The effect of competition between two spatially separated markets - An investigation of two interlinked Bak-Sneppen models.

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Abstract— **This paper investigates the effect of competition in a market consisting of interlinked economic agents. In particular, the effect of increased competition from the surrounding markets is demonstrated. The presented work is an extension of the Bak-Sneppen model (Bak and Sneppen 1993). Here are two Bak-Sneppen models interlinked such that if the lowest fitness value of one market exceeds the fitness values of the other market minus transportation cost, all cells lower than this band will receive a new random value. The model shows that interdependency between markets has a strong effect on the competitiveness of the least competitive market. The external competition is able to make the least competitive market perform better as well as worse than on its own.**

Keywords— **Bak-Sneppen model, interdependency, competition.**

I. INTRODUCTION

Economic interdependence can be a consequence of specialization and the division of labour. The interdependence in an economic system has been known to economic theory at least since A.A. Cournot (1838) (beginning of Chapter XI) wrote:" *but in reality the economic system is a whole of which the parts are connected and react on each other. An increase in the incomes of the producers of commodity A will affect the demand for commodities B,C, etc., and the incomes of their producers, and, by its reaction will change the demand for commodity A*". The early recognition has however not had the consequence that the theoretical understanding of the dynamic effects of interdependence in economic systems are fully understood. Simultaneous interdependent actions are difficult to capture.

Supply chain networks can work as an example where economic interdependency is evident. These networks within the agricultural sector have received a lot of attention (Dooley and Akridge 1998; Darroch et al. 2002; Ioannou 2005). The exchange within the supply chain will often be between different companies that will seek to maximize their revenue within their sphere of interest, but may have little or no knowledge or interest in the remaining players in the supply chain. The competitiveness of the individual companies within a supply chain will however to some degree depend upon the actions and efficiency of other participants in the supply chain. The individual economic agent is entangled with the performance of the other agents in the chain. This paper investigates the effect of competition in a market consisting of interlinked economic agents. Particularly, the effect of increased competition from the surrounding markets is demonstrated.

II. THE MODEL

This work is an extension of a model originally used by Bak and Sneppen (Bak and Sneppen 1993) to model biological evolution. The nature of the BS model has sparked an interest in the scientific community beyond the area of biological evolution (Boettcher and Percus 1999; 2001; Danila et al. 2007; Simkin and Roychowdhury 2007) and a number of groups have applied the model to the study of economic systems (Cuniberti et al. 2001; Ausloos et al. 2004; Bartolozzi et al. 2006; Lu and Du 2007).

The BS model utilizes a circular lattice consisting of L sites to represent an ecological system. Each of the L sites represents a species and is assigned a parameter called fitness to describe its degree of adaptation with respect to the external environment. The notion "fitness" shall in this connection be understood in an economic context. This means that it does not refer to a specific economic notion such as the surplus of the farms but is an abstract expression of the units' economic ability (an analogy is the valuation of a firm through its stock price). The fitness values used in the model are defined in such a way that ascending values correspond to better fitness. The model consist in this case of a one-dimensional linear set of cells $l = 1,\ldots, L$

This geometry is chosen in order to have a simple and specific way of defining who is interacting with whom, however it is not important for the general principles behind the model.

Another property of the BS model is the spatial interdependency.

Each cell is assigned a stochastical independent number h_l (the fitness value) uniformly distributed on the interval $F = [V_{min} ; V_{max}]$, that is:

$$h_l \in U_n \left|\left[V_{\min}, V_{\max}\right]\right. \qquad (1)$$

At each time step, the cell with the smallest fitness value is found. Such as:

$$h_{l_{\min}} = \min_l h_l \qquad (2)$$

This cell and the two neighbouring cells (l_{min} -1), (l_{min} +1) are assigned a new fitness value.

The argument for also changing the two neighbouring cell values originates from its biological background (the different species are dependent on each other either in form of prey or predator). In the economic interpretation of the model, it is assumed that the farms are depending on each other as suppliers or buyers. The complex business relationship from real life is simplified to a foreseeable fixed structure, namely the two neighbours. The small communities are interlinked with the surrounding society through a small number of channels. Instead of trying to capture the structure of the real interactions taking place and thereby complicating the model considerably, this abstraction makes it possible to follow the development of structures caused by the interaction among the units.

The BS model is in spite of its rather minimalist composition, a model showing complex behaviour. After an extensive transient period, the distribution becomes (statistically) stationary. From numerical (Bak and Sneppen 1993) and analytical (Flyvbjerg et al. 1993) studies it has been shown that the values of the fitness evolve to a step function, characterized by a single value, the critical threshold value h^c. For $h< h^c$ the distribution of $P(h)$ is uniformly equal to zero while for $h>h^c$ we have $P(h)= 1/(1- h^c)$, determined by the normalization condition (Bartolozzi et al. 2006).

This does not mean that any value of h can be considered to be preserved. Even values higher than the critical threshold value will, because of the interdependency in the model, eventually become neighbour to a l_{min} -value and thereby change. The BS model helps investigate the dynamic evolution of interdependent economic agents subject to competitive pressure. The entanglement of firms in a supply chain network means that the development of a single firm is linked to the development of its network of interaction.

At the same time as individuals are competing internally on a local market is the local market as a whole subject to market forces from the surroundings. For simplicity we will assume that the influence of the other markets on the one studied can be seen as the effect of a single market. It is possible to estimate the boundaries of price fluctuations that the single market is subject to, by plotting the transportation cost from that market to similar trading points. Only when the price difference is large enough to cover the transportation cost will it be feasible to enter the market (here we are overlooking any strategic consideration which could justify price dumping). If only looking at a single market the transportation cost is therefore directly proportional to the room of manoeuvre relating to the price. However, because all the markets are interlinked, domino effects will appear, and the simple direct proportionality is loosing its grip.

To investigate the dynamic effect of competition between two spatially separated markets, the following model proposed: a model consisting of two separate BS models. $\text{Market}_{\text{high}}$ with a better point of origin than the $\text{market}_{\text{low}}$.

The fitness values ascribed to $\text{market}_{\text{high}}$ are random values uniformly distributed

$$h_{l(high)} \in U_n \left|\left[V_{\min} + X_{high} ; V_{\max} + X_{high}\right]\right. \qquad (3)$$

as $\text{market}_{\text{low}}$ consists of random values uniformly distributed

$$h_{l(low)} \in U_n \left| \left[V_{\min}; V_{\max} \right] \right. \qquad (4)$$

Besides the internal updating rule there is an external linking rule between the two markets.

$$\text{If} \qquad h_{l\,(market\ a)} < (h_{l(min)\,(market\ b)} - Y) \qquad (5)$$

then $h_{l(market\ a)}$ will receive a new fitness value. Where *a* and *b* can have the values *low* and *high* and *Y* is the span between the markets (covering e.g. transportation cost).

This means, if the lowest fitness value of one market exceeds the fitness values of the other market minus the value *Y* will all cells lower than this band receive a new random value. See illustration 1.

This simulates two markets developing on their own; however once the difference between them exceeds an entrance cost (*Y*) the competitive market oust the less competitive farms in the other market. This model can be used to understand the influence external markets can have on local development. By making comparative studies between the normal development of the single BS-model and a BS-model submitted to forces from outside. It is then possible to investigate the influence of the values X_{high}.

III. ANALYSIS OF THE "TWO-MARKET"-MODEL

First is a single BS-model with the size of the lattice *L*=100, and the fitness values uniformly distributed on the interval *F*=[1;100] presented.

In the single BS-model the average fitness value will as a result of the internal competition (selection process) rise to a given level, around which the system will fluctuate. The system can at this level be described as a punctuated equilibrium. At each time step is the economic ability of all the involved farms clear and it will seem possible for the individual farm to foresee its survival capacity. Some of the farms will because of the high competitiveness they have obtained feel secure. If there were no interdependency would their interpretation be right, however as interdependency plays a part the future competitiveness is less clear. The average fitness reached after the initial phase is interesting as the level of the (statistically) stationary state expresses the economic ability in a system only exposed to the internal competition.

This means that in an economic interpretation of the model the economic competitiveness in a market only influenced by the Bak-Sneppen selection process will the average fitness shift upwards from the random average value, and settle around an average fitness

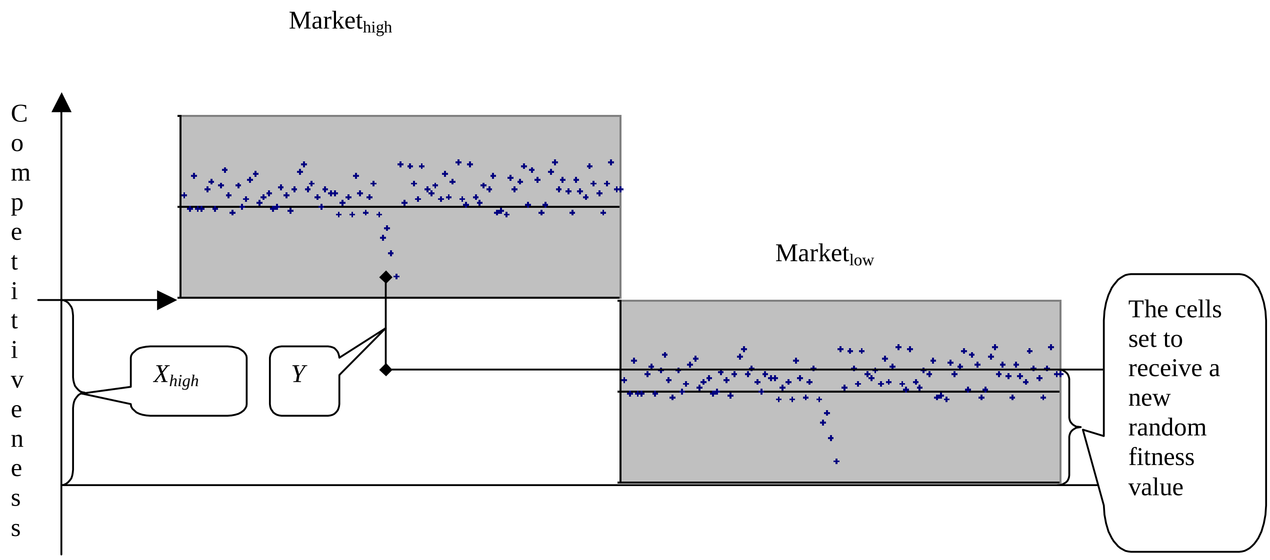

Illustration 1. Illustration of the two-market model

value of 78,586; see figure 1 (Damgaard 2002). The average value of the less competitive farm is 20,881.

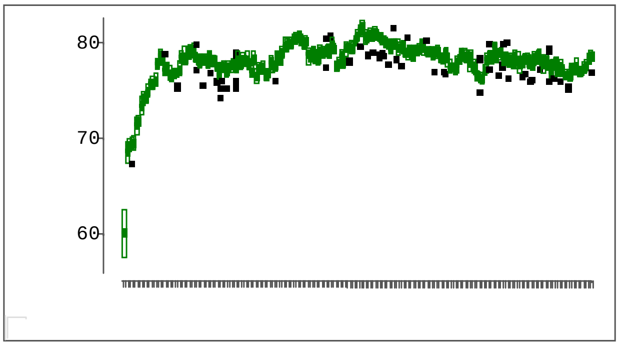

Figure 1. Mean value of the classic BS-model running for 10000 periods

The figures 1 to 8 is composed of the same elements that is: The x-axis represent time and y-axis is the fitness value (please note that the y-axis has different scales in the individual figures). The mean fitness of the *L* farms is represented with a green dot and the standard deviation is shown as bars (however hardly visible in figure 1). The black dots are outliers.

The two-market model investigates the effect competition between two spatially separated markets has on each other. The competition from the leading market will set a lower limit of economic fitness below which a unit is unable to survive. All farms with a lower competitiveness will be ousted by the market-leads.

The effect of different values of X_{high} in equation 3 will be investigated. Especially the development of market$_{low}$ as X_{high} varies will be shown. The margin *Y* that determines the band between the lowest unit of one market and the lowest of the other before receiving new random values, (see equation 5), is kept constant.

In this numerical experiment is the size of the lattice *L*=100, and the fitness values uniformly distributed on the interval *F*=[1;100]. The margin *Y*=25. The effect of X_{high} is presented by using the values $X_{high} \in \{40,50,60,70,80\}$. Market$_{high}$ is only presented in the two extremities as the interdependency has a very limited effect on its development. The expected values mentioned in the numerical explanation is referring to the results from the analysis of a single BS-model (Damgaard 2002) and the values found when no interaction from neighbouring markets took place.

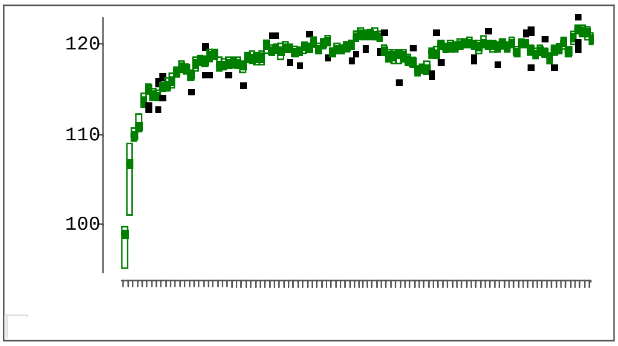

Figure 2. Two market model, market$_{high}$ X_{high}= 40, *Y*= 25, periods=5000

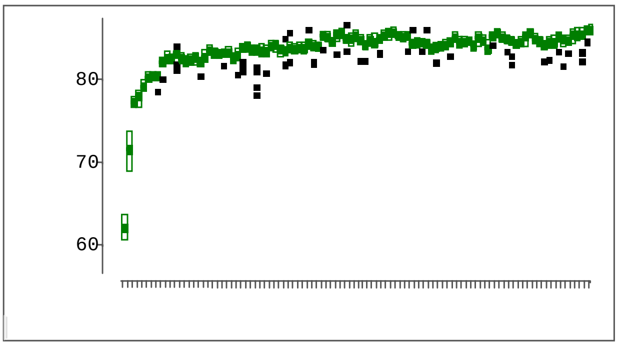

Figure 3. Two market model, market$_{low}$ X_{high}= 40, *Y*= 25, periods=5000

At X_{high}= 40 appears the development of the two markets to have the same structure as the pure BS-model (see figure 2 and 3). Market$_{high}$ is fluctuating around the expected level (with an average of 117, see figure 2). The value market$_{low}$ is fluctuating around a significantly higher level (85,031, see figure 3) than 78,586 as the internal selection process can account for. So Market$_{high}$ is pulling market$_{low}$ up and making it able to perform better than on its own. Market$_{low}$ meets the competition from market$_{high}$ and have through the interaction with market$_{high}$ achieved a better average competitiveness. The competition has strengthened market$_{low}$.

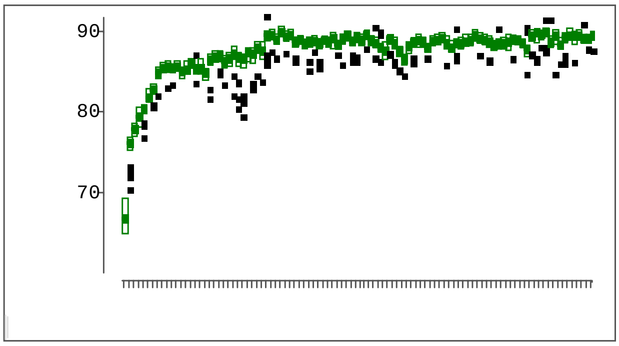

Figure 4. Two market model, $market_{low}$, X_{high}= 50, Y= 25, periods=5000

When X_{high} =50 (see figure 4) $market_{low}$ is approaching an even higher level, though there is also slightly larger variation in the points. The losing value of $Market_{high}$ will have an average on 45,881 for $market_{low}$.

The competition from $market_{high}$ is at a level where it generally helps $market_{low}$ to perform better (with an average value of 88,466, see figure 4). The growing variation indicates that parts of $market_{low}$ are starting to have difficulties to maintain the competitive capacity continuously.

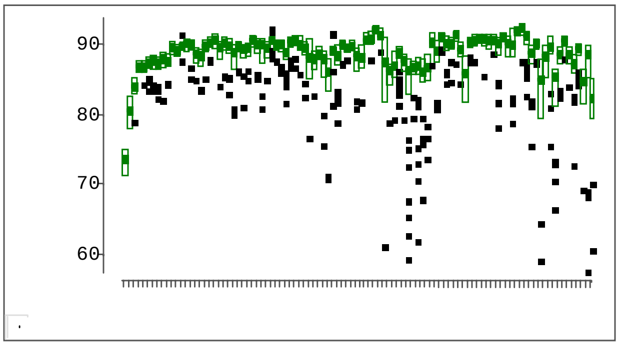

Figure 5. Two market model, $market_{low}$, X_{high}= 60, Y= 25, periods=5000

Figure 5 show $market_{low}$ when X_{high} =60. Again, $market_{low}$ is fluctuating on a higher level (88,214), but is conspicuously stressed by $market_{high}$'s influence, and unable to maintain the high level of competitiveness throughout the sequence. It is the low probability for cells to ascribe values high enough to be unaffected by $market_{high}$'s interventions that stresses $market_{low}$.

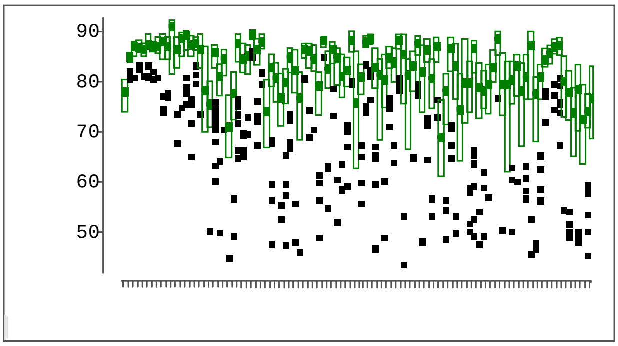

Figure 6. Two market model, $market_{low}$, X_{high}= 70, Y= 25, periods=5000

Figure 6 show $market_{low}$ when X_{high} =70. $Market_{low}$ is obviously hit hard by the influence of the dominating market. The average is now only a bit higher than the unaffected BS-model (80.004) and even in times fluctuating below this level.

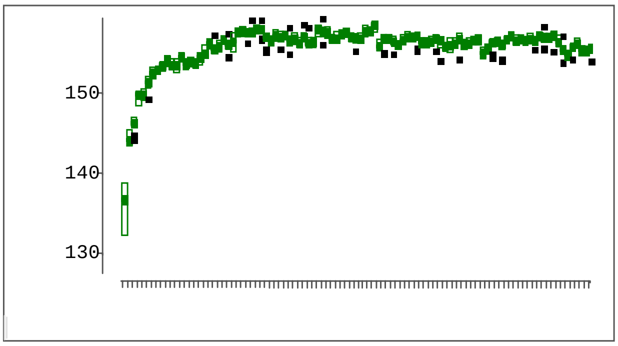

Figure 7. Two market model, $market_{high}$ X_{high}= 80, Y= 25, periods=5000

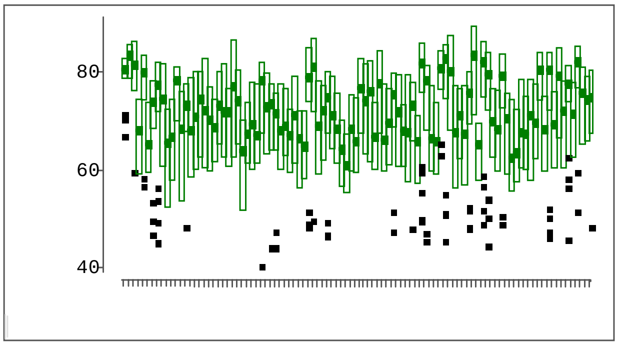

Figure 8. Two market model, $market_{low}$, X_{high}= 80, Y= 25, periods=5000

Figure 7 shows the development in $market_{high}$ for X_{high} =80 and figure 8 the same development for $market_{low}$. The development of $market_{high}$ is, as can be seen, having the same structure as the single market in the BS-model. Market$_{low}$ is not able to meet the competition from market$_{high}$. The average value of the looser of market$_{high}$ is too high for market$_{low}$ to develop their market without being interrupted. Market$_{low}$ is maintained underdeveloped at a level (70,611) obviously below the one expected in the single market version of BS-model. This sequence is showing how economic competition among markets is able to both lift markets up to a higher level of competitive capacity through their interdependency as well as maintain markets underdeveloped and at level of competitiveness below their actual potential.

In order to better demonstrate and quantify the effect of market$_{high}$'s influence on market$_{low}$'s development is the average value that the two markets are fluctuating around after entering the stationary state shown below. The precision of the individually calculated values is presented in (Damgaard 2002). All are based on 20000 periods and the span is kept constantly Y= 25.

Table 1. The average value that the two markets are fluctuating around after entering the stationary state at different points of origin, X_{high} for market$_{high}$. The values of the marked cells originate from figure 2 to 8.

X_{high}	Low	High
0	79,223	78,133
10	77,974	88,398
20	80,018	99,887
30	80,334	108,14
40	85,031	117
50	88,466	129,04
60	88,214	137,78
70	80,004	149,29
80	70,611	158,12
90	62,313	169,47

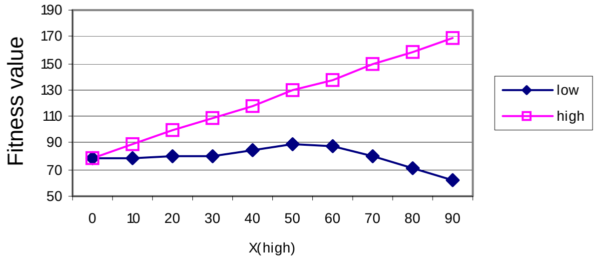

Figure 9. The average value that the two markets are fluctuating around after entering the stationary state at different points of origin, X_{high} for market$_{high}$.

The average fitness values of market$_{high}$ increase linearly with the interval of X_{high}. The competitiveness of market$_{high}$ is not at any point clearly affected by the interrelationship with market$_{low}$. By zooming in on market$_{low}$ it is seen in the following figure that the effect of this interaction on the other hand is having a strong influence on the average fitness value in market$_{low}$. The interdependency between the two markets is determining the level of competitiveness market$_{low}$ is able to obtain. The competition from market$_{high}$ needs to be at a high level (X_{high} [40;60]) for market$_{low}$ to reach its maximal potential. Market$_{low}$ is at this level sensitive for rising competition and if the competition gets stronger is market$_{low}$ unable to meet the challenge from market$_{high}$. The interdependency between the markets will for X_{high} higher than 70, cripple market$_{low}$.

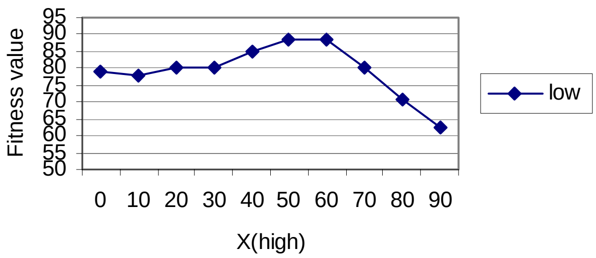

Figure 10. The average value market$_{low}$ is fluctuating around after entering the stationary state at different points of origin, X_{high} for market$_{high}$.

Not only is the average value affected by the competition from market$_{high}$ however also the internal structure of market$_{low}$ is affected. The change in the internal characteristics of market$_{low}$ gives a better understanding of the way the competition from market$_{high}$ hits market$_{low}$.

The spatio-temporal distribution of the selected l_{min} –the time dependent position of the loser, is affected by the interrelation among the markets. The number of periods the loser is found among the three cells that in previous period had their fitness value changed (the loser (l_{min}) and its neighbours (l_{min} -1) and (l_{min} +1)) is also dependent on the interrelation. The effect market$_{high}$ has on the interconnection among the cells in market$_{low}$ is evident when the length of the longest domino effect among cells are found for the different values of X_{high}.

Table 2. The maximum number of periods the loser is found among the three cells that in the previous period had their fitness value changed at different value of $X_{(high)}$

X_{high}	Low	High
0	28	26
10	29	32
20	34	29
30	41	32
40	66	32
50	39	25
60	22	32
70	13	32
80	7	26
90	4	33

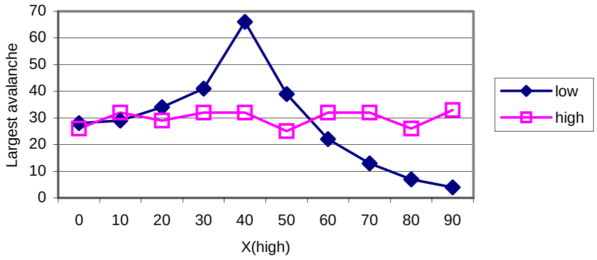

Figure 11. The maximum number of periods the loser is found among the three cells that in the previous period had their fitness value changed at different value of $X_{(high)}$

The maximum number of periods the loser is found among the three cells that in the previous period is reflecting the importance interdependency has for the development of the market. The maximum number of periods for $market_{high}$ is seen to fluctuate around the same level for all values of X_{high}, showing that the expected size of the maximum number of periods is almost constant for markets unaffected of outer influence.

The interdependency is manifesting itself in a more interesting manner for $market_{low}$. In $market_{low}$ the maximum number of periods the loser is found among the mentioned cells is obviously becoming longer as the competition from $market_{high}$ is raising. The higher the maximum number of periods is, the more spatially correlated is the selection of the worst performing farm.

Once the competition gets perceptible is the influence of $market_{high}$ reducing the value found in table 2. The importance of the interdependency among the farms in $market_{low}$ is sifted towards the interdependency between the two markets to play the most significant role. At $X_{high} > 40$ is the external market relation taking control over the development of $market_{low}$.

It is important to notice the shift, where the maximum on the curve is. The highest level of average fitness value was found for X_{high}=50 (X_{high}=60 had an almost similarly high fitness value), whereas the highest value in table 2 is found for X_{high}=40. This displacement exhibits that the internal structural changes in $market_{low}$ is taking place at a lower level of competition than it is reflected in the mean fitness values. The market will reorganise its focus point towards the competition coming from external markets before it will be reflected in its competitiveness. It seems natural to presume that the second derivative is zero close to X_{high}=40 corresponding to the point of inflection.

IV. SUMMARY

The "two-market"-model shows that interdependency between markets has a strong effect on the competitiveness of the least competitive market. The external competition is able to make the least competitive market perform better as well as worse than on its own. The interdependency among the markets affects the structure of the selection process in the least competitive market. The internal interdependency in the market increases with the markets competitiveness until the second derivative is zero. Then the interdependency falls off to a lower level than the market on its own.

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