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VECTOR AUTOREGRESSIONS, POLICY ANALYSIS, AND DIRECTED ACYCLIC GRAPHS: AN APPLICATION TO THE U.S. ECONOMY

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The paper considers the use of directed acyclic graphs (DAGs), and their construction from observational data with PC-algorithm TETRAD II, in providing over-identifying restrictions on the innovations from a vector autoregression. Results from Sims' 1986 model of the US economy are replicated and compared using these data-driven techniques. The directed graph results show Sims' six-variable VAR is not rich enough to provide an unambiguous ordering at usual levels of statistical significance. A significance level in the neighborhood of 30 % is required to find a clear structural ordering. Although the DAG results are in agreement with Sims' theory-based model for unemployment, differences are noted for the other five variables: income, money supply, price level, interest rates, and investment. Overall the DAG results are broadly consistent with a monetarist view with adaptive expectations and no hyperinflation.

JEL classification codes: C1, E1
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I. Introduction

Vector autoregressions (VARs) are widely used in empirical research because of their humility with respect to zero restrictions and assumed knowledge of the way the world actually works. Some (Cooley and Dwyer,

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1998, Cooley and LeRoy, 1985, Leamer, 1985) have argued that, while VAR models may be useful for forecasting, they are not appropriate for policy analysis. As VARs (as usually applied) represent summaries of the correlation structure embedded in observational data (non-experimental data), they cannot be interpreted independently of a maintained structural model. In other words, for policy interpretations, the humility referred to in our opening sentence must be forgone in favor of explicit zero-type restrictions on at least some components of the VAR. In this paper we consider identifying restrictions on relationships among contemporaneous innovations.[1] The now common use of the Choleski decomposition to provide such restrictions is sometimes deemed inadequate because it imposes a just-identified contemporaneous structure that is not necessarily supported by economic theory or by the causal structure embedded in the data. Accuracy of policy inferences drawn from such analysis is therefore conditional on the validity of the maintained hypothesis of a particular just-identified structural form.

Sims (1986) and others have noted that when there is contemporaneous correlation among variables, the choice of an ordering in the Choleski decomposition may make a significant difference for interpretation of impulse responses and forecast error variance decompositions**.** As an alternative to the Choleski decomposition, some researchers (Sims, 1986; Bernanke, 1986; Blanchard and Quah, 1989; Leeper, Sims and Zha, 1996; Hess and Lee, 1999 and Kim, 2001) suggest the use of orthogonalizations that allow the researcher to impose over-identifying restrictions on the model. We follow the literature and label these models as Structural Vector Autoregressions (SVARs) as they rely on prior theory as the source of their identifying restrictions. Bernanke's approach achieves identification via the assumption that distinct, mutually orthogonal, behavioral shocks drive the model, and that lagged relationships among the variables are not restricted. The "Bernanke decomposition" relaxes

[1] More general identification restrictions could be considered on both contemporaneous innovations, as well as (subset restrictions) on lagged values of the variables in the VAR. The approach used in this paper follows that of Sims (1986) and Bernanke, where we focus on restrictions on the relationships among contemporaneous innovations. We should point out that directed graphs could be used for the more general identification problem, as well as for the restricted case considered here (see Pearl, 2000, for a discussion of identification and directed acyclic graphs).

the assumption of a just-identified structure for the VAR innovations; it requires imposing a particular causal ordering of the variables. This imposition may itself be arbitrary, as theory may not always yield a clear identifying structure.

In an often-cited paper, Sims (1986) showed that a VAR model on the U.S. economy could be used for policy analysis if appropriate identifying restrictions are imposed. He achieved identification by using two factorizations. First, he used a Choleski decomposition that imposed a just-identified structure. Second, he applied a more flexible identification method based on economic theory that relaxes the assumption of a just-identified structure for the economy.

Cooley and Dwyer (1998) argue that although VARs are attractive research tools for characterizing the dynamic relationships among variables without having to invoke economic theory restrictions, SVARs "are certainly not invariant to the identifying assumptions and may not be reliable as vehicles for identifying the relative importance of shocks." Sims' (1986) work is not exempt from this observation, as (apparently) he based his identifying constraints on subjective (non data-based) considerations. Here we investigate whether Sims' (1986) results continue to hold when a less subjective, more data-driven approach is applied to achieve an identifying interpretation of his six variable VAR on the U.S. economy. Specifically, identification is achieved by modeling the contemporaneous innovations from Sims' (1986) VAR model with directed acyclic graphs, as recently presented in Spirtes, Glymour, and Scheines (1993). These models are based on screening-off (to be explained below) characteristics present in correlations and partial correlations involving three or more variables.

The approach investigated here is one extreme, of allowing the data to provide motivation behind the over-identifying restrictions in structural VAR models. The approach is very much in the spirit of one of several uses of VARs discussed by Cooley and LeRoy (1985) and others. Cooley and LeRoy (1985, p. 288) write: "One can, of course reverse the sequence of theorizing and empirical testing. That is, econometricians can use VAR models to generate stylized facts about the causal orderings of macroeconomic variables that seem to be robust empirically. Then theorists would try to explain these patterns." This is not to say that DAGs have nothing to offer for more theoretically-based hypothesis testing with VAR models. Only that, at a minimum, understanding the "screening-off" characteristics present in a set of VAR innovations may be helpful in thinking about the mechanism that generated the data and in planning for future policy modeling with that data.

Results indicate that achieving model identification through the use of directed acyclic graphs can yield plausible and theoretically consistent impulse response functions that can be used in policy analysis. The paper is presented as follows. The next section examines a standard VAR model and the implications of the identification restrictions. We follow this with a brief introduction to directed acyclic graphs and recent algorithmic results of Spirtes, Glymour, and Scheines (1993). Sims' (1986) policy model is then summarized and we offer a reconsideration of his model using directed acyclic graphs. A conclusion follows.

II. VAR Models and Identification

For a given vector of historical data X_t, a VAR can be expressed as:

$$X_t = \sum_{i=1}^{k} B_i X_{t-i} + C Z_t + u_t \tag{1}$$

where X_t and u_t are both (m x 1) random vectors, Z_t is a (q x 1) vector of non-stochastic (or strictly exogenous) variables, and B_i and C are appropriately dimensioned matrices of coefficients. The innovation term u_t is assumed to be white noise, where $E(u_t) = 0$, $\Sigma_u = E(u_t u_t')$ is an (m x m) positive definite matrix. The innovations u_t and u_s are independent for $s \neq t$. Although serially uncorrelated, contemporaneous correlation among the elements of u_t is possible. These observed innovations are mongrel, as they are combinations of more basic "structural" or driving sources of variation in the data. Following Bernanke, these driving sources of variation are themselves orthogonal and can be written as:

$$e_t = A u_t \tag{2}$$

Here zero restrictions on A are investigated to obtain an identified structural VAR.

Generally speaking, there are no easy counting rules for identifying *A*, but for a VAR in m variables if we leave more than m (m - 1) / 2 parameters free (to be estimated) the model is not identified. Doan (1993, pp. 8-10) suggests the following rule: if there is no combination of i and j ($i \neq j$) for which both A_{ij} and A_{ji} are nonzero, the model is identified. Usual innovation accounting procedures (impulse response, forecast error decompositions and historical decompositions) can be carried-out on the transformed VAR:

$$A X_t = \sum_{i=1}^{k} A B_i X_{t-i} + A C Z_t + A u_t \tag{3}$$

This paper's contribution is in the application of the directed acyclic graphs as an aid to identifying structural VAR models. Before discussing model specification and estimation, a brief overview of directed acyclic graphs is presented.

III. Directed Acyclic Graphs (DAGs)

Directed acyclic graphs exploit a non-time sequence asymmetry in causal relations. Consider a causally sufficient set of three variables *X*, *Y*, and *Z*. We illustrate a causal fork, *X* causes both *Y* and *Z*, as: $Y \leftarrow X \rightarrow Z$. Here the unconditional association between *Y* and *Z* is nonzero (as both *Y* and *Z* have a common cause in *X*), but the conditional association between *Y* and *Z*, given knowledge of the common cause *X*, is zero: a common cause screens-off association between its joint effects. Illustrate the inverted causal fork, both *X* and *Z* cause *Y*, as: $X \rightarrow Y \leftarrow Z$. Here the unconditional association between *X* and *Z* is zero, but the conditional association between *X* and *Z* given the common effect *Y* is not zero: a common effect does not screen-off association between its joint causes. These screening-off attributes of causal relations are captured in the literature of directed graphs.[2]

A directed graph is a picture representing the causal flow among a set of variables. More formally, it is an ordered triple $< V, M, E >$ where *V* is a

[2] Orcutt (1952), Simon (1953), Reichenbach (1956) and Papineau (1985) offer more detailed discussion of these screening-off asymmetries in causal relations. For a description of other causal asymmetries see Hausman (1998).

non-empty set of vertices (variables), M is a non-empty set of marks (symbols attached to the end of undirected edges), and E is a set of ordered pairs. Each member of E is called an edge. Vertices connected by an edge are said to be adjacent. If we have a set of vertices $\{A, B, C, D\}$: (i) the undirected graph contains only undirected edges (e.g., A—B); (ii) a directed graph contains only directed edges (e.g., $B \rightarrow C$); (iii) an inducing path graph contains both directed edges and bi-directed edges ($C \leftrightarrow D$); (iv) a partially oriented inducing path graph contains directed edges ($\rightarrow$), bi-directed edges ($\leftrightarrow$), non-directed edges (o — o) and partially directed edges (o $\rightarrow$). A directed acyclic graph is a directed graph that contains no directed cyclic paths (an acyclic graph contains no vertex more than once). Only acyclic graphs are used in the paper.

Directed acyclic graphs are designs for representing conditional independence as implied by the recursive product decomposition:

$$Pr\,(x_1, x_2, x_3, \ldots x_n) = \prod_{i=1}^{n} Pr\,(x_i \mid pa_i) \tag{4}$$

where Pr is the probability of vertices $x_1, x_2, x_3, \ldots x_n$ and pa_i the realization of some subset of the variables that precede (come before in a causal sense) X_i in order ($X_1, X_2, \ldots, X_n$). Pearl (1995) proposes d-separation as a graphical characterization of conditional independence. That is, d-separation characterizes the conditional independence relations given by equation (4). If we formulate a directed acyclic graph in which the variables corresponding to pa_i are represented as the parents (direct causes) of X_i, then the independencies implied by equation (4) can be read off the graph using the notion of d-separation (defined in Pearl, 1995):

Definition: Let X, Y, and Z be three disjoint subsets of vertices in a directed acyclic graph G, and let p be any path between a vertex in X and a vertex in Y, where by "path" we mean any succession of edges, regardless of their directions. Z is said to block p if there is a vertex w on p satisfying one of the following: (i) w has converging arrows along p, and neither w nor any of its descendants are on Z, or, (ii) w does not have converging arrows along p, and w is in Z. Further, Z is said to d-separate X from Y on graph G, written $(X \perp\!\!\!\perp Y \mid Z)_G$, if and only if Z blocks every path from a vertex in X to a vertex in Y.

Geiger, Verma, and Pearl (1990) show that there is a one-to-one correspondence between the set of conditional independencies, $(X \perp\!\!\!\perp Y \mid Z)$, implied by equation (4) and the set of triples (*X*, *Y*, *Z*) that satisfy the d-separation criterion in graph *G*. Essential for this connection is the following result: if *G* is a directed acyclic graph with vertex set *V*, *A* and *B* are in *V*, and *H* is also in *V*, then *G* linearly implies the correlation between *A* and *B* conditional on *H* is zero if and only if *A* and *B* are d-separated given *H*.

Spirtes, Glymour, and Scheines (1993) have incorporated the notion of d-separation into an algorithm (PC algorithm) for building directed acyclic graphs, using the notion of sepset (defined below).

The PC Algorithm is an ordered set of commands which begins with a general unrestricted set of relationships among variables and proceeds step-wise to remove edges between variables and to direct "causal flow." The algorithm is described in Spirtes, Glymour, and Scheines (1993, p. 117). Refinements are described as the Modified PC Algorithm (Spirtes, et al., p. 166), the Causal Inference Algorithm (p. 183), and the Fast Causal Inference Algorithm (p.188). We restrict our discussion to PC algorithm, since the basic definition of a sepset is used in all and PC Algorithm is the most basic.

Briefly, one forms a complete undirected graph *G* on the vertex set *V*. The complete undirected graph shows an undirected edge between every variable of the system (every variable in *V*). Edges between variables are removed sequentially based on zero correlation or partial correlation (conditional correlation). The conditioning variable(s) on removed edges between two variables is called the sepset of the variables whose edge has been removed (for vanishing zero order conditioning information the sepset is the empty set). Edges are directed by considering triples *X*—*Y*—*Z*, such that *X* and *Y* are adjacent, as are *Y* and *Z*, but *X* and *Z* are not adjacent. Edges between triples: *X*—*Y*—*Z* are directed as: $X \rightarrow Y \leftarrow Z$, if *Y* is not in the sepset of *X* and *Z*. If $X \rightarrow Y$, *Y* and *Z* are adjacent, *X* and *Z* are not adjacent, and there is no arrowhead at *Y*, then orient *Y*—*Z* as $Y \rightarrow Z$. If there is a directed path from *X* to *Y*, and an edge between *X* and *Y*, then direct (*X*—*Y*) as: $X \rightarrow Y$.

In applications, Fisher's *z* is used to test whether conditional correlations are significantly different from zero. Fisher's *z* can be applied to test for significance from zero; where $z(\rho(i, j/k)\, n) = 1/2\, (n - |k| - 3)^{1/2}\, ln \{(|1 + \rho(i, j/k)|)\, (|1-\rho(i, j/k)|)^{-1}\}$ and *n* is the number of observations used to estimate the correlations, $\rho(i, j/k)$ is the population correlation between series *i* and *j* conditional on series k (removing the influence of series k on each i and j), and $|k|$ is the number of variables in k (that we condition on). If i, j, and k are normally distributed and $r(i, j/k)$ is the sample conditional correlation of i and j given k, then the distribution of $z(\rho(i, j/k)\, n) - z(r(i, j/k)\, n)$ is standard normal.

PC Algorithm can commit type I and type II errors on both edge existence (it can fail to include an edge when it should include it and can include an edge when it should not) and edge direction (it may fail to put an arrowhead at vertex *A* when it should put it at vertex *A* and it may put an arrowhead at *A* when, in fact, it should not have put an arrowhead there). Spirtes, Glymour, and Scheines (1993) have explored several versions of PC Algorithm on simulated data with respect to errors on both edge inclusion (yes or no) and direction (arrowhead at *A* or not). They conclude that there is little chance of the algorithm including an edge that is not in the "true" model. However, there is, with small sample sizes (less than say 200 observations) considerable chance that the algorithm will omit an edge that belongs in the model. Further, arrowhead commission errors (putting an arrowhead where it does not belong) appear to be more likely than edge commission errors (putting an edge where it does not belong). Accordingly, the authors conclude: "In order for the method to converge to correct decisions with probability 1, the significance level used in making decisions should decrease as the sample size increases, and the use of higher significance levels (e.g. 0.2 at sample sizes less than 100, and 0.1 at sample sizes between 100 and 300) may improve performance at small sample sizes." (Spirtes, Glymour, and Scheines, 1993, p. 161).

Applications of directed graphs to VAR model identification are not commonplace. A similar procedure has been suggested in Swanson and Granger (1997). Their procedure considers only first order conditional correlation, and involves more subjective insight by the researcher to achieve a "structural recursive ordering." One advantage of using this method of analysis is that results based on properties of the data can be compared to a priori knowledge of a structural model suggested by economic theory or subjective intuition.

IV. Illustration Using Sims' (1986) Model

To examine the importance of using a data-determined method for achieving identification of a VAR model, we estimated Sims' (1986) six variable quarterly model of the U.S. economy using two different identification methods. One model uses the standard Sims' (1980) VAR methodology where identification is achieved via use of Choleski factorization procedure. The second model uses a modification of the Bernanke factorization where contemporaneous causal path of the model innovations is determined via use of directed graphs.

The model is estimated in log levels (except interest rates and unemployment rate, which are in levels) over the period 1948/1-1979/3. The estimation period is truncated at 1979/3 to avoid the likely need for modeling the shift in money supply behavior around 1979/4, and to allow for direct comparisons of current results with Sims' (1986). The variables in the VAR system are real GNP (*Y*), real business investment (*F*), GNP price deflator (*P*), the M1 measure of money (*M*), unemployment (*U*), and Treasury-bill rates (*R*). All measures are the same as those used in Sims (1986). Four quarterly lags on each variable and a constant term are used.

The lower triangular elements of the correlation matrix (*corr*) on innovations (errors) from the four-lag VAR, fit to 127 data points, are given as equation (5). Here we list, in lower case letters, the equation innovations for each column across the top of the matrix: y = innovations in income, f = innovations in investment, p = innovations in price, m = innovations in money, u = innovations in unemployment, and r = innovations in interest rates.

$$
\begin{array}{cc}
 & \begin{array}{cccccc} y & f & p & m & u & r \end{array} \\
corr = & \begin{bmatrix}
1.000 & & & & & \\
.518 & 1.000 & & & & \\
.004 & .002 & 1.000 & & & \\
.355 & .146 & .209 & 1.000 & & \\
-.647 & -.452 & -.194 & -.329 & 1.000 & \\
.045 & .162 & -.022 & -.039 & -.173 & 1.000
\end{bmatrix}
\end{array}
\qquad (5)
$$

It is this matrix that drives the TETRAD II search for underlying restrictions on contemporaneous innovations.

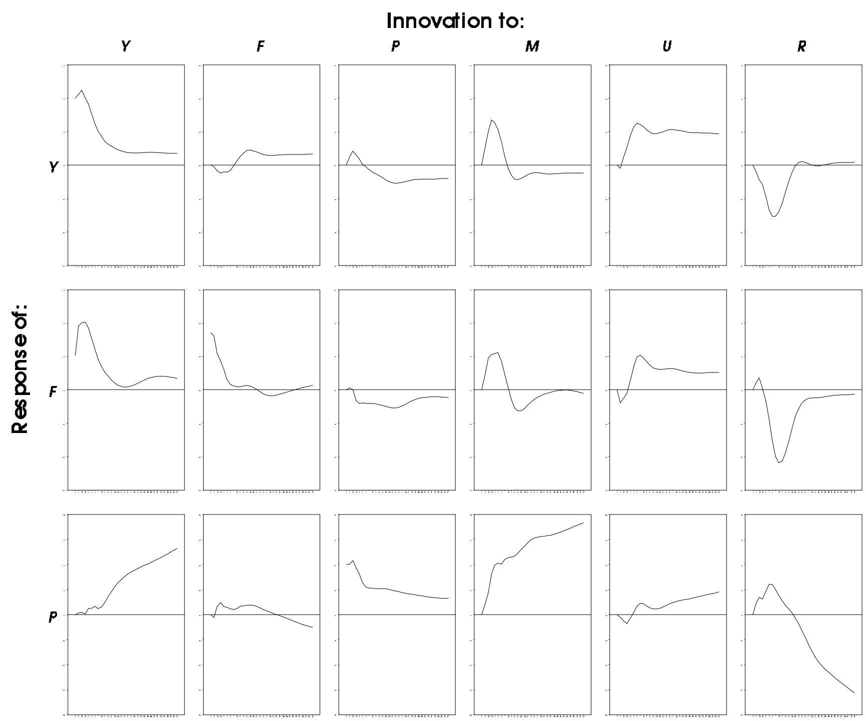
A. Model Identification Assuming a Just Identified Structure

In the first model the VAR is specified as in Sims' (1986). This allows us to replicate his impulse response functions based on a Choleski factorization (see Sims, 1986, chart 1). The variables are ordered as follows: output, investment, prices, money, unemployment rate, and interest rate. The impulse response functions obtained from this first model are presented in Figures 1.A and 1.B. Our VAR model's results and Sims' Choleski results are essentially identical. However, as we do not place instrumental priors on our VAR, responses from our Choleski decomposition will not be identical to those found in Sims (1986). Each small graph represents the response of a variable in a given row to a one-standard-deviation innovation in a variable in a given column over 32 consecutive quarters.

The dynamic effects of a (non-monetary) shock in output on real and nominal variables are presented in column 1. Positive output innovations increase output, investment, and interest rates, but decrease unemployment for about 10 quarters. An unemployment shock, column 5, is interpreted as a labor supply disturbance by Sims, capturing the complex dynamics of varying labor-force participation rate. Labor supply innovations have positive effects on output with steady increase in the first four quarters; thereafter, output remains at the higher level. While the level of unemployment rises temporarily, it returns to normal in about 8 quarters. Investment response is similar to that of output, while growth in prices is moderate. Money stock increases smoothly and remains at the higher level. The short-term interest rate is approximately constant, initially declining for a brief period then quickly returning to equilibrium levels.

Responses to money innovations are given in column 4. Real variables, income, investment, and unemployment show short-run responses, which do not persist over the long run. Money and prices show persistent long-run responses to money innovations. The delayed positive response of prices appears to be consistent with either adaptive expectations behavior or sticky prices, a point which, apparently, led Sims to suggest that commodity prices (prices set in auction markets) be added to the model to help sort-out the alternative expectations hypotheses. The weak response of real variables,

Figure 1. A. Impulse Response Functions Based on Choleski Decomposition

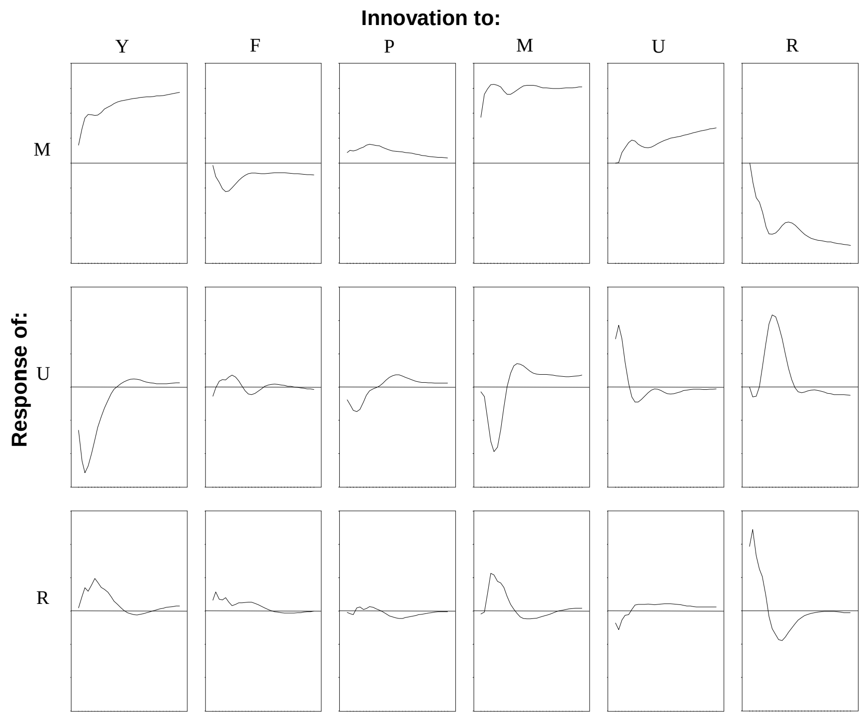


however, leads Sims (1986) to question whether these responses are consistent with a rational expectations monetarist theory. He notes: "the weakness of the real responses does not fit rational expectations monetarist theory well."

A positive shock to interest rates yields a notable temporary decline in output, which returns to its normal level after about 12 quarters. Prices temporarily increase for about 6 quarters, and thereafter decline persistently. A strong and persistent negative response of money stock is also observed in response to innovation in interest rates. The unemployment rate momentarily declines then rises sharply for about 12 quarters before finally returning to normal.

Overall the impulse responses summarized in Figures 1.A and 1.B appear to be generally consistent with a monetarist's view of the economy with

Figure 1. B. Impulse Response Functions Based on Choleski Decomposition



adaptive expectations (with no hyperinflation). Real variables show weak responses to money supply shocks; while prices show a persistent positive lagged response. Output responds positively and most strongly to shocks in employment.

The Choleski-generated responses are based on the contemporaneous causal ordering: innovations in output cause innovations in investment, innovations in investment cause innovations in prices, innovations in prices cause innovations in money, innovations in money cause innovations in unemployment, and innovations in unemployment cause innovations in interest rates. As an alternative to the Choleski-based responses, Sims (1986) considers theory-based interactions among innovations using the Bernanke factorization of contemporaneous correlations. Below we consider interrelations among these innovations based on directed graphs.

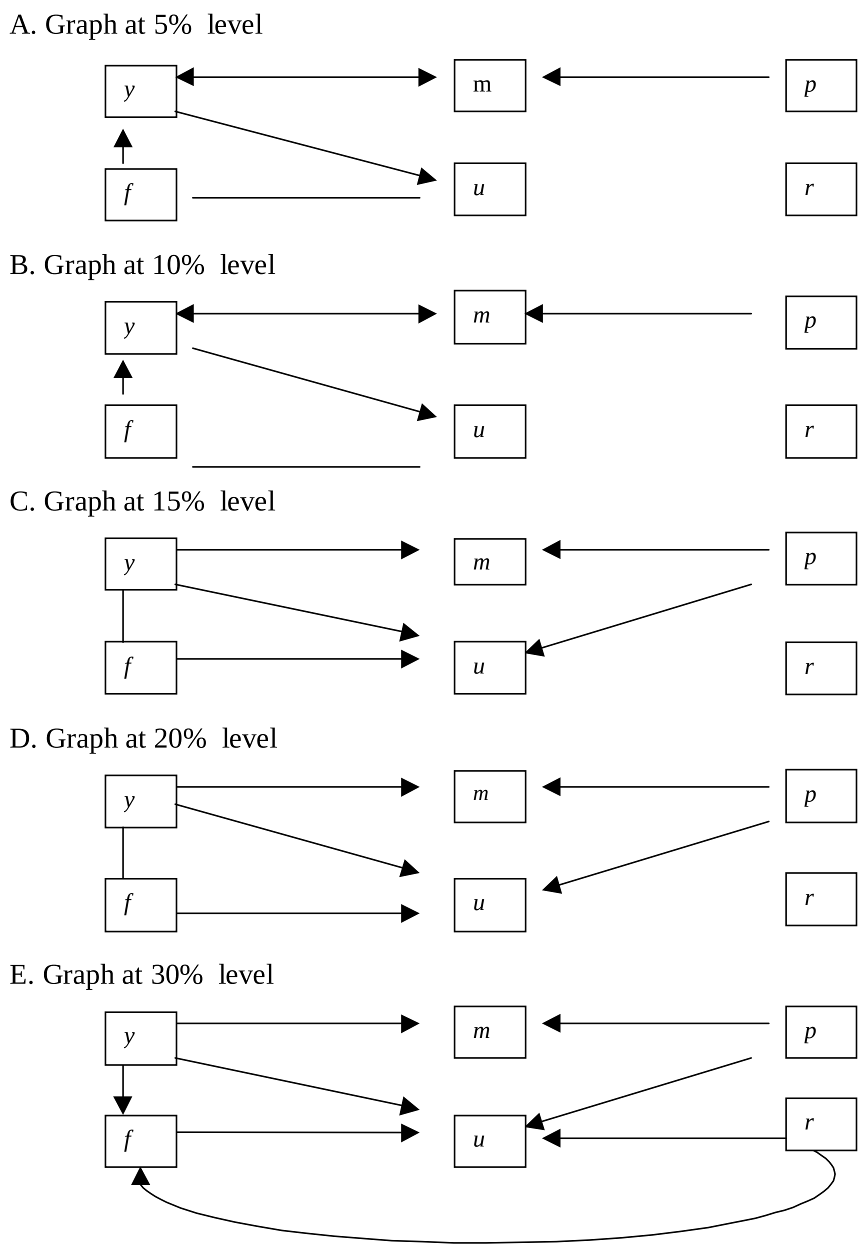
B. Directed Graph Results

The innovation correlation matrix given by equation (5) is used as the starting point for our analysis of the innovations from Sims' six-equation VAR. TETRAD II is applied to these correlations. As suggested by Spirtes, Glymour, and Scheines (1993), various levels of significance are considered in an attempt to achieve an unambiguous causal structure of the variables in contemporaneous time. Figure 2 presents graphs on innovations from Sims' (1986) six variable VAR at the following nominal levels of significance: 0.05, 0.10, 0.15, 0.20, and 0.30. As the TETRAD II search algorithm involves multiple hypothesis testing for edge removal, the final significance level is generally larger than that reported as nominal. At the 5 % and 10 % significance levels the directed edges are found as given in Panels A and B. The resulting graphs are identical, indicating directed edges from investment and money to output, and from output to money and unemployment. Directed edges are also observed running from prices to money and from money to output. However, the relationship between investment and unemployment is ambiguous, since there is an undirected edge connecting these variables (there is a relationship between investment and unemployment, but we cannot say which variable is causal).

Given the ambiguity in results at these low levels of significance, higher levels of significance of 15 % and 20 % are considered. These are given in Figure 2, Panels C and D. Although a directed edge from investment to unemployment is obtained at both of these higher levels, there is now an undirected edge between investment and output. Economic theory could be used as in Sims (1986) to direct this ambiguous causal path, but the approach will then be subject to the earlier criticism of arbitrariness. Interestingly, interest rates do not enter the system in any of the directed graphs in Panels A-D. The directed edges between prices and money, output and money, output and unemployment, and prices and unemployment seem to be stable across the 15-20 % significance levels.

Finally, as reported in Panel E, an unambiguous causal ordering is found at the 30 % level of significance. Innovations in output cause innovations in money, investment, and unemployment. Innovations in prices cause innovations in money and unemployment, while innovations from investment

Figure 2. Specification Search Using TETRAD II



cause innovations in unemployment. Innovations in interest rates cause innovations in investment.

Although 30 % is a rather high significance level, it does merit discussion, as it is the lowest significance level considered which gives us an unambiguous directed graph. The alternative of using levels of say 5 % or 10 % is to conclude that the data on this six variable model are not rich enough to sort out a clear causal graph. This alternative is certainly worth considering as it is a contribution to demonstrate that Sims' (1986) six variable model does not yield a definite ordering using our directed graph techniques. However, offering the "first" unambiguous ordering in a search over alternative levels of significance allows the researcher to quantitatively assess the robustness of his/her results with respect to significance levels.

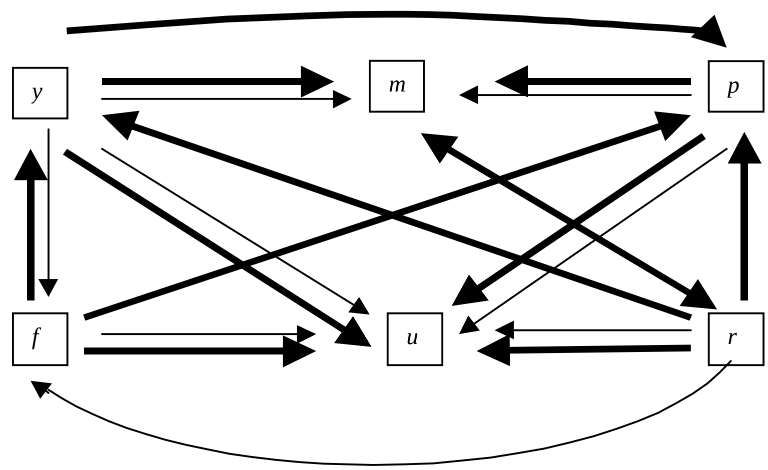
Further, Scheines et al. (1994) recommend that users of their algorithm should "vary the significance level to obtain an idea of how robust the results are. The program tends to underfit -that is, to include too few edges- at small samples. Increasing the significance level makes it easier for the program to retain edges between variables" (Scheines et al. 1994, p. 105). Given that only 127 quarterly data observations are used for this study, the suggestion to use higher significance levels is relevant in this case (although readers may suggest that our stretching their suggestion to 30 % is a priori unreasonable).

In addition to the Choleski-generated responses, Sims (1986) considers restrictions to produce theory-based impulse responses. Here he considers two models where innovations in interest rates, investment, money, prices, and output are components of the demand and supply for money. Figure 3 presents the directed graph representation of these two alternative identifications used by Sims (1986). Panel A outlines his first identification, while Panel B represents his second case. For ease of comparison, Sims' (1986) two identification scenarios, in thick bold lines, are superimposed on our model identification from Figure 2 (Panel E).

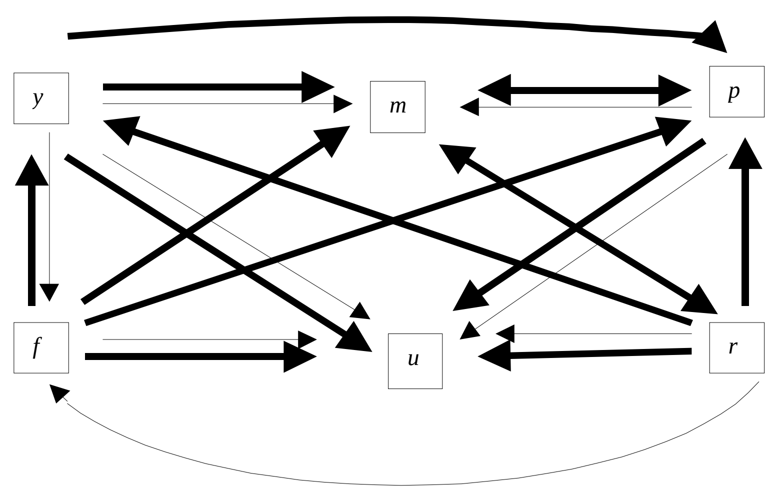
Although Sims' (1986) identification restrictions are based on economic theory and those for this study are based on data patterns, both approaches have similarities in the resulting causal structure. From Figure 3, it can be seen that both identifications allow innovations in money to respond to innovations in output and prices. The unemployment equation allows unemployment to depend on output, investment, interest rates, and prices.

Figure 3. Sims'Identifications (Thick Lines) and DAG Identification (Thin Lines)

A. Sims' 1st Identification Chart and DAG at 30% level



B. Sims' 2nd Identification Chart and DAG at 30% level



However, in both panels, Sims' (1986) theory-based identifications offer several extra causal connections that seem to lack support from the data. For instance, in both identification cases, Sims (1986) suggests that innovations from interest rates cause innovations in all other variables, except investment. In contrast, the TETRAD II-based identification finds innovations in interest rates cause innovations in investment and unemployment only. Recall that a fairly high level of significance had to be used to find theses edges. Notice too that TETRAD II finds an edge running from output to investment; whereas, Sims' (1986) two alternative identifications yield the opposite causal flow; investment cause income in contemporaneous time.

Further, Sims (1986) specifies bi-directional arrows between *m* and *p* (second identification, Figure 3) and between *m* and *r* (first and second identification, Figure 3). Recall that our TETRAD II-based directed graphs too (Figure 2) resulted in bi-directional arrows (at the 5 % and 10 % levels of significance we saw *y* and *m* were bi-directed), which suggests the possibility of an omitted variable(s) or an equilibrium or feedback process.[3]

The directed edge which Sims (1986) places between innovations in interest rates (*r*) and income (*y*) does not show-up using TETRAD II, as the zero-order correlation (unconditional correlation) between innovations in interest rates and income is 0.04, with an associated p-value of 0.62 –more than double the highest-level p-value entertained in our application of TETRAD II–. Furthermore, the edge between innovations in income (*y*) and price (*p*), which Sims (1986) includes in his structural identification, does not appear in the TETRAD II model as the p-value on this edge is 0.97. In addition, Sims (1986) places edges between innovations in prices (*p*) and innovations in interest rates (*r*) and innovations in money supply (*m*) and interest rates (*r*). Zero-order correlations between these have p-values of 0.81 and 0.67, respectively, suggesting little data-generated support for these edges.

The identifying restrictions suggested by TETRAD II's graph in Figure 2, Panel E, were tested using the likelihood ratio test for over-identification as given in Doan (pp. 8-10). Given a six variable system, there are 15 lower triangular elements which can be non-zero in a just identified model, i.e.,

[3] We do not model feedback or equilibrium processes. The reader is directed to Richardson and Spirtes (1999) for a computational algorithm that can handle such cyclic graphs.

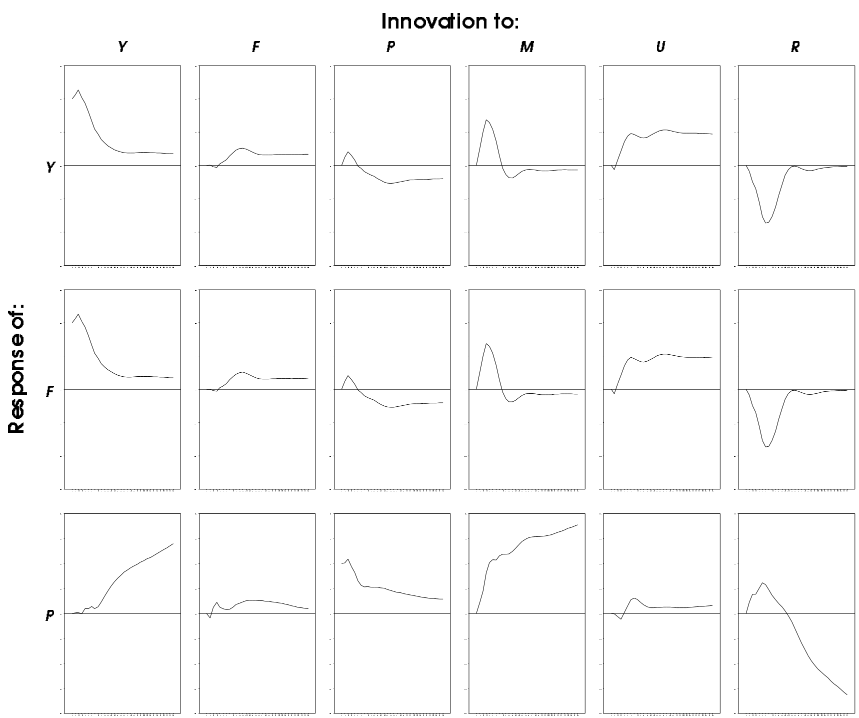
with m equal to the number of series in the VAR, we have m (m - 1) / 2 free parameters. The directed graph restrictions result in a chi-squared statistic of 2.37. With 7 degrees of freedom, we reject these zero restrictions at a p-value of 0.94, suggesting that the restrictions are consistent with the data. (We did not test Sims' (1986) ordering as it does not meet the simple Doan condition for identification).

While the above analysis suggests that several of the edges in Sims' identified model are questionable, the TETRAD II results are not without ambiguity. Probably most noticeable from Figure 2 are the reversal of causal direction as we change level of significance. At low levels of significance (0.05 and 0.10) we see that investment innovations (*f*) cause income innovations (*y*); while at the higher level of significance (0.30) we see just the opposite, innovations in investment (*f*) cause innovations in income (*y*). Furthermore, we see a bi-directed edge between innovations in income (*y*) and innovations in money (*m*) at low levels of significance; while at higher levels of significance the edge between *y* and *m* is directed as: $y \rightarrow m$. Such edge reversals are of course unsatisfying and point us in two directions. First, if we want to maintain the posture, outlined at the beginning of the paper, of relying primarily on data-based identifications, the ambiguity suggests additional data points to provide more precision on estimates of correlation and partial correlation structure. A second direction, which moves us away from our focus on databased identifications, is to rely on prior theory. Swanson and Granger (1997, p. 360) note in a discussion of their similar "structural identification procedure" that the issue of "reversibility" of causal direction among variables is "just an artifact of the contemporaneous nature of the correlation constraints that are tested." To resolve such ambiguities they suggest the use of prior knowledge based on economic theory to choose between two alternate models (1997, p. 363).

C. Innovation Accounting with the TETRAD II Suggested Structure

Figures 4.A. and 4.B. present the impulse response functions for the model identified via directed graph results. A positive shock in output (column 1) results in persistent increases in prices and money and a short-term negative response in unemployment. Comparing these responses with the responses

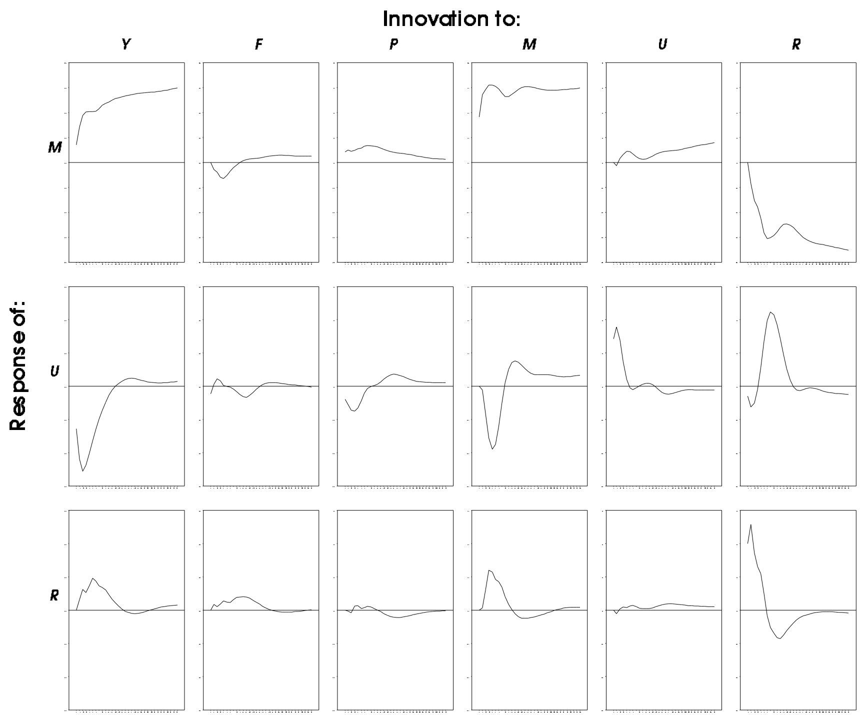
Figure 4.A. Impulse Response Functions Based on DAG at 30% Significance Level



of the same variables to innovations in investment (column 2), we see different patterns, suggesting that other components of output (consumption and government spending) may be responsible for the persistent long-run movements in prices and money and the short-term negative response of unemployment. Although, not having their measures (consumption and government spending) in this study, we cannot say more than the differences in responses are suggestive.

Positive money innovations (column 4) increase investment and output for the first 8 quarters then returning to normal within two years. Innovations in money result in sustained positive response in prices. Interest rates also respond positively in the first 3 quarters or so, thereafter returning to normal levels. Unemployment initially declines in the first year, then increases for about 6 quarters before it returns to normal levels.

Figure 4.B. Impulse Response Functions Based on DAG at 30% Significance Level



Innovations in prices and investment are not major movers of the other variables in our six variable VAR. Innovations in each of the other four variables have sustained or lasting influence on at least one other variable in our six variable system. Innovations in income have a strong and persistent impact on prices and money and considerable short-term influence on unemployment and investment. Money innovations appear to have their strongest lasting impact on prices, showing only short-term impacts (delayed by one or two periods) on the other four variables (excluding itself). Innovations in unemployment appear to be the strongest lasting influence on output. Interest rate innovations have a strong persistent influence on money and prices, both negative in the long run.

Surprisingly, the responses generated from the DAG look similar to those generated from Sims' (1986) initial Choleski factorization (Figures 1.A and

1.B). The few differences are primarily in the responses to innovations in investment. For example, consider the response of output to innovations in investment. Under the Choleski decomposition, in response to a positive shock to investment, output declines in the first eight quarters and thereafter is positive. This impulse response is (perhaps) not reasonable as we expect, a priori, that an increase in investment should result in expansion in output. Under the DAG-based decomposition, however, output responds positively to innovations in investment. This latter response is more consistent with our priors. This difference between Sims' Choleski results and our directed graph results apparently is due to differences in our respective treatments of interest rates. Sims has interest rates ordered last on the results of Figures 1.A and 1.B, while the directed graphs (Figure 2) shows interest rates as a causal factor for investment in contemporaneous time. Otherwise, the Choleski ordering used by Sims is very similar to the information flows summarized by the directed graph given in Figure 2, Panel E.

V. Concluding Remarks

The vector autoregression has found favor among many in applied econometrics for study of observational data. Among the reasons for its attractiveness is its reliance on data and avoidance of strong zero-one-type restrictions, as the VAR represents an efficient summary of the covariance patterns in historical data. However to make policy recommendations additional identifying restrictions have to be put on the VAR representation. Heretofore research workers have relied on either a Choleski factorization or theory to provide such restrictions. Both methods are subjective in the sense that the data are not given a strong role in providing explicit zero-type restrictions required for identification. This paper has asked whether results from a VAR model offered in Sims (1986) continue to hold when a less subjective, more data-driven approach, is applied to achieve an identifying interpretation of a six variable VAR on the U.S. economy.

The motivation for proceeding in this fashion was offered in the early paper by Cooley and LeRoy (1985). They suggest that one valid use of the VAR is to summarize regularities in the data, which in turn, may then motivate additional theoretical work. To date, not much work along this line has been forthcoming although Cooley and LeRoy (1985, p. 288) do cite papers by Ashenfelter and Card (1982), and Litterman and Weiss (1985) as examples of the type of research for which VAR results do generate additional theoretical work. Perhaps the reason for the lack of more studies in this vein is that both the Choleski factorization and the structural factorization involve considerable amounts of judgment on the part of the research worker. Thus it becomes problematic for analysts to know just what parts of their results are based on data and what parts based on assumed identifications. Directed acyclic graphs, while still subject to the charge of subjectivity (as we have seen here, for example, in terms of the choice of significance level), are a move in the direction in which Cooley and LeRoy point us.

Here we replicate the VAR results of an important model of Sims, where identification was achieved using a Choleski factorization. Subsequently, a second model was estimated where a contemporaneous causal ordering on the model's innovations was determined using TETRAD II's representation in terms of a directed acyclic graph. The directed graph results show Sims' six variable model not rich enough to provide an unambiguous ordering at usual levels of statistical significance. We required a significance level in the neighborhood of 30 % to find a clear structural ordering. At this rather high level of significance we found impulse response functions to be quite similar to the Choleski generated responses found by Sims (1986). These responses appear to be broadly consistent with a monetarist's view of the economy with adaptive expectations with no hyperinflation.

Additional work on type I and II errors, the possibility of multiple causal structures, and feedback and cyclic graphs is certainly warranted. Here we varied significance levels from 0.01 to 0.30 and found a number of causal patterns, one of which was a directed acyclic graph (the result found at the 30 % significance level). Questions on multiple graphs at each significance level have not been addressed. Further, we have not considered the possibility of feedback in contemporaneous time. Investigations with this algorithm (TETRAD II) and other work on cyclic graphs is now underway (see Richardson and Spirtes, 1999, for discussion of a recent algorithm for modeling cyclic graphs).

References

Ashenfelter, O., and D. Card (1982), "Time Series Representations of Economic Variables and Alternative Models of the Labour Market," *Review of Economic Studies* **49**: 761-781.

Bernanke, B.S. (1986), "Alternative Explanations of the Money-Income Correlation," *Carnegie-Rochester Conference Series on Public Policy* **25**: 49-99.

Bernanke, B.S., Gertler, M., and M. Watson (1997), "Systematic Monetary Policy and the Effects of Oil Price Shocks," *Brookings Papers on Economic Activity* **1**: 91-157.

Blanchard, O. J., and D. Quah (1989), "The Dynamic Effects of Aggregate Demand and Supply Disturbances," *American Economic Review* **79**: 655-673.

Cooley, T.F., and M.D. Dwyer (1998), "Business Cycle Analysis Without Much Theory: A Look at Structural VARs," *Journal of Econometrics* **83**: 57-88.

Cooley, T.F., and S.F. LeRoy (1985), "Atheoretical Macroeconomics: A Critique," *Journal of Monetary Economics* **16**: 283-308.

Doan, T. (2000), *RATS: User's Manual*, *Version 5.0*, Evanston, Illinois, Estima.

Geiger, D., Verma, T., and J. Pearl (1990), "Identifying Independence in Bayesian Networks," *Networks* **20**: 507-533.

Hausman, D.M. (1998), *Causal Asymmetries*, New York, Cambridge University Press.

Hess, J.H., and B.S. Lee (1999), "Stock Returns and Inflation with Supply and Demand Disturbances," *The Review of Financial Studies* **12**:1203-1218.

Kim, S. (2001), "International Transmission of U.S. Monetary Shocks: Evidence from VAR's," *Journal of Monetary Economics* **48**: 339-372.

Leamer, E.E. (1985), "Vector Autoregression for Causal Inference?" *Carnegie-Rochester Conference Series on Public Policy* **22**: 225-304.

Leeper, E.M., Sims, C.A., and T. Zha (1996), "What Does Monetary Policy Do?" *Brookings Papers on Economic Activity* **2**:1-63.

Litterman, R., and L.Weiss. (1985). "Money, Real Interest Rates and Output: A Reinterpretation of Postwar U.S. Data," *Econometrica* **53**:129-56.

Orcutt, G. (1952), "Toward a Partial Redirection of Econometrics," *Review of Economic and Statistics* **34**:195-213.

Papineau, D. (1985), "Causal Asymmetry," *British Journal of the Philosophy of Science* **36**: 273-89.

Pearl, J. (1995), "Causal Diagrams for Empirical Research," *Biometrika* **82**: 669-710.

Pearl, J. (2000), *Causality*, Cambridge, Cambridge University Press.

Richardson, T., and P. Spirtes (1999), "Automated Discovery of Linear Feedback Models," in C. Glymour and G. Cooper, eds., *Computation, Causation and Discovery*, Menlo Park, AAAI Press.

Reichenbach, H. (1956), *The Direction of Time*, Berkeley, University of California Press.

Scheines, R., Spirtes, P., Glymour, C., and C. Meek (1994), *TETRAD II Tools for Causal Modeling: User's Manual and Software*, New Jersey, Lawrence Erlbaum Associates, Inc.

Simon, H.A. (1953), "Causal Ordering and Identifiability," in W.C. Hood and T.C. Koopmans, eds., *Studies in Econometric Method*: 49-74, New York, Wiley.

Sims, C.A (1980), "Macroeconomics and Reality," *Econometrica* **48**: 1-48.

Sims, C.A (1986), "Are Forecasting Models Usable for Policy Analysis," *Federal Reserve Bank of Minneapolis Quarterly Review* **10**: 2-16.

Spirtes, P., Glymour, C., and R. Scheines (1993), *Causation, Prediction, and Search*, New York, Springer-Verlag.

Swanson, N.R., and C.W.J. Granger (1997), "Impulse Response Functions Based on a Causal Approach to Residual Orthogonalization in Vector Autoregressions," *Journal of the American Statistical Association* **92**: 357-367.