include the facility to restrict variables to be binary.

The model includes the facility that any of the N netput (x_1, …, x_N) may either be bought at prices (w_1, …, w_N) or produced within the firm applying the production processes (D^1,…, D^N). In a number of cases, production of input at the firm level is not an option. For instance fertilizers and pesticides are typically not produced on farms. In those cases the relevant production processes are excluded from the matrix **D** and the corresponding elements from the production vector **u**.

4. Demonstration of the farm LP model

4.1. Farm type and technology

This section demonstrates the LP model presented in general terms in Section 3.4. The model solves the problem (10) shown in Section 3.1, to estimate the cost function for an individual farm.

The demonstration farm used here is a model dairy farm (main product milk) with 50 hectares of ordinary farm land, 2 hectares of meadow in permanent pasture, a cattle stable with space for 70 dairy cows and required young stock, and a fixed labour force of 2,500 hours per year.

In addition to milk, the farm produces young stock for replacement (heifers) or sale (new born calves and heifers), roughage, grain and other cash crops. All machine operations are carried out using contractors.

The production technology is spanned by a set of four milk production processes represented by four different levels of feed (measured in Feed Units (FE)) and corresponding milk production per cow (decreasing marginal productivity). The quality of feed is controlled by restrictions on level of protein, fat acid, sugar, starch, digestibility, fill, and chewing time. Feed is provided in the form of 14 different roughage products, and 4 different concentrates. The roughage production technology is for each of the 14 types of roughage spanned by 4 different levels of nitrogen (and other) fertilizers. Economies of scale is modelled by 3 scale de-

pendent levels of labour per cow, and 3 scale dependent levels of contractor costs per hectare of green feed. Production and price data are based on contemporary norms or standard data.[9]

4.2. Results

The results of optimising production for seven different levels of milk production on this model farm are shown in detail in FOI Working Paper No. 10/2007 (not shown here due to space limitations).

The results show that total cost increases from € 6,722 to € 58,156 when production increases from 340,000 kg to 560,000 kg milk. This corresponds to an increase in average cost from € 0.02 to € 0.10 per kg milk. Marginal cost increases from € 0.10 to € 0.30 when production increases from 340,000 kg to 560,000 kg.

At the low (340,000 kg) production level the production is based on 45 dairy cows all being fed according to the low intensity feed plan 2 (5,011 FE per cow). When production increases, the feed intensity increases gradually, and with a production of 500,000 kg or more, all cows are being fed according to the high intensity feed plan 4 (5,976 FE per cow).

The main results in the form of the numerically estimated marginal cost and the average cost curves are illustrated in Figure 1.

[9] Details can be found in FOI Working Paper No. 7/2007

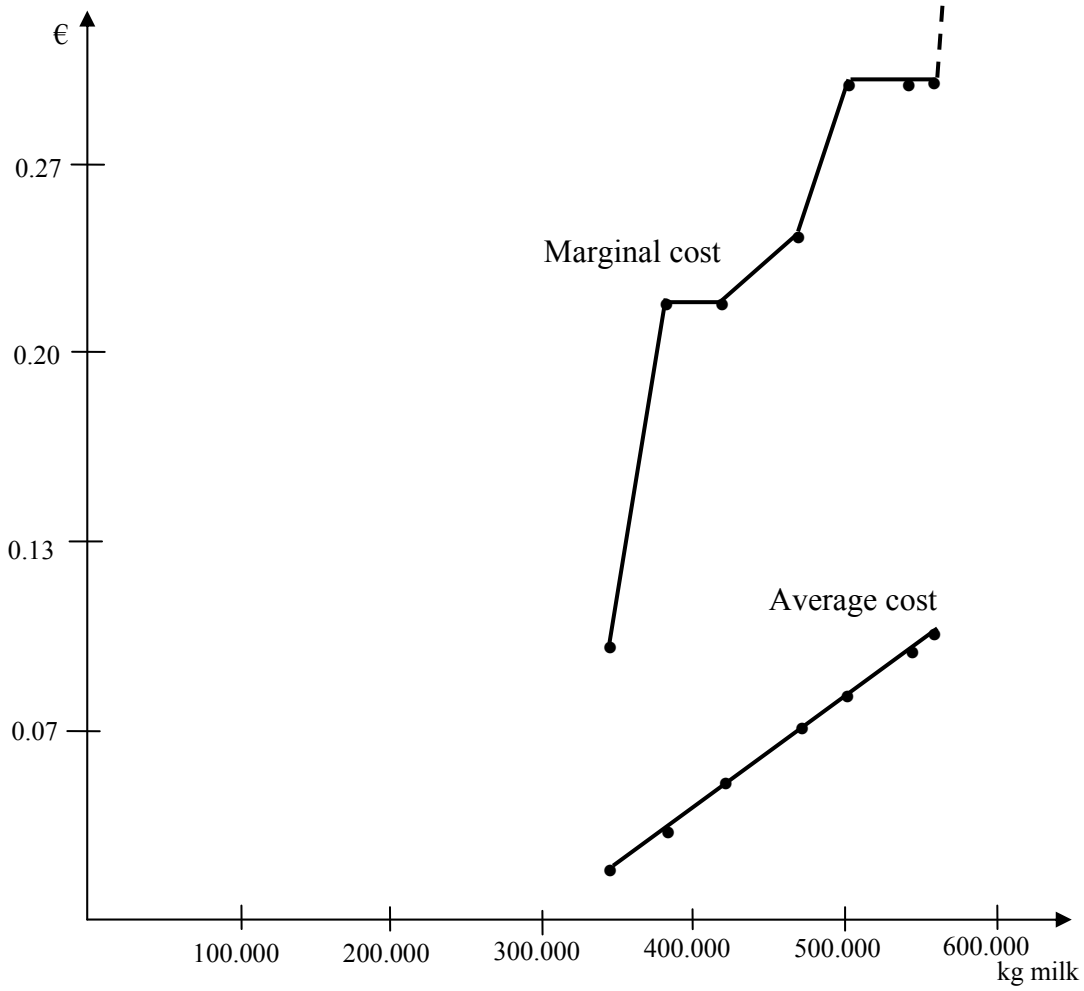

Figure 1. Estimated marginal and average cost functions. € per kg milk

The maximum potential maximum production is 570,010 kg milk (70 cows each yielding the maximum of 8,143 kg milk per year. At around this production level the marginal cost rises to infinity (indicated by the dotted vertical line at the end of the marginal cost curve in Figure 1).

The results show that with the current milk price of € 0.30 per kg milk, it just pays to produce the maximum amount of milk (marginal cost at this production level is € 0.30 per kg). However, should the milk price decrease to, for example € 0.27, then the farm should reduce production to a level between 470,000 and 500,000 kg where marginal cost ranges from € 0.24 to € 0.30 per kg milk.

The results from running the LP-model also include shadow prices for each restriction. The shadow price of land is € 517 per hectare of land when milk production is 340,000 kg and € 578 per hectare when it is 560,000 kg milk.

5. Discussion and perspectives

The farm LP-model presented in the previous section may be seen as a generic model that can be used to generate supply functions for any individual farm type. Thus the model may be used at the micro level to estimate consequences for individual farm types.

The model may also be used as the basic building block when constructing agricultural sector models. The production and netput use of each of these representative farms can be aggregated according to the indication in Section 3.1, and the total model complex can therefore be used as a tool for policy analysis and technology assessment related to the whole agricultural sector.

The power of the model approach presented in this paper is the explicit modelling of the production technology as a convex production frontier spanned by a set of production processes. By including scale specific production processes and the use of integer programming to control economies of scale, this provides an unprecedented degree of flexibility.

The theoretical justification of a farm based sector model is an important background for any sector model construction. Mathematical programming techniques such as linear programming are widely used tools in agricultural economics. An often used option is to represent a whole region or sector as a single hypothetical farm that is optimised as a single entity (Schuler and Kachele 2003; Zander 2003; Kerselaers et al. 2007). Other models describe the behaviour of individual farms as e.g. in agent-based models (Balmann 1997; Berger 2001; 2006; Happe 2004).

Despite the approach, further work is needed both concerning the technical aspects (choice of farm types, modelling technologies, programming of aggregation, and iterative procedure for generating partial equilibrium) and modelling the dynamics of adjustment over time. In this context, formation of price expectations and modelling of investment and changes in farm structure over time are important topics. Also the question of model calibration needs to be reconsidered.

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