Production risk in multi-output industries: estimates from Norwegian dairy farms

Tveteras R.[1], Flaten O.[2] and Lien G.[2,3]

[1] Department of Industrial Economics, Risk Management and Planning, University of Stavanger, Norway
[2] Norwegian Agricultural Economics Research Institute (NILF), Oslo, Norway
[3] Faculty of Life Science, Lillehammer University College, Lillehammer, Norway

Abstract— **Farmers who produce multiple outputs are portfolio managers in the sense that they use inputs to balance expected economic return and variance of return. This paper estimates the structure of the stochastic multi-output production technology in Norwegian dairy farming, allowing for a more flexible specification of the technology than previous studies. We find that an increase in input levels leads primarily to higher output variability, and that inputs also influence the covariance of shocks between outputs. Risk-reducing effects of inputs on outputs are primarily present in the covariance functions. Technical change leads to shifts in the profit distribution over the data period, but no welfare improvement for risk-averse farmers.**

Keywords— **Multi-output technologies, production risk, dairy farming**

I. INTRODUCTION

Farmers face substantial risks caused by biophysical factors such as weather, soil, and diseases. Since farmers are generally risk averse (e.g. Chavas and Holt, 1996; Lence, 2000; Isik and Khanna, 2003; Kumbhakar and Tveteras, 2003), and adverse outcomes can have large consequences for their welfare, they will try to mitigate these risks through input and output choices. Many farmers produce several outputs, and can be regarded as portfolio managers who, through input and output choices, balance the expected economic return and risk (e.g. by using variance of return as a risk metric) of their farm assets. But farmers' choice sets are much more limited than is generally the case for a financial investor. Many farmers, especially small-scale farmers, also typically face severe financing and liquidity constraints. Thus, the relative risk will generally be higher for many farmers than for well-diversified financial investors. Consequently, one could expect farmers to be concerned about on-farm decision variables that influence risk, primarily the effects of their input choices on economic risk. They should benefit from increased knowledge about risk effects of their input choices and technological change. Thus, econometric estimates of the relationships between input use and production risk would be useful. In this paper we will provide estimates of the structure of production risk. Our case study is the Norwegian dairy farming sector, where small farms produce several risky outputs, face significant economic risks, and where there is limited knowledge about the structure of production risk.

After Just and Pope (1978) in a seminal paper characterised production functions with flexible risk properties, several econometric studies of production risk have appeared in the literature. The primary focus has been on agriculture, while a few have studied aquaculture.[1] Just and Pope's stochastic production function is useful in analysis of the structure of production in industries in which there are frequent stochastic exogenous shocks on the production technology, but where the mix of input levels or the choice of technology can influence the effect of these shocks on output.[2] A central issue in econometric studies of production risk is the marginal effect of increasing or reducing input use on output risk, often measured by output variance. Some studies have also investigated the effects of technological changes on

[1] Econometric studies of production risk in agriculture using a primal model approach include Antle (1983), Antle and Goodger (1984), Di Falco *et al.* (2007), Groom *et al.* (2008), Griffiths and Anderson (1982), Kumbhakar (1993), Just and Pope (1979), Nelson and Preckel (1989), Regev *et al.* (1997), Roberts *et al.* (2004), Serra *et al.* (2006), Traxler *et al.* (1995) Wan and Anderson (1990), and Wan *et al.* (1992), while aquaculture has been studied by Asche and Tveteras (1999) and Tveteras (1999, 2000).

[2] The state contingent approach (Chambers and Quiggin, 2001) has been introduced as an alternative to the traditional parametric stochastic production function approaches such as the Just-Pope specification belongs. Full application of the state contingent approach is very data demanding (Just, 2003), and empirical work based on this approach has only recently appeared in the agricultural economics literature (O'Donnell and Griffiths, 2006; Chavas, 2008).

the structure of production risk (Traxler et al., 1995; Tveteras, 1999, 2000).

Agricultural production is often a multi-output technology, but econometric analyses of production risk have predominantly specified single-output technologies. A few exceptions are Hallam et al. (1989), Wan et al. (1992) and Isik and Devadoss (2006), who all estimated multi-output Just-Pope (JP) production functions. Hallam et al. (1989) estimated their function using a cross-section of Sudanese crop farms, Wan et al. (1992) estimated the structure of production risk using Chinese regional crop production data, and Isik and Devadoss (2006) used Idaho district-level crop data. This study of Norwegian dairy farming is also concerned with estimating the structure of a stochastic multi-output technology. We extend the model framework of Wan et al. (1992) to a more flexible multi-output technology in which we allow elasticities of output means, variances and covariances of outputs to vary by input levels. Further, several studies using the JP model framework, including Wan et al. (1992) and Isik and Devadoss (2006), used aggregated data at the level of a region or nation in their estimation. This leads to biased estimates when there are producer-specific shocks and producer heterogeneity. Since we have available a large unbalanced farm-level panel data set, we can test a rich set of hypotheses on the structure of production risk. We allow input levels not only to influence the levels of output risk as measured by conditional output variances, but also to influence covariances between outputs. For example, we test for time-effects on production risk. Using our estimated production functions we also extend the analysis to the first two moments of profit to get a better understanding of how the means, variances and covariances of outputs influence the profit distribution.

The paper is organised as follows: Section 2 discusses the theoretical and econometric framework for analysis of production risk. Econometric specifications of Just-Pope production functions are provided in section 3. In section 4 we describe the data set on Norwegian dairy farms. Section 5 presents the empirical results and discusses their implications. Finally, in section 6 we provide a concluding discussion.

II. MODELLING PRODUCTION RISK

A. Production risk in the single-output case

Just and Pope (1978, 1979) proposed a stochastic production function which is general enough to accommodate both increasing and decreasing output variance in inputs. The single-output JP production function has the general form

$$y = f(\mathbf{x}) + u = f(\mathbf{x}) + h(\mathbf{x})^{0.5}\varepsilon \qquad (1)$$

where $f(\cdot)$ is the mean function (or deterministic component of production), $h(\cdot)$ is the variance function that captures the relationship between input use and output variability, and ε is an index of exogenous production shocks with zero mean and variance σ_ε^2. With this formulation we see that inputs x influence mean output and output variance independently, since

$$E(y) = f(\mathbf{x}) \text{ and } Var(y) = Var(u) = h(\mathbf{x})\sigma_\varepsilon^2 \qquad (2)$$

One of the requirements JP propose for specifications of risky production technologies is that the production function should be general enough to accommodate both increasing and decreasing output risk (variance) in inputs, i.e.

$$\partial \mathrm{Var}(y)/\partial x_k = \partial \mathrm{h}(\cdot)/\partial x_k <=> 0 \qquad (3)$$

should all be possible.[3]

Theoretical models of the competitive firm under production risk generally use the expected utility (EU) framework, where risk averse producers choose the input vector **x** which maximises their expected utility based on observed (or expected) output and input prices (p, w) and a priori knowledge of the structure of the risky production technology. The solution to the EU maximisation problem $\max_x$ EU(π(**x**; p, **w**) is the indirect utility function

$$U^* = U^*[E(\pi), Var(\pi)],\ dU^*/dE(\pi) > 0,$$
$$dU^*/dVar(\pi) < 0 \qquad (4)$$

where U^* represents the solution to the maximisation problem, E(π) is mean profit and Var(π) is the variance of profit. The indirect utility function

[3] See Just and Pope (1978) for other requirements for a risky production technology.

represents the producer's subjective trade-off between mean profit (output) and variance of profit (output) on producer welfare. There is a positive linear relationship between the moments of output and the moments of profit under JP production risk (1), with the mean and variance of profit given by

$$E(\pi) = p\cdot E(y) - \mathbf{w'x} = p\cdot f(\mathbf{x}) - \mathbf{w'x} \tag{5a}$$

$$Var(\pi) = p^2\cdot Var(y) = p^2\cdot h(\mathbf{x})\sigma_\varepsilon^2 \tag{5b}$$

In the extreme case of risk neutrality (i.e. $dU^*/dVar(\pi) = 0$), the producer is only concerned about mean profit (output), and completely ignores the profit variance effects of input choices.

B. Production risk in the multi-output case

Many farmers produce several outputs. Risk management is mentioned as a possible reason for multi-output production, for example, by Mundlak (2001, p. 41). In the multi-output nonjoint case the JP production function can be specified as

$$y_m = f_m(\mathbf{x}_m) + u_m \tag{6}$$

for each output $m = 1,..., M$. Inputs can be *allocable* or *nonallocable*.[4] In the case of nonallocable inputs, which we will assume here, the subscript m disappears from the input levels $\mathbf{x}$. The means, variances and covariances of outputs are given by:

$$E(y_m) = f_m(\mathbf{x}),\ m = 1,..., M \tag{6a}$$

$$Var(y_m) = Var(u_m) = h_m(\mathbf{x};\ \sigma^2_{\varepsilon m}),\ m = 1, \ldots, M \tag{6b}$$

$$Cov(y_l, y_m) = Cov(u_l, u_m)$$
$$= g_{lm}(\mathbf{x};\ \sigma^2_{\varepsilon lm}),\quad l, m = 1,..., M, l \neq m \tag{6c}$$

where $h_m(\mathbf{x})$ is the variance function for output m, $g_{lm}(\mathbf{x})$ is the covariance function, and $\sigma_{\varepsilon m}$ and $\sigma_{\varepsilon lm}$ are the variances of exogenous random shocks ε_m and ε_{lm} in the variance and covariance functions, respectively.

The mean and variance of profit in the nonjoint, nonallocable multi-output case are:

$$E(\pi) = \Sigma_m\, p_m\cdot E(y_m) - \mathbf{w'x} = \Sigma_m\, p_m\cdot f_m(\mathbf{x}) - \mathbf{w'x} \tag{7a}$$

$$Var(\pi) = \Sigma_m\, p_m{}^2\cdot Var(y_m)$$
$$= \Sigma_m\, p_m{}^2\cdot h_m(\mathbf{x}) + 2\Sigma_l\Sigma_m\, p_l p_m\, g_{lm}(\mathbf{x}) \tag{7b}$$

respectively. The effects of a change in the level of input k on mean and variance of profit are:

$$\partial E(\pi)\ /\partial x_k = \Sigma_m p_m\cdot\partial f_m(\mathbf{x})/\partial x_k - w_k \tag{8a}$$

$$\partial Var(\pi)\ /\partial x_k =$$
$$\Sigma_m p_m{}^2\cdot\partial h_m(\mathbf{x})/\partial x_k + 2\Sigma_l\Sigma_m\, p_l p_m\, \partial g_{lm}(\mathbf{x})/\partial x_k \tag{8b}$$

We see from equations (4) and (8a and b) that in the nonallocable case the risk averse producer will take into account the aggregate marginal effect on all M outputs of changing input levels.[5]

There may also be time-specific effects which influence both means, variances and covariances of output, $E(y_m) = f_m(\mathbf{x};\ \mathbf{t})$, $Var(y_m) = h_m(\mathbf{x};\ \mathbf{t})\sigma_{\varepsilon m}$, $Cov(y_l,y_m) = g_{lm}(\mathbf{x};\ \mathbf{t})\sigma_{\varepsilon lm}$, where $\mathbf{t}$ is a vector of time dummy variables. Time-specific effects can include technological changes which affect $f_m(\cdot)$, $h_m(\cdot)$ and $g_{lm}(\cdot)$. Technological changes may be both risk-increasing and risk-reducing. Again, when the risk averse producer compares technologies a trade-off will be made between the effects on mean and variance of output (profit). One additional challenge when one moves from a single-output to a multi-output technology is that the risk effects, as measured by output variances and covariances, associated with $\mathbf{x}$ and $\mathbf{t}$ may have different signs across outputs m, thus leading to ambiguous results for an analysis of producer welfare based on the primal JP model specification. In order to be able to analyse the effects on producer welfare it is then necessary to employ the predicted effects on conditional means, variances and covariances of outputs in the moments of profit (7a and 7b).

One approach to the analysis technical change under production risk which is devoid of risk preferences, is to use the concept of first-order stochastic dominance (SD) (Hadar and Russell, 1969; Hanoch and Levy, 1969). We define the first two moments of profit associated with production technology 0, $E(\pi_0(\cdot))$ and $Var(\pi_0(\cdot))$, which are

[4] An input is *allocable* when the amount of the input used in producing output y_j can be distinguished from the amount of the same input used in producing y_k ($j \neq k$) (Beattie and Taylor, 1985).

[5] However, the subjective weighting of the marginal effects is determined by the producer's degree of risk aversion, i.e. the size of $dU^*/dVar(\pi)$.

determined by $f_{m0}(\cdot)$, $h_{m0}(\cdot)$, $g_{lm0}(\cdot)$, $m = 1, \ldots, M$, and moments of profit associated with production technology 1, $E(\pi_1(\cdot))$ and $Var(\pi_1(\cdot))$, determined by $f_{m1}(\cdot)$, $h_{m1}(\cdot)$, $g_{lm1}(\cdot)$, $m = 1, \ldots, M$. If the first two moments fully describe the distribution of profit, then the two technologies give rise to two distinct probability density functions $g_0(\cdot)$ and $g_1(\cdot)$ with associated cumulative density functions (CDFs) $G_0(\cdot)$ and $G_1(\cdot)$, respectively. Assume that $G_0(\cdot)$ and $G_1(\cdot)$ are continuous and monotonic. If technology 1 first-order stochastically dominates technology 0 globally, i.e.,

$$G_1(\pi \mid p,\mathbf{w},\mathbf{x}) \le G_0(\pi \mid p,\mathbf{w},\mathbf{x})$$

for all nonnegative p, $\mathbf{w}$, $\mathbf{x}$, with strict inequality for some p, $\mathbf{w}$, $\mathbf{x}$, then all producers with utility functions U such that $U' \ge 0$ will prefer technology 1 to technology 0. In words, if the CDF of technology 1, $G_1(\cdot)$, lies to the right of the CDF of technology 0, $G_0(\cdot)$, both risk averse, risk neutral and risk loving producers will adopt technology 1. When $E\pi_1(\cdot) > E\pi_0(\cdot)$ and $Var\pi_1(\cdot) \le Var\pi_0(\cdot)$ for given values of $(p,\mathbf{w},\mathbf{x})$, then technology 1 is preferred by all producers. However, when $E\pi_1(\cdot) > E\pi_0(\cdot)$ and $Var\pi_1(\cdot) > Var\pi_0(\cdot)$, then technology 1 may not first-order stochastically dominate technology 0.

III. ECONOMETRIC SPECIFICATIONS OF MULTI-OUTPUT JP PRODUCTION FUNCTIONS

The Cobb-Douglas (CD) form has been used in many econometric specifications, including the multi-output approach of Wan *et al.* (1992), both for the mean functions $f_m(\cdot)$ and variance and covariance functions $h_m(\cdot)$ and $g_m(\cdot)$ of the JP technology. Yet it is well-known fact that choice of a CD function imposes very strong restrictions on the production technology. For this reason, more flexible functional forms have been used in many econometric studies of production technologies and productivity. Log-linearization of the Cobb-Douglas or the more flexible translog cannot be used in a JP model framework since the error term is not specified in the usual multiplicative form $y_m = f_m(\cdot)e^{u_m}$, but in the additive manner $y_m = f_m(\cdot) + u_m$ to ensure consistency with the JP postulates. If the u_m term is normally distributed then the multiplicative form will give rise to a log-normal output distribution, while the additive form will give rise to a normally distributed output. Hence, the model has to be estimated by nonlinear methods with these functional forms. The less flexible Cobb-Douglas has probably been preferred relative to the translog in previous studies due to the difficulties of obtaining convergence of nonlinear estimates with the latter functional form.

We use a generalised Leontief (GL) (Diewert, 1971; Driscoll *et al.*, 1992) specification of the mean function $f(\cdot)$ in this paper. Earlier studies have compared the GL with other functional forms, such as the popular translog (TL).[6] The generalised Leontief (GL) mean function for output m is given by

$$\begin{aligned} y_{mit} &= f_m(\mathbf{x}_{mit};\mathbf{D}_t,\boldsymbol{\mu}_m,\boldsymbol{\alpha}) + u_{mit} \\ &= \alpha_{m0} + \sum_k \alpha_{mk} x_{k,it}^{0.5} + 0.5\sum_j \sum_k \alpha_{mjk} x_{j,it}^{0.5} x_{k,it}^{0.5} \qquad (9) \\ &\quad + \sum_t \alpha_{mt} D_t + \sum_k \sum_t \alpha_{mkt} D_t x_{k,it}^{0.5} + \mu_{mi} + u_{mit} \\ m &= 1, 2, 3. \end{aligned}$$

where the subscripts j, $k = 1,\ldots, K$ refers to inputs, subscripts i and t refer to firms and years, respectively and where D_t is a dummy for time. We avoid the conventional time trend straitjacket by allowing the rate of technical change to vary from year to year using time dummy variables. The time trend model is convenient when there is only a single time series data set available to the researcher, since few degrees of freedom are lost. However, neither theory nor empirical observations provide any support for a constant rate of technical progress. Farm-specific effects on mean output are given by $\mu_{m,i}$, and are treated as fixed effects.

[6] Driscoll *et al.* (1992) compare the flexibility properties of the TL and GL. Tveteras (1999, 2000) estimate both translog and GL mean production functions. Several studies have employed the more restrictive Cobb-Douglas specification for the mean function, e.g. Hallam *et al.* (1989) and Wan *et al.* (1992) in their multi-output Just-Pope functions. They were then forced to estimate the Cobb-Douglas nonlinearly in order not to violate the Just-Pope postulates.

We make the following assumptions on the error term u_{mit}: (a) $E[u_{mit}] = 0$, i.e. expectation is zero; (b) $\mathrm{Var}(u_{mit}) = h_m(\mathbf{z}; \boldsymbol{\beta}, \sigma^2_{\varepsilon m})$, i.e. variances are functions of explanatory variables $\mathbf{z}$, which may include input levels $\mathbf{x}$, and associated parameters $\boldsymbol{\beta}$, and the variance of exogenous stochastic shocks $\sigma^2_{\varepsilon m}$; (c) $\mathrm{Cov}(u_{mit}, u_{lit}) = g_{lm}(\mathbf{z}; \boldsymbol{\lambda}, \sigma^2_{\varepsilon ml})$, i.e. covariances between two random output shocks are functions of explanatory variables $\mathbf{z}$, which may include input levels $\mathbf{x}$, associated parameters $\boldsymbol{\lambda}$, and the variance of exogenous stochastic shocks $\sigma^2_{\varepsilon ml}$; (d) $\mathrm{Cov}(u_{mit}, u_{ljs}) = 0$ for ($i \neq j$ and $t = s$) or ($i = j$ and $t \neq s$), i.e. there is no stochastic interdependence between output shocks for different farms or over time.

The marginal mean of output m with respect to input k is given by

$$\frac{\partial E(y_m)}{\partial x_k} = \frac{\partial f_m}{\partial x_k} = 0.5\alpha_k x_k^{\,0.5} + 0.5\alpha_{kk}\sum_j \alpha_{jk} x_j^{0.5} x_k^{\,0.5} \quad (10)$$

Returns to scale for product m is given by $RTS_m(\mathbf{x}) = \Sigma_k E_{m,k}(\mathbf{x}) = \Sigma_k(\partial f_m/\partial x_k)(x_k / f_m(\mathbf{x}))$, i.e. it is equal to the sum of the K input elasticities for output m (the $E_{m,k}$'s). If the estimate of $RTS_m(\mathbf{x})$ is greater than, equal to, or less than unity, the returns to scale are increasing, constant, or decreasing, respectively.

The econometric specifications of the variance functions estimated here are special cases of Harvey's (1976) variance function specification $\mathrm{Var}(u_m) = h_m(\mathbf{z}) = \exp[\mathbf{z}\boldsymbol{\beta}_m]$, where the vector $\mathbf{z}$ represents input levels or transformations of input levels, e.g., logarithms of inputs and second-order terms.[7] A nice property of the variance function in Harvey's formulation is that positive output variances are always ensured. Note that in the JP model $\mathrm{Var}(y_m) = \mathrm{Var}(u_m)$.

The argument of the exponent is a generalised Leontief function given by:

$$\begin{aligned} Var(u_m) &= h_m(\mathbf{x}_{mit}; \mathbf{D}_t, \beta) \\ &= exp(\beta_0 + \textstyle\sum_k \beta_k x_k^{0.5} + 0.5\sum_j\sum_k \beta_{jk} x_j^{0.5} x_k^{0.5} \\ &\quad + \textstyle\sum_t \beta_t D_t) \\ m &= 1,2,3. \end{aligned} \quad (11)$$

This specification satisfy the flexibility requirements of JP. The marginal output risk in input k is given by

$$\begin{aligned} \frac{\partial Var(u_m)}{\partial x_k} = \frac{\partial h_m}{\partial x_k} &= exp(\beta_0 + \sum_k \beta_k x_k^{0.5} \\ &\quad + 0.5\sum_j\sum_k \beta_{jk} x_j^{0.5} x_k^{0.5} + \sum_t \beta_t D_t) \\ &\quad \cdot \left(0.5\beta_k x_k^{-0.5} + 0.5\beta_{kk}\sum_j \beta_{jk} x_j^{0.5} x_k^{-0.5}\right) \end{aligned} \quad (12)$$

The elasticity measures for the variance function $h(\cdot)$ are analogous to the $E_{m,k}$, and the RTS elasticities for the mean function $f(\cdot)$. The output variance elasticity of product m with respect to input k is given by

$$VE_{m,k} = \frac{\partial h_m}{\partial x_k}\frac{x_k}{h_m} \quad (13)$$

If input k is risk-increasing (risk-decreasing), then $VE_{m,k}$ is greater (less) than zero. For our GL specification of the variance function $VE_{m,k}$ is

$$VE_{m,k} = \left(0.5\beta_k x_k^{-0.5} + 0.5\sum_j\sum_{k\neq j} \beta_{jk} x_j^{0.5} x_k^{-0.5}\right) x_k \quad (14)$$

The total output variance elasticity (TVE) of product m in inputs is defined as

$$TVE_m(\mathbf{x};\beta) = \sum_k VE_{m,k}(\mathbf{x}) = \sum_k \frac{\partial h_m}{\partial x_k}\frac{x_k}{h_m(\mathbf{x})} \quad (15)$$

i.e. the sum of the K output variance elasticities with respect to inputs. TVE is the analogue of the RTS elasticity measure derived from the mean function. If TVE is greater (smaller) than zero, then a factor-neutral expansion of input levels will lead to an increase (decrease) in the variance of output m.

[7] The first element of $\mathbf{z}$, z_0, is taken as unity. This implies that $\mathrm{Var}(\varepsilon) = \exp(\beta_0)$.

The econometric specification of the covariance functions for outputs (l, m) also employs a generalised Leontief functional form:

$$\begin{aligned}\text{Cov}\,(u_l, u_m) &= g_{lm}(\mathbf{x}_{mit}; \mathbf{D}_t, \lambda) \\ &= \lambda_0 + \sum_k \lambda_k x_k^{0.5} + \\ &\quad 0.5 \sum_j \sum_k \lambda_{jk} x_j^{0.5} x_k^{0.5} + \\ &\quad \sum_t \lambda_t D_t \end{aligned} \tag{16}$$

where l, m = 1, 2, 3, $l \neq m$, and λ is a vector of parameters. This specification is flexible enough to allow both positive and negative effects of input changes on output covariances, and to allow the effects of inputs to vary over their levels.

The marginal covariance for outputs (l, m) in input k is given by

$$\frac{\partial \text{Cov}(u_l, u_m)}{\partial x_k} = \frac{\partial g_{lm}}{\partial x_k} = 0.5 \lambda_k x_k^{0.5} + 0.5 \lambda_{kk} \sum_j \lambda_{jk} x_j^{0.5} x_k^{0.5} \tag{17}$$

The covariance elasticity of outputs (l, m) in input k is given by

$$CE_{lm,k}(\mathbf{x}) = \frac{\partial g_{lm}}{\partial x_k} \frac{x_k}{g_{lm}(\mathbf{x})} \tag{18}$$

The total output covariance elasticity (TCE_{lm}) for outputs (l, m) in inputs is defined as

$$TCE_{l,m}(\mathrm{x}; \lambda) = \sum_k CE_{lm,k}(\mathrm{x}) = \sum_k \frac{\partial g_{lm}}{\partial x_k} \frac{x_k}{g_{lm}(\mathrm{x})} \tag{19}$$

i.e. the sum of the K output covariance elasticities with respect to inputs, $CE_{lm,k}$. If TCE is greater (smaller) than zero, then a factor-neutral expansion of input levels will lead to an increase (decrease) in the covariance between outputs l and m.

The stochastic Just-Pope production functions are estimated in two stages. In the first stage we handle the potential covariance between errors and heteroskedasticity by estimating the mean production functions simultaneously by Zellner's SURE, where White's heteroskedasticity-consistent covariance matrix of the parameter estimates is computed (White, 1980). Since production risk is a special form of heteroskedasticity, as noted by Asche and Tveteras (1999), the first-stage White estimates provide consistent estimates and valid inference. In the second stage the variance and covariance functions are estimated by SURE using the predicted residuals from the first stage. When the production model is estimated by SURE, the error terms (i.e. the exogenous shocks to production) are assumed to be correlated. In the context of dairy farms this means, for example, that when there are shocks to milk production there also tend to be shocks to meat production (e.g. diseases or weather conditions which affect both milk and meat production).

IV. DATA

The data source is the Norwegian Farm Accountancy Survey. This is an unbalanced set of farm-level panel data, collected by the Norwegian Agricultural Economics Research Institute (NILF). It includes farm production and economic data collected annually from about 1000 farms, divided between different regions, farm size classes, and types of farms. Participation in the survey is voluntary. There is no limit on the numbers of years a farm may be involved in the survey. Approximately 10% of the survey farms are normally replaced every year. The farms are classified according to their main category of farming, defined in terms of the standard gross margins of the farm enterprises.

The data set used in the analysis is a unbalanced panel with 4624 observations on 651 dairy farms from 1993 to 2003. We distinguish between three outputs; milk measured in litres (y_1), meat measured in kg (y_2) and an aggregate of other outputs measured in Norwegian kroner (NOK) (y_3).[8] The aggregate of other outputs includes revenues from additional farm enterprises and direct payments (mainly paid per livestock head or per hectare, with rates varying according to type of livestock and crops). Dairy farms

[8] 5.10 NOK ≈ 1 USD.

use both allocable and nonallocable inputs. The six inputs included in our econometric models are farm land (x_1), labour (x_2), purchased feed (x_3), materials (x_4), number of cattle (x_5) and machinery capital (x_6). Labour is to some extent allocable, for example, labour used in meat production instead of e.g., milking the cows. But to distinguish between the amount of labour input used in producing milk and the amount of the same input used in producing meat is difficult. The same applies to inputs such as fixed capital (e.g. buildings and machinery), purchased feed, miscellaneous materials and services purchased (e.g. veterinary services), and the stock of cattle. In our case we also have a measurement problem, since we do not have data on, e.g., labour-hours used in meat production and labour-hours used for managing the dairy cows. Hence, we have to use the total amount of each input in the production functions for each output. This means that we should be somewhat cautious in drawing too strong conclusions from individual input elasticities.

Table 1 presents summary statistics for all farms in the estimating sample. All monetary values have been deflated by the consumer price index.

Table 2 presents correlation coefficients for outputs and inputs. The correlations were moderately positive. Materials and number of cattle exhibited the strongest correlation (0.79) between inputs.

Table 1 Summary statistics for sample 1993-2003

		Mean	SD	Minimum	Maximum
y_1	Milk (litres)	80797	32944	9005	273929
y_2	Meat (kg)	3558	2102	-78.0	29140
y_3	Other outputs (NOK)	284189	77720	56749	1935202
x_1	Farm land (ha)	196	81.8	36.0	595
x_2	Labour (hours)	3385	863	669	7460
x_3	Purchased feed (NOK)	144926	67904	3704	793298
x_4	Materials (NOK)	97776	43153	10986	366333
x_5	Cattle (number of animals)	20.6	8.1	2.6	80.2
x_6	Machinery capital (NOK)	87532	46084	587	411321
	Number of farms	651			
	Number of observations	4624			

Table 2 Correlation coefficients for inputs and outputs

	y_1	y_2	y_3	x_1	x_2	x_3	x_4	x_5	x_6
y_1	1.00								
y_2	0.65	1.00							
y_3	0.58	0.53	1.00						
x_1	0.67	0.64	0.71	1.00					
x_2	0.52	0.43	0.46	0.45	1.00				
x_3	0.75	0.69	0.61	0.57	0.49	1.00			
x_4	0.75	0.71	0.71	0.76	0.51	0.70	1.00		
x_5	0.88	0.82	0.66	0.73	0.53	0.75	0.79	1.00	
x_6	0.51	0.51	0.49	0.52	0.34	0.51	0.61	0.54	1.00

Outputs: Milk (y_1), meat (y_2) and other outputs (y_3). Inputs: farm land (x_1), labour (x_2), purchased feed (x_3), materials (x_4), number of cattle (x_5) and machinery capital (x_6).

V. EMPIRICAL RESULTS

A. Test for heteroskedasticity

Before we proceeded to the estimation of production risk using multi-output JP production functions, we determined whether heteroskedasticity was present in dairy farming production technology. A Breusch-Pagan (BP) test was first undertaken (Breusch and Pagan, 1979) based on the GL mean production functions. The BP test statistic, when the explanatory variables in the GL models are used, was distributed as chi-square with 44 degrees of freedom. The estimated BP values were 3073 ($p < 0.001$) for milk output , 5237 ($p < 0.001$) for meat output, and 8440 ($p < 0.001$) for other outputs. Hence, the null hypothesis of homoskedasticity can be firmly rejected for all three outputs at conventional significance levels, implying that White's heteroskedasticity-consistent covariance matrix should be estimated to ensure valid inference.

B. Just-Pope model results

Table 3 presents estimated elasticities for a JP model with farm-specific fixed effects in the mean function. We evaluated the mean, standard error and t-values of the elasticities in the sample mean value of each variable. The delta method was used for estimation of the standard errors (Oehlert, 1992).

Most inputs had a positive effect on the mean function, as measured by the input elasticities ($E_{m,k}$), with stock of cattle having the highest elasticity for milk and meat. As expected, the sum of the input elasticities, returns to scale (RTS), was positive for all outputs. Only meat had returns to scale around one, while milk and other outputs had an RTS around 0.5.

The estimated variance functions predict that there are both risk-increasing and risk-decreasing inputs, as measured by the output variance elasticities ($VE_{m,k}$) in Table 3. Furthermore, the estimates show that the same input can have opposite risk effects on the three outputs (e.g., materials). For both milk and meat output land had the most positive variance elasticity, while labour had the most negative variance elasticity for milk and meat output, implying that labour input plays a risk-reducing role. The total variance elasticity, which is the sum of all $VE_{m,k}$ values, was positive for all three outputs, implying that the variance function contributed to increased variability as the scale of the farm increases.

To get a more complete picture of production risk in dairy farming, the covariance functions must also be investigated. Inputs had both significant positive and significant negative marginal effects on covariances, as measured by the covariance elasticities ($CE_{lm,k}$) in Table 3. For example, machinery capital had a significantly positive effect at the 10% confidence level on the covariance between milk and meat output ($CE_{12,6}$), but significantly negative effects on the covariances between milk and other outputs ($CE_{13,6}$) and meat and other outputs ($CE_{23,6}$). We found that the total covariance elasticity tended to be negative between milk and meat outputs, and between milk and other outputs, but positive between meat and other outputs. But none of the elasticities were significantly different from zero at the 10% confidence level.

C. Time-specific effects

Next, we examined time-specific effects on the distribution of output. We found statistically significant differences between several years in mean and variance of output for each of the three outputs, according to the estimated parameters associated with the time-dummy variables of the mean, variance and covariance functions.[9] However, it is difficult to assess from the parameters what the total effect is on the risky production technology. As a first step to understanding the total effect, we present the predicted conditional means, variances and covariances of outputs in Table 4. The estimated models are evaluated at the overall sample average input levels.

1. [9] The estimated parameters are not reported here due to space considerations, but are available from the authors upon request.

Table 3 Elasticity estimates JP model with farm-specific effects in mean function

Function	Symbol	Estimate	St.err.	*t*-value	*p*-value
Mean function					
Output 1 (milk)					
Farm land	E_{11}	0.014	0.011	1.320	0.188
Labour	E_{12}	0.022	0.009	2.540	0.011
Purchased feed	E_{13}	0.129	0.007	17.640	0.000
Materials	E_{14}	0.042	0.008	5.290	0.000
Number of cattle	E_{15}	0.200	0.014	14.260	0.000
Machinery capital	E_{16}	0.019	0.004	5.040	0.000
Return to scale	RTS_1	0.427	0.017	25.860	0.000
Output 2 (meat)					
Farm land	E_{21}	0.145	0.030	4.830	0.000
Labour	E_{22}	-0.025	0.025	-1.010	0.310
Purchased feed	E_{23}	0.272	0.020	13.380	0.000
Materials	E_{24}	0.016	0.022	0.710	0.475
Number of cattle	E_{25}	0.708	0.039	18.180	0.000
Machinery capital	E_{26}	-0.003	0.011	-0.290	0.772
Return to scale	RTS_2	1.114	0.046	24.240	0.000
Output 3 (other outputs)					
Farm land	E_{31}	0.139	0.020	7.090	0.000
Labour	E_{32}	0.050	0.016	3.080	0.002
Purchased feed	E_{33}	0.107	0.013	8.050	0.000
Materials	E_{34}	0.074	0.015	5.100	0.000
Number of cattle	E_{35}	0.202	0.026	7.920	0.000
Machinery capital	E_{36}	-0.017	0.007	-2.500	0.013
Return to scale	RTS_3	0.555	0.030	18.470	0.000
Variance function					
Output 1 (milk)					
Farm land	VE_{11}	0.933	0.185	5.050	0.000
Labour	VE_{12}	-0.491	0.185	-2.650	0.008
Purchased feed	VE_{13}	0.344	0.165	2.080	0.037
Materials	VE_{14}	-0.432	0.192	-2.250	0.024
Number of cattle	VE_{15}	0.072	0.250	0.290	0.774
Machinery capital	VE_{16}	-0.057	0.094	-0.600	0.547
Total variance el.	TVE_1	0.368	0.164	2.260	0.024
Output 2 (meat)					
Farm land	VE_{21}	0.478	0.173	2.760	0.006
Labour	VE_{22}	-0.454	0.173	-2.620	0.009

Purchased feed	VE_{23}	0.184	0.154	1.190	0.233
Materials	VE_{24}	0.393	0.179	2.190	0.029
Number of cattle	VE_{25}	0.398	0.234	1.700	0.089
Machinery capital	VE_{26}	0.290	0.088	3.300	0.001
Total variance el.	TVE_{2}	1.289	0.153	8.440	0.000
Output 3 (other outputs)					
Farm land	VE_{31}	0.313	0.178	1.760	0.079
Labour	VE_{32}	0.390	0.179	2.190	0.029
Purchased feed	VE_{33}	0.120	0.159	0.760	0.449
Materials	VE_{34}	0.485	0.185	2.620	0.009
Number of cattle	VE_{35}	-0.037	0.241	-0.150	0.877
Machinery capital	VE_{36}	0.149	0.091	1.640	0.100
Total variance el.	TVE_{3}	1.421	0.158	9.020	0.000
Covariance function					
Milk – meat					
Farm land	$CE_{12,1}$	0.104	0.070	1.490	0.137
Labour	$CE_{12,2}$	-0.325	0.070	-4.640	0.000
Purchased feed	$CE_{12,3}$	0.098	0.062	1.570	0.117
Materials	$CE_{12,4}$	-0.082	0.073	-1.130	0.257
Number of cattle	$CE_{12,5}$	0.048	0.095	0.510	0.613
Machinery capital	$CE_{12,6}$	0.064	0.036	1.790	0.074
Total cov. elasticity	TCE_{12}	-0.094	0.062	-1.530	0.127
Milk - other outputs					
Farm land	$CE_{13,1}$	-0.386	0.284	-1.360	0.174
Labour	$CE_{13,2}$	-0.183	0.285	-0.640	0.521
Purchased feed	$CE_{13,3}$	0.646	0.254	2.540	0.011
Materials	$CE_{13,4}$	0.434	0.295	1.470	0.142
Number of cattle	$CE_{13,5}$	-0.466	0.385	-1.210	0.226
Machinery capital	$CE_{13,6}$	-0.385	0.145	-2.660	0.008
Total cov. elasticity	TCE_{13}	-0.341	0.252	-1.350	0.176
Meat - other outputs					
Farm land	$CE_{23,1}$	-0.037	0.351	-0.110	0.915
Labour	$CE_{23,2}$	0.177	0.352	0.500	0.614
Purchased feed	$CE_{23,3}$	0.799	0.313	2.550	0.011
Materials	$CE_{23,4}$	0.529	0.365	1.450	0.147
Number of cattle	$CE_{23,5}$	-0.902	0.476	-1.900	0.058
Machinery capital	$CE_{23,6}$	-0.458	0.179	-2.560	0.010
Total cov. elasticity	TCE_{23}	0.107	0.311	0.350	0.729

Table 4 Predicted means, variances and covariances of outputs from the estimated model with farm-specific fixed effect

Year	E(y_1)	Var(y_1)	E(y_2)	Var(y_2)	E(y_3)	Var(y_3)	Cov(u_1,u_2)	Cov(u_1,u_3)	Cov(u_2,u_3)
1993	64,579	3,076,810	7,944	57,718	37,484	137,101,550	-302,502	56,196,501	1,453,941
1994	63,669	2,767,340	8,140	48,375	30,712	93,906,470	-310,470	61,124,243	2,371,017
1995	63,829	2,921,096	8,173	41,750	8,338	99,229,831	145,964	36,967,316	158,133
1996	63,232	3,464,287	8,229	54,240	12,557	101,832,457	-74,001	37,167,316	986,799
1997	63,869	2,974,168	8,228	40,599	10,390	80,735,921	-116,242	38,867,316	774,445
1998	64,325	3,068,561	8,312	57,818	20,828	85,175,215	-105,672	35,767,316	-1,450,991
1999	65,482	3,075,124	8,375	63,618	15,807	100,518,480	290,917	10,067,316	-1,979,941
2000	66,980	2,388,094	8,278	53,713	23,621	110,720,675	166,959	25,167,316	-972,383
2001	67,369	3,141,941	8,346	47,677	7,144	99,226,200	410,624	39,167,316	-2,397,461
2002	69,705	3,676,869	8,372	71,830	20,923	113,302,404	172,329	56,911,337	-2,310,220
2003	73,276	5,879,484	8,476	74,727	4,715	119,202,162	-887,664	208,267,316	428,625

Table 4 shows that there are no clear trends in conditional means, variances and covariances of outputs over time. For milk and meat the trend is increasing mean output, but there are negative shifts in some years. For other outputs (output 3) there were sharp drops in the mean in some years, in particular the last year, 2003. The variances of outputs do not exhibit a clear trend, with negative and positive shifts from year to year. For milk and meat output the variance increased in the last years. However, the picture is not complete without the covariances between outputs. The table shows that the covariance between milk and meat, Cov(*u1*,*u2*), takes both negative and positive values over time. The covariance between milk and other outputs, Cov(*u1*,*u3*), is always positive and is highest in the final year, while the covariance between meat output and other outputs, Cov(u2, u3), exhibits large shifts between positive and negative values.

Table 4 provides ambiguous results with respect to which technology is the preferred one for the average farm. In order to rank these technologies we need to employ the estimated JP production functions in the profit mean and variance equations (7a and b). Next, we examine the effects of these individual moments of output on the moments of profit. Table 5 presents predicted mean and standard deviation of profit evaluated at the overall sample mean output prices, input prices and input levels. What emerges from Table 5 is that technological change is not a smooth process, and that it has not necessarily led to welfare improvements for farmers when economic risk is taken into account. Fig. 1 plots the associated CDF of profit for selected years, using the conditional means and standard deviations from Table 5, and assuming a normal distribution for profit. This figure shows that the 2003 profit CDF, having the highest mean, does not stochastically dominate by first or second degree the profit CDFs for the previous years since it has a longer left tail than the other CDFs.

Table 5 Predicted mean and standard deviation of profit from the estimated model with farm-specific fixed effect

Year	E(π)	SD(π)
1993	-385164	27746
1994	-388059	28597
1995	-408456	24272
1996	-404577	24789
1997	-404100	23393
1998	-388411	19555
1999	-386095	16725
2000	-375741	20688
2001	-387923	22537
2002	-363220	25427
2003	-360280	42880

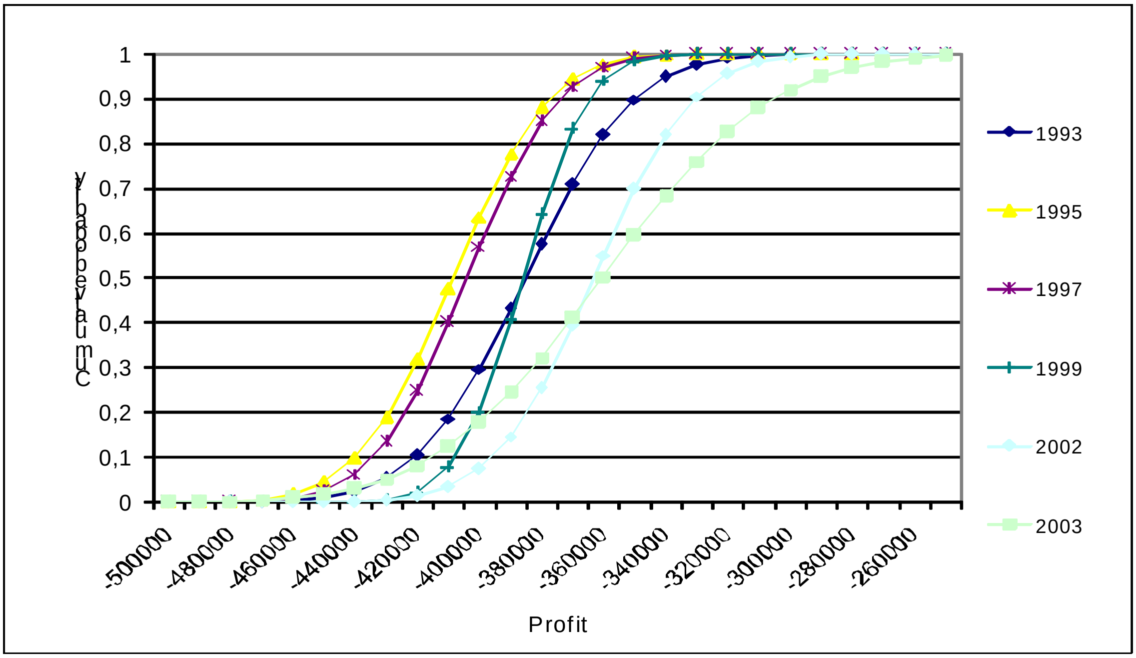

Fig. 1 Cumulative density function of profit in various years assuming a normal distribution

VI. CONCLUDING DISCUSSION

Multi-output farmers are portfolio managers who balance expected economic return and variance of return through input choices. Hence, farmers and policy makers can benefit from a better understanding of the structure of risk. This paper has demonstrated that econometric analyses becomes much richer when one accounts for both the multi-output character of production and output risk. But at the same time it becomes more difficult to predict the effects of input changes, technical change etc. on the welfare of risk-averse producers only by examining the estimated production functions. We have attempted to provide implications for risk-averse producers by estimating not only the predicted effects on moments of output from multi-output Just-Pope technologies, but also by using predicted moments of outputs to predict moments of profit.

Our estimated econometric model can be used by farmers to evaluate the effects of changes in input use, for example, an input-neutral scale expansion or a change in the use of one of the inputs, on both the expected profit and variability in profit. This would be valuable for risk averse farmers who are concerned about effects on both the mean and variance to assess riskiness of profits. Furthermore, it increases the understanding of the effects of technological change over time on the structure of economic risk.

We find that inputs primarily increase output variance, and that the size of the effect of an input may differ across outputs. Our results show that input levels also influence the covariance of shocks between outputs, and that risk-reducing effects of inputs primarily are present in the covariance functions. Technical change leads to shifts in the profit distribution over the data period. Mean profit is higher in the last two data years compared to the previous nine years, but profit variance is also higher. Hence, there is no evident welfare improvement for risk averse farmers in the sense that the technology in the last year does not first-order stochastically dominate earlier years.

ACKNOWLEDGEMENTS

Comments from participants at a seminar at the Norwegian Agricultural Economics Research Institute and from J. Brian Hardaker and Subal C. Kumbhakar are gratefully acknowledged. This research was funded by the Research Council of Norway.

REFERENCES

Antle, J. M. (1983) Testing the stochastic structure of production: A flexible moment-based approach, *Journal of Business & Economic Statistics*, **1**, 192-201.

Antle, J. M. and Goodger, W. J. (1984) Measuring stochastic technology: The case of Tulare milk production, *American Journal of Agricultural Economics*, **66**, 342-50.

Asche, F. and Tveteras, R. (1999) Modeling production risk with a two-step procedure, *Journal of Agricultural and Resource Economics*, **24**, 424-39.

Beattie, B. R. and Taylor, C. R. (1985) *The Economics of Production*, John Wiley & Sons, New York.

Breusch, T. and Pagan, A. (1979) A simple test for heteroskedasticity and random coefficient variation, *Econometrica*, **47**, 1287-94.

Chambers, R G. and Quiggin, J. (2001) *Uncertainty, Production, Choice, and Agency: The State-Contingent Approach*, Cambridge University Press, Cambridge.

Chavas J. P. (2008) A cost approach to economic analysis under state-contingent production uncertainty, *American Journal of Agricultural Economics*, **90**, 435-46.

Chavas, J. P. and Holt, MT (1996) Economic behavior under uncertainty: A joint analysis of risk preferences and technology, *Review of Economics and Statistics*, **78**, 329-35.

Diewert, W. E. (1971) An application of the Shephard duality theorem: A generalized Leontief production function, *Journal of Political Economy*, **79**, 481-507.

Di Falco, S., Chavas, J. P. and Smale, M. (2007) Farmer management of production risk on degraded lands: the role of wheat variety diversity in the Tigray region, Ethiopia, *Agricultural Economics*, **36**, 147-56.

Driscoll, P., McGuirk, A. and Alwang, J. (1992) Testing hypotheses of functional structure: Some rules for determining flexibility of restricted production models, *American Journal of Agricultural Economics*, **74**, 100-8.

Griffiths, W. E. and Anderson, J. R. (1982) Using time-series and cross-section data to estimate a production function with positive and negative marginal risks, *Journal of the American Statistical Association*, **77**, 529-36.

Groom, B., Koundouri, P., Nauges, C. and Thomas, A. (2008) The story of the moment: Risk averse Cypriot farmers respond to drought management, *Applied Economics*, **40**, 315-26.

Hallam, A., Hassan, R. M. amd D'Silva, B. (1989) Measuring stochastic technology for the multi-product firm: The irrigated farms of Sudan, *Canadian Journal of Agricultural Economics*, **37**, 495-512.

Hadar, J. and Russell, W. R. (1969) Rules for ordering uncertain prospects, *American Economic Review*, **49**, 25–34.

Hanoch, G. and Levy, H. (1969) Efficiency analysis of choices involving risk, *Review of Economic Studies*, **36**, 335–45.

Harvey, A. C. (1976) Estimating regression models with multiplicative heteroskedasticity, *Econometrica*, **44**, 461-65.

Isik, M. and Devadoss, S. (2006) An analysis of the impact of climate change on crop yields and yield variability, *Applied Economics*, **38**, 835-44.

Isik, M. and Khanna, M. (2003) Stochastic technology, risk preferences, and adoption of site-specific technologies, *American Journal of Agricultural Economics*, **85**, 305-17.

Kumbhakar, S. C. (1993) Production risk, technical efficiency, and panel data, *Economics Letters*, **41**, 11-6.

Kumbhakar, S. C. and Tveteras, R. (2003) Risk preferences, production risk and firm heterogeneity, *Scandinavian Journal of Economics*, **105**, 275-93.

Just, R. E. (2003) Risk research in agricultural economics: opportunities and challenges for the next twenty-five years, *Agricultural Systems*, **75**, 123-59.

Just, R. E. and Pope, R. D. (1978) Stochastic specification of production functions and economic implications, *Journal of Econometrics*, **7**, 67-86.

Just, R. E. and Pope, R. D. (1979) Production function estimation and related risk considerations, *American Journal of Agricultural Economics*, **61**, 276-84.

Lau, L. J. (1972) Profit functions of technologies with multiple inputs and outputs, *Review of Economics and Statistics*, **54**, 281-89.

Lence, S. H. (2000) Using consumption and asset return data to estimate farmers' time preferences and risk attitudes, *American Journal of Agricultural Economics*, **82**, 934-47.

Mundlak, Y. (2001) Production and supply, in *Handbook of Agricultural Economics, Volume 1A Agricultural Production* (Eds.) B. L. Gardner and G. C. Rausser, Elsevier Science B.V., Amsterdam, pp. 4-85.

Nelson, C. H. and Preckel, P. V. (1989) The conditional beta distribution as a stochastic production function, *American Journal of Agricultural Economics*, **71**, 370-8.

Oehlert, G. W. (1992) A note on the delta method, *American Statistician*, 46, 27–9.

O'Donnell, C. J. and Griffiths, W. E. (2006) Estimating state-contingent production frontiers, *American Journal of Agricultural Economics*, **88**, 249-66.

Regev, U., Gotsch, N. and Rieder, P. (1997) Are fungicides, nitrogen and plant growth regulators risk-reducing? Empirical evidence from Swiss wheat production, *Journal of Agricultural Economics*, **48**, 167-178.

Roberts, R. K., Walters, J. T., Larson, J. A., English, B. C. and Howard, D. D. (2004) Effects of disease, nitrogen source, and risk on optimal nitrogen fertilization timing in winter wheat production, *Agronomy Journal*, **96**, 792-99.

Serra, T., Zilberman, D., Goodwin, B. K. and Featherstone, D. (2006) Effects of decoupling on the mean and variability of output, *European Review of Agricultural Economics*, **33**, 269-88.

Traxler, G., Falck-Zepeda, J., Ortiz-Monasterio R. J. I. and Sayre, K. (1995) Production risk and the evolution of varietal technology, *American Journal of Agricultural Economics*, **77**, 1-7.

Tveteras, R. (1999) Production risk and productivity growth: Some findings for Norwegian salmon aquaculture, *Journal of Productivity Analysis*, **12**, 161-79.

Tveteras, R. (2000) Flexible panel data models for risky production technologies with an application to salmon aquaculture, *Econometric Reviews*, **19**, 367-89.

Wan, G. H. and Anderson, J. R. (1990) Estimating risk effects in Chinese foodgrain production, *Journal of Agricultural Economics*, **41**, 85-93.

Wan, G. H., Griffiths, W. E. and Anderson, J. R. (1992) Using panel data to estimate risk effects in seemingly unrelated production functions, *Empirical Economics*, **17**, 35-49.

White, H. (1980) A heteroscedasticity-consistent covariance matrix estimator and a direct test for heteroskedasticity, *Econometrica*, **48**, 817-38.

The corresponding author:

- Author: Gudbrand Lien
- Institute: NILF
- Street: P.O. Box 8024 Dep.
- City: NO-0030 Oslo
- Country: Norway
- Email: gudbrand.lien@nilf.no