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Peter Berck
University of California, Berkeley

William R. Bentley
Winrock, International

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DEPARTMENT OF AGRICULTURAL AND RESOURCE ECONOMICS
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by

Peter Berck and William R. Bentley

California Agricultural Experiment Station
Giannini Foundation of Agricultural Economics
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I. Introduction

The United States used its power of eminent domain to take a considerable fraction of all remaining old growth redwood for inclusion in the Redwood National Park. At the second of the two takings that were to be the Park, there were claims by the forest industry that the taking would ruin the economy of the redwood region[1]. The industry mounted a massive campaign to discourage the expansion of the Park with such publicity stunts as driving logging trucks through the streets of San Francisco (Champion). Although it is hard to see how the Park could be in the interest of wood products workers, it is also hard to see how the Park could fail to be in the interest of the owners of the forest resource. They would receive just compensation, including payment for their timber and severance damages for the loss of economic usefulness of their mills, roads, etc. In addition, they (at least as a group) would be left with a unique resource in shorter supply.

The addition to the value of the remaining forest property caused by a taking is called "enhancement." Insofar as the enhancement affects the same operating unit as the taking affects and occurs at the time of the taking, the compensation for the taking can be offset by the enhancement. It is fair to assume that the industry captured the lion's share of this enhancement effect since, (1) a company (Pacific Lumber) uninvolved in the taking holds a large portion of the remaining redwood; (2) much of the rest of the redwood is in units different from the units partially taken; and (3) the enhancement probably occurred before the actual day of the taking.

The purpose of this paper is to estimate the size of this enhancement. Doing so requires a model of price formation. Since old-growth redwood is widely regarded as an exhaustible resource--its beauty comes from a growing process that takes many centuries--such a model should be consistent with Hotelling's resource model. The restrictions that the theory places on estimation are examined from a new perspective, that of short- and long-run changes in demand side variables. From this perspective, the theory is actually much less restrictive than one would otherwise have thought.

The paper begins with this introduction. Section II describes a class of estimable models that includes the Hotelling resource model and describes the restrictions that Hotelling's model implies. Section III contains the estimation of this model for the redwood stumpage market. Section IV contains tests for constancy of coefficients, for consistency with the theory, and for distinguishing long- and short-run effects. Section V contains an estimate of the dollar value of the enhancement in each of the Park takes. Section VI is the conclusions. Appendix I contains a list of the variables and their sources, while Appendix II contains the estimation of an annualized price series.

II. The Model

The market for redwood stumpage depends upon the demand for redwood products and the producers' supply decisions. The demand side of the model leads to the simple estimation of a demand curve. The supply side is not so easy. Berck (1979) shows how these choices might be made with rational expectations by a present value maximizer. The choice facing the redwood producers is which year to sell their material. To make that choice, the producers need to make price forecasts for future years. These forecasts are themselves dependent upon the demand for the product in future years and the producers' view of their own behavior. With these forecasts, the producers could be profit maximizers, mean variance utility maximizers, or any of a number of other possible behavioral types. Regardless of what the correct behavioral model is, the agents must base their decisions on the observable, relevant information. This information includes those variables that act as demand shifters, which we shall denote as z. To keep the discussion manageable, z will be discussed as if it were a scalar, but the actual estimation will involve three variables of this type. The variables, z, might include forecasts of z or lags of z, a point we shall return to later. The information also includes the inventory of the resource, x. In the case of old-growth redwood, the values of this variable were known by all with reasonable precision. Finally, the agents need a measure of the return available on other assets, which we will take to be an interest rate, r. Thus, describing their behavior (observable quantity, q, and price) as functions of this information gives reduced form equations that do not place great demands on behavioral assumptions about the agents. It is also common.[2]

The three equations of this model, assuming a logarithmic functional form, are:

$$\ln(Q_0) = c_1 + c_2 \ln(z) - \alpha \ln(p_0) \qquad \text{Demand equation} \qquad (1)$$

$$\ln(Q_0) = a_1 + a_2 \ln(z) + a_3 \ln(r) + a_4 \ln(x) \qquad \text{Quantity equation} \qquad (2)$$

$$\ln(p_0) = b_1 + b_2 \ln(z) + b_3 \ln(r) + b_4 \ln(x) \qquad \text{Price equation.} \qquad (3)$$

One possible model for the agent's behavior is the Hotelling model. This model, which we shall present in two forms, places testable restrictions on the coefficients of the reduced form equations. The next subsection describes the restrictions that Hotelling's model places on equations of this form and interprets the variables, z, as being used for short- or long-term forecasts.

Consistency with the Hotelling Model

The well-known Hotelling model analyzes a natural resource model as equilibrium in a product and capital market. The resource, being a capital asset, must earn the same rate of return as any other capital asset. The flow of the resource is an ordinary product whose supply must equal its demand. These two conditions and the condition that the resource stock must be exactly exhausted give the equations for Hotelling's model. A solution of this model can be written as price is a function of stock, interest rate, and exogenous variables from the demand equation. Similarly, quantity sold at any time, q, is a function of the same variables. These two equations are a reduced form but contain restrictions on the parameters.

Since the variables, z, contain information about the future, there are two obvious submodels. In one, the expectations about future values of the demand variables change as current values change.[3] Most crudely, a 1 percent increase in demand today is viewed as being likely to persist for all time. This sort of expectation would come from a variable that was believed to be a Gauss-Wiener process. In the other submodel, expectations about the distant future do not change (much) with changes in the present value of the variables. A variable believed to have short trendless cycles would be of this sort.

Letting Q(z, p) be the demand curve and making use of Hotelling's rule, $p = p_0 e^{rt}$, the equation for exactly exhausting the stock is

$$x = \int_0^T Q(z, p_0 e^{rt})\, dt \,. \qquad (4)$$

Letting the demand curve have the constant elasticity form, $Q = f(z)p^{-\alpha}$, where f is continuously differentiable, this integral becomes

$$x = \int_0^T f(z)\, p_0^{-\alpha}\, e^{-r\alpha t}\, dt \qquad (5)$$

which can be solved for the estimating equation for price,

$$\ln(p_0) = \frac{\ln(f(z)) + \ln(1 - e^{-\alpha r T}) - \ln(x) + \ln(\alpha r)}{\alpha}. \tag{6}$$

In log form the demand curve is $\ln(Q_0) = \ln(f(z)) - \alpha \ln(p_0)$, so the estimating equation for quantity is

$$\ln(Q_0) = -\ln(1 - e^{-r\alpha T}) + \ln(x) + \ln(\alpha r). \tag{7}$$

The demand curve and equations (3) and (4) are Hotelling's model with all changes being long-run changes[4].

The price and quantity reduced form equations derived from this model have a number of exclusion restrictions and cross-equation constraints. First, the constant terms in the price and quantity equations differ only by division by the price elasticity, α, from the demand equation. This restriction should not be tested for the usual reason--departures from the exact functional form show up as changes in the intercept. Second, the stock (and interest) effect in the price equation is $1/\alpha$ times the stock (and interest) effect in the quantity equation. Third, and last, the demand shift variables are completely missing from the quantity equation. These restrictions on the reduced form and demand equations define a version of the Hotelling model in which changes in current demand shift variables are believed to affect future demand shift variables in the same way.

A more flexible view of the Hotelling model would make a distinction between short-run and long-run changes in the demand shift variables. Consider a partition of the exogenous demand variables into z_s for the immediate term or short-run values and z_L for the long-run values. Letting Q(p) be the demand curve and making use of Hotelling's rule, $p = p_0 e^{rt}$, the equation for exhausting the stock is

$$x = \int_0^S Q(z_s, p_0 e^{rt})\, dt + \int_S^T Q(z_L, p_0 e^{rt})\, dt. \tag{8}$$

The first integral is the sum of demand in the short run and the second integral is the sum of demand in the long run. To find the effect of changing the demand shifters in the long and short run, totally differentiate the exhaustion equation with respect first to z_s and p_0 and then with respect to z_L and p_0.

$$0 = \int_0^s (Q_z \, dz_s + Q_p \, dp) \, dt + \int_s^T (Q_p \, dp) \, dt \tag{9}$$

and

$$0 = \int_0^s (Q_p \, dp) \, dt + \int_s^T (Q_z \, dz_L + Q_p \, dp) \, dt. \tag{10}$$

These two equations are enough to derive limiting results--for the case when the short run is very short--that are not dependent on functional form. On taking the limit of equation (9) as $s \to 0$, one finds that $dp/dz_s = 0$, or price does not respond to transient changes in the demand for the resource. Taking the limit as $s \to 0$ in equation (10) shows that price does respond to permanent changes in demand. The situation for quantity is somewhat less clear. Since transient changes in demand do not affect price, the partial derivative of demand with respect to z_s is also the total derivative. The effect of a long-run change on quantity could, however, be zero. All that is needed is for the second integral in equation (10) to be zero which, in fact, happens for constant elasticity demand--a case to which we now return.

Substituting the functional form of the demand curve into equation (8) gives:

$$x = \int_0^s f(z_s) \, p_0^{-\alpha} \, e^{-r\alpha t} \, dt + \int_s^T f(z_L) \, p_0^{-\alpha} \, e^{-r\alpha t} \, dt. \tag{11}$$

Assuming z_s and z_L are constants, carrying out the integration, taking logarithms, and solving for $\ln(p_0)$ gives

$$\ln(p_0) = \frac{\ln[f(z_s)\,(1 - e^{-\alpha rs}) + f(z_L)\,(e^{-\alpha rs} - e^{-\alpha rT})] - \ln(x) + \ln(\alpha r)}{\alpha}. \tag{12}$$

On substituting (12) into the demand equation, one gets the equation for Q_0:

$$\ln(Q_0) = \ln(f(z_s)) - \ln[f(z_s)\,(1 - e^{-\alpha rs}) + f(z_L)\,(e^{-\alpha rs} - e^{-\alpha rT})] + \ln(x) - \ln(\alpha) - \ln(r). \tag{13}$$

Equations (11) and (12) would be the reduced form equations for the more general form of Hotelling's model except that it is z that is observable and not z_s and z_L.

To see what restrictions are placed on the estimable equations by the expanded Hotelling model, take the derivatives of $\ln(p_0)$ with respect to the z's:

$$\frac{dp_0/dz_S}{p_0} = \frac{f'(z_S)\,(1- e^{-\alpha rs})}{\alpha[f(z_S)\,(1- e^{-\alpha rs}) + f(z_L)\,(e^{-\alpha rs} - e^{-\alpha rT})]} \tag{14}$$

and

$$\frac{dp_0/dz_L}{p_0} = \frac{f'(z_L)\,(e^{-\alpha rs} - e^{-\alpha rT})}{\alpha[f(z_S)\,(1- e^{-\alpha rs}) + f(z_L)\,(e^{-\alpha rs} - e^{-\alpha rT})]}\ . \tag{15}$$

As $s \to 0$, one gets $dp_0/dz_S = 0$, which is expected, and

$$\frac{dp_0/dz_L}{p_0} = \frac{f'(z_L)}{\alpha\, f(z_L)} \tag{16}$$

which says that the long term-price equation elasticity of a demand shifter is just $1/\alpha$ times its demand elasticity.

Let $Q_0 = Q(z_s,p_0)$. Since $\ln(Q_0) = \ln(f(z_s)) - \alpha \ln(p_0)$ and $d(\ln(p_0)/dz) = 1/p_0\ dp_0/dz$, then one can use the chain rule to find

$$\frac{dQ_0/dz_S}{Q_0} = \frac{f'(z_S)}{f(z_S)} - \frac{f'(z_S)\,(1-e^{-r\alpha s})}{f(z_S)\,(1- e^{-\alpha rs}) + f(z_L)\,(e^{-\alpha rs} - e^{-\alpha rT})} \tag{17}$$

and

$$\frac{dQ_0/dz_L}{Q_0} = \frac{f'(z_L)}{f(z_L)} - \frac{f'(z_L)\,(e^{-\alpha rs} - e^{-\alpha rT})}{f(z_S)\,(1- e^{-\alpha rs}) + f(z_L)\,(e^{-\alpha rs} - e^{-\alpha rT})} \tag{18}$$

On taking the limits as $s \to 0$ of (17) and (18), one gets

$$\frac{dQ_0}{dz_S} = \frac{Q_0 f'(z_S)}{f(z_S)}$$

and

$$\frac{dQ_0}{dz_L} = 0.$$

Thus, in the limiting case, short-run changes in demand conditions have no effect on price and change quantity in exactly the way one would predict from looking at the demand curve, holding price constant. Long-run changes in demand condition, on the other hand, do change price but have no effect on quantity at all.

The estimable equations (1)-(3) can be tested for the restrictions implied by the various forms of the Hotelling model. Because it is the current inventory and not its expectation at some future time that appears in the equations, all forms of the model contain restrictions across equations on the inventory variable. As the model is written, the same is true of the interest rate variable, though one could certainly conceive of the case in which there was a different interest rate used for the two subperiods. In terms of equations (1)-(3), the inventory and interest rate restrictions are

$$-a_3 = \alpha\, b_3 \text{ and } -a_4 = \alpha\, b_4. \qquad (19)$$

In the simple Hotelling model, which is also the case if $s \to 0$, the parameter restrictions are that z is missing from the quantity equation and its elasticity in the price equation is a multiple of its demand elasticity:

$$a_2 = 0 \text{ and } b_2 = c_2/\alpha. \qquad (20)$$

In the short-run-long-run form of the model, the restriction that a change in z changes only z_s and not z_L is

$$c_2 - \alpha\, b_2 = a_2. \qquad (21)$$

Finally, in the short-run-long-run model, the restriction that a change in z changes only z_L and not z_s is

$$\alpha b_2 = a_2. \qquad (22)$$

In summary, Hotelling's theory (and the choice of functional form) imply the restrictions in equation (19). When an observed change in a demand shifter has the same effect in the short and long run, equation (20) is true. When the change is short run only, (21) is true. When the change is long run only, (22) is true. One could also construct cases intermediate between (21) and (22) but, the statistical tests below will show that this effort is not really necessary.

In our discussion so far we have treated s and T as fixed numbers and, more importantly, the difference T - s as fixed. If T were very far into the future (or in the limit, ∞), this would pose no problem for estimation. A more general view of T and s would be that T is a calendar date, D is the current date, and S is the time interval for the short run, so its calendar date, s, is S + D. Time series estimation occurs at a sequence of calendar dates, so the time between the end of the short run and the terminal time, T, is T - S - D. The weighting of the short- and long-run effects in equations (12) and (13) depends on the terms $(1 - e^{-\alpha rs})$ and $(e^{-\alpha rs} - e^{-\alpha rT})$ which generalize to $(1 - e^{-\alpha rS})$ and $(e^{-\alpha rS} - e^{-\alpha r(T-D)})$ when one considers many possible starting dates for the estimation. Since the weighting of the long- and short-term effects varies over the sample period, one might expect the coefficients in the estimated equations to systematically change in their value over the sample. The other possibility is that T is so far in the future that the reweighting of the long- and short-term effects makes no difference whatsoever. A test for the constancy of coefficients, reported below, resolves this question.

In summary, Hotelling's model and the functional form always imply the testable restrictions on the coefficients on inventory and interest rates. Assumptions about elasticity of expectations--whether observable changes affect the short or long run or both--are testable. So is the assumption that the terminal time is far enough in the future so that it does not affect the estimation. Before performing these tests of constancy and of the restrictions, we first turn to the estimation of equations (1)-(3).

III. Estimation

The data to estimate a reduced form system for old growth redwood consist of information on the housing related variables, interest rates, inventories of redwood and a competing species, and price information. In this section we present the estimation of the three equations.

The price information was records of 162 sales from 1953 to 1977. The time span was chosen to begin after the Korean conflict and the post-World War II adjustment and to end with the second taking of the Park. With that second taking, the quantities of old growth redwood left in private hands was too small to constitute much of a market for these statistical purposes. Worse, the bidding on the few sales that did occur might well have been influenced by a desire to increase the value of the Park take. Appendix II details our use of this price information.

A. Demand

The demand for redwood stumpage is a derived demand and as such depends upon its price and the demand for the items for which it is used. Although there is little direct evidence of the final uses of redwood, earlier studies (McKillop; Kidder Peabody and Co.) list construction, fences, saunas, outdoor furniture, framing, paneling, and exterior siding among other uses. These uses are largely captured in housing starts and additions and maintenance expense statistics. Demand should also be influenced by the price of competing species. These prices are also functions of housing starts and additions and maintenance expenses. Additionally, the competing species prices depend upon the stock of the competing species. Substituting these three variables for the price of the competing species gives the equation below.

log(quantity) =	-1.802	-.361 log(price)	+ .468 log($starts_{-1}$)
s.e.	9.577	0.143	0.144
t-statistic	0.19	-2.54	3.26
	+.599 log($additions_{-1}$)		-.114 log(fir-stock)
s.e.	0.242		0.716
t-statistic	2.47		0.156

Based on this equation, the own price elasticity of demand is .36.[5] The asymptotic t-ratio indicates that it is unlikely the true elasticity is of the wrong sign. The sum of the demand elasticities for housing starts and additions and maintenance are 1.06, which is nearly one. This is sensible since one expects a doubling of the demand for the things where redwood is used to double the demand for redwood. The t-ratios on these variables show that they are significantly different from zero at the .95% level. The growing stock of the competing species has a coefficient near zero, indicative of an inability to measure this effect or of an actual lack of substitution possibilities. Insofar as redwood is used for decking, hot tubs, and decoration, the substitutio`n effects may indeed be small, but the construction uses should be easily substitutable with treated lumber. Thus, we favor the conclusion that the effects are simply difficult to measure.

The R^2 of this equation is 0.50 and the Durbin-Watson statistic is 2.12, indicating an acceptable fit and that we cannot reject the hypothesis of no autocorrelation or obvious misspecification.

B. Reduced Form Price Equation

The reduced form price equation is the regression of the log of price on all the exogenous variables in the system. Housing starts, additions and maintenance expenses, and the stock of competing species, fir, are assumed to influence demand. Supply depends directly upon stock and interest rate as well as on all of the demand side variables. Thus, the reduced form price equation is the regression of the log of price on the log of these five variables.

log(price) =	-11.424	-1.60 log(inventory)	+.128 log(interest)
s.e.	22.947	.527	.160
t-statistic	.498	3.04	.80
	+.476 log($starts_{-1}$)		+.248 log($additions_{-1}$)
s.e.	.275		.534
t-statistic	1.73		.465
	+1.27 log(fir-stock)		
s.e.	1.86		
t-statistic	.686		

The $R^2 = 0.91$ and the Durbin-Watson is 1.81 indicating a very good fit with little evidence of autocorrelation. (The upper limit of the "grey" area for the Durbin-Watson statistic at the 5 percent significance level is 1.89).

The outstanding result in this equation is the very strong effect of stock on current price. The elasticity is 1.6, so a 1 percent decrease in the remaining stock increases the price by 1.6 percent. The standard error on this coefficient is .53, so the hypothesis that the coefficient has the wrong sign can be rejected at the 95 percent significance level while the hypothesis that the true elasiticity is unity cannot be rejected. Of the other coefficients, only that on housing starts is statistically significantly different from zero. Any portfolio theory that has redwood and bonds as assets would predict that increased interest rates, *ceteris paribus*, would lead to a decrease in the demand for redwood as an asset and thus a decrease in its price. The real interest rate variable is of the wrong sign but has a negligible coefficient and is not statistically significantly different from zero. The problem may simply be that the long-term real interest rate is indeed constant.[6]

C. Reduced Form Quantity

Finally, the reduced form equation for quantity:

log(quantity) =	+0.708	+0.484 log(inventory)	+.0165log(interest)
s.e.	6.890	.158	.048
t-statistic	.10	3.06	.34
	.+.273 log(starts$_{-1}$)		+.439 log(additions$_{-1}$)
s e.	.0825		.160
t-statistic	3.31		.2.74
	-.406 log(fir-stock)		
s e.	.557		
t-statistic	.73		

The Durbin-Watson is 2.35 and the R^2 = 0.82.

This equation follows the pattern on the price equation except that both of the housing related demand shifters are significantly different from zero.

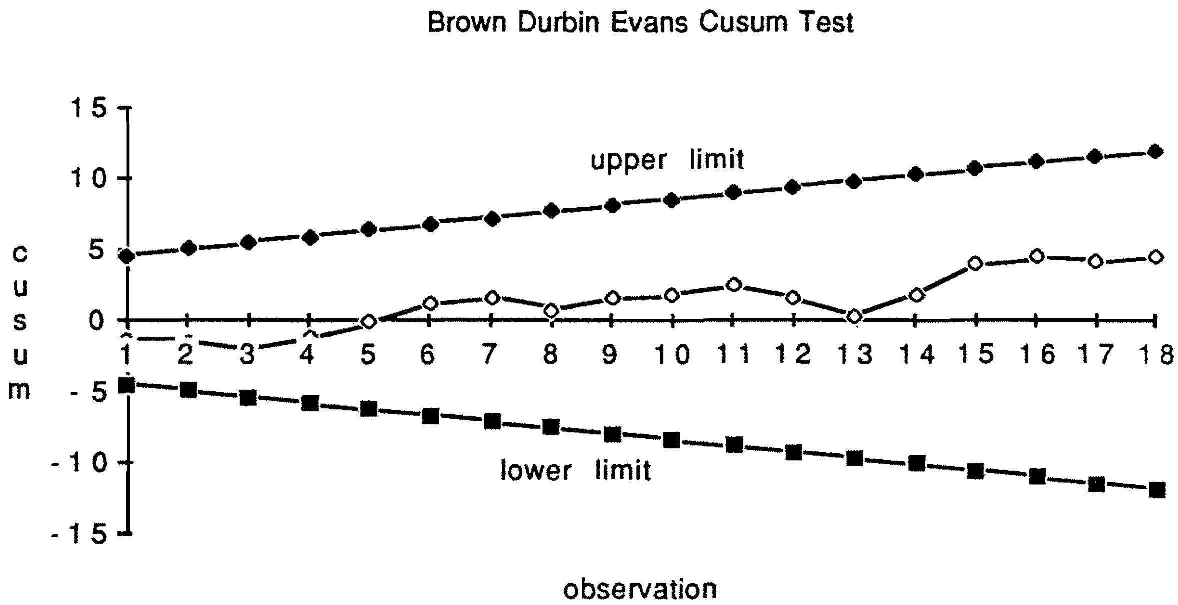
Constancy of Coefficients

When the terminal time is near, the short and long run become the same, which is to say, everything is the long run. Thus, if the terminal time were near enough to influence choices very much, one would expect that the responsiveness of price to demand shift variables would increase over time.

One simple test for this sort of behavior is to allow the coeffcient on housing starts to take on one value in the first years and a second value in the later years. Testing equation by equation for a difference in the values of this coefficient yields the conclusion that it changed in no equation. The values of the coefficient are less than 2 percent different in any equation, and the greatest t-statistic for a difference is .8. Thus, it is highly unlikely that the coefficient on housing starts changed very much over the time period.

The second test performed was that there were two distinct time periods for all the coefficients. The $F_{.95}(6,13)$ statistics were 1.56 for the price equation and 2.38 for the quantity equation. The critical value is 2.63, so we conclude that the coefficients did not change radically over time and the effect of the approach of the terminal time was not important.

Finally, a Brown-Durbin-Evans (1975) test of the constancy of coefficients was performed. The cusum test is meant to detect systematic changes in the coefficients over the time period, exactly the case at hand. The test statistic consists of the sums of suitably normalized one-period-ahead forecast residuals. For each subperiod of the form years r through T- 1, where r is at least the number of regressors plus one and T is the number of observations, one runs the indicated regression and uses it to predict one period ahead. The error in prediction is then divided by its standard error and multiplied by the ratio of the standard error of the *rth* equation to the whole sample. These standardized residuals for the price equation are summed and the sums are plotted in figure 1. The upper and lower lines in that figure bound the 95 percentile critical region. The result is that, since the cusums do not enter the critical region, the hypothesis of constancy cannot be rejected. The same is true of the quantity equation. Thus, we conclude that there is no reason to be concerned with the approach of the terminal time--long- and short-run effects have equal weight over the time period.



Brown Durbin Evans Cusum Test
15
10
5
0
-5
-10
-15
c
u
s
u
m
upper limit
lower limit
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18
observation

IV. Testing the Hotelling Theory

In the classic Hotelling case the long and short run are identical, or s = 0 and T = ∞. Letting the elasticity of demand be α and the demand elasticity of housing starts be β, the theory implies that the housing start elasticity of price is β/α and the housing start elasticity of quantity is zero. Using a Wald test, this hypothesis can be rejected quite soundly. The test statistic is 11.52 which is the realization of a χ^2 with 2 degrees of freedom. Since the critical point (95 percent confidence) for $\chi^2_{(2)}$ is 5.99, we conclude that a change in housing starts does not imply an equal change in the whole time path of housing starts.

Inventory is special because a change in inventory certainly changes one's expectations of future inventory. Thus, it is not affected by the distinction between the long and the short run. The test is to compare the inventory coefficient on the price equation with $1/\alpha$ from the demand equation. The test statistic is 1.96 with 1 degree of freedom. This χ^2 is not big enough to reject at the 95 percent level.

The last test is for the changes in housing starts reflecting only a short term effect. Here the test statistic is 4.43 with 2 degrees of freedom. Again, the $\chi^2_{(2)}$ statistic does not reject the hypothesis that long-term expectations of housing starts are unaffected by short-term changes. Equivalently, current values of housing starts do not affect the agent's long term expectations about housing starts.

V. The Effects of the Taking

The Redwood National Park was created by taking 3.1 million thousand board feet (MBF) from the private sector. The first taking in January of 1968 accounted for 1.7 million MBF, while the March 27, 1978 taking was 1.4 million MBF. After these two takings, private inventories of old-growth redwood were 7.2 million MBF. Thus, the two takes together removed 30 percent of the available old-growth redwood supply.

The calculation for the effects of the second Park take, evaluated at the time of the taking, are straightforward. The take was 16.2 percent of the available inventory. The price elasticity of inventory estimated above was 1.6, so the effect of the taking was to increase price by 26.0 percent. The market price predicted from the sales data for 1978 was $310, so the Park take was responsible for raising prices by $81 per MBF. Multiplying this quantity through by the 7.2 million MBF remaining in private hands gives an increase of $583 million accruing to the remaining redwood holders. It is in addition to the amounts paid by the government to the companies whose timber they took, which was $689,527,000.[7] The timber taken for the Park was not, of course, average timber and that, as well as a host of ancillary issues, is why the timber taken appears to be worth so much more per MBF than the remaining timber.

Similar calculations can be made for the 1968 take. The price at that time was $57.7 per MBF; the inventory was 19.1 million MBF. Thus, the price change was $5.4 per MBF for a final value of $103 million accruing to holders of private timber beyond the value paid by the government for the Park, which was $155 million.

The standard error for the elasticity estimate is one-half so, even at the lower edge of a 95 percent confidence interval, the increased values from the Park takes are in the hundreds of millions of dollars. Similarly, if one alters the specification of the equation by using a price series predicted by a linear rather than log-log regression, one gets an elasticity of .8, still leading to increased values in the hundreds of millions of dollars.

The bottom line of this exercise is that the timber companies were very well served by the Park takings, and their protestations to the contrary are less than comprehensible. Put differently, the companies could well have donated the Parks if they could have found a way to share the gains.

VI. Conclusions

The removal of the stock of an exhaustible will drive up the price of the remaining stock, and when the stock is small to begin with, the effect can be quite dramatic. Redwood National Park was purchased by the government for the precise purpose of saving some of the last remaining redwoods, so the effect of the taking on price should be, and was, large. The companies holding remaining timber benefitted by $583 million, so the consumers of redwood must have lost at least this amount. Excluding interest, the cost to non redwood owning Americans of the second park take was about $1.2 billion--the higher cost of wood plus the price of the taking-- which was a far cry from the approximately $300 million Congress set aside to buy the Park. One wonders whether the Park would still have been taken if the full cost to the public had been known.

On the methodological front, the Hotelling model has rather fewer restrictions on the reduced form than one might first have thought. Only the interest rate and stock variables are necessarily constrained by the theory. The estimation does not reject these constraints. Constraints on housing starts and other demand side variables depend upon whether one views changes in these variables as being temporary or permanent. Our results show that they are percieved closer to temporary than permanent. Thus, Redwood behaves as Hotelling predicted an exhaustible resource would, if one believes that changes in todays housing starts have little bering on what housing starts are expected in the future.

Appendix 1

Sources of Variables

Housing Starts.: Total new housing units started. Historical Statistics and Statistical Abstract.

Additions and Maintainance: Value of new residential buildings put in place less the value of new housing units. Historical Statistics and Stat. Abstract

Interest Rates: Three month bills, market yield. Historical Statistics and Statistical. Abstract.

Gross National Product Deflator: National Income and Product Accounts, 1929-1976 and 1976-1979.

Redwood Production: Production of redwood lumber, in millions of bd ft. Statistical Abstract. various years.

Growing Stock of Other Species:. Net volume of sawtimber greater than 29 inches diameter at breast height in the Pacific Northwest, westside, 1953-1977. Values for 52, 62, 70, 77 are actual; all others are by linear interpolation. An Analysis of the Timber Situation in the United States, 1952-2000. USDA, Forest Service. Forest Resource Report No. 23. December, 1982.

Inventory of Redwood: Actual value for 1978. For earlier years, cumulative cut times percent of cut that is old growth times overrun factor is subtracted from inventory. Sources for 1978 inventory, overrun factors and percent of cut that is old-growth is by personal communcation with U. S. Department of Justice

Sales Data: Personal communication with U.S. Department of Justice.

Appendix 2.

Annualized Price Series

The price of stumpage should be a function of the location, quality, and time of the stumpage sale. A regression of price on these characteristics would recover the relationship which could then be used to construct a price series for constant quality sales. The constant quality price series was constructed for a sale with average volume and percent uppers sold in Humbolt county between private parties. This section describes the construction of that series.

The data consist of 162 sales of timber from 1953 to 1977. For each sale, there are data on price, volume, type of buyer or seller, quality, and location. The price (PRI) is recorded as dollars per MBF. Over the period of this study, price ranged from a low of $6 per MBF to a high of $329 per MBF. Upper grades, such as clear all heart, command a large market premium over grades like construction, so a sale that is expected to yield a higher percentage of upper grades is certainly worth more. The percent of upper grades (PUPP) varies between 40 percent and 60 percent with an average of 47 percent. Sale volumes (VOL) range from 100 MBF to 240,000 MBF, with a mean of 12,000 MBF. To give a sense of scale to this, the largest redwood tree has a volume of 285 MBF (Maita 1987) while the total cut of the Pacific Lumber Company was 140,000 MBF in 1981, so these sales range from the very large to the miniscule. Not all sellers or buyers have the same terms of sale, and the data include dummy variables for the agent type. The U. S. Forest Service (USFS), for instance has sale terms that include an escalator clause. Other recorded types of agents are the California Department of Forestry (CDF) as a seller or the State of California (STATE) as a buyer. The sales come from three counties, each of which has different accessibilities to the mills and, therefore, different costs of harvesting. Dummy variables are used to indicate the county: Del Norte (DELNO) and Mendocino (MENDO), while all other sales come from Humbolt County. Finally, dummy variables for the year of the sale (e.g., D53) were included to account for the changes in price over time.

A regression, in log-log form, of price on these other variables gave a reasonable fit--an R^2 of 91 percent--and most of the yearly dummies significantly different from zero at the 95 percent level.

Based on these regression results, we estimate that a 1 percent increase in the sale volume increases price by .04 percent, and this is statistically significantly different from zero. If one believes that large sales can costly be broken down into small sales and small sales costlessly aggregated into large sales, this coefficient should be zero, which it is not. Although this may at first seem like a small elasticity, the largest sale is 2,400 times the smallest one, resulting in a near doubling of the price because of the volume effect. A 1 percent change in percent uppers changes price by .28 percent. Since percent uppers is on the order of 50, this works out that a change in percent uppers from 50 percent to 60 percent results in a 5.6 percent change in price. The standard error on this coefficient is large, however. The seller or buyer type and county of sale dummies hold only the surprise that, when the state is the buyer, price is appreciably higher. Since the state may be buying trees different from those of other buyers--larger and more scenic--this is explainable.

The prediction performance of this equation was measured with a jacknife (Efron 1982). The root mean square error of prediction was $33, which is trivial by comparison with the prices in the later part of the period but very large compared to the early prices. The root mean square percent error was 46 percent, which gives a much better feel for the accuracy of prediction for an individual sale. The average of many such sales would have a prediction error very similar to that part of the prediction error caused by uncertainty in the locus of the regression line, which would give numbers about half of the above. Thus, for the purposes of deriving a yearly price, the errors should be in the 20-30 percent range. Table 1 gives the coefficients, their standard errors, and t-statistics. The predicted series for price was prepared by antilogging the predictions of the log of price plus one half the standard error of the regression, which accounts for the difference between the expectation of a log normal and a normal distribution. Table 2 gives the predicted prices.

TABLE 1
Hedonic Regression

Variable Name[a]	Estimated Coefficient	Standard Error	T-ratio 130 df
PUPP	0.288	0.239	1.21
VOL	0.047	0.019	2.44
MENDO	-0.037	0.078	-0.48
DELNO	-0.068	0.097	-0.70
STATE	0.494	0.250	1.97
CDF	-0.002	0.096	-0.02
USFS	0.032	0.115	0.28
D53	1.117	0.919	1.22
D54	0.859	0.939	0.92
D55	1.331	0.934	1.43
D56	1.498	0.946	1.58
D57	1.690	0.940	1.80
D58	1.509	0.937	1.61
D59	1.714	0.948	1.81
D60	1.790	0.954	1.88
D61	1.743	0.958	1.82
D62	1.516	0.952	1.59
D63	1.836	0.946	1.94
D64	2.052	0.948	2.16
D65	2.224	0.948	2.35
D66	2.291	0.968	2.37
D67	1.899	0.935	2.03
D68	2.447	0.918	2.67
D69	2.492	0.914	2.73
D70	2.774	0.921	3.01
D71	2.557	0.926	2.76
D72	2.619	0.956	2.74
D73	3.416	0.940	3.63
D74	3.738	0.923	4.05
D75	3.508	0.968	3.62
D76	3.644	0.915	3.98
D77	4.130	0.919	4.49

R-SQUARE = 0.9101
DURBIN-WATSON = 2.1497

[a]PUPP is percent uppers, VOL is volume, MENDO is 1 if the sale is in Mendocino County, DELNO is 1 if the sale is in Del Norte County, STATE is 1 if the State of California was the buyer, CDF is 1 if the California Dept of Forestry was the seller, and USFS was 1 if the U.S. Forest Service was the seller. The regression was run in log-log form and the standard errors were corrected for the heteroscedasticity.

TABLE 2
Yearly Price

Year	Price	Year	Price
1953	15	1966	49
1954	12	1967	33
1955	19	1968	58
1956	22	1969	60
1957	27	1970	80
1958	23	1971	64
1959	28	1972	69
1960	30	1973	152
1961	29	1974	210
1962	23	1975	167
1963	31	1976	191
1964	39	1977	311
1965	46		

Source: Computed. Price is dollars per MBF.

[1]See, for instance,Western Council of Lumber Production, "Additions to the park would cause severe econoimic impacts," including the "displacement of hundreds more workers from the industry," erosion of the tax base and "deterioration of government services."

[2]Fisher, Cootner and Bailey, model production from scrap as depending on the stock of scrap and the price. Adams and Haynes, model stumpage supply as depending on stock and price. Pindyck models the supply of fringe firms as depending on price and cumulative production, which is just total quantity less remaining reserves. All of these models are silent as to the behavioral model for the agent, and all these models lead to reduced forms similar to the form chosen in this paper.

[3]Hicks (1939), p 204 *et seq.*, discusses the elasticity of expectations in terms of whether a one percent price change today would be expected to also cause a price change tomorrow. He distinguishes the pivotal cases of 0 and 1.

[4]The model could be expanded by making demand grow at an exponential rate,γ, less than $r\alpha$ without changing the qualitative results. Such an expansion would simply have the effect of replacing the $r\alpha$ terms with γ-$r\alpha$.

[5] In McKillop's (1969) earlier redwood work, using different demand shift variables and a monthly rather than a yearly time frame, the estimated elasticity was .42.

[6] Fama (1970) is of this opinion, but the matter is far from settled.

[7]The sum is the total of all the Judgments and Orders for civil cases C78-0879TEH, C78-0868 TEH, and C78-0874 TEH, in the United States District Court for the Northern District of California, August 10, 1987

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