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# Estimation in a long-run short-run model

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# ESTIMATION IN A LONG-RUN SHORT-RUN MODEL

by

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Often, production decisions must be made under conditions of uncertainty regarding key variables influencing supply and demand conditions. In this paper, it is demonstrated that long-run and short-run profit-maximizing decisions under uncertainty may result in the familiar econometric result (usually ascribed to multicollinearity): The estimated betas of current and lagged variables tend to have opposite signs.

## ESTIMATION IN A LONG-RUN SHORT-RUN MODEL

Inputs to a production process, particularly in resource and agricultural problems where the production process is quite lengthy, are not all chosen at the same time. Many inputs are chosen well before output prices are known, while others are chosen with very good knowledge of price. This is, of course, the standard model of the long and short run. This paper addresses estimation when Muth's (1961) rational expectations model is expanded to encompass a long run and short run.

The motivation for addressing this issue is derived from the problem of estimating a reduced-form equation for the price of stumpage (standing trees ready to be cut). The major demand-shift variable in such a model is housing starts. When both starts and lagged starts are included in such an equation, the coefficient on lagged starts is negative while that on starts is positive. Of course, these variables have much in common, so the phenomenon could be written off to multi-collinearity. There is, however, another explanation that depends upon a long- and short-run model. This paper develops a rational expectations long-run short-run model and shows why a forecast of good news has the opposite effect of the good news itself.

In the case of a forest in the American West, there are time-consuming bureaucratic hurdles to be overcome before timber may be harvested. In the private sector in California, one must file a timber harvest plan with the California Department of Forestry and have it approved. In the public sector, the forest must be cruised (surveyed for trees) and the stumpage put out to public bid. In all ownerships roads must be built or improved. Rain, mud, fire danger, and snow also create strong seasonal constraints for logging. After logging, there is milling and drying, which are also time consuming. For the better grades of redwood, the air-drying itself can take about two years. The sum of all of these processes is a one- to three-year time scale for the provision of lumber. Stumpage owners must commit to cutting their timber well before the state of the market is known. Other aspects of the process, such as shipping, are done after the state of the market is known. There is also an opportunity to cut from previously approved and roaded but uncut areas. The ability (and need) to act at two separate times creates a long-run short-run model.

To see these consequences, it is best to abstract the situation somewhat. Consider a resource whose shadow value, *in situ*, is known and nearly constant from year to year. The shadow value should be nearly constant because it depends upon long-term demand conditions—the forecast of which (for instance an average of 1.5 million starts) changes very little with current information. The first input in the production process is chosen when only the distribution of the demand-shift variable, housing starts, is known. It is some amalgam of road building, filing plans, and such. The second input is chosen after the starts are known. It is some amalgam of milling, drying, and such and cutting from areas that are already permitted and roaded.

If all of the information in this model were known at the beginning, the firms would know the demand and supply curves for lumber. They would equate them and find price. Lumber price would be a function of the things that shift those curves—the price of the two inputs and housing starts. An econometrician would have a simple job: regress lumber price on the input prices and housing starts.

When the situation is that the information regarding, for the sake of argument, starts is not available, the firms have a much more difficult procedure. As described above, they can set supply equal to demand and solve for price for any level of starts. The uncertainty in the starts then induces an uncertainty in price. The firms must then make a two-stage decision, given that uncertain, and later certain, price.

The model could be viewed as an expansion the Muth rational expectations idea to beliefs about a whole parameter vector rather than just a mean. It is also close in spirit to Wolak's (1991) estimation of utility-cost functions when the firms are heterogenous and Stavins and Jaffe's (1990) estimation of wetland response when land is heterogenous. In all of these cases, an underlying heterogeneity parameter—relating to cost, land quality, or demand—plays a key role in the estimation.

## The Pure Theory

At time t-1, a risk-neutral, expected-profit maximizing, price-taking agent chooses inputs, $x_1$, to a production process. At time t, the remaining inputs, $x_2$, are chosen and the output is y. The prices of the inputs are $w_1$, and $w_2$, respectively.

At time t-1, the demand function at time t, Q(p,h), depends upon the realized price at time t and the unknown value of h (which may as well be housing starts). The distribution, F, and density, f, of h are known by all agents, and the parameters of the distribution are $\beta$.

Consider period t first. By t, the value of $x_1$ has been chosen. Everything known about the representative firm is embodied in its restricted profit function, $\pi(p,w_2; x_1)$, the most amount of money it can make when prices are p, $w_2$ and $x_1$ has already been chosen. Equilibrium is simply that supply is equal to demand. Letting $D_x$ be the differential operator, differentiate with respect to x and make use of Hotelling's lemma,

$$D_p\pi(p,w_2; x_1) = Q(p,h).$$

For any given value of h, this equation has a solution of the form $p= R(w_2, x_{1,}h)$, and that solution is the reduced form for price, given $x_1$. We assume this equation is invertible for $h = R^{-1}(w_2, x_{1,}p)$. Since h is a random variable, p is also a random variable. Its density, g, is $f(R^{-1})$ dh/dp. Its distribution is G.

In period t-1, the representative agent maximizes expected profits. Since the representative agent cannot affect price, G, its distribution, is taken as fixed. The first-period problem is to choose $x_1$ to maximize expected profit:

$$\max_{x_1} \int \pi - w_1 x_1 \, dG.$$

The first-order conditions for a maximum are

$$0 = \int D_{x_1}\pi - w_1 \, dG(\beta,p),$$

and $x_1^*$ is its solution, the factor demand for $x_1$ as a function of the known parameters of G, $\beta$, and factor prices, w.

Plugging $x_1^*$ into the restricted profit function and subtracting the cost of $x_1^*$ gives profits at t:

$$\Pi = \pi(x_1^*(\beta,w_1,w_2),p,w_2) - w_1 x_1^*.$$

Now setting demand equals to supply gives

$$D_p\Pi = Q(p,h)$$

with solution

$$p = R''(w_1,w_2,\beta,h).$$

The true reduced-form R" includes the distribution parameter of F as well as the realized value, h. Leaving out the distribution parameter $\beta$, for example mean and variance, will bias the regression coefficients whenever $\beta$ is correlated with the outcome. In other words, any time an informative prediction can be made about h, and agents act on that prediction, the prediction must be included in the reduced-form regression.

The reduced-form has the (at first) peculiar property that changes in beliefs about h work opposite of changes in h. To be more precise, consider an element of $\beta$, $\beta_0$, that shifts F in the manner of first-degree stochastic dominance. Let e be the unit vector with a one in the place corresponding to the position of $\beta_0$, then: $F(\beta + e\,\beta_0) \leq$

$F(\beta)$. All agents (not only risk-averse agents) agree that an increase in $\beta_0$ means more h.

The logic of the exercise is fairly simple, though the calculus is not. Since demand has shifted out (ex-hypothesis), the agents will increase $x_1$ and, *in their view*, the distribution of price will be less favorable. To begin, recall that (at time t-1) p is distributed as $F(R^{-1},\beta)$ and the first-order condition for profit maximization, $\int D_x\pi \, dF = w_1$. Totally differentiating the first-order condition with respect to $\beta_0$ and $x_1$ gives

$$(\int D_{xx}\pi \, dF)\, dx + [D_t \, (\int D_x \, dF(R^{-1}(x_1 + t,p,w_2),\beta) \,]\, dx$$
$$+[D_{\beta_0} \int D_x \, dF(R^{-1},\beta)] \, d\beta_0 = 0.$$

The first term is negative because $D_{xx}\pi$ is negative. The second term is

$$\lim_{t\text{->}0} \{(1/t) \int D_x\pi \, dF(R^{-1}(x_1+t,...)...) - \int D_x\pi \, dF(R^{-1}(x_1,...)...)\}.$$

Since F is a CDF it is non-decreasing in $h=R^{-1}$; the properties of the reduced-form R give $dh/dx \geq 0$. Thus, $F(R^{-1}(x_1 + t \, ...)...) \geq F(R^{-1}(x_1 \; ...)...)$ which is exactly first-degree stochastic dominance. Since $D_x\pi$ is positive, the stochastic dominance theorem makes the expression with lim negative for all t and the second term of the total differentiation is non-positive. The third term is positive by the original assumption on $\beta_0$. Thus, $dx_1/ \, d\beta_0 > 0$.

With an increase in $\beta_0$, the price distribution shifts up, but only in the sense that the expected value of the quasi-rents to $x_1$ grows. Rearranging the total derivative and dropping the first term which is negative gives

$$[D_t \, (\int - D_x\pi \, dF(R^{-1}(x_1 + t,p,w_2),\beta) \,]\, dx/d\beta_0 < [D_{\beta_0} \int D_x \, \pi dF(R^{-1},\beta)].$$

Since the first term is positive, so is the second, which is the same as $D_{\beta_0} \int D_x \pi dG >$ 0, which was to be shown.

To summarize, increasing h in the sense of FSD increases $x_1$ and the distribution of p in the sense that the new distribution has higher quasi-rents to $x_1$ than the old distribution of p.

**A Cobb-Douglas Log-Normal Example**

The steps to create the example start with a careful consideration of the problem of a representative firm. In the second period, the price of output, P, will be known. The second-period problem for the firm is the ordinary one of maximizing profits, given whatever first-period choice, $x_1$, was made. After a little algebra, the supply curve of such a firm in the second period is derived. By setting that supply curve equal to demand, one can find the distribution of price given $x_1$. As mentioned above, it is the uncertainty in housing starts that induces the distribution in price. The last step is to have the firms maximize expected profits, given the distribution of future prices and to be sure that the choice of $x_1$ accomplishes that maximization.

Let the production function for the lumber be Cobb-Douglas with decreasing returns to scale:

$$(1) \qquad y = A\, x_1^{a_1}\, x_2^{a_2}$$

where x is input and y is output.

The restricted profit function, profits given $x_1$, is

$$(2) \qquad \Pi(x_1, P) = \max_{x_2} (PA\, x_1^{a_1}\, x_2^{a_2} - w_2\, x_2)$$

where $w_i$ is the factor price for $x_i$ and P is output price. Restricted profits are the most money that can be made given the prices and given that the level of $x_1$ has already been chosen. It is well known that the derivative of a restricted profit function with respect to an input price is the factor demand for that input—the amount of the input to be purchased to maximize profits. Here the amount of $x_2$ is chosen after the price, P, is known.

On taking the derivative and solving for $x_2$,

$$(3) \qquad x_2 = \left[\left(\frac{w_2}{a_2}\right)(PA)^{-1}\, x_1^{-a_1}\right]^{\frac{1}{a_2-1}}.$$

By substituting for $x_2$ in (2), one derives another expression for restricted profits—this one in terms of the second-period price. So the restricted profit function is

$$(4) \qquad \Pi(x_1, P) = (1-a_2)\left[\left(\frac{w_2}{a_2}\right)^{\frac{a_2}{a_2-1}} P^{\frac{1}{1-a_2}}\, A^{\frac{1}{1-a_2}}\, x_1^{\frac{a_1}{1-a_2}}\right].$$

Equation (4) gives profits as a function of the uncertain price, P, and the first-period choice of $x_1$. The first-period choice of $x_1$ is made to maximize expected profits;

$$(5) \qquad \max_{x_1}\; E\Pi(x_1, P) - w_1\, x_1.$$

Solving the first-order condition gives

$$(6) \qquad x_1^* = \left[\left(\frac{w_1}{a_1}\right)\left(\frac{w_2}{a_2}\right)^{\frac{a_2}{1-a_2}} E\left[P^{\frac{1}{1-a_2}}\right]^{-1} A^{\frac{-1}{1-a_2}}\right]^{\frac{1-a_2}{s-1}}$$

where $s = a_1 + a_2$. Substituting back into the restricted profit function and subtracting the factor cost for $x_1$,

(7) $$\Pi(P, EP) = \left[ \left(\frac{w_1}{a_1}\right)^{\frac{-a_1}{1-s}} \left(\frac{w_2}{a_2}\right)^{\frac{-a_2}{1-s}} A^{\frac{1}{1-s}} \right] \bullet$$

$$\left[ (1-a_2)\, P^{\frac{1}{1-a_2}}\, E\left( P^{\frac{1}{1-a_2}} \right)^{\frac{a_1}{1-s}} - a_1\, E\left( P^{\frac{1}{1-a_2}} \right)^{\frac{1-a_2}{1-s}} \right].$$

(When P = EP, this reduces to

$$\Pi(P) = (1-s)\,(PA)^{\frac{1}{1-s}} \left(\frac{w_1}{a_1}\right)^{\frac{-a_1}{1-s}} \left(\frac{w_2}{a_2}\right)^{\frac{-a_2}{1-s}},$$

the familiar form for the Cobb-Douglas profit function.) The short-run supply curve is

(8) $$\frac{\partial \Pi\left(x_1^*, P\right)}{\partial P} = P^{\frac{a_2}{1-a_2}}\, E\left( P^{\frac{1}{1-a_2}} \right)^{\frac{a_1}{1-s}} K$$

where

$$K = \left[ \left(\frac{w_1}{a_1}\right)^{\frac{-a_1}{1-s}} \left(\frac{w_2}{a_2}\right)^{\frac{-a_2}{1-s}} A^{\frac{1}{1-s}} \right].$$

Let demand be log-linear and let the demand shift variable, h, be log-normally distributed; the short-run equilibrium is given by

(9)
$$GP^{-\gamma} h^{\beta} = P^{\frac{a_2}{1-a_2}} E\left(P^{\frac{1}{1-a_2}}\right)^{\frac{a_1}{1-s}} K,$$

where G, $\gamma$, and $\beta$ are positive constants.

Solving this for $\ln P^{\frac{1}{1-a_2}}$,

(10)
$$\ln P^{\frac{1}{1-a_2}} = \frac{1}{a_2 + \gamma - \gamma a_2}\left(\ln G/K + \beta \ln h - \frac{a_1}{1-s} \ln E\left(P^{\frac{1}{1-a_2}}\right)\right).$$

Since $\ln h \sim N(\mu, \sigma^2)$, $\ln\left(P^{\frac{1}{1-a_2}}\right)$ is also normal. Let $\delta = a_2 + \gamma - \gamma a_2$, which is positive because $a_2 < 1$.

(11)
$$\ln P^{\frac{1}{1-a_2}} \sim N\left(\frac{\ln G/K + \beta\mu - \frac{a_1}{1-s} \ln E\left(P^{\frac{1}{1-a_2}}\right)}{\delta}, \frac{\beta^2 \sigma^2}{\delta^2}\right).$$

From the usual formulas, $P^{\frac{1}{1-a_2}}$ is normally distributed with mean

(12) $$E\,P^{\frac{1}{1-a_2}} = \exp\left[\frac{\ln (G/K) + \beta\mu - \frac{a_1}{(1-s)} \ln E\left(P^{\frac{1}{1-a_2}}\right)}{\delta} + \frac{\beta^2 \sigma^2}{2\,\delta^2}\right].$$

Solving,

(13) $$E\,P^{\frac{1}{1-a_2}} = \left\{(G/K)^{1/\delta} \exp\left[\beta\mu/\delta + \beta^2 \sigma^2 / 2\,\delta^2\right]\right\}^{\frac{1-s}{1-a_2}}.$$

By the same argument, let $\theta = \delta/(1 - a_2)$.

(14) $$E\,P = \left\{(G/K)^{\frac{1}{\theta}} \exp\left[\beta\mu/\theta + \beta^2 \sigma^2 / 2\,\theta^2\right]\right\}^{\frac{1-s}{1-a_2}}.$$

The ex-post reduced form is

(15) $$\ln P = \delta^{-1}(\ln G/K + \beta \ln h) - \frac{a_1}{1-a_2}\left\{\frac{1}{\delta} \ln(G/K) + \beta\mu/\delta + \beta^2 \sigma^2 / 2\,\delta^2\right\}.$$

Equation (15) is what should be estimated. It differs from the naive specification in including the parameters ($\mu$, $\delta$) of the distribution of housing starts as well as including the realized values. From (15) and (14), it is clear that expected price increases with $\mu$, but realized price decreases in $\mu$, ln h held constant. In (15) h and $\mu$ have different signs: a surprise in housing starts—high when $\mu$ is low—is what gives a high price. Since the two variables have opposite signs, it accounts for a frequent observation: Running a regression with current and lagged starts gives opposite signs to the two variables. The lagged variable in that regression is simply a proxy for

$\mu$. It belongs in the regression, and the opposite signs are expected. There is more that can be dragged from (15), but we shall desist. When the decision-making agents do not know the values of variables, the information that they have must be used to supplement the ordinary variables in a reduced-form equation. Estimation is not so simple after all.

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