Journal of Agricultural and Applied Economics, 37,3(December 2005):619–634

Estimating Indirect Production Functions with a More General Specification: An Application of the Lewbel Model

Christiana E. Hilmer and Matthew T. Holt

Whereas consumer theory employs several different empirical specifications for estimating indirect utility functions, producer theory has relied on the Translog specification to estimate the indirect production function. In this paper, we apply Lewbel's more general functional specification and investigate its implications for the estimation of indirect production functions in productivity analysis. An attractive feature of the Lewbel model is that it nests both the Translog and the almost ideal supply system, offering a method to assess the empirical validity of all three specifications. Aggregate U.S. production data are used to examine the performance of the three models in an empirical application.

Key words: duality, indirect production function, nested test

JEL Classifications: C32, C52, Q12

A traditional undertaking in applied economics has been to understand and adequately characterize the structure of production for various industries at various stages of aggregation. Indeed, the motivation underlying some of the earliest quantitative work in economics was to relate productive inputs to output(s) in an empirically meaningful yet tractable way. Specifications such as the Cobb-Douglas and CES production functions were developed primarily to assist researchers in this process. Of course, the limitations of these models in terms of placing potentially unwarranted *a priori* restrictions on the structure of the production process are now well known. In response, attempts have been made to posit functional specifications, such as Christensen, Jorgenson, and Lau's Translog model and Diewert's (1971) generalized Leontief, that place comparatively few *a priori* restrictions on the production function. These and similar functional specifications comprise the class of what are referred to as second-order (locally) flexible functional forms and have become the foundation of modern empirical production analysis (Chambers).

At much the same time that research on developing more realistic functional forms was taking place, another revolution was occurring in microeconomics. Specifically, the ability to adequately characterize production processes through the specification of appropriate dual formulations, such as the cost or profit functions, was fully realized and exploited (Blackorby, Primont, and Russell;

Christiana E. Hilmer is assistant professor, Department of Economics, San Diego State University, San Diego, CA. Matthew T. Holt is professor, Department of Agricultural and Resource Economics, North Carolina State University, Raleigh, NC.

The authors thank two anonymous reviewers of the *Journal* for helpful comments and suggestions. C.E.H. acknowledges the support of the Virginia Tech Agricultural Experiment Station. M.T.H. acknowledges the support of the North Carolina State Agricultural Experiment Station.

Diewert 1971, 1974; Jorgensen and Lau 1974a,b). The advent of modern duality theory has also greatly affected the manner in which empirical production-related research has occurred, the result being that the cost function has become a mainstay of applied production economics research (see, e.g., Berndt and Khaled; Berndt and Wood; Christensen and Green).

Naturally the cost function is not the only dual relationship that can be fruitfully employed in production analysis. In recent years, the indirect production function, a function that is the inverse of the cost function, that is dual to the direct production function, and that has as its analog the indirect utility function in consumer theory, has also been explored. Empirical applications of the indirect production function have been reported (Appelbaum; Chambers and Lee; Gajanan and Ramaiah; Garofalo and Malhotra; Kim; Yuhn).

Although the assumptions embedded in an indirect production function (maximization of output subject to an expenditure constraint) differ from those implied by a cost function (minimization of expenditure subject to a production constraint), one advantage of the former is that both compensated and uncompensated factor demand elasticities can be readily obtained (Gajanan and Ramaiah). Another advantage of the indirect production function, at least where agricultural production is concerned, is that problems with including realized output in the cost function are effectively circumvented. As Pope and Just argue, the combination of biological lags and stochastic production inherent to agriculture makes including realized output in a cost function problematic. They propose a sophisticated scheme wherein the distance function that is dual to the cost function is utilized to obtain consistent estimates of expected output in a cost function framework. Their methodology is elegant and has appeal; however, production risk could also be handled directly by instead specifying a (stochastic) indirect production function, in which, as opposed to cost minimization, output is endogenous and expenditure is exogenous.[1]

Although progress has been made, all prior applications of indirect production functions have relied on the Translog specification, with the exception of Appelbaum.[2] This general lack of functional "exploration" is, indeed, surprising. As noted previously, the indirect production function is the supply analog to the indirect utility function. As opposed to the single specification used to estimate most indirect production functions, numerous functional forms have been used to estimate indirect utility functions, including the Translog model, the almost ideal demand system (AIDS) of Deaton and Muellbauer, and Lewbel's hybrid model. The AIDS and Lewbel models have been embraced as appealing alternatives to the Translog in the consumer demand literature, in part because of their second-order flexibility, compatibility with demand theory, and relative ease of estimation. The properties that make these specifications appealing for estimating indirect utility functions should also make them appealing for estimating indirect production functions.

In this paper, we explore using Lewbel's model as an alternative specification for the indirect production function. An attractive feature of the Lewbel model is that it nests two additional models: the Translog and the supply analog to the AIDS, the almost ideal supply system (AISS). The nesting property of the Lewbel specification offers a method to assess the empirical validity of the Lewbel model relative to the Translog and the AISS. Similar to

[1] As Chambers and Lee suggest, there may be other valid reasons for using an indirect production function when agricultural data are employed. Specifically, because of the possibility of credit constraints, imperfect credit markets, or both, it may well be the case that agricultural producers maximize output subject to an input expenditure constraint, at least in the short run. See Kim for additional arguments in favor of using an indirect production function in lieu of a cost function.

[2] Appelbaum estimated a generalized Box-Cox specification of the indirect production function which, among other things, nests the generalized Leontief and the Translog as alternatives. This said, Appelbaum did not report any key parameter, elasticity, or other "structure of production" (e.g., total factor productivity) estimates for his model.

Chambers and Lee, and Pope and Just, we use aggregate U.S. time-series data for agricultural production in the empirical application. Although we find that the structural implications are similar across models, important discrepancies do exist, especially for expenditure elasticities (and, consequently, the elasticity of scale) and for implied rates of technical change, suggesting that in future, researchers might need to pay more attention to functional form when estimating indirect production functions.

The Indirect Production Function and Alternative Specifications

Consider a representative firm that uses an n^* $\times$ 1 vector of productive inputs z in combination with a production function, $f(\cdot)$, to produce scalar (maximal) output y. Furthermore, assume that z can be portioned according to $z = (\boldsymbol{x}, K)$, in which $\boldsymbol{x}$ is an $n \times 1$ vector ($n = n^* - 1$) of short-run variable inputs, and K denotes a scalar input (capital) that is fixed in the short run. The production process can then be more formally stated as

$$(1) \quad y = f(\boldsymbol{x}, K, t),$$

in which t is an index used to represent technological change. Some of the relevant regularity conditions associated with $f(\boldsymbol{x}, K, t)$ include that it is a nondecreasing, twice continuously differentiable, and quasi-concave function of $\boldsymbol{x}$. Now, assume the vector of input prices associated with variable input vector $\boldsymbol{x}$ is given by $\boldsymbol{w}$, a conformable $n \times 1$ vector. The linear relation

$$(2) \quad e = \boldsymbol{w}^{\mathrm{T}}\boldsymbol{x}$$

then defines total expenditure e on short-run variable inputs $\boldsymbol{x}$.[3]

The indirect production function is obtained by maximizing output from Equation (1) subject to the outlay constraint in Equation (2). Specifically, the indirect production function dual to the direct production function of Equation (1) is given by

$$(3) \quad y = y(\boldsymbol{w}, e, K, t) = \max_{\boldsymbol{x}} \, [f(\boldsymbol{x}, K, t) \,|\, \boldsymbol{w}^{\mathrm{T}}\boldsymbol{x} \leq e].$$

Properties of $y(\boldsymbol{w}, e, K, t)$ include that it is continuous, nonincreasing in $\boldsymbol{w}$ and nondecreasing in e, and homogeneous of degree zero in $\boldsymbol{w}$ and e. By applying Roy's identity to the indirect production function of Equation (3), we obtain

$$(4) \quad x_i = -\frac{\partial y/\partial w_i}{\partial y/\partial e} = x_i^{\mathrm{m}}(\boldsymbol{w}, e, K, t), \quad i = 1, \ldots, n,$$

which is the set of Marshallian (uncompensated) factor demands. Among other things, the demands in Equation (4) will be homogeneous of degree zero in $\boldsymbol{w}$ and e. As Kim and others note, the cost function can be recovered from the indirect production function of Equation (3) by inversion—that is, by solving the indirect production function $y = y(\boldsymbol{w}, e, K, t)$ for e as

$$(5) \quad e = e(\boldsymbol{w}, y, K, t).$$

By substituting Equation (5) for the expenditure term in demand system Equation (4), we obtain

$$(6) \quad x_i = x_i^{\mathrm{h}}(\boldsymbol{w}, y, K, t), \qquad i = 1, \ldots, n,$$

which is the system of Hicksian (compensated) factor demands, which are consistent with cost minimization. By exploiting the relationship between the system in Equation (4) and that in Equation (6), it is possible to obtain Marshallian and Hicksian elasticities of factor demands, as well as expenditure elasticities.

For purposes of econometric estimation, it is necessary to specify a functional form for indirect production function $y(\boldsymbol{w}, e, K, t)$. Roy's identity may then be applied, and the system of Marshallian factor demands can be derived and estimated. As noted previously, prior research on indirect production functions

[3] A superscripted "T" denotes vector (matrix) transposition.

has relied almost exclusively on the Translog form. Here, we use Lewbel's model as the elemental specification for the indirect production function—a model, as we shall show, that has nested within its specification the Translog and AISS models.

A Lewbel-type indirect production function can be defined as the logarithmic transformation

$$(7)\quad y(\boldsymbol{w}, e, K, t) = \prod_{k=1}^{n} w_k^{-B_k}\left[\ln e\left(1 + \sum_{i=1}^{n}\sum_{j=1}^{n}\beta_{ij}\ln w_j\right) - \ln g(\boldsymbol{w}, e, K, t)\right].$$

In Equation (7), $\ln g(\boldsymbol{w}, e, K, t)$ is defined by

$$(8)\quad \ln g(\boldsymbol{w}, e, K, t) = \alpha_0 + \sum_{i=1}^{n}\alpha_i \ln w_i + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\beta_{ij}\ln w_i \ln w_j + V_t t + t\sum_{i=1}^{n}V_i \ln w_i + B_{te}t\ln e + \frac{1}{2}V_{tt}t^2 + H_k K + H_{kK}K^2 + K\sum_{i=1}^{n}J_i \ln w_i + \chi_k K\ln e.$$

The indirect production function should, of course, be homogeneous of degree zero in factor prices and expenditure and symmetric in second-order price terms as well. Additionally, because we will be estimating share equations, adding-up restrictions are needed. These properties are satisfied by imposing the following parametric restrictions on Equations (7) and (8):

$$(9a)\quad \sum_{i=1}^{n}\alpha_i = 1,\quad \sum_{i=1}^{n}B_i = 0,\quad \sum_{i=1}^{n}\sum_{j=1}^{n}\beta_{ij} = 0,$$

$$(9b)\quad \beta_{ij} = \beta_{ji}\quad \text{for all } i \text{ and } j,$$

$$(9c)\quad \sum_{i=1}^{n}V_i = B_t,\quad \text{and}\quad \sum_{i=1}^{n}J_i = \chi_k.$$

Finally, to guarantee convergence, we use the additional identifying restriction that $\alpha_0 = 0$.

By imposing the restrictions in Equation (9) on the model in Equations (7) and (8) and by applying the logarithmic version of Roy's identity to the resulting model, the following share equation system is obtained:[4]

$$(10)\quad S_i = \left\{\alpha_i + \sum_{j=1}^{n}\beta_{ij}\ln w_j + tV_i + KJ_i - \ln e\sum_{j=1}^{n}\beta_{ij} + B_i\left[\ln e\left(1 + \sum_{j=1}^{n}\sum_{k=1}^{n}\beta_{jk}\ln w_j\right) - \ln g(\boldsymbol{w}, e, K, t)\right]\right\} \div \left(1 - B_t t - \chi_k K + \sum_{j=1}^{n}\sum_{k=1}^{n}\beta_{jk}\ln w_j\right),$$

in which $i = 1, \ldots, n$, $\ln g(\boldsymbol{w}, e, K, t)$ is as defined in Equation (8), and S_i is the expenditure share for the ith input.[5] Because of adding up, $\Sigma_{i=1}^{n} S_i = 1$. The share equation system of Equation (10), the indirect production function in Equations (7) and (8), and the parameter restrictions in Equation (9) comprise the Lewbel indirect production function model. This system provides a flexible way of modeling factor demands that, moreover, allows for nonhomothetic technology.

As already noted, the Lewbel model nests within its parameterization both the AISS and Translog models. Specifically, the share equation system for the AISS is expressed as

$$(11)\quad S_i = \left\{\alpha_i + \sum_{j=1}^{n}\beta_{ij}\ln w_j + tV_i + KJ_i + B_i[\ln e - \ln g(\boldsymbol{w}, e, K, t)]\right\} \div (1 - B_t t - \chi_k K),\quad i = 1, \ldots, n.$$

The AISS in Equation (11) is derived from the Lewbel-type indirect production function in Equations (7) and (8) by imposing, along with the restrictions in Equation (9), the additional restrictions that $\Sigma_{j=1}^{n}\beta_{i,j} = 0$ for all i. Alternatively, the Translog model can be derived

[4] Share equations are derived in Appendix 1.

[5] Note that $S_i = w_i x_i/e = -[\partial \ln g(\boldsymbol{w}, C, K, t)/\partial \ln w_i]/[\partial \ln g(\boldsymbol{w}, C, K, t)/\partial \ln C]$.

from Equations (7) and (8) by imposing the additional restrictions that $B_i = 0$ for all i. The resulting share equation system is then

$$(12)\quad S_i = \left(\alpha_i + \sum_{j=1}^{n} \beta_{ij}\ln w_j + tV_i + KJ_i - \sum_{j=1}^{n} \beta_{ij}\ln e\right) \div \left(1 - B_t t - \chi_k K + \sum_{j=1}^{n}\sum_{k=1}^{n} \beta_{jk}\ln w_j\right), \quad i = 1, \ldots, n.$$

The share equation system in Equation (12) is therefore similar to the Translog inverse production function system estimated by Chambers and Lee, Kim, and Gajanan and Ramaiah, among others.

To assess the implications of each model with respect to the underlying "structure of production," it is useful to examine various elasticity measures. As previously noted, it is possible to obtain from the inverse production function model estimates of Marshallian (uncompensated), Hicksian (compensated), and expenditure elasticities. We can relate expenditure elasticities to the elasticity of scale as well. The Hicksian (uncompensated) price elasticities of input demand are

$$(13)\quad \varepsilon_{ij}^{u} = -\delta_{ij} + \frac{\beta_{ij} + B_i\left(\sum_k \beta_{jk}\ln e - \alpha_j - \sum_k \beta_{jk}\ln w_k - tV_j - KJ_j\right) - S_i \sum_k B_{jk}}{S_i\left(1 - B_T t - X_k K + \sum_k \sum_l B_{kl}\ln w_k\right)},$$

$$(14)\quad \varepsilon_{ij}^{u} = -\delta_{ij} + \frac{B_{ij} - B_i\left(\alpha_j + \sum_k B_{jk}\ln w_k + tV_j + KJ_j\right)}{S_i(1 - B_T t - X_k K)},$$

$$(15)\quad \varepsilon_{ij}^{u} = -\delta_{ij} + \frac{B_{ij} - S_i \sum_k B_{jk}}{S_i\left(1 - B_T t - X_k K + \sum_k \sum_l B_{kl}\ln w_k\right)},$$

for, respectively, the Lewbel, AISS, and Translog models, in which δ_{ij} is a Kronecker delta such that its value equals 1 if $i = j$ and 0 otherwise. The expenditure elasticities of input demand for the Lewbel, AISS, and Translog models are

$$(16)\quad \varepsilon_{iE} = 1 + \frac{B_i\left(1 + \sum_k \sum_j B_{jk}\ln w_j - B_T t - X_K K\right) - \sum_k B_{ik}}{S_i\left(1 - B_T t - X_k K + \sum_k \sum_j B_{kj}\ln w_k\right)},$$

$$(17)\quad \varepsilon_{iE} = 1 + \frac{B_i}{S_i},$$

$$(18)\quad \varepsilon_{iE} = 1 + \frac{\sum_k B_{ik}}{S_i\left(1 - B_T t - X_k K + \sum_k \sum_l B_{kl}\ln w_k\right)}.$$

With the use of the Slutsky decomposition to combine these elasticities and the relevant factor share, the compensated own- and cross-price elasticities are

$$(19)\quad \varepsilon_{ij}^{c} = \varepsilon_{ij}^{u} + S_j\varepsilon_{iE}.$$

Turning to the effect of technology on production, the rate of technical change for the Lewbel and AISS production functions is measured by

$$(20)\quad \varepsilon_{YT} = \prod_k w_k^{-B_k}\left(-V_t - \sum_k V_k \ln w_k - B_t \ln E - W_{tt}t\right)\frac{t}{y},$$

and the rate of technical change for the Translog production function is measured by

$$(21)\quad \varepsilon_{YT} = \left(-V_t - \sum_k V_k \ln w_k - B_t \ln E - V_{tt}t\right)\frac{t}{y}.$$

The input bias in technical change for the three models is measured by

$$(22)\quad b_i = \frac{t\left[V_i - B_i\left(V_T + \sum_k V_k \ln w_k + B_t \ln E + V_{TT}t\right)\right] + S_i B_T}{S_i\left(1 - B_T t - X_k K + \sum_k \sum_l B_{kl} \ln w_k\right)},$$

$$(23)\quad b_i = \frac{t\left[V_i - B_i\left(V_t + \sum_k V_k \ln w_k + B_t \ln E + V_{tt}t\right)\right] + S_i B_T t}{S_i(1 - B_T t - X_k K)},$$

$$(24)\quad b_i = \frac{tV_i + S_i B_T t}{S_i\left(1 - B_T t - X_k K + \sum_k \sum_l B_{kl} \ln w_k\right)}.$$

According to Kim, if technical change is occurring, ε_{YT} will be positive. Furthermore, if $b_i > 0$, input i is technology intensive; if $b_i = 0$, input i is technology neutral; and if $b_i < 0$, input i is technology saving.

Data Description and Empirical Results

To explore the potential usefulness of the three functional forms, we estimate the Lewbel, AISS, and Translog models with an updated version of the familiar data on U.S. agricultural production found in Ball et al.[6] These data consist of time series observations on U.S. aggregate agricultural output that combine fuel and electricity (F); feed, seed, and livestock (S); chemicals (C); other intermediate inputs (I); labor (L); and fixed input capital (K) for the period 1948–2002.[7] Data are aggregated by the Divisia indexing method, with the base year defined as 1996.

Each model was estimated by maximum likelihood estimation subject to the parameter restrictions imposed by symmetry and homogeneity. Each variable also was normalized to unity at its respective mean. Because the full set of share equations sums to one, a singular variance-covariance matrix would result if all five equations were included in the estimation procedure. Consequently, during estimation, we dropped the labor equation so that five equations, the indirect production function,

[6] We are indebted to Eldon Ball for providing us with additional years of data beyond that available in the 1997 paper.

[7] See Ball et al. for an excellent description of the original data sources as well as the methods used to construct the subcategory indices.

Table 1. Estimated Compensated Elasticities[a]

Lewbel Model			AISS Model			Translog Model		
Elasticity	Estimate	Standard Error	Elasticity	Estimate	Standard Error	Elasticity	Estimate	Standard Error
ε_{FF}	−0.3729*	0.0469*	ε_{FF}	−0.3493*	0.0426*	ε_{FF}	−0.2900*	0.0528
ε_{FS}	−0.0274	0.0945	ε_{FS}	−0.0573	0.0572	ε_{FS}	0.1662*	0.0698
ε_{FC}	0.0481	0.1024	ε_{FC}	0.0247	0.0445	ε_{FC}	0.0647	0.0535
ε_{FI}	0.3396*	0.0546*	ε_{FI}	0.3083*	0.0565*	ε_{FI}	0.2598*	0.0574
ε_{FL}	0.0126	0.1886	ε_{FL}	0.0736	0.0694	ε_{FL}	0.1317	0.0837
ε_{SF}	0.0179	0.0187	ε_{SF}	−0.0095	0.0094	ε_{SF}	−0.0271*	0.0114
ε_{SS}	−0.6447*	0.0543*	ε_{SS}	−0.4488*	0.0274*	ε_{SS}	−0.4267*	0.0356
ε_{SC}	−0.2044*	0.0491*	ε_{SC}	−0.0053	0.0190	ε_{SC}	−0.0599*	0.0264
ε_{SI}	0.4401*	0.0463*	ε_{SI}	0.4781*	0.0205*	ε_{SI}	0.5303*	0.0252
ε_{SL}	0.3912*	0.0958*	ε_{SL}	−0.0144	0.0289	ε_{SL}	−0.0166	0.0501
ε_{CF}	0.0679	0.0470	ε_{CF}	0.0144	0.0309	ε_{CF}	0.0443	0.0366
ε_{CS}	−0.3559	0.1702	ε_{CS}	−0.0122	0.0783	ε_{CS}	−0.2516*	0.1109
ε_{CC}	−1.1768*	0.3169*	ε_{CC}	−0.7218*	0.0821*	ε_{CC}	−1.0244*	0.1160
ε_{CI}	0.1574	0.1423	ε_{CI}	0.5902*	0.0695*	ε_{CI}	0.3892*	0.0934
ε_{CL}	1.3074*	0.4988*	ε_{CL}	0.1295*	0.1055*	ε_{CL}	0.8425*	0.1838
ε_{IF}	0.0395*	0.0073*	ε_{IF}	0.0268*	0.0050*	ε_{IF}	0.0229*	0.0050
ε_{IS}	0.2229*	0.0263*	ε_{IS}	0.2595*	0.0111*	ε_{IS}	0.2865*	0.0136
ε_{IC}	−0.0351	0.0416	ε_{IC}	0.0751*	0.0085*	ε_{IC}	0.0501*	0.0120
ε_{II}	−0.5037*	0.0319*	ε_{II}	−0.4469*	0.0148*	ε_{II}	−0.4652*	0.0158
ε_{IL}	0.2764*	0.0830*	ε_{IL}	0.0857*	0.0192*	ε_{IL}	0.1057*	0.0274
ε_{LF}	−0.0461	0.0327	ε_{LF}	0.0133	0.0115	ε_{LF}	0.0212	0.0135
ε_{LS}	0.3575*	0.0924*	ε_{LS}	−0.0189	0.0298	ε_{LS}	−0.0164	0.0495
ε_{IC}	0.5724*	0.1385*	ε_{IC}	0.0339	0.0237	ε_{IC}	0.1984*	0.0433
ε_{LI}	0.4292*	0.1317*	ε_{LI}	0.1565*	0.0367*	ε_{LI}	0.1933*	0.0502
ε_{LL}	−1.3131*	0.2737*	ε_{LL}	−0.1848*	0.0602*	ε_{LL}	−0.3965*	0.1106

[a] An asterisk (*) denotes significance at the 5% level.

and four remaining share equations are jointly estimated for each of the three models. As autocorrelation was found in the initial estimates, the models were re-estimated by Berndt-Savin, Moschini-Moro, and symmetric *R* autocorrelation correction methods. The results presented below are from the asymmetric *R* specification, which had the highest log likelihood values.[8]

Looking first at overall goodness of fit, we find that each of the three models have R^2 values above .94 and system R^2 values due to Buse ranging from .9626 for the AISS system to .9715 for the Lewbel system. Given that the measures of fit are very similar, it is instructive to look for potential differences and similarities in the three functional forms with the various elasticities mentioned above. Compensated own- and cross-price elasticities calculated at the data means by Equation (19) for each of the models are presented in Table 1.[9] Estimated own-price elasticities for all three models are negative and significant, suggesting that the inputs all follow the law of factor demand. At the same time, the estimated cross-price elasticities are nearly all positive, suggesting that the five inputs are substitutes. The two statistically significant exceptions are found in the Translog specification, in which the demand for fuels and electricity decreases

[8] Parameter estimates for each of the three models can be found in Appendix 2.

[9] Allen elasticities of substitution were also estimated. However, these values are not reported because they contain the same information as the compensated elasticities.

Table 2. Estimated Own-Price Compensated Elasticities at Different Normalizations[a]

Lewbel Model			AISS Model			Translog Model		
Elasticity	Estimate	Standard Error	Elasticity	Estimate	Standard Error	Elasticity	Estimate	Standard Error
Mean of 1948–1952								
ε_{FF}	−0.2643*	0.0559	ε_{FF}	−0.1828*	0.0644	ε_{FF}	−0.1472	0.0773
ε_{SS}	−0.5861*	0.0432	ε_{SS}	−0.4525*	0.0246	ε_{SS}	−0.4268*	0.0314
ε_{CC}	−1.5640*	0.7777	ε_{CC}	−0.4899*	0.1851	ε_{CC}	−1.2631*	0.2685
ε_{II}	−0.5357*	0.0383	ε_{II}	−0.4797*	0.0193	ε_{II}	−0.4925*	0.0189
ε_{LL}	−1.3138*	0.2678	ε_{LL}	−0.2361*	0.0562	ε_{LL}	−0.4247*	0.0983
Mean of 1973–1977								
ε_{FF}	−0.2822*	0.0536	ε_{FF}	−0.2658*	0.0492	ε_{FF}	−0.1762*	0.0621
ε_{SS}	−0.6051*	0.0441	ε_{SS}	−0.4551*	0.0246	ε_{SS}	−0.4273*	0.0307
ε_{CC}	−1.1200*	0.2626	ε_{CC}	−0.7384*	0.0716	ε_{CC}	−0.9908*	0.0995
ε_{II}	−0.4991*	0.0313	ε_{II}	−0.4492*	0.0144	ε_{II}	−0.4629	0.0160
ε_{LL}	−1.3295*	0.2984	ε_{LL}	−0.1515*	0.0664	ε_{LL}	−0.3460*	0.1225
Mean of 1998–2002								
ε_{FF}	−0.3775*	0.0459	ε_{FF}	−0.3498*	0.0414	ε_{FF}	−0.3220*	0.0493
ε_{SS}	−0.7215*	0.0761	ε_{SS}	−0.4248*	0.0375	ε_{SS}	−0.4131*	0.0489
ε_{CC}	−1.0844*	0.2201	ε_{CC}	−0.7531*	0.0624	ε_{CC}	−0.9586*	0.0802
ε_{II}	−0.4917*	0.0312	ε_{II}	−0.4330*	0.0149	ε_{II}	−0.4496*	0.0142
ε_{LL}	−1.3235*	0.2716	ε_{LL}	−0.2077*	0.0741	ε_{LL}	−0.4102*	0.1112

[a] An asterisk (*) denotes significance at the 5% level.

with respect to the price of feed, seed, and livestock, and the demand for feed, seed, and livestock decreases with respect to the price of fuels and electricity. In addition to testing whether the results conform to production theory, the compensated elasticities can be used to determine whether the indirect production function is quasi-convex. The eigenvalues of the matrix of substitution elasticities (calculated at the data means) for the Lewbel, AISS, and Translog models indicate that this matrix is negative semidefinite, suggesting that all three specifications satisfy the curvature restriction implied by supply theory (Chalfant, Gray, and White).

As already mentioned, the elasticity results discussed so far have been obtained at the data means. Because it is possible that the calculated elasticities differ at different points in the distribution, we renormalized the data at different points and recalculated the elasticities to check for potential cross-distribution differences. Specifically, instead of the mean, we normalized on the first five, the middle five, and the last five data points. In interest of space, only the own-price compensated elasticities are presented in Table 2. As expected, the own-price elasticities are all negative and statistically significant with one exception, the own-price compensated elasticity for fuels and electricity at the beginning of the data for the Translog model. The magnitudes of the own-price elasticities across the different specifications are similar.

A well-known aspect of compensated elasticities is that they isolate input price effects without considering the expenditure effects that accompany changes in input prices. To jointly consider both effects, we calculate the uncompensated elasticities in Table 3. As expected, the estimated own-price elasticities are all negative for each of the three models. Turning to cross-price elasticities, the estimated uncompensated values in Table 3 differ markedly from their compensated counterparts in Table 1. Many of the uncompensated elasticities are negative, suggesting that when the expenditure effect is accounted for, the majority of inputs

Table 3. Estimated Uncompensated Elasticities[a]

Lewbel Model			AISS Model			Translog Model		
Elasticity	Estimate	Standard Error	Elasticity	Estimate	Standard Error	Elasticity	Estimate	Standard Error
ε_{FF}	−0.4179*	0.0447	ε_{FF}	−0.3882*	0.0417	ε_{FF}	−0.3323*	0.0525
ε_{FS}	−0.3036	0.1607	ε_{FS}	−0.2963*	0.0727	ε_{FS}	−0.4263*	0.0941
ε_{FC}	−0.0177	0.1179	ε_{FC}	−0.0322	0.0442	ε_{FC}	0.0028	0.0576
ε_{FI}	−0.1715	0.1553	ε_{FI}	−0.1339*	0.0558	ε_{FI}	−0.2216*	0.0738
ε_{FL}	−0.2667*	0.1304	ε_{FL}	−0.1681*	0.0800	ε_{FL}	−0.1314	0.0735
ε_{SF}	−0.0439*	0.0168	ε_{SF}	−0.0496*	0.0090	ε_{SF}	−0.0668*	0.0109
ε_{SS}	−1.0241*	0.0778	ε_{SS}	−0.6951*	0.0340	ε_{SS}	−0.6710*	0.0443
ε_{SC}	−0.2948*	0.0539	ε_{SC}	−0.0639*	0.0189	ε_{SC}	−0.1181*	0.0279
ε_{SI}	−0.2619*	0.0661	ε_{SI}	0.0222	0.0258	ε_{SI}	0.0782*	0.0362
ε_{SL}	0.0076	0.0803	ε_{SL}	−0.2636*	0.0303	ε_{SL}	−0.2637*	0.0432
ε_{CF}	−0.0034	0.0370	ε_{CF}	−0.0539	0.0295	ε_{CF}	−0.0313	0.0374
ε_{CS}	−0.7938*	0.3133	ε_{CS}	−0.4317*	0.1049	ε_{CS}	−0.7155*	0.1474
ε_{CC}	−1.2811*	0.3521	ε_{CC}	−0.8217*	0.0808	ε_{CC}	−1.1349*	0.1255
ε_{CI}	−0.6531*	0.3507	ε_{CI}	−0.1861*	0.0925	ε_{CI}	−0.4692*	0.1500
ε_{CL}	0.8646*	0.3604	ε_{CL}	−0.2948*	0.1187	ε_{CL}	0.3733*	0.1473
ε_{IF}	−0.0101	0.0062	ε_{IF}	−0.0157*	0.0048	ε_{IF}	−0.0149*	0.0051
ε_{IS}	−0.0816	0.0513	ε_{IS}	−0.0012	0.0165	ε_{IS}	0.0548*	0.0203
ε_{IC}	−0.1076*	0.0477	ε_{IC}	0.0130	0.0088	ε_{IC}	−0.0051	0.0132
ε_{II}	−1.0672*	0.0703	ε_{II}	−0.9294*	0.0201	ε_{II}	−0.8941*	0.0246
ε_{IL}	−0.0315	0.0617	ε_{IL}	−0.1780*	0.0201	ε_{IL}	−0.1288*	0.0226
ε_{LF}	−0.0313	0.0315	ε_{LF}	−0.0080	0.0113	ε_{LF}	−0.0068	0.0132
ε_{LS}	0.4485*	0.1450	ε_{LS}	−0.1498*	0.0408	ε_{LS}	−0.1884*	0.0696
ε_{IC}	0.5941*	0.1516	ε_{IC}	0.0027	0.0249	ε_{IC}	0.1574*	0.0480
ε_{LI}	0.5976*	0.2074	ε_{LI}	−0.0858	0.0539	ε_{LI}	−0.1250	0.0871
ε_{LL}	−1.2211*	0.2263	ε_{LL}	−0.3173*	0.0602	ε_{LL}	−0.5704*	0.0898

[a] An asterisk (*) denotes significance at the 5% level.

are complements. The difference in sign between the compensated and uncompensated elasticities is consistent with previous findings by Gajanan and Ramaiah and Kim.[10] There are three statistically significant positive cross-price uncompensated elasticities in the Lewbel specification and two statistically significant positive cross-price uncompensated elasticities in the Translog specification. As with the compensated elasticities, we obtained uncompensated elasticities for three different normalizations of the data. The results are recorded in Table 4. All of the own-price uncompensated elasticities are negative and statistically significant, which implies that the Marshallian demands slope downward.

[10] On the basis of the sign differences, Gajanan and Ramaiah caution policy makers to consider uncompensated rather than compensated elasticities.

Table 5 presents the estimated expenditure elasticities for the Lewbel, AISS, and Translog models of Equations (16), (17), and (18), respectively. Economic theory suggests that most of these elasticities should be positive because firms should increase the amount of all inputs used in response to increases in expenditure. Over each of the three models, all inputs have a positive sign and are generally statistically significant. The one exception is for labor in the Lewbel specification, which is not statistically different from zero. The expenditure elasticity for labor is less than one in the AISS and Translog specification, suggesting that labor is expenditure inelastic. Conversely, the expenditure elasticities for fuels and electricity; feed, seed, and livestock; and chemicals are all greater than one, suggesting that those inputs are expenditure elas-

Table 4. Estimated Own-Price Uncompensated Elasticities at Different Normalizations[a]

Lewbel Model			AISS Model			Translog Model		
Elasticity	Estimate	Standard Error	Elasticity	Estimate	Standard Error	Elasticity	Estimate	Standard Error
Mean of 1948–1952								
ε_{FF}	−0.3048*	0.0542	ε_{FF}	−0.2135*	0.0634	ε_{FF}	−0.1833*	0.0774
ε_{SS}	−1.0218*	0.0706	ε_{SS}	−0.7345*	0.0317	ε_{SS}	−0.7019*	0.0389
ε_{CC}	−1.6408*	0.8199	ε_{CC}	−0.5594*	0.1836	ε_{CC}	−1.3458*	0.2794
ε_{II}	−1.0811*	0.0825	ε_{II}	−0.9166*	0.0231	ε_{II}	−0.8800*	0.0282
ε_{LL}	−1.2156*	0.2089	ε_{LL}	−0.4399*	0.0558	ε_{LL}	−0.6434*	0.0770
Mean of 1973–1977								
ε_{FF}	−0.3220*	0.0519	ε_{FF}	−0.2998*	0.0482	ε_{FF}	−0.2133*	0.0621
ε_{SS}	−1.0165*	0.0680	ε_{SS}	−0.7276*	0.0317	ε_{SS}	−0.6983*	0.0389
ε_{CC}	−1.2333*	0.2987	ε_{CC}	−0.8464*	0.0702	ε_{CC}	−1.1110*	0.1088
ε_{II}	−1.0666*	0.0700	ε_{II}	−0.9335*	0.0204	ε_{II}	−0.8924*	0.0250
ε_{LL}	−1.1975*	0.2523	ε_{LL}	−0.2599*	0.0656	ε_{LL}	−0.4880*	0.1016
Mean of 1998–2002								
ε_{FF}	−0.4211*	0.0441	ε_{FF}	−0.3885*	0.0405	ε_{FF}	−0.3646*	0.0487
ε_{SS}	−1.0357*	0.0962	ε_{SS}	−0.6278*	0.0435	ε_{SS}	−0.6134*	0.0577
ε_{CC}	−1.2061*	0.2493	ε_{CC}	−0.8708*	0.0605	ε_{CC}	−1.0851*	0.0886
ε_{II}	−1.0612*	0.0651	ε_{II}	−0.9377*	0.0195	ε_{II}	−0.9040*	0.0227
ε_{LL}	−1.2746*	0.2303	ε_{LL}	−0.3325*	0.0789	ε_{LL}	−0.5863*	0.0914

[a] An asterisk (*) denotes significance at the 5% level.

tic. The expenditure elasticity for other inputs is expenditure elastic for the Lewbel and AISS models, whereas it is unit elastic in the Translog model. These results differ from those of Chambers and Lee, who find that all expenditure elasticities are positive and close to one.[11]

Table 6 investigates the effect of technological innovation by computing the rates of technical change in Equations (20) and (21) and the input bias terms in Equations (22), (23), and (24). The estimated rate of technical change is positive and significant for the Lewbel and Translog specification, implying that, on average, technology increased during the period from 1948 to 2002, whereas it is not statistically different from zero for the AISS specification. The bottom panel of Table 6 suggests that fuels and electricity were technology neutral and that labor was technology saving for all three models. Finally, the AISS

[11] Our results are not directly comparable to the findings in Chambers and Lee because we treat capital as a fixed input and estimate a short-run model, whereas they allow capital to vary and hence estimate a long-run model.

Table 5. Estimated Expenditure Elasticities[a]

	Estimate		
Elasticity	Lewbel Model	AISS Model	Translog Model
Fuels and electricity	1.1774* (0.3132)	1.0188* (0.1122)	1.1089* (0.1502)
Feed, seed, and livestock	1.6170* (0.1181)	1.0500* (0.0486)	1.0413* (0.0542)
Chemicals	1.8669* (0.6547)	1.7882* (0.1901)	1.9775* (0.2519)
Other inputs	1.2979* (0.1167)	1.1113* (0.0355)	0.9881* (0.0361)
Labor	−0.3878 (0.2801)	0.5582* (0.0883)	0.7332* (0.1084)

[a] An asterisk (*) denotes significance at the 5% level. Standard errors are in parentheses.

Table 6. Estimated Technical Change and Technical Bias Elasticities[a]

	Estimate		
Elasticity	Lewbel Model	AISS Model	Translog Model
Technical change	0.3070* (0.0590)	0.0434 (0.1971)	0.2128* (0.0671)
Bias			
Fuels and electricity	0.0757 (0.2495)	0.0230 (0.1005)	−0.1033 (0.2579)
Feed, seed, and livestock	0.0739 (0.0909)	0.1510* (0.0455)	0.2574 (0.1309)
Chemicals	−0.9862 (0.6558)	0.2958* (0.1506)	−0.6277 (0.9228)
Other inputs	−0.0249 (0.1246)	0.2421* (0.0448)	0.7155* (0.1359)
Labor	−0.8214* (0.3110)	−0.7315* (0.2787)	−1.2108* (0.3693)

[a] An asterisk (*) denotes significance at the 5% level. Standard errors are in parentheses.

model indicates that feed, seed, and livestock; chemicals; and other inputs were technology intensive, whereas the Lewbel model indicates that these inputs were technology neutral. The Translog model suggests that fuels and electricity; feed, seed, and livestock; and chemicals were technology neutral, whereas other inputs were technology intensive. Table 7 presents rates of technical change and input bias for the Lewbel, AISS, and Translog models that have been normalized on the beginning, middle, and end sections of the data. The three models generally do not agree regarding statistical significance over the different normalizations. In this case, the conclusions drawn about the effect of technology depend critically on which specification is chosen.

The empirical results from the three potential indirect production functions are generally consistent with production theory. This suggests that the Lewbel and AISS models are indeed potentially attractive alternatives to the Translog model for future work in production economics. As previously mentioned, the Lewbel model nests both the AISS and the Translog indirect production functions, thereby providing a simple likelihood ratio test of the relative goodness of fit of the three models. This test is based on comparing the log-likelihood functions for each of the three models to determine whether any are statistically superior to the others. Evaluated at the data means, the log-likelihood functions for the Lewbel, AISS, and Translog models, respectively, are 1,111.25, 1,102.16, and 1,105.6. Because the Lewbel specification has four more parameters than the AISS and the Translog, there are four degrees of freedom for the likelihood ratio tests. Hence, likelihood ratio tests reject the null hypothesis of equality between the Lewbel and AISS models at the 1% significance level and the Lewbel and Translog models at the 5% significance level. In other words, our results provide evidence to suggest that although all three models are consistent with production theory, the Lewbel model might be statistically superior to the AISS and Translog models. These results are consistent with expectations given the additional flexibility of the Lewbel specification and that studies in the demand literature find that the Lewbel model is statistically superior to the AIDS and Translog models (see Eales; Wang, Halbrendt, and Johnson; Yen and Chern).

Conclusion

Although consumer theory employs several different empirical specifications for estimating indirect utility functions, producer theory has relied on the Translog specification to estimate the analogous indirect production function. In this study, we are the first to employ two alternative specifications, the Lewbel and AISS, to estimate a standard indirect production function. Using time series data on U.S. agricultural production, we find that all three models produce estimated elasticities that generally conform to production theory. Thus, we conclude that all three are potentially valuable approaches to estimating indirect production functions. However, likelihood ratio tests, because the Lewbel model nests the remaining

Table 7. Estimated Technical Change and Technical Bias Elasticities at Different Normalizations[a]

Lewbel Model			AISS Model			Translog Model		
Elasticity	Estimate	Standard Error	Elasticity	Estimate	Standard Error	Elasticity	Estimate	Standard Error
Mean of 1948–1952								
ε_{YT}	0.0860*	0.0296	ε_{YT}	0.1335*	0.0401	ε_{YT}	0.0230	0.0280
b_F	0.0217	0.0561	b_F	0.0055	0.0154	b_F	−0.0215	0.0448
b_S	0.0103	0.0134	b_S	0.0180*	0.0040	b_S	0.0308	0.0177
b_C	−0.3567	0.2506	b_C	0.2286*	0.0511	b_C	−0.1963	0.2806
b_I	−0.0023	0.0263	b_I	0.0408*	0.0076	b_I	0.1122*	0.0193
b_L	−0.1329*	0.0439	b_L	−0.0958*	0.0322	b_L	−0.1501*	0.0444
Mean of 1973–1977								
ε_{YT}	0.3389*	0.0665	ε_{YT}	0.0401	0.2025	ε_{YT}	0.2250*	0.0711
b_F	0.1131	0.2969	b_F	0.0274	0.1154	b_F	−0.1254	0.3034
b_S	0.0576	0.0844	b_S	0.1350*	0.0415	b_S	0.2375*	0.1190
b_C	−0.8960	0.5829	b_C	0.2536	0.1317	b_C	−0.5489	0.8103
b_I	−0.0243	0.1278	b_I	0.2409*	0.0449	b_I	0.7271*	0.1374
b_L	−0.9667*	0.3707	b_L	−0.8324*	0.3152	b_L	−1.4194*	0.4287
Mean of 1998–2002								
ε_{YT}	0.4023*	0.1203	ε_{YT}	−0.9821*	0.3765	ε_{YT}	0.4424*	0.1365
b_F	0.0998	0.3622	b_F	0.0152	0.2953	b_F	−0.1493	0.4737
b_S	0.1436	0.2053	b_S	0.2587	0.1673	b_S	0.6028*	0.2953
b_C	−1.1441	0.7465	b_C	−0.4801	0.3307	b_C	−0.8167	1.2596
b_I	−0.0564	0.1712	b_I	0.2718*	0.0947	b_I	1.2695*	0.2915
b_L	−1.1740*	0.5307	b_L	−0.7119	0.4439	b_L	−2.1706*	0.6879

[a] An asterisk (*) denotes significance at the 5% level.

specifications, suggest that the Lewbel specification might be the statistically superior specification in this application.

Although the Lewbel, AISS, and, Translog models conform to production theory, the policy implications drawn from the results differ depending on which specification is chosen. One example occurs in the estimation of the compensated elasticities in which the Translog model suggests that demand for feed, seed, and livestock and the price of fuel are complements, whereas the Lewbel and AISS specifications suggest that the demand for feed, seed, and livestock is not related to the price of fuel. Another example occurs in the estimated expenditure elasticities in which the AISS and Translog specifications suggest that labor is inelastic, whereas the Lewbel specification suggests labor is not statistically different from zero. The most notable difference between specifications occurs when examining the rate of technical change by year. The AISS Lewbel and Translog models suggest that technology has been slowly increasing from 1948 to 2002, whereas the AISS model suggests that technology has been neutral. These differences suggest that it is important to consider alternative specifications of the indirect production function in case the implications drawn from the results differ between model choices.

[Received November 2003; Accepted August 2005.]

References

Appelbaum, E. "On the Choice of Functional Forms." *International Economic Review* 2(June 1979):449–58.

Ball, V.E., J.C. Bureau, R. Nehring, and A. Somwaru. "Agricultural Productivity Revisited." *American Journal of Agricultural Economics* 79(November 1997):1045–63.

Berndt, E.R., and M.S. Khaled. "Parametric Productivity Measurement and Choice Among Flexible Functional Forms." *Journal of Political Economy* 87(December 1979):1220–45.

Berndt, E.R., and D.O. Wood. "Technology, Prices, and the Derived Demand for Energy." *Review of Economics and Statistics* 57(August 1975): 259–68.

Blackorby, C., D. Primont, and R. Russell. *Duality, Separability, Functional Structure: Theory and Economic Applicatons and the Structure of Utility Functions.* New York: American Elsevier, 1978.

Buse, A. "Goodness-of-Fit in the Seemingly Unrelated Regression Model." *Journal of Econometrics* 10(April 1979):109–13.

Chalfant, J.A., R.S. Gray, and K.J. White. "Evaluating Prior Beliefs in a Demand System: The Case of Meats Demand in Canada." *American Journal of Agricultural Economics* 73(May 1991):476–90.

Chambers, R.G. *Applied Production Analysis: A Dual Approach.* Cambridge: Cambridge University Press, 1988.

Chambers, R.G., and H. Lee. "Constrained Output Maximization and U.S. Agriculture." *Applied Economics* 18(April 1986):347–57.

Christensen, L.R., and W.H. Green. "Economies of Scale in U.S. Electric Power Generation." *Journal of Political Economy* 84(August 1976):655–76.

Christensen, L.R., D.W. Jorgenson, and L.J. Lau. "Transcendental Logarithmic Utility Functions." *American Economic Review* 65(June 1975):367–83.

Deaton, A.D., and J. Muellbauer "An Almost Ideal Demand System." *American Economic Review* 70(June 1980):312–26.

Diewert, W.E. "An Application of the Shepherd Duality Theorem: A Generalized Leontief Production Function." *Journal of Political Economy* 79(May–June 1971):481–507.

———. "Functional Forms for Revenue and Factor Requirements Functions." *International Economic Review* 15(Febuary 1974):119–30.

Eales, J.S. "The Inverse Lewbel Demand System." *Journal of Agricultural and Resource Economics* 19(July 1994):173–82.

Gajanan, S.N., and K.C. Ramaiah. "An Econometric Estimation of Hicksian and Marshallian Elasticities in Indian Manufacturing." *Southern Economic Journal* 63(October 1996):406–17.

Garofalo, G.A., and D.M. Malhotra. "A Regional Comparison of the Impact of Changes in Input Prices on Input Demand for U.S. Manufacturing." *Regional Science and Urban Economics* 22(June 1992):213–28.

Jorgenson, D.W., and L.J. Lau. "The Duality of Technology and Economic Behavior." *Review of Economic Studies* 41(April 1974a):181–200.

———. "Duality and Differentiability in Production." *Journal of Economic Theory* 9(September 1974b):23–42.

Kim, H.Y. "Analyzing the Indirect Production Function for U.S. Manufacturing." *Southern Economic Journal* 55(October 1988):494–504.

Lewbel, A. "Nesting and the AIDS and Translog Demand Systems." *International Economic Review* 30(May 1989):349–56.

Pope, R.D., and R. Just. "Empirical Implementation of Ex Ante Cost Functions." *Journal of Econometrics* 72(May 1996):231–49.

Wang, Q., C. Halbrendt, and S.R. Johnson. "A Non-Nested Test of the AIDS vs. the Translog Demand System." *Economics Letters* 51(May 1996):139–43.

Yen, S.T., and W. Chern. "Flexible Demand Systems with Serially Correlated Errors: Fat and Oil Consumption in the United States." *Journal of Agricultural Economics* 74(August 1992): 525–32.

Yuhn, K.H. "Functional Separability and the Existence of Consistent Aggregates in U.S. Manufacturing." *International Economic Review* 32(February 1991):229–50.

Appendix 1. Parameter Estimates[a]

Lewbel Model			AISS Model			Translog Model		
Parameter	Estimate	Standard Error	Parameter	Estimate	Standard Error	Parameter	Estimate	Standard Error
α_F	−0.0454	0.0297	α_F	−0.0010	0.0097	α_F	0.0032	0.0283
α_S	0.4183*	0.0510	α_S	0.0841*	0.0261	α_S	0.3362*	0.0476
α_C	0.5197*	0.0640	α_C	−0.0671*	0.0315	α_C	0.4480*	0.0589
α_I	0.5054*	0.0996	α_I	0.1287	0.0704	α_I	0.2588*	0.0925
B_{FF}	0.0223*	0.0016	B_{FF}	0.0239*	0.0020	B_{FF}	0.0209*	0.0019
B_{FS}	−0.0109*	0.0035	B_{FS}	−0.0114*	0.0024	B_{FS}	−0.0137*	0.0028
B_{FC}	−0.0002	0.0022	B_{FC}	−0.0013	0.0017	B_{FC}	−0.0016	0.0019
B_{FI}	−0.0050	0.0026	B_{FI}	−0.0050	0.0021	B_{FI}	−0.0068*	0.0023
B_{SS}	0.0230	0.0120	B_{SS}	0.0756*	0.0078	B_{SS}	0.0617*	0.0110
B_{SC}	−0.0451*	0.0085	B_{SC}	−0.0152*	0.0045	B_{SC}	−0.0334*	0.0071
B_{SI}	−0.0217	0.0097	B_{SI}	0.0098	0.0051	B_{SI}	0.0161*	0.0091
B_{CC}	−0.0165	0.0112	B_{CC}	0.0106*	0.0047	B_{CC}	−0.0087	0.0063
B_{CI}	−0.0363*	0.0099	B_{CI}	0.0062	0.0039	B_{CI}	−0.0214*	0.0072
B_{II}	−0.0037	0.0188	B_{II}	0.0498*	0.0079	B_{II}	0.0397*	0.0141
B_F	−0.0050	0.0067	B_F	0.0008	0.0045			
B_S	−0.0753*	0.0157	B_S	0.0123	0.0121			
B_C	−0.0043	0.0208	B_C	0.0461*	0.0102			
B_I	−0.0684*	0.0275	B_I	0.0506*	0.0148			
V_F	0.0078	0.0073	V_F	0.0012	0.0042	V_F	−0.0040	0.0081
V_S	0.0612*	0.0213	V_S	0.0376*	0.0139	V_S	0.0454	0.0272
V_C	−0.0389	0.0284	V_C	0.0153	0.0108	V_C	−0.0299	0.0427
V_I	0.0556	0.0383	V_I	0.1087*	0.0308	V_I	0.2479*	0.0460
V_T	−0.6158*	0.1841	V_T	−1.1788*	0.3025	V_T	−0.4591*	0.2033
V_{TT}	0.3300	0.1773	V_{TT}	1.0805*	0.2839	V_{TT}	0.1283	0.1933
B_T	−0.1000*	0.0273	B_T	−0.0080	0.0319	B_T	0.0184	0.0267
J_F	0.0323*	0.0095	J_F	0.0395*	0.0074	J_F	0.0240*	0.0101
J_S	−0.0708*	0.0187	J_S	0.0830*	0.0192	J_S	−0.0384*	0.0193
J_C	−0.1095*	0.0263	J_C	0.0599*	0.0236	J_C	−0.1030*	0.0220
J_I	−0.0364	0.0383	J_I	0.1271*	0.0474	J_I	−0.0101	0.0355
X_K	0.2276*	0.0240	X_K	−0.0146	0.0488	X_K	0.1460*	0.0238
H_K	−1.1390*	0.2285	H_K	−3.0244*	0.7735	H_K	−1.0495*	0.2223
H_{KK}	1.1219*	0.2903	H_{KK}	4.8114*	1.1480	H_{KK}	0.8451*	0.2643
B_{FL}	−0.0062	0.0034				B_{FL}	−0.0021	0.0023
B_{SL}	0.0084	0.0151				B_{SL}	−0.0387*	0.0127
B_{CL}	0.0533*	0.0147				B_{CL}	0.0201*	0.0075
B_{IL}	0.0267	0.0193				B_{IL}	−0.0234	0.0149

[a] An asterisk (*) denotes significance at the 5% level.

Appendix 2: Proof of Specification

A Lewbel-type indirect production function can be defined as the logarithmic transformation

$$\text{(A.1)} \quad y(\mathbf{w}, e, K, t) = \prod_{k=1}^{n} w_k^{-B_k}\left[\ln e\left(1 + \sum_{i=1}^{n}\sum_{j=1}^{n} \beta_{ij}\ln w_j\right) - \ln g(\mathbf{w}, e, K, t)\right].$$

In Equation (7), $\ln g(\mathbf{w}, e, K, t)$ is defined by

$$\text{(A.2)} \quad \ln g(\mathbf{w}, e, K, t) = \alpha_0 + \sum_{i=1}^{n} \alpha_i \ln w_i + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} \beta_{ij}\ln w_i \ln w_j + V_t t + t\sum_{i=1}^{n} V_i \ln w_i + B_{te} t \ln e$$
$$+ \frac{1}{2}V_{tt}t^2 + H_K K + K\sum_{i=1}^{n} J_i \ln w_i + \chi_k K \ln e + \frac{1}{2}H_{KK}K^2.$$

To obtain the input demand function, we use Roy's identity:

$$\text{(A.3)} \quad \frac{\partial y(\mathbf{w}, e, K, t)}{\partial w_i} = -B_i \frac{1}{w_i}\prod_{k=1}^{n} w_k^{-B_k}\left[\ln e\left(1 + \sum_{i=1}^{n}\sum_{j=1}^{n} \beta_{ij}\ln w_j\right) - \ln g(\mathbf{w}, e, K, t)\right]$$
$$- \frac{1}{w_i}\prod_{k=1}^{n} w_k^{-B_k}\left(\alpha_i + \sum_{j=1}^{n} \beta_{ij}\ln w_j + V_i t + J_i K - \ln e \sum_{j=1}^{n} \beta_{ij}\right),$$

$$\text{(A.4)} \quad \frac{\partial y(\mathbf{w}, e, K, t)}{\partial e} = \frac{1}{e}\prod_{k=1}^{n} w_k^{-B_k}\left(1 + \sum_{i=1}^{n}\sum_{j=1}^{n} \beta_{ij}\ln w_j - B_{te}t - \chi_k K\right).$$

The Lewbel factor demands are

$$\text{(A.5)} \quad x_i = -\frac{\partial y/\partial w_i}{\partial y/\partial e} = \frac{\frac{1}{w_i}\prod_{k=1}^{n} w_k^{-B_k}\left(\alpha_i + \sum_{j=1}^{n} \beta_{ij}\ln w_j + V_i t + J_i K - \ln e \sum_{j=1}^{n} \beta_{ij}\right)}{\frac{1}{e}\prod_{k=1}^{n} w_k^{-B_k}\left(1 + \sum_{i=1}^{n}\sum_{j=1}^{n} \beta_{ij}\ln w_j - B_{te}t - \chi_k K\right)}$$
$$- \frac{B_i \frac{1}{w_i}\prod_{k=1}^{n} w_k^{-B_k}\left[\ln e\left(1 + \sum_{i=1}^{n}\sum_{j=1}^{n} \beta_{ij}\ln w_j\right) - \ln g(\mathbf{w}, e, K, t)\right]}{\frac{1}{e}\prod_{k=1}^{n} w_k^{-B_k}\left(1 + \sum_{i=1}^{n}\sum_{j=1}^{n} \beta_{ij}\ln w_j - B_{te}t - \chi_k K\right)}, \quad i = 1, \ldots, n.$$

The share equations are obtained by multiplying Equation (A.5) by w_i/e and canceling obvious terms,

$$\text{(A.6)} \quad S_i = \frac{\alpha_i + \sum_{j=1}^{n} \beta_{ij}\ln w_i + V_i t + J_i K - \ln e \sum_{j=1}^{n} \beta_{ij} + B_i\left[\ln e\left(1 + \sum_{j=1}^{n}\sum_{k=1}^{n} \beta_{jk}\ln w_j\right) - \ln g(\mathbf{w}, e, K, t)\right]}{1 + \sum_{j=1}^{n}\sum_{k=1}^{n} \beta_{jk}\ln w_j - B_{te}t - \chi_k K},$$
$$i = 1, \ldots, n.$$

The AISS is derived from the Lewbel-type indirect production function in Equation (A.1) by imposing, along with the adding up and homogeneity restrictions, the additional restrictions that $\Sigma_{j=1}^{n} \beta_{ij} = 0$ for all i. Imposing this restriction yields the AISS indirect production function

$$\text{(A.7)} \quad y(\mathbf{w}, e, K, t) = \prod_{k=1}^{n} w_k^{-B_k}[\ln e - \ln g(\mathbf{w}, e, K, t)],$$

where $\ln g(\mathbf{w}, e, K, t)$ is defined in Equation (A.2). To obtain the input demand functions, we follow the same procedure used above for

$$\text{(A.8)} \quad \frac{\partial y(w, e, K, t)}{\partial w_i} = -B_i \frac{1}{w_i} \prod_{k=1}^{n} w_k^{-B_k}[\ln e - \ln g(w, e, K, t)] - \frac{1}{w_i} \prod_{k=1}^{n} w_k^{-B_k}\left(\alpha_i + \sum_{j=1}^{n} \beta_{ij} \ln w_j + V_i t + J_i K\right),$$

$$\text{(A.9)} \quad \frac{\partial y(w, e, K, t)}{\partial e} = \frac{1}{e} \prod_{k=1}^{n} w_k^{-B_k}(1 - B_{te}t - \chi_k K).$$

The AISS factor demands are

$$\text{(A.10)} \quad x_i = -\frac{\partial y/\partial w_i}{\partial y/\partial e} = \frac{\frac{1}{w_i} \prod_{k=1}^{n} w_k^{-B_k}\left\{\alpha_i + \sum_{j=1}^{n} \beta_{ij} \ln w_j + V_i t + J_i K + B_i[\ln e - \ln g(w, e, K, t)]\right\}}{\frac{1}{e} \prod_{k=1}^{n} w_k^{-B_k}(1 - B_{te}t - \chi_k K)},$$

$$i = 1, \ldots, n.$$

The share equations are obtained by multiplying Equation (A.10) by w_i/e and canceling obvious terms

$$\text{(A.11)} \quad S_i = \frac{\alpha_i + \sum_{j=1}^{n} \beta_{ij} \ln w_j + tV_i + KJ_i + B_i[\ln e - \ln g(w, e, K, t)]}{1 - B_{te}t - \chi_k K}, \qquad i = 1, \ldots, n.$$

Alternatively, the Translog model can be derived from Equation (A.1) by imposing the restrictions that $B_i = 0$ for all i along with the homogeneity and adding up restrictions. The indirect production function is

$$\text{(A.12)} \quad y(w, e, K, t) = \ln e\left(1 + \sum_{i=1}^{n} \sum_{j=1}^{n} \beta_{ij} \ln w_j\right) - \ln g(w, e, K, t),$$

where $\ln g(w, e, K, t)$ is defined in Equation (A.2). To obtain the Translog input demand functions, we yet again use Roy's identity

$$\text{(A.13)} \quad \frac{\partial y(w, e, K, t)}{\partial w_i} = \frac{1}{w_i}\left[\ln e \sum_{j=1}^{n} \beta_{ij} - \left(\alpha_i + \sum_{j=1}^{n} \beta_{ij} \ln w_j + V_i t + J_i K\right)\right],$$

$$\text{(A.14)} \quad \frac{\partial y(w, e, K, t)}{\partial e} = \frac{1}{e}\left(1 + \sum_{i=1}^{n} \sum_{j=1}^{n} \beta_{ij}\, w_j - B_{te}t - \chi_k K\right).$$

The Translog factor demands are

$$\text{(A.15)} \quad x_i = -\frac{\partial y/\partial w_i}{\partial y/\partial e} = \frac{\frac{1}{w_i}\left(\alpha_i + \sum_{j=1}^{n} \beta_{ij} \ln w_j + V_i t + J_i K - \ln e \sum_{j=1}^{n} \beta_{ij}\right)}{\frac{1}{e}\left(1 - B_{te}t - \chi_k K + \sum_{i=1}^{n} \sum_{j=1}^{n} \beta_{ij} \ln w_j\right)}, \qquad i = 1, \ldots, n.$$

The resulting share equation system is then

$$\text{(A.16)} \quad S_i = \frac{\alpha_i + \sum_{j=1}^{n} \beta_{ij} \ln w_j + V_i t + JK_i - \ln e \sum_{j=1}^{n} \beta_{ij}}{1 - B_t t - \chi_k K + \sum_{i=1}^{n} \sum_{j=1}^{n} \beta_{ij} \ln w_j}, \qquad i = 1, \ldots, n.$$