An Econometric Analysis of Price Dynamics in the Presence of a Price Floor: The Case of American Cheese

Jean-Paul Chavas and Kwansoo Kim

In this paper, we present an econometric analysis of the effects of a price floor on price dynamics and price volatility. A price floor (implemented as a part of government pricing policy) provides a censoring mechanism for price determination. We specify and estimate a dynamic Tobit model under time-varying volatility. The model is applied to analyze the effects of a price support program on price dynamics and price volatility in the U.S. American cheese market. The econometric analysis provides useful insights on price dynamics in the presence of a government-determined price floor.

Key Words: censored regression, market liberalization, model selection, price dynamics, price volatility

JEL classifications: Q0, D4, C5

Government pricing policy involves a variety of policy instruments, including tariffs, subsidies, and price floors. Government-set price floors mean that the price distribution is censored at the price support level. This is typically implemented by a government agency set to purchase an unrestricted quantity of the targeted commodity at the price floor. This generates two possible regimes: a "market regime," in which the market price is higher than the support price and the government does not intervene, and a "government regime," in which government purchases take place, preventing the market price from falling

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below the support price. Price support programs have been an important part of agricultural policy since the 1930s. They have contributed to both increasing market prices and reducing price volatility. This has stimulated empirical investigations of the effects of such programs on agricultural markets (e.g., Holt and Johnson; Shonkwiler and Maddala). However, previous analyses have focused on a static approach. This suggests a need to better understand the dynamic implications of price floors. Also, over the last decade, U.S. agricultural policy has seen a decreased reliance on price support programs. The effects of such changes on price levels and price volatility in agricultural markets need to be evaluated.

The objective of this paper is to investigate the effect of price floor on price dynamics and price volatility. The analysis focuses on price dynamics in the U.S. American cheese market, a market that has been the subject of a government price support program. As illustrated in Figure 1, the American cheese price was at
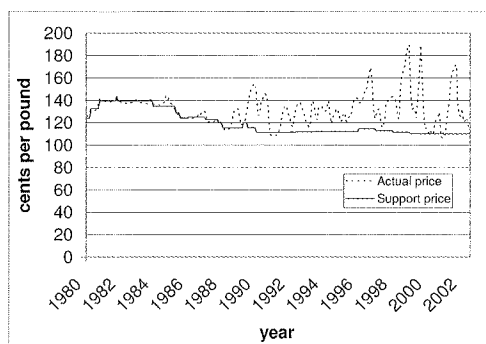
Figure 1. Actual and Support Prices of American Cheese

the support price level during most of the 1980s. However, the 1990s saw market liberalization associated with lower support prices and with market prices being higher most of the time than the support prices (see Figure 1). A main motivation for this paper is to better understand the effect of such market liberalization on the evolution of prices and price volatility. The analysis relies on a reduced-form model of market price determination in the presence of price floor. This leads to the specification and estimation of a dynamic censored regression model. We examine empirically the dynamic aspects of price adjustment in the U.S. American cheese market under market liberalization. We also investigate whether a price support program can still be effective in stabilizing market prices under a market regime in which the role of government is minimal. Finally, the model specification allows for time-varying volatility. This provides a basis for investigating the effects of price floor on market price dynamics and the interactions between changing price volatility and a price support program.

The paper is organized as follows. First, we develop a dynamic reduced-form model of market price determination in the presence of price floor. This involves specifying an econometric model of prices that are censored at the price support level under time-varying volatility. We use Vuong's nonnested test to investigate the relative performance of two distributional specifications: normality (Amemiya; Tobin) and lognormality (Amemiya and Bos-

kin). The analysis is then applied to the U.S. American cheese market. The econometric results show how the price support program and stock holding affect both expected prices and the volatility of prices. Implications of censoring for price determination and price dynamics are discussed. Finally, concluding remarks are presented, along with suggestions for future study.

The Model

In this section, we develop an econometric model investigating the process of market price determination in the presence of a government price support program. Under market equilibrium, the interaction between supply and demand determines market prices. In the absence of storage, supply and demand shocks (such as weather effects and changes in consumer income) affect market price. Over time, these shocks create fluctuations in prices, and when unanticipated, they generate price uncertainty. However, in the presence of stocks, there is an incentive to hold inventory in response to changes in prices. For example, under risk neutrality, an active storage firm would choose inventory such that the discounted expected price next period is equal to the current price plus storage cost. As a result, as long as stocks are positive, competitive storage behavior affects price dynamics and contributes to reducing price volatility (e.g., Deaton and Laroque 1992, 1996; Williams and Wright). Then, the market price is determined by the interactions between supply, demand, and storage behavior.

Let y_t^* be the market price for a commodity at time t in the absence of government intervention. Denote the supply function for that commodity at time t by $S(y_t^*, \cdot)$ and the demand function (including both final demand and demand for stocks) by $M(y_t^*, \cdot)$. As illustrated in Figure 2, the market equilibrium price y_t^* satisfies $S(y_t^*, \cdot) = M(y_t^*, \cdot)$. Solving this market equilibrium condition for y_t^* gives the reduced-form equation for price

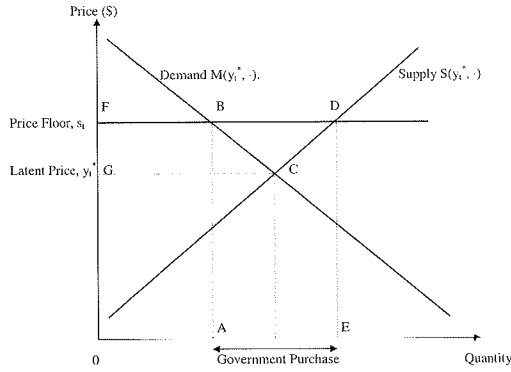$$y_t^* = f(\mathbf{X}_t, \beta) + e_t,$$

Figure 2. Market Equilibrium under a Price Support Program

where $\mathbf{X}_t$ is a vector of explanatory variables (including past prices), β is a vector of unknown parameters, and e_t is an error term assumed to be independently distributed across time periods.

First, we introduce a government price support scheme in this market. The price support is characterized by the floor price s_t reflecting government policy at time t. The observed market price at time t, y_t, then becomes

$$y_t = \begin{cases} y_t^* & \text{if } y_t^* > s_t \\ s_t & \text{if } y_t^* \le s_t, \end{cases}$$

where y_t^* is now the latent price that would be observed in the absence of government intervention. When $y_t^* > s_t$, the price support is inactive and $y_t = y_t^*$. However, if $y_t^* \le s_t$, then a government agency intervenes in the market and buys (and usually stores) the commodity at a price s_t. Figure 2 illustrates where government purchase corresponds to the quantity BD or AE. This effectively creates a perfectly elastic demand at price s_t, thus preventing any decrease in the market price below s_t. The observed market price y_t is then determined according to the model

$$(1a) \quad y_t = \begin{cases} y_t^* & \text{if } y_t^* > s_t \\ s_t & \text{if } y_t^* \le s_t, \end{cases}$$

$$(1b) \quad y_t^* = f(\mathbf{X}_t, \beta) + e_t.$$

Equations (1a) and (1b) constitute a Tobit or censored regression model (Amemiya; Tobin).

The dependent variable y_t is censored at s_t at time t. From Equation (1a), the latent variable y_t^* is observed only if $y_t^* > s_t$. This corresponds to the market regime, in which the latent price is the market price ($y_t = y_t^*$). Alternatively, y_t^* is censored and unobserved if $y_t^* \le s_t$. This corresponds to the government regime, for which the price support program determines the market price (with $y_t = s_t$). Equations (1a) and (1b) provide a generic model of price determination in the presence of a price support program, allowing for endogenous regime switching between the market regime and the government regime.

The latent price y_t^* reflects what the market price would be in the absence of a government price support program. A key issue in Equation (1b) is the specification representing the factors affecting the latent price y_t^*. Zellner and Palm propose alternative ways of specifying $f(\mathbf{X}_t, \beta) + e_t$ in Equation (1b). This provides some flexibility for econometric modeling. To see that, consider the aggregate excess demand function (i.e., aggregate demand plus demand for stocks minus production) for the commodity in the absence of censoring. At time t, let this excess demand function take the form $[b_0 + b_w \mathbf{w}_t + \mathbf{B}(L)y_{t-1} + \mathbf{D}(L)y_t^* + \mathbf{A}(L)\varepsilon_t]$, where $\mathbf{w}_t$ is a vector of exogenous variables (supply–demand shifters), $\mathbf{B}(L)$, $\mathbf{D}(L)$, and $\mathbf{A}(L)$ are polynomial lag operators (satisfying $L^i y_t = y_{t-i}$), and ε_t is an error term at time t. This is a flexible linear specification that allows for dynamic effects through lagged latent prices (y_{t-1}^*, y_{t-2}^*, . . .), lagged actual prices (y_{t-1}, y_{t-2}, . . .), and lagged error terms (e_{t-1}, e_{t-2}, . . .). In the absence of a government price support program, market equilibrium must satisfy a zero-aggregate excess demand.

$$(2a) \quad b_0 + b_w \mathbf{w}_t + \mathbf{B}(L)y_{t-1} + \mathbf{D}(L)y_t^*$$
$$+ \mathbf{A}(L)e_t = 0.$$

Equation (2a) corresponds to a structural model for the determination of latent price y_t^* (e.g., see Holt and Johnson; Shonkwiler and Maddala). However, estimating the structural model in Equation (2a) can be difficult for at least two reasons. First, data might not

be available for all relevant variables (e.g., with monthly data, all supply–demand shifters might not be measured on a monthly basis). Second, in the absence of strong *a priori* information on the dynamics of price determination, some of the parameters in Equation (2a) might not be identified. This suggests a need to explore alternative specifications to Equation (2a).

Consider the case in which the supply–demand shifters $\mathbf{w}_t$ are generated by the stochastic process: $\mathbf{F}(L)\mathbf{w}_t = c_0 + \mathbf{C}(L)\mathbf{u}_t$, where $\mathbf{F}(L)$ and $\mathbf{C}(L)$ are polynomial lag matrices and $\mathbf{u}_t$ is a vector of error terms. If $\mathbf{F}(L)$ is invertible, substituting this expression into Equation (2a) yields

$$(2b) \quad b_0 + b_w\mathbf{F}(L)^{-1}[c_0 + \mathbf{C}(L)\mathbf{u}_t] + \mathbf{B}(L)y_{t-1}$$
$$+ \mathbf{D}(L)y_t^* + \mathbf{A}(L)e_t = 0.$$

Assume that the stochastic process $[b_w\mathbf{F}(L)^{-1}\mathbf{C}(L)\mathbf{u}_t + \mathbf{A}(L)e_t]$ has a Wold representation. Then, it can be written as a moving average process: $b_z\mathbf{F}(L)^{-1}\mathbf{C}(L)\mathbf{u}_t + \mathbf{A}(L)e_t = \mathbf{G}(L)v_t$, where v_t is a white noise error term and $\mathbf{G}(L)$ is a polynomial lag operator. Substituting this expression into Equation (2b) gives

$$(2c) \quad b_0 + b_w\mathbf{F}(L)^{-1}c_0 + \mathbf{B}(L)y_{t-1} + \mathbf{D}(L)y_t^*$$
$$+ \mathbf{G}(L)v_t = 0.$$

In the case where $\mathbf{D}(L)$ is invertible, Equation (2c) yields

$$(2d) \quad y_t^* = a_0 + \mathbf{H}(L)y_{t-1} + \mathbf{K}(L)v_t,$$

where $a_0 = -\mathbf{D}(L)^{-1}[b_0 + b_w\mathbf{F}(L)^{-1}c_0]$, $\mathbf{H}(L) = -\mathbf{D}(L)^{-1}\mathbf{B}(L)$, and $\mathbf{K}(L) = -\mathbf{D}(L)^{-1}\mathbf{G}(L)$. Equation (2d) is a reduced-form equation for the latent price y_t^*. Following Zellner and Palm, it is also a "final equation" with several important characteristics. First, it does not include lagged latent prices as right-hand-side variables. Second, the supply–demand shifters $\mathbf{w}_t$ do not appear as explanatory variables in Equation (2d) (they have been substituted out). Third, because it is a reduced form, Equation (2d) neglects structural information.

But this confers one advantage: its parameters do not suffer from identification problems. Finally, as emphasized by Zellner and Palm, Equation (2d) remains compatible with the structural model in Equation (2a).

Below, we focus our attention on the specification in Equation (2d) as a dynamic representation of the latent price y_t^*. As just noted, this final equation provides a simple but flexible specification of latent price determination in a way consistent with market price determination. For that reason, the empirical analysis presented in this paper is based on Equation (2d). More specifically, we consider the case where $a_0 = \beta_0 + \mathbf{x}_t\bar{\beta}$ (where the $\mathbf{x}_t$'s are variables behaving as intercept shifters), $\mathbf{H}(L)y_{t-1} = \Sigma_{k=1}^{m} \beta_k y_{t-k}$, and $\mathbf{K}(L) = I$. It implies that market dynamics are captured by the m lagged actual prices ($y_{t-1}, y_{t-2}, \ldots, y_{t-m}$). In the context of Equation (1b), this means that $f(\mathbf{X}_t, \beta) = \beta_0 + \Sigma_{k=1}^{m} \beta_k y_{t-k} + \mathbf{x}_t\bar{\beta}$ and $e_t = v_t$.[1] Then, Equations (1a) and (1b) provide a convenient and flexible representation of dynamics in the presence of censoring (e.g., Pesaran and Samiei 1992a,b). In addition, to examine possible changes in price volatility, we also allow for a time-varying standard deviation, σ_t. This establishes a heteroscedastic Tobit model.

Alternative specifications can be used for the error term e_t in Equation (1b). We consider two cases: normal distribution and lognormal distribution. First, we consider the case in which y_t^* is normally distributed with mean $f_N(\mathbf{X}_t, \beta)$ and variance σ_{Nt}^2. This is the normal

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[1] An alternative dynamic Tobit specification is to express y_t^* as a function of lagged latent prices (y_{t-1}^*, y_{t-2}^*, ...) (see Lee; Morgan and Trevor; Wei 1999, 2002). As noted by Lee, this includes as a special case the Tobit model under autocorrelated error terms (Zeger and Brookmeyer). We did not rely on this specification for three reasons: (1) as noted above, the use of lagged actual price is compatible with a structural specification with flexible dynamics, (2) the use of lagged latent variables means that the likelihood function involves multiple integrals (which could require switching from the standard maximum likelihood method to simulated estimation methods), and (3) estimating time-varying σ_t becomes more difficult in this context (Lee).

censored regression model in which the log likelihood function of the sample is (Maddala)

$$(3) \quad \ln(L_N) = \sum_{t \in N_0} \ln(\Phi_t) + \sum_{t \in N_1} \ln[1/(2\pi\sigma_{Nt}^2)^{1/2}]$$
$$- \sum_{t \in N_1} [y_t - f_N(\mathbf{X}_t, \beta)]^2/(2\sigma_{Nt}^2),$$

where N_0 is the set of observations for which $y_t^* = s_t$, N_1 is the set for which $y_t^* > s_t$, and $\Phi_t = \text{Prob}(y_t^* \leq s_t)$ is the distribution function of the standard normal evaluated at $[s_t - f_N(\mathbf{X}_t, \beta)]/\sigma_{Nt}$. After parameterizing σ_{Nt}, the censored regression model can be estimated by the maximum likelihood method. It is well known that, in general, the maximum likelihood estimation method generates consistent and asymptotically efficient estimates of the parameters.

Second, we consider the lognormal case in which $\ln(y_t^*)$ is normally distributed with mean $E(\ln y_t^*) = \ln[f_L(\mathbf{X}_t, \gamma)] - 0.5\sigma_{Lt}^2$ and $\text{Var}(\ln y_t^*) = \sigma_{Lt}^2$ (Amemiya and Boskin). Under lognormality, the log likelihood function of the censored sample is

$$(4) \quad \ln(L_L) = \sum_{t \in N_0} \ln(\mathbf{F}_t) + \sum_{t \in N_1} \ln\{1/[(2\pi)^{1/2}\sigma_{Lt}y_t]\}$$
$$+ \sum_{t \in N_1} \{\ln(y_t) - \ln[f_L(\mathbf{X}_t, \gamma)]$$
$$+ \sigma_{Lt}^2/2\}^2/(2\sigma_{Lt}^2),$$

where $F_t = \text{Prob}(\ln y_t^*) \leq \ln s_t$ is the distribution function for the standard normal evaluated at $\{\ln(s_t) - \ln[f_L(\mathbf{X}_t, \gamma)] + \sigma_{Lt}^2/2\}/\sigma_{Lt}$. Again, in the absence of misspecification, consistent and asymptotically efficient parameter estimates can be obtained by the maximum likelihood method.

Data and Model Specification

Applying our approach to U.S. American cheese prices, we investigate the dynamics of American cheese price and its volatility, with a special focus on the role of the government price support program. Application to the American cheese market is motivated by the considerable changes in government involvement in this market: from extensive government intervention during most of the 1980s to a reduction in support prices for American cheese during the 1990s. These changes in government intervention provide an empirical setting in which the role of the price support program in the dynamics of price and its volatility can be examined. We rely on the heteroscedastic Tobit model in Equations (1a) and (1b) that allows for endogenous regime switching and time-varying volatility. We also investigate the case in which stocks affect both the mean and the variance of prices.

The data used in this study are monthly American cheese price and stock data for the period January 1980–June 2002. Monthly American cheese prices (cents/lb.) were obtained from the USDA. We use Wisconsin assembly point prices measured in 40-pound blocks. Monthly American cheese stock data were obtained from the National Agricultural Statistics Service and Agricultural Stabilization and Conservation Service, USDA. This stock series is measured in thousands of pounds at the beginning of every month.

Figure 1 shows American cheese price and the corresponding support price during the sample period. It identifies two different periods in terms of government involvement. The first period is the 1980s, when the market price was always very close to the corresponding support price (see Figure 1). This period is characterized by the consistent presence of the government regime: government purchases were required to prevent the market price from falling below the support price. As a result, the market price was basically determined by the support price and government stocks were positive throughout the period (see Figure 3). The second period covers the years 1990–2002, when the market price was almost always above the support price (see Figure 1). This corresponds to the consistent presence of the market regime. During this period, in the absence of significant government purchases, government stocks were consistently very low (see Figure 3). These changes in the degree of government intervention provide a basis for an econometric analysis of the effects of the government price support program on price dynamics and price volatility.

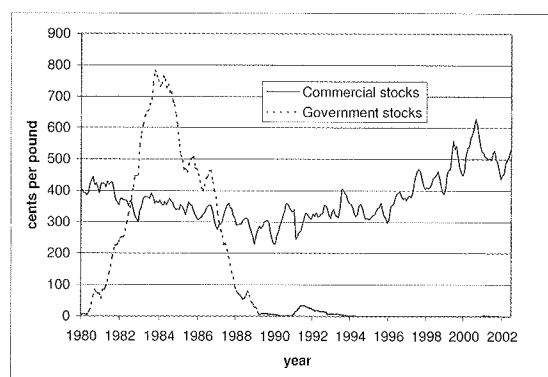As discussed above, we consider two sto-

Figure 3. Commercial and Government Stocks of American Cheese

chastic Tobit specifications for Equations (1a) and (1b). First, we assume the error term e_t in Equation (1b) to be distributed $N(0, \sigma^2_{Nt})$. We let $f_N(\cdot) = \beta_0 + \Sigma^m_{j=1} \beta_j y_{t-j} + \mathbf{x}_t \bar{\beta} + e_{Nt}$ and $\sigma_{Nt} = \delta_0 + \mathbf{z}_t \bar{\delta} > 0$, where the parameters β_0, β_j, $\bar{\beta}$, δ_0, and $\bar{\delta}$ are to be estimated and $\mathbf{x}_t$ and $\mathbf{z}_t$ are vectors of explanatory variables affecting the level of price and price volatility (captured by σ_{Nt}), respectively. When $\bar{\delta}$ differs from zero, this specification allows for heteroscedasticity. Second, we consider a censored lognormal model in which $\ln(y^*_t)$ is normally distributed $N[f_L(\cdot) - \sigma^2_{Lt}/2, \sigma^2_{Lt}]$. We let $f_L(\cdot) = \beta_0 + \Sigma^m_{j=1} + \mathbf{x}_t \bar{\beta} + e_{Lt}$ and $\sigma_{Lt} = \delta_0 + \mathbf{z}_t \bar{\delta} > 0.$[2]

To investigate the effects of stocks on the conditional mean and variance of American cheese price, we introduce lagged stocks of American cheese in $\mathbf{x}_t$ and $\mathbf{z}_t$. We allow the stock effects to differ between commercial

stocks and government stocks. As shown in Figure 3, the patterns of each stock vary over the sample period. As expected, government stocks are high (low) when the price support and government purchases are active (inactive) in the market. From the economic literature on storage (e.g., Deaton and Laroque 1992, 1996; Williams and Wright), higher (lower) commercial stocks are expected to reduce (increase) the market price level while lowering (increasing) price volatility. However, commercial stocks and government stocks play a different market function: the former is motivated by anticipated price increases (e.g., Williams and Wright) and the latter is the key policy instrument used in implementing the government price support program (which prevents price decrease below the support price). This suggests that commercial stocks and government stocks would have different effects on prices. On that basis, we included separately lagged commercial stocks (CS_{t-1}) and lagged governments stocks (GS_{t-1}) in $\mathbf{x}_t$ and $\mathbf{z}_t$. This provides a framework to document the differential effects of commercial versus governments stocks on price levels and price volatility.

In addition, competitive storage theory indicates that a higher (lower) interest rate provides a disincentive (incentive) to hold private stocks, which is expected to affect both price level and price volatility. On that basis, the interest rate[3] (IR_t) is also included in $\mathbf{x}_t$ and $\mathbf{z}_t$ to capture its effects on the conditional mean and variance of American cheese price. Finally, a time trend T is introduced in $\mathbf{x}_t$ and $\mathbf{z}_t$, reflecting possible long-term trend effects. Monthly dummy variables D_{it} (where $D_{it} = 1$ in the ith month and 0 otherwise) are included in $\mathbf{x}_t$ to capture seasonality effects in the American cheese market.[4]

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[2] For simplicity, we focus on linear specifications of the reduced-form equation for latent price y^*_t. Note that Deaton and Laroque and Ng have argued that private storage generates nonlinear price dynamics with regime switching between stockout and speculative stockholding. The investigation presented by Ng presents empirical evidence of strong persistence in the stockout regime that is inconsistent with the theory. This suggests that a ''convenience yield'' might smooth out the differences across regimes. This makes it unclear what nonlinearities arise in price dynamics. In the absence of strong *a priori* information about nonlinearities, a linear specification is convenient and parsimonious for our purpose. Exploring nonlinearity issues appears to be a good topic for further investigation.

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[3] The interest rate IR is measured by the 6-month Treasury Bill Rate.

[4] We also investigated whether a risk premium possibly affected the expected value of American cheese prices (as in ARCH-M models; see Engle et al.). For that purpose, we introduced the variance σ^2_t in $\mathbf{x}_t$. We found that the corresponding coefficient was not significantly different from zero. As a result, the analysis presented below ignores such risk premium effects.

We investigate the issue of changes in price volatility. As illustrated in Figure 1, although observed price volatility was low in the 1980s (under the government regime), it increased significantly in the 1990s (under market liberalization). We would like to address two questions: (1) How much of these changes are due to the censoring effects of the price support program? and (2) Did other factors contribute to the changing volatility? Answering these questions requires considering a heteroscedastic model with a time-varying variance σ_t^2. The following variables are included among the variance shifters $\mathbf{z}_t$: lagged price y_{t-1}, lagged commercial stocks CS_{t-1}, lagged government stocks GS_{t-1}, the interest rate IR_t, and a time trend T. Introducing the lag price y_{t-1} allows price volatility to vary with market conditions (as reflected by the last-period price level). As noted above, the inclusion of commercial stocks, government stocks, and interest rate will allow us to evaluate their effects on price volatility. A time trend is included to capture possible changes in market instability during the sample period.

These considerations generate the following models of American cheese price at time t.

Censored Normal

$$(3a) \quad y_t = \max\{y_t^*, s_t\}$$

$$(3b) \quad y_t^* = \beta_0 + \sum_{k=1}^{m} \beta_k y_{t-k} + \beta_{CS}CS_{t-1} + \beta_{GS}GS_{t-1}$$
$$+ \beta_{IR}IR_t + \beta_T T_t + \sum_{j=1}^{11} \beta_{Mj}D_{jt} + e_{Nt}$$

$$(3c) \quad [Var(y_t^*)]^{1/2} \equiv \sigma_{Nt}$$
$$= \delta_0 + \delta_1 y_{t-1} + \delta_{CS}CS_{t-1}$$
$$+ \delta_{GS}GS_{t-1} + \delta_{IR}IR_t + \delta_T T_t$$

Censored Lognormal

$$(4a) \quad y_t = \max\{y_t^*, s_t\}$$

$$(4b) \quad y_t^* = \beta_0 + \sum_{k=1}^{m} \beta_k y_{t-k} + \beta_{CS}CS_{t-1} + \beta_{GS}GS_{t-1}$$
$$+ \beta_{IR}IR_t + \beta_T T_t + \sum_{j=1}^{11} \beta_{Mj}D_{jt} + e_{Lt}$$

$$(4c) \quad \{Var[\ln(y_t^*)]\}^{1/2}$$
$$\equiv \sigma_{Lt} = \delta_0 + \delta_1 y_{t-1} + \delta_{CS}CS_{t-1}$$
$$+ \delta_{GS}GS_{t-1} + \delta_{IR}IR_t + \delta_T T_t$$

where y_t^* is the latent American cheese price at time t, and e_{Nt} and e_{Lt} are error terms. In the censored normal model, e_{Nt} is distributed $N(0, \sigma_{Nt}^2)$. In the censored lognormal model, $\ln(y_t^*)$ is distributed normally with variance σ_{Lt}^2. Equations (3a)–(3c) and (4a)–(4c) represent the determination of American cheese price in the presence of censoring and conditional heteroscedasticity. They provide the econometric specification used below in the empirical investigation of the effect of the price support program and stocks on price dynamics and price volatility in the U.S. American cheese market.

Econometric Results

The empirical analysis consists of estimating the two competing models in Equations (3a)–(3c) and (4a)–(4c) by maximum likelihood as applied to the U.S. American cheese market. The maximum likelihood estimation method produces consistent and asymptotically efficient parameter estimates, given a correct distributional assumption. The econometric estimates provide information on the determinants of American cheese price and price volatility.

First, the order of the AR process (m) must be chosen in Equations (3b) and (4b). This was done with the Schwarz criterion (Judge et al., p. 426) in which m is chosen to maximize $[\ln(\text{maximum likelihood}) - K \ln(T)/2]$, where K is the number of parameters and T is the number of observations. The Schwarz criterion suggested $m = 3$. Thus, the following analysis is based on the dynamic Tobit specifications with 3 months lagged prices ($m = 3$). The maximum likelihood estimates are presented in Table 1 under normality as well as lognormality.

The performance of the two models (normal versus lognormal) was compared with the nonnested test proposed by Vuong. The value of the log likelihood function was -623.60 for the censored normal model and -724.45 for

Table 1. Parameter Estimates for Heteroscedastic Dynamic Tobit: U.S. American Cheese Price, January 1980–June 2002 (Normal vs. Lognormal)

		Normal		Lognormal	
	Parameter	Estimate	Standard Error	Estimate	Standard Error
Mean parameters					
β_0	Intercept for mean	8.1291**	3.9176	−12.9354	8.7064
β_1	Price of American cheese at time $t-1$	1.4590***	0.0716	1.4301***	0.0904
β_2	Price of American cheese at time $t-2$	−0.7586***	0.1126	−0.7916***	0.1142
β_3	Price of American cheese at time $t-3$	0.2008***	0.0674	0.1539***	0.0845
β_{CS}	Commercial stocks (CS_{t-1})	0.0026	0.0049	−0.0014	0.0100
β_{GS}	Government stocks (GS_{t-1})	−0.0004	0.0009	−0.0033	0.0060
β_{IR}	Interest Rate (IR_t)	0.2844**	0.1317	0.3949	0.4072
β_T	Time trend (T)	0.0578	0.0888	0.4171*	0.2270
β_{M1}	Dummy for January (M_1)	−0.3298	0.9001	1.1215	1.5824
β_{M2}	Dummy for February (M_2)	−1.1429	0.7619	0.3989	1.1447
β_{M3}	Dummy for March (M_3)	1.8917**	0.8912	4.6721***	1.4488
β_{M4}	Dummy for April (M_4)	−0.1777	1.1302	10.7490***	1.4200
β_{M5}	Dummy for May (M_5)	−0.3247	1.1111	2.2316*	1.2089
β_{M6}	Dummy for June (M_6)	0.8501	1.2136	4.1985***	1.4698
β_{M7}	Dummy for July (M_7)	0.7865	1.0731	6.1952***	1.1596
β_{M8}	Dummy for August (M_8)	2.0875**	0.9926	6.0458***	1.0606
β_{M9}	Dummy for September (M_9)	1.7329**	0.8439	6.5351***	1.2342
β_{M10}	Dummy for October (M_{10})	1.3158*	0.7868	2.6386**	1.0710
β_{M11}	Dummy for November (M_{11})	0.2368	1.0068	3.4553***	1.1159
Standard deviation parameters					
δ_0	Intercept for standard deviation	19.1296***	3.0326	0.7827***	0.0628
δ_1	Price of American cheese at time $t-1$	−0.0978***	0.0223	−0.0035***	0.0003
δ_{CS}	Commercial stocks (CS_{t-1})	−0.0072**	0.0033	−0.0004***	0.0001
δ_{GS}	Government stocks (GS_{t-1})	0.00157**	0.0007	0.00003*	0.00001
δ_{IR}	Interest Rate (IR_t)	0.01234	0.1206	−0.0008	0.0007
δ_T	Time trend (T)	−0.4180***	0.0586	−0.0095***	0.0007
	Log likelihood function	−623.60		−724.45	

Note: Asterisks indicate statistical significance at the * 10%, ** 5%, and *** 1% levels.

the censored lognormal model. Under the null hypothesis that the models are equivalent, the Vuong test rejected the null hypothesis in favor of the normal model at the 5% significance level. This provides statistical evidence that the normal distribution provides a better fit to the sample data (compared with the lognormal distribution). Thus, conditional on the explanatory variables in $\mathbf{x}_t$ and $\mathbf{z}_t$, the error term ε_t appears to be better represented by the symmetric normal distribution than by the skewed lognormal distribution. On that basis, the rest of the paper focuses on the censored normal regression model.

Next, we investigate the empirical rele-vance of introducing heteroscedasticity in the normal model. This was done by testing the null hypothesis that $\delta_1 = \delta_{CS} = \delta_{GS} = \delta_{IR} = \delta_T = 0$ in Equation (3c). The likelihood ratio test statistic for this hypothesis was 17.84. Under the null hypothesis of homoscedasticity, the statistic has an asymptotic chi-square distri-bution with five degrees of freedom. At a 5% significance level, the critical value of the test is 11.07. This leads us to reject the null hy-pothesis of homoscedasticity for American cheese prices. Put differently, we find strong statistical evidence of time-varying volatility during the sample period. Note that this is un-related to the censoring effects of the price

support program (because the Tobit specification already captures the censoring effects associated with the program).

As discussed before, both commercial stocks (CS_{t-1}) and government stocks (GS_{t-1}) are allowed to affect the mean as well as the variance of American cheese price. We investigated whether commercial stock effects and governments stock effects are the same on price formation. This corresponds to the null hypothesis: $\beta_{CS} = \beta_{GS}$ and $\delta_{CS} = \delta_{GS}$. The likelihood ratio test statistic was 17.56. With two degrees of freedom, the critical value is 5.99 at the 5% level. Thus, we strongly reject the null hypothesis and conclude that private and public stocks have different effects.

The parameter estimates presented in Table 1 show the factors affecting the dynamic determination of American cheese price. The lagged price effects are all statistically significant. This presents evidence of significant market price dynamics. In the censored normal model, the coefficient of y_{t-1}, β_{t-1}, is 1.4590, showing an initial overreaction to recent price changes.[5] However, in the absence of censoring, the roots of the estimated AR(3) are in the unit circle.[6] This suggests that the model is stationary in the absence of censoring. Table 1 shows that stocks (both commercial CS_{t-1} and government GS_{t-1}) have insignificant effects on the mean of latent price. However the interest rate IR_t is found to have a positive and significant effect on the mean latent price: a one-point increase in IR_t generates a 0.2844 cent/lb. increase in $E(y^*)$. This indicates that, by increasing the opportunity cost of inventory, a higher interest rate provides a disincentive to hold commercial stocks, which induces a rise in market price. In the normal regression model, the time trend parameter is positive but not statistically significant. Finally, a number of monthly dummy variables (M_i's) are significantly different from zero at the 5% level (see Table 1). This reflects the

presence of seasonality in the American cheese market.

In the standard deviation specification, except for the effect of the interest rate (IR_t), all parameters δ's are highly significant. This is consistent with the previous heteroscedasticity test result: there is strong evidence of time-varying latent volatility. The coefficient of lagged price δ_1 is negative and highly significant, indicating that latent volatility varies with recent market conditions. The stock effects δ_{CS} and δ_{GS} differ between commercial and government stocks: whereas higher commercial stocks reduce the latent standard deviation, higher government stocks tend to increase it. Both effects are significantly different from zero at the 5% level. The elasticities of the latent standard deviation σ_N with respect to stocks vary over the sample period. With respect to commercial stocks, $\partial \ln(\sigma_N)/\partial \ln(CS_{t-1})$ was -1.40 in 1982 and -0.46 in 1995. With respect to government stocks, $\partial \ln(\sigma_N)/\partial \ln(GS_{t-1})$ was 0.30 in 1982 and 0.01 in 1995. This provides strong evidence that commercial stocks tend to reduce price volatility. This underlines the role of private stock management in stabilizing market prices. It is interesting to find that government stocks tend to increase latent price volatility. However, such stock holding being closely associated with the government price support program, the net effects of price floor policy also need to consider the variance-reducing effects of price censoring. This issue is discussed further later. Note that although stocks can affect both mean latent price and latent volatility (see Table 1), it is only the latter volatility effect that exhibits statistical significance.

Implications of Censoring

This section explores the implications of the censored normal model in Equations (3a) and (3b) for the distribution of prices. In particular, the expected value of y_t is (Maddala)

$$
\begin{aligned}
(5a) \quad E(y_t) = {} & \text{Prob}(D_t = 1) \\
& \times \{ f_N(\mathbf{X}_t, \beta) + E[e_{Nt} | e_{Nt} > s_t \\
& - f_N(\mathbf{X}_t, \beta)] \} + \text{Prob}(D_t = 0)s_t,
\end{aligned}
$$

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[5] Note that the coefficients of lagged prices follow a similar pattern in both the normal regression model and the lognormal regression model (see Table 1).

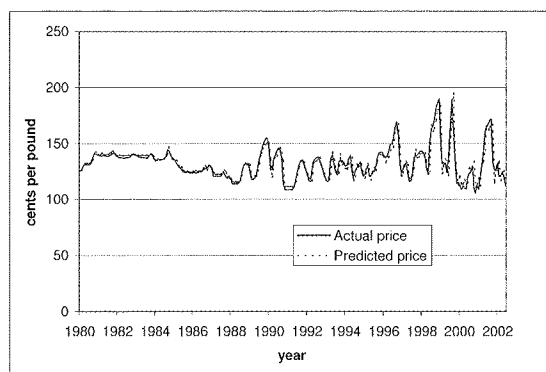[6] For the normal censored model, the dominant root is 0.8408.

Figure 4. Actual and Predicted Prices of American Cheese

Figure 5. Estimated Standard Deviation of American Cheese Price

$$= [1 - \Phi(h_t)]f_N(\mathbf{X}_t, \beta) + \sigma_{Nt}\phi(h_t)$$
$$+ \Phi(h_t)s_t,$$

where $D_t = 1$ (=0) if $y_t = y_t^*$ ($<y_t^*$), $\phi(\cdot)$ and $\Phi(\cdot)$ are the density and distribution functions for the standard normal random variable, and $h_t = [s_t - f_N(\mathbf{X}_t, \beta)]/\sigma_{Nt}$. Equation (5a) indicates that expected price $E(y_t)$ is a weighted average of the support price s_t and of the expected market price conditional on $D_t = 1$ and the weights involve the probability of censoring, $\Phi(h_t)$, that is, the probability of facing the government regime at time t. With Equation (5a), we examine the performance of the estimated model by comparing the expected prices with actual prices. This is shown in Figure 4. It illustrates that the model has a high explanatory power during the sample period.

Similarly, one can derive the variance of y_t as (see the proof in the Appendix)

$$(5b) \quad V(y_t) = \sigma_{Nt}^2[1 - \Phi(h_t) + h_t\phi(h_t) + h_t^2\Phi(h_t)$$
$$- [h_t\Phi(h_t) + \phi(h_t)]^2],$$

where $h_t = [s_t - f_N(\mathbf{X}_t, \beta)]/\sigma_{Nt}$ and the probability that the censored variable y_t^* is unobserved is denoted by $\text{Prob}(D_t = 0) = \text{Prob}[e_{Nt} < s_t - f_N(\mathbf{X}_t, \beta)] = \Phi(h_t)$. Equation (5b) provides some useful insights on the effects of the price support program on price volatility. It shows that $V(y_t)/\sigma_{Nt}^2 \equiv 1 - \Phi(h_t) + h_t\phi(h_t) + h_t^2\Phi(h_t) - [h_t\Phi(h_t) + \phi(h_t)]^2$ measures the effect of censoring on price variance. For ex-
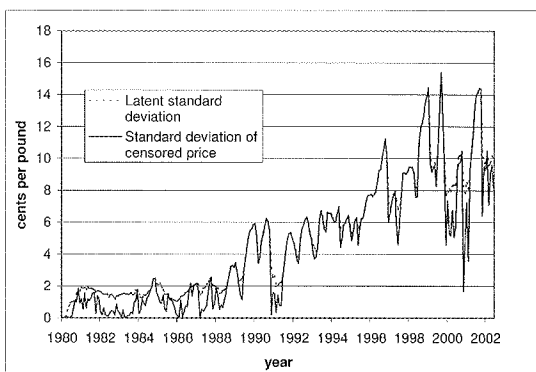
ample, in the absence of censoring, $V(y_t)/\sigma_{Nt} = 1$. Alternatively, under censoring, $V(y_t)/\sigma_{Nt}$ is reduced. The magnitude of the reduction (toward zero) measures the risk-reducing effects of the price support program.

By Equation (5b), the estimated standard deviation of American cheese price $V(y_t)^{1/2}$ is simulated over the sample period. Together with the latent standard deviation σ_t, the results are reported in Figure 5. Figure 5 shows large changes in price instability during the sample period. First, the latent standard deviation of American cheese price σ_{Nt} was the smallest in the early 1980s. This is also the period when the government price support program was active, contributing to a further reduction in price volatility through censoring effects. From Equation (5b), the censoring effects can be seen when $V(y_t)^{1/2}$ is much smaller than σ_{Nt} (e.g., as found in 1983). As a result, Figure 5 shows that the standard deviation of American cheese price $V(y_t)^{1/2}$ was quite small during the 1980s. A combination of low latent variability and volatility-reducing effects of the price support program practically eliminated price uncertainty in this market in the 1980s.

Figure 5 also shows that the standard deviation of American cheese price increased in the 1990s. Again, this was because of two factors: (1) a larger and increasing latent standard deviation σ_{Nt} and (2) smaller censoring effects because of low support prices. The trend to-

ward a rise in the latent standard deviation σ_{Nt} can be attributed to changing market conditions during the 1990s (e.g., changing average price and stock holding patterns). Also, through most of the 1990s, σ_{Nt} and $V(y_t)^{1/2}$ were similar in magnitude: from Equation (5b), the censoring effects of the price support program were small or negligible. In other words, over the last decade, both higher latent variability and market liberalization policy (through a lower support price) contributed to an increase in price uncertainty in the American cheese market.

Implications for Price Dynamics

Given the significant changes in price instability just documented and their relationship with policy changes concerning the price support program, this section investigates further implications of our censored normal model for price dynamics. With the use of dynamic multipliers, we simulated the effects of changes in selected variables on the path of expected price and the variance of price given in Equations (5a) and (5b). One should note that all dynamics are "local" in the sense that they depend on the particular path being evaluated. This is because Equations (5a) and (5b) involve nonlinear dynamics: the functions ϕ and Φ are nonlinear functions of lagged price. To account for this, we focus our analysis on simulated results under two scenarios: one covering the period 1982–1983 and the other covering the period 1995–1996. These two periods correspond to two extreme situations with respect to the price support program in the American cheese market. The first scenario (1982–1983) represents the government regime, when the government price support program is always active. The second scenario (1995–1996) represents the market regime, when the price support program is inactive.

First, we simulated the effects of a temporary shock in the price of American cheese. Two shocks were investigated: a 1 cent/lb. increase in the December 1981 price (simulated through 1982–1983) and a 1 cent/lb. increase in the December 1994 price (simulated 1995–1996). Under these two scenarios, Figure 6
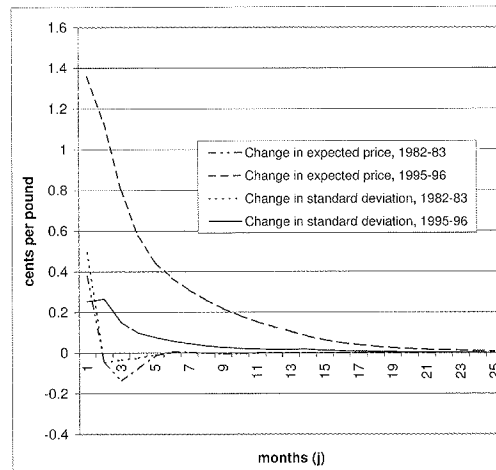

Figure 6. The Effects of a Temporary Shock in American Cheese Price on the Expected Future Prices $E(y_{t+j})$ and the Standard Deviation of Future Prices $[V(y_{t+j})]^{1/2}$

shows the dynamic effect of an exogenous change in American cheese price y_t on the expected future prices $E(y_{t+j})$ and the standard deviation of future prices $V(y_{t+j})^{1/2}$, $j = 0, 1, 2, \ldots$. Figure 6 indicates that under the government regime scenario (1982–1983), market price changes have only a small effect on price dynamics and price volatility. This is intuitive because the price support is the key determining factor for the market price in this situation. However, under the market regime scenario (1995–1996), the simulations show that short-term price dynamics are important, implying significant dynamic adjustments in the American cheese market in the absence of government intervention.

Second, the effects of a permanent shock in the support price of American cheese were simulated. Again, a 1 cent/lb. price increase is simulated, covering two periods: 1982–1983 (government regime) and 1995–1996 (market regime). The simulation results are presented in Figure 7, which shows the dynamic effect of a permanent change in the support price s_t on the expected future prices $E(y_{t+j})$ and the standard deviation of future prices $V(y_{t+j})^{1/2}$, $j = 1, 2, 3, \ldots$. As expected, under the government regime (1982–1983), the support
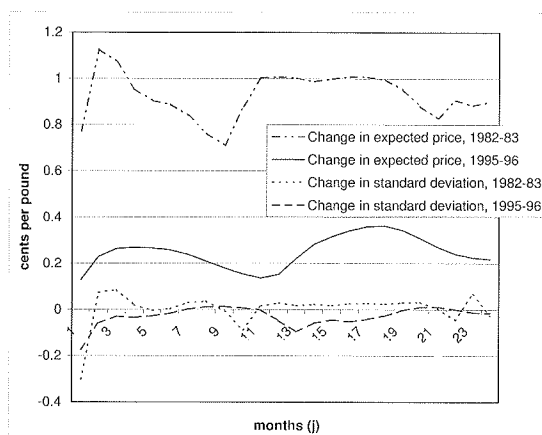
Figure 7. The Effects of a Permanent Shock in the Support Price of American Cheese on the Expected Future Prices $E(y_{t+j})$ and the Standard Deviation of Future Prices $[V(y_{t+j})]^{1/2}$

price is found to have large effects on price dynamics and price volatility. A permanent increase in the price support generates an almost parallel increase in the American cheese price in both the short and long run. Under the government regime scenario, the large and negative decrease in the initial effect ($j = 1$) on the standard deviation suggests that the censoring effect of the price support program effectively decreases short-term price instability. However, as shown in Figure 7, the long-term effects of a permanent increase in the price support on $V(y_{t+j})^{1/2}$ are found to be negligible. This means that under the government regime scenario, although the price support program reduces short-term price instability, it does not appear to contribute to a reduction in long-term price instability.

Figure 7 also shows the effect of the price support on price dynamics and price volatility under the market regime scenario (1995–1996). In the short run, the effect on price volatility is found to be negative but relatively small. This implies that, even if the support price is set relatively low, it can still contribute to a (small) reduction in price volatility. However, Figure 7 shows that such effects become negligible in the long run. Again, it appears that the support price program would not con-

tribute to a significant reduction in long-term price instability. However, under the market regime, the effects of the support price on expected market price are found to be more important. In the 1995–1996 scenario, a 1 cent/lb. permanent increase in the support price contributes to an increase in expected price that varies between 0.18 and 0.38 cents/lb. (see Figure 7). And such effects do not decline in the long run. It suggests that the cumulative effect of an increase in support price on expected market price is significant even when price support is not binding. This indicates that long-term price dynamics can be significantly affected by government policy even under limited government intervention. Clearly, such effects are generated through the censoring mechanism of the price support program. This raises the question: Is it possible for government policy to affect market prices without involving a large cost to the taxpayers?

To address this issue, we used our estimated model to evaluate further the difference between market price y_t and latent price y_t^*. This difference is zero under the market regime, but positive under the government regime. In this latter case, government purchase is needed to prevent the market price from falling below the support price. As shown in Figure 2, having information on supply–demand elasticities, the simulated difference ($y_t - y_t^*$) can be used to evaluate government purchase (AE or BD) and its welfare effects. The welfare effect of the price support program for American cheese can be evaluated as follows: producer welfare increases (as measured by the change in producer surplus; Figure 2, area FDCG), consumer welfare declines (as measured by the change in consumer surplus; Figure 2, area FBCG), whereas government expenditures amount to the area ABDE (Figure 2). Thus, area ABCDE can be taken as a measure of net welfare loss to society. We rely on such a measure later. However, it should be kept in mind that this measure can be biased in several ways.[7] It can be downward biased because it

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[7] Measuring the extent of such biases is empirically difficult. This appears to be a good topic for future research.

neglects the efficiency loss associated with taxes supporting the program. It can be upward biased because it implicitly assumes that all government purchases are destroyed. Some of the government purchases of American cheese actually generate revenue (e.g., by selling the commodity on the world market at subsidized prices) as well as social benefit (e.g., from government purchases in school breakfast and school lunch programs). Finally, it can be upward biased by neglecting possible benefits of market stabilization (e.g., under risk aversion).

The elasticity of demand for American cheese has been estimated to be -0.25 (Huang). Taking the short-run elasticity of supply to be $+0.25$, government purchases were simulated (with Monte Carlo simulation and 2,000 replications). The simulation was conducted for the 12 months of 1995 when the market price (varying between $1.21 and $1.42/lb.) was consistently higher than the support price ($1.12/lb.). From the simulation exercise for 1995, the expected increase in producer surplus was $15.17 million, supported by $7.53 million of expected government expenditures and $7.31 million of expected net welfare loss to society. As long as the probability of price censoring is positive, this shows that, on average, the price support program still generates positive welfare effects for producers and negative welfare effects for taxpayers. The results also indicate that both the cost and the benefit of the price support program are small when the support price is set relatively low.[8] To evaluate the welfare effects of the support price, the 1995 simulation was repeated under two alternative scenarios: a 10-cent increase in the support price (to $1.22/lb.) and a 20-cent increase (to $1.32/lb.). Under a $1.22/lb. support price for 1995, the expected increase in producer surplus was $44 million, supported by $21.62 million of ex-

pected government expenditures, and generated $20.92 million of expected net welfare loss to society. Under a $1.32/lb. support price, the expected increase in producer surplus was $94.93 million, supported by $43.79 million of expected government expenditures, and $41.75 million of expected net welfare loss to society. As expected, increasing the support price does increase average producer surplus, government purchases, and net welfare loss to society. The ratio (average change in producer surplus)/(net welfare loss) varies from 2.04 under a $1.12/lb. support price, to 2.10 under a $1.22/lb. support price, to 2.27 under a $1.32/lb. support price. This result indicates that, although lower support prices reduce taxpayer cost as well as aggregate welfare loss, they might not improve the relative economic efficiency of transferring income from consumers and taxpayers to producers. However, it should be kept in mind that these welfare measurements neglect possible benefits from market stabilization. Measuring such benefits empirically is a good topic for future research.

Concluding Remarks

We have presented an econometric analysis of the effects of a price support program on price dynamics and price volatility. Focusing on the price support providing a censoring mechanism to price determination, we specified and estimated two competing models with dynamic Tobit specification under time-varying volatility: a normal censored model and a log-normal censored model. We applied these models to analyze econometrically the effects of a price support program on price dynamics and price volatility in the U.S. American cheese markets. A Vuong test indicated that the normal censored Tobit model performed better.

Our econometric analysis uncovered several important findings. First, we documented how the price support program contributed to reducing price volatility in the U.S. cheese market over the last two decades. Second, we uncovered evidence that such volatility-reducing effects are much stronger in the short run

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[8] It should be kept in mind that these results apply only to the price support program for American cheese. Government dairy policy also involves price supports for butter and nonfat dry milk, import quotas, export subsidies, and classified pricing implemented by the Federal Milk Marketing order. The welfare effects of these programs are not being evaluated here.

than in the long run. Third, we found that, even under the market regime scenario (in which the support price is below the market price), the support price program can still have some significant positive effects on long-run expected prices.

In addition, our econometric analysis provided empirical evidence on the dynamics of American cheese prices and their changing volatility in connection with market liberalization policy introduced in the 1990s. First, our empirical results suggest that market conditions in the 1990s have been associated with a significant increase in price volatility. Second, we found evidence of significant effects of both private and public stocks on price volatility. This stresses the importance of how both commercial stocks and government stocks can affect market price instability. Third, our econometric analysis has identified some important dynamic aspects of price adjustment in the U.S. American cheese market under market liberalization. This stresses the need to differentiate between the short run versus the long run in the analysis of the effects of a price support program on market price dynamics. Finally, we evaluated the welfare effects of changing the price support level. Our analysis indicates that, although lower support prices reduce taxpayer cost and aggregate welfare loss, they might not improve the relative economic efficiency of transferring income from consumers and taxpayers to producers.

Although we focused our empirical attention on the U.S. American cheese market, it would be of interest to investigate the role of government intervention in other markets. Future research is also needed to extend our analytical framework and further investigate the role of stocks in price dynamics.

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Appendix

Consider the standardized residual $\varepsilon_t = [y_t - f_N(\mathbf{X}_t, \beta)]/\sigma_{Nt}$. With $h_t = [s_t - f_N(\mathbf{X}_t, \beta)]/\sigma_{Nt}$, we get

$$(A1) \quad E(\varepsilon_t) = [E(y_t) - f_N(\mathbf{X}_t, \beta)]/\sigma_{Nt}$$
$$= h_t \Phi(h_t) + \phi(h_t),$$

from Equation (3a). In addition,

$$E(\varepsilon_t^2) = \int_{-\infty}^{h_t} h_t^2 \phi(u)\, du + \int_{h_t}^{\infty} \varepsilon_t^2 \phi(u)\, du.$$

From Maddala (p. 365), we have $\int_{h_t}^{\infty} \varepsilon_t^2 \phi(u)\, du = [1 - \Phi(h_t)] E(\varepsilon_t^2 | \varepsilon_t > h_t)] = [1 - \Phi(h_t)][1 + h_t E(\varepsilon_t | \varepsilon_t > h_t)] = [1 - \Phi(h_t)]\{1 + h_t \phi(h_t)/[1 - \Phi(h_t)]\}$. It follows that

$$(A2) \quad E(\varepsilon_t^2) = 1 - \Phi(h_t) + h_t \phi(h_t) + h_t^2 \Phi(h_t).$$

With $V(y_t) = \sigma_{Nt}^2 V(\varepsilon_t) = \sigma_{Nt}^2 \{E(\varepsilon_t^2) - [E(\varepsilon_t)]^2\}$, Equations (A1) and (A2) yield Equation (3b).