Journal of Agricultural and Applied Economics, 38,2(August 2006):249–253

Price Endogenous Mathematical Programming Models and Trade Analysis

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Takayama and Judge introduced the price endogenous mathematical programming model as an alternative to the traditional econometric approach to sector-level policy analysis. McCarl and Spreen provided a review of price endogenous mathematical programming models. In that paper, they showed how price endogeneity can be introduced into a standard firm-level linear programming model. The introduction of price endogeneity allows expansion of the firm-level specification to a market-level analysis. At the time of publication of McCarl and Spreen, however, the application of price endogenous mathematical programming models was limited by the availability of software packages that could directly solve such models. The typical application used linear supply and/or demand relationships, which resulted in a quadratic programming (QP) specification. The advent of MINOS in the 1980s and then its incorporation into GAMS has lifted the computation constraint. In the present day, numerous price endogenous models have been developed. I can lay claim to six such models.

In this paper, I develop the spatial equilibrium model that I have frequently applied to policy-relevant problems in agricultural and applied economics. First, I show how a spatial equilibrium model can be derived from a standard transportation model. Second, I discuss price equilibria across time, and alternative product forms are discussed. Third, I extend this result to the multiproduct form of the model. This extension includes a discussion of the integrability of demand systems. The paper closes with observations regarding the models developed at the University of Florida, including the world orange juice model, the Florida grapefruit model, and the world banana market model.

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Transportation Models

The transportation model is used in varied applications. In the standard formulation it is assumed there are $i = 1, \ldots, I$ supply points and $j = 1, \ldots, J$ demand points. A single homogeneous product is to be shipped from the supply points to the demand points. Per unit shipping costs are known and are given by C_{ij}. A fixed amount of the product (S_i) is available at each supply point, and a fixed amount (d_j) is demanded at each destination. In this formulation, X_{ij} is the amount shipped from supply point i to demand point j. The objective of the problem is to minimize the total cost of transporting the product from the supply points to the demand points. Mathematically, the problem is to find X_{ij} such that

$$\text{(1a)} \quad \min \sum_{i=1}^{J} \sum_{j=1}^{J} C_{ij} X_{ij}$$

$$\text{(1b)} \quad \text{s.t.} \quad \sum_{j=1}^{J} X_{ij} \le S_i, \qquad i = 1, \ldots, I$$

$$\text{(1c)} \quad \sum_{i=1}^{I} X_{ij} \le d_j, \qquad j = 1, \ldots, J$$

$$\text{(1d)} \quad X_{ij} \ge 0.$$

The solution of this problem is straightforward, via either the simplex method or one of the many special-purpose transportation algorithms that have been developed.

Spatial Equilibrium Models

Even though the transportation problem has many applications in agricultural economics, the assumption of fixed supply and demand imposes limitations. Suppose that the assumption is modified so that quantity supplied at each supply point j depends upon price and the quantity demanded at each demand point I also depends upon price. In fact, if S_i and d_j are endogenous, a particular point (or region or country) may have excess supply (exporter) or demand (importer). Therefore, the *a priori* distinction between exporters and importers is eliminated. Suppose there are $n = I + J$ regions and the inverse supply function in region k and $k = 1, \ldots, n$ is

$$P_k = S_k(Y_k), \qquad k = 1, \ldots, n$$

where P_k is the supply price in region k and Y_k is the quantity supplied and it is assumed that

$$\frac{dS_k(Y_k)}{dY_k} > 0; \qquad k = 1, \ldots, n$$

that is, the supply function is upward sloping. Next assume that the inverse demand function in region k is

$$P^k = d_k(Z_k), \qquad k = 1, \ldots, n$$

where P^k is the demand price[1] in region k and Z_k is the quantity demanded. Also assume that the demand functions are downward sloping; that is,

$$\frac{dd_k(Z_k)}{dZ_k} < 0.$$

Maintain the assumption of constant per unit transportation costs (C_{ij}) and that the unknowns of the problem are X_{ij}, the amount shipped from region i to region j. With no exogenous delineation of exporters and importers, however, a variable X_{ii} has been introduced. Interpret X_{ii} as the amount produced and consumed in region i.

To shorten the discussion, the spatial equilibrium formulation is presented based upon Takayama and Judge. The formulation is

$$\text{(2a)} \quad \max \sum_{i=1}^{n} \int d_i(Z_i)\, dZ_i - \sum_{i=1}^{n} S_i(Y_d)\, dY_i - \sum_{i=1}^{n}\sum_{j=1}^{n} C_{ij}X_{ij}$$

$$\text{(2b)} \quad \text{s.t.} \sum_{j=1}^{n} X_{ij} \leq Y_i, \qquad i = 1, \ldots, n$$

$$\text{(2c)} \quad \sum_{i=1}^{n} X_{ij} \leq Z_j, \qquad j = 1, \ldots, n$$

$$\text{(2d)} \quad Z_i, Y_i, X_{ij} \geq 0.$$

By forming the Lagrangian of the problem and deriving the first-order conditions, it can be established that if region i exports to region j ($X_{ij} > 0$), then $P^j = P^i + C_{ij}$; that is, the price in region j is equal to the price in region i plus the cost of transport from i to j.

In essence, the mathematical programming model given by (2a)–(2d) is the linear programming problem given by (1a)–(1d) where S_i and d_j are now endogenous. Solution of (2a)–(2d) establishes a spatial price equilibrium and identifies importers and exporters. If the supply and demand relationships are linear, then (2a)–(2d) is a QP problem and can be readily solved by a software package such as GAMS.

One major shift in how these models are constructed and used has occurred over the past 30 years. In the 1970s, these models took on a prescriptive nature and were used to identify apparent inefficiencies in markets. In the last 15 years, however, the typical assumption is that markets are rational. Therefore, the model must be calibrated to replicate a base year. In other words, the data are logical and the model should be consistent with the data. The process of making the model consistent is called validation and is generally accomplished

[1] The term "supply price" refers to the price associated with the supply curve; similarly "demand price" is the price from the demand curve.

by calibrating parameters to replicate a particular year or average across several years.

In a study of the European Union (EU) banana trade regime, Spreen et al. developed a QP model of the world banana market. The EU banana trade regime included preferences for selected importers through the imposition of tariff-rate quotas on other importers. The QP framework facilitated modeling of this system of preferences. Constraints were placed on selected X_{ij} variables to mimic the tariff quota.

Extension to Equilibrium over Time and Form

Once the concept of price equilibrium over space is established, it can be easily extended to equilibrium over time and form. Although these formulations are not as widely found in the literature compared to spatial equilibrium, they do represent potentially useful modeling techniques. For example, Busby and Spreen used a model originally developed by Pana-Cryan of the world market for Florida grapefruit to assess the implications of possible withdrawal of a fungicide used to extend the shelf life of fresh grapefruit. Given Florida's dominance as a supplier of fresh grapefruit to the major importing markets, a price endogenous model was developed.

The four major markets for Florida grapefruit are the United States, Canada, the EU, and Japan. Inverse demand equations were estimated for these four markets. Because Florida grapefruit is sold in both fresh and processed form, the price of grapefruit juice is endogenously determined. In the supply side of the model, fruit is allocated to fresh and processed form and also allocated across the four markets, which differ in terms of quality standards. Differing quality is represented by the proportion of fruit that meets the fresh market standard (this is known as packout in the industry). The model is then completed by the estimation of inverse demand equations for each market.

Implicit Supply Models

Among market equilibrium models, price endogenous mathematical programming models offer the option of implicit supply. In this case, a direct relationship between price and quantity supplied is not available. This may be because data are not available to enable estimation of an explicit supply function. Another possibility is that the relationship between current price and current supply does not represent a true supply function because of the production process embedded in the market equilibrium. For example, the supply function for a tree crop such as citrus is embedded because of the impact of tree age on citrus yields. Producers respond to price signals by planting trees. Once a tree is planted, however, it takes 3 years before it begins to bear fruit. The tree will not reach full production until it reaches 10 to 12 years of age.

In Spreen, Brewster, and Brown, a model of the world orange juice market was developed in which Sao Paulo, Brazil, and Florida are treated as the two endogenous supply regions.[2] In each region, a vintage model of orange production is utilized. In Florida, the inventory of orange trees by variety and age is known (FASS). Annual yield by variety and age is reported at the close of each season. Using USDA-FAS data, a similar tree age profile can be estimated for Sao Paulo. By multiplying the number of trees in an age category by the average yield of trees in that age category and summing across all age categories, production of fruit for the current season can be estimated. A similar procedure is used in both Florida and Sao Paulo. After fruit production is estimated, historic processed utilization rates are used to estimate the proportion of fruit processed into juice.[3] Average juice yields convert tons of fruit into gallons of juice.

Multiproduct Models

In the United States and Canadian orange juice markets, two product forms of orange juice

[2] Sao Paulo and Florida collectively account for approximately 85% of the world supply of orange juice.

[3] In Florida, approximately 95% of the orange crop is processed. In Sao Paulo, this percentage is more variable, but averages about 80%.

each account for nearly one half of consumption. These are not from concentrate (NFC) and from concentrate, also called reconstituted orange juice (RECON). Although these products are similar, NFC commands a substantial price premium over RECON. Another important distinction is that the NFC consumed in North America is primarily produced in Florida. Sao Paulo dominates the supply of frozen concentrated orange juice (FCOJ) to markets in the EU and Asia.

In order to incorporate both NFC and FCOJ into the world orange juice model, two steps must be completed. First, a two-product demand system must be estimated. To simplify the model, demand is estimated at the processor (free on board) level so that NFC and FCOJ, the main input into the production of RECON, are included. Second, the model must be able to endogenously produce NFC and FCOJ in both Florida and Sao Paulo. Juices from different varieties of oranges differ in color and taste. Using data obtained from selected Florida processors, quality standards for each product were obtained. A blending model was developed in which the ingredients were juice from different varieties and the quality standards were color, Brix,[4] and ratio.[5]

Because NFC and FCOJ are close substitutes, the demand system is interdependent. This relates to an assumption required in multiproduct programming models known as integrability. In order to satisfy integrability, the demand system must have symmetric cross-quantity effects. Because the demand equations used in price endogenous mathematical programming models are price-dependent demand equations, then symmetry requires that

$$\frac{\partial P_i}{\partial q_j} = \frac{\partial P_j}{\partial q_i} \quad \text{for all } i \neq j.$$

In general, Marshallian demand equations do not have symmetric cross effects in either quantity-dependent or price-dependent form. Compensated demand equations exhibit symmetric Jacobians and these are suitable for use in mathematical programming models.

In the world orange juice model, symmetric demand systems were estimated for both the United States and Canada. With NFC not yet a significant proportion of orange juice consumption in the EU and Asia, a single-product inverse demand equation was estimated. By merging the blending model used to simulate NFC and FCOJ production in Florida with the interdependent demand systems, the model was able to replicate the 2001–2002 season in terms of the volume and price of both NFC and FCOJ produced in Florida.

After spatial price equilibrium is established, prices are adjusted for processing and harvest costs to give grower prices. Growers use these prices in their new planting decisions. New plantings are predicted and existing trees are aged 1 year and adjusted for normal death loss. The model moves forward one season and is solved again.

The world orange juice model offers two interesting extensions of price endogenous mathematical programming models. First, an implicit supply formulation is used to account for the lag between price signals, new plantings, and production response. Second, an interdependent demand system is used to more appropriately model the United States and Canadian orange juice markets.

Other examples of implicit supply models are Spreen, McCarl, and White, a model of the beef cattle sector of Guyana, and Deepak, Spreen, and VanSickle, who developed a model of the U.S. fresh winter vegetable market in North American. Both of these models had intertemporal dimensions but neither included intertemporal price equilibrium conditions.

Concluding Remarks

Price endogenous mathematical programming models have been valuable tools in a wide array of applications related to both trade policy and other issues, such as the possible impact of proposed pesticide bans. The advantages of these models include the ability to impose policy restrictions directly in the model and the ability to model supply when inadequate time

[4] Brix is a measure of sugar content.

[5] Ratio is the sugar/acid ratio, which relates to sweetness.

series data are available to allow estimation of supply functions. The biological process of production may also be directly incorporated in implicit supply models. The primary disadvantage is that such models do not possess statistical properties, and therefore statistical inference is not possible. These models generate point estimates of production, consumption, and price, and confidence intervals cannot be constructed about these point estimates. Nevertheless, these models can be valuable to decision makers because they attempt to assess the implications of proposed policy changes.

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