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Extending Theil's Inequality Index: Addressing Dynamic Convergence in the OECD

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Theil's inequality index is used to measure convergence in 14 Organization for Economic Cooperation and Development (OECD) countries in terms of per capita income, per capita government and investment expenditures, and industrial employment. Results indicate that all four variables have converged over the sample period, 1950–1988. Next, the indices of the four variables are made dynamic by using pairwise cointegration and Johansen's $I(2)$ multi-cointegration tests. These tests indicate that the four inequalities are cointegrated; that is, there exists a long-run equilibrium between the four inequalities of the 14 OECD countries. However, the inequality in per capita government expenditure has no effect on the G-7 equilibrium when analyzed without the Other 7.

Key Words: cointegration, convergence, G-7, inequality, OECD

Whether countries are becoming more similar (convergence) in terms of per capita income and the identification of the factors that contribute to income convergence are important economic issues. One way for convergence to occur is for relatively poor countries to grow faster than relatively rich ones (Barro). The empirical literature has taken two main approaches in studying convergence issues: the construction of inequality measures (e.g., Ahluwalia, Carter, and Chenery; Berry, Bouruignon, and Morrison; Gao et al.; Theil and Deepak; Ram 1988, 1989a; Seale, Theil, and Deepak; Theil 1989, 1996; Theil and Deepak 1994, 2002; Theil and Seale; Wright), and regression analysis (e.g., Baradaran-Shoraka; Barro; Barro and Sali-i-Martin; Branco and Williamson; Deepak, Seale, and Moss; Grier and Tullock; Ram 1988). The evidence supports the idea that high-income countries are converging; however, the reasons why are less clear (Grier and Tullock; Goa et al.). Unlike these former studies which have focused on the short-run or on static models, this study focuses on the long-run by determining a method of measuring convergence, testing convergence on a group of countries, and determining the long-run relationships among selected macroeconomic variables.

Specifically, two questions are posed. The first is whether 14 Organization for Economic Cooperation and Development (OECD) countries are converging in terms of income, government expenditure, investment expenditure,

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and industrial employment.[1] Theil's inequality measure is used to answer this question. The second is whether the inequality of income has a long-run relationship with the three other inequalities. This question is explained using both pairwise cointegration analysis and Johansen's multiple cointegration technique.

Data

This study focuses on 14 OECD countries because of their increased importance in recent years, and the fact that the data for these countries are readily available and relatively accurate. The data used in this study are from two sources: Summers and Heston data set, and the OECD (1963, 1969, 1989, 1991a, and 1991b). The Summers and Heston data are constructed based on purchasing power parity.[2] The variables in this study from the Summers and Heston data are income per capita, government expenditure per capita, investment expenditure per capita, and population.[3] The industrial employment variable is taken from the OECD data set. The criteria for choosing these variables are based on macroeconomic relationships (De Long; Grossman and Helpman; Lucas; Romer), results from empirical investigations (Adams; Barro; Barro and Sala-i-Martin; De Long and Summers; Glomm and Ravkumar; Wright; Zind), and the availability of data.

Measuring Convergence

To dynamically measure convergence and interpret the changes, a consistent and decomposable index is needed. Theil's inequality index meets this criteria and the criteria for an inequality index.[4] Theil's inequality index has been used in numerous international comparison studies (e.g., Ahluwalia, Carter, and Chenery; Berry, Bouruignon, and Morrison; Ram 1989; Theil 1989, 1996). A major strength of Theil's inequality index is that it meets all four criteria for an inequality index, yields a consistent index, and is additively decomposable.[5] The derivation of Theil's inequality index can be found in the Appendix.

Inspection of the inequality measures reported in columns 2, 6, 10, and 14 of Table 1 clearly indicates that total inequality for all four variables has decreased considerably.[6] Income inequality (column 2) decreased from 0.21 in 1950 to 0.03 in 1988, a decrease of more than 86%, while the inequality of government expenditure (column 6) decreased from 0.17 to 0.05, a decrease of 67% during the same time period. Total investment inequality (column 10) decreased by 92% (0.25 to 0.02) while the inequality in industrial employment (column 14) decreased by 48% (0.03 to 0.01). Thus, on average, the OECD countries became much more affluent and more similar. These countries are converging in terms of income, government expenditure, investment expenditure, and industrial employment.

The decomposability of Theil's index can be used to determine whether the driving force behind the strong tendency toward convergence among the OECD countries is due to changes in regional inequality or changes in within-region inequality. Accordingly, the 14 countries are grouped into two regions: G-7 (United States, Canada, France, Italy, Japan, U.K., and West Germany); and the Other 7

[1] The OECD countries that are included in this analysis are the G-7 (United States, Canada, Japan, U.K., West Germany, France, Italy) and the Other 7 (Austria, Belgium, Denmark, The Netherlands, Norway, Ireland, and Spain).

[2] Purchasing power parity (PPP) is the number of currency units required to buy goods equivalent to what can be bought with one unit of the currency of the base country (Kravis, Heston, and Summers 1982, p. 383). Cross-country comparisons based on PPPs are generally thought to be superior to those based on official exchange rates (Kravis, Heston, and Summers 1978).

[3] Summers and Heston define government expenditure as public consumption and investment expenditure as private and public expenditure.

[4] The four requirements for an index are symmetry, mean-independence, population homogeneity, and the Pigou-Dalton condition (Bourguignon; Osberg).

[5] Bourguignon defines additive decomposability as a measure that the total inequality of a population can be broken down into a weighted average of the inequality existing within subgroups of the population and the inequality existing among them.

[6] The inequality measure has a lower bound of 0 but no upper bound; zero represents the case where no inequality exists.

(Austria, Belgium, Denmark, Ireland, The Netherlands, Norway, and Spain). The regional inequalities, J_g, are displayed in columns 3, 7, 11, and 15 of Table 1.

The Other 7 has not substantially narrowed the income gap between themselves and the G-7 (40% reduction, column 3). So what is driving the decline in income inequality of the 14 countries? Within-region inequality has decreased by 90% for the G-7 as shown in column 4 (primarily due to fast growth in Japan and slow growth in the U.S.) and only 57% for the Other 7 (column 5). Although the Other 7 countries are converging, they are converging within themselves at a slower rate than the G-7. Hence, total income inequality for the 14 OECD countries has dramatically declined due to the decline in inequality within the G-7.

Part of the explanation for the major change in inequality in the G-7 can be attributed to investment expenditures. Total investment inequality (column 10) decreased by 92% but the majority of the decline is within the G-7 (96% decline). Table 2 details the initial and ending level of investment expenditures and the respective growth rates for the OECD countries. The initial value for Japan is significantly lower than for the other countries; however, Japan's increase in the rate of expenditure on investment per capita (21 times the initial value) has boosted its rank to one of the top countries in terms of investment expenditure per capita. Japan's increase in investment expenditure and the United State's relatively slow increase heavily influence convergence within the G-7, which essentially drives the convergence of investment in the OECD countries.

On average, government-expenditure inequality is also converging at a slower rate than income and investment inequality. Regional inequality accounts for only 10%–15% of total government inequality over the 39 years of this study. Within-regional inequality is driving the convergence. Specifically, within-regional inequality among the Other 7 has decreased 71% while the G-7 decreased by 65%.[7] The G-7 has increased government expenditures 2.3 times over the 39-year period while the Other 7 has increased government expenditures by 3.4 times. The convergence in the Other 7 is due to increased expenditures on public goods by Spain and Norway. The slow rate of government expenditure in The Netherlands also helped the Other 7 to converge by allowing the other countries to catch up. The convergence in the 14 OECD countries is due to the slow growth of the United States and the U.K. and the faster growth of the Other 7 countries.

The inequality in industrial employment is the only complex category. Total inequality has decreased 47% from 1950 to 1988 (Table 1, column 14). The lowest level of inequality among the 14 countries occurred in 1979. Regional inequality increased even though total inequality decreased. The regional inequality increased 88% (column 15), which means the two regions have grown further apart over time; however, on average, regional inequality accounted for less than 5% of total industrial inequality until the late 1970s. Then regional inequality jumped to almost 22% of total inequality. Once again, within-region inequality is responsible for the reduction in total inequality. Inequality among the G-7 decreased 54% (column 16) and decreased 62% for the Other 7 (column 17). The driving force behind the reduction in the G-7 is the rate of increase in the number of people employed by industry in Japan. The relatively fast rate of increase in industrial employment by Ireland and Spain has influenced the convergence of the Other 7.

In summary, convergence is supported for all four variables and generally occurred at a faster rate in the income and investment variables. However, there was little inequality among the countries in terms of industrial employment even in 1950. One of the strongest driving forces behind the decreases in all of the inequalities is Japan's behavior. The next section extends the analysis to address the dynamic issues of convergence in the 14 OECD countries.

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[7] Within-region inequality, J_g, calculates the inequality among the countries within a given region.

Table 1. Income, Government, Investment, and Industry Inequality in OECD Countries

	Income Inequality				Government Inequality				Investment Inequality				Industrial Inequality			
Year 1	J 2	J_R 3	$J_{G\text{-}7}$ 4	$J_{Other\text{-}7}$ 5	J 6	J_R 7	$J_{G\text{-}7}$ 8	$J_{Other\text{-}7}$ 9	J *10*	J_R 11	$J_{G\text{-}7}$ 12	$J_{Other\text{-}7}$ 13	J *14*	J_R 15	$J_{G\text{-}7}$ 16	$J_{Other\text{-}7}$ 17
1950	0.2143	0.0146	0.2182	0.0725	0.1699	0.0264	0.1374	0.1848	0.2489	0.0072	0.2579	0.1302	0.0266	0.0003	0.0246	0.0381
1951	0.1975	0.0145	0.2023	0.0498	0.2446	0.0370	0.2152	0.1555	0.1647	0.0081	0.1675	0.0813	0.0348	0.0015	0.0297	0.0576
1952	0.1856	0.0148	0.1888	0.0460	0.2612	0.0435	0.2254	0.1645	0.1669	0.0087	0.1690	0.0829	0.0308	0.0015	0.0269	0.0457
1953	0.1812	0.0154	0.1811	0.0589	0.2700	0.0416	0.2333	0.1945	0.1715	0.0070	0.1736	0.1015	0.0306	0.0013	0.0281	0.0382
1954	0.1633	0.0121	0.1660	0.0479	0.2300	0.0322	0.2009	0.1765	0.1394	0.0026	0.1451	0.0787	0.0325	0.0006	0.0318	0.0329
1955	0.1610	0.0127	0.1628	0.0457	0.2170	0.0294	0.1893	0.1757	0.1497	0.0055	0.1561	0.0606	0.0325	0.0004	0.0329	0.0266
1956	0.1502	0.0118	0.1520	0.0418	0.2107	0.0280	0.1849	0.1670	0.1258	0.0042	0.1298	0.0635	0.0280	0.0004	0.0285	0.0210
1957	0.1387	0.0113	0.1396	0.0407	0.2142	0.0282	0.1900	0.1573	0.0915	0.0028	0.0918	0.0662	0.0271	0.0004	0.0276	0.0208
1958	0.1254	0.0108	0.1260	0.0330	0.2062	0.0288	0.1799	0.1596	0.0887	0.0031	0.0919	0.0407	0.0290	0.0002	0.0305	0.0168
1959	0.1237	0.0122	0.1209	0.0437	0.1954	0.0286	0.1685	0.1551	0.0923	0.0058	0.0874	0.0798	0.0267	0.0002	0.0278	0.0171
1960	0.1095	0.0118	0.1036	0.0553	0.1848	0.0271	0.1568	0.1643	0.0646	0.0040	0.0553	0.0984	0.0292	0.0003	0.0301	0.0203
1961	0.0914	0.0101	0.0867	0.0427	0.1809	0.0278	0.1527	0.1566	0.0349	0.0022	0.0291	0.0593	0.0316	0.0002	0.0332	0.0186
1962	0.0876	0.0095	0.0840	0.0353	0.1752	0.0271	0.1474	0.1529	0.0404	0.0017	0.0387	0.0388	0.0292	0.0001	0.0308	0.0164
1963	0.0799	0.0091	0.0767	0.0280	0.1609	0.0243	0.1352	0.1466	0.0318	0.0022	0.0302	0.0256	0.0284	0.0001	0.0302	0.0148
1964	0.0745	0.0086	0.0705	0.0317	0.1562	0.0240	0.1297	0.1509	0.0233	0.0010	0.0200	0.0389	0.0210	0.0004	0.0215	0.0143
1965	0.0761	0.0082	0.0735	0.0273	0.1496	0.0235	0.1240	0.1414	0.0301	0.0006	0.0301	0.0251	0.0199	0.0004	0.0204	0.0126
1966	0.0722	0.0083	0.0695	0.0228	0.1595	0.0256	0.1328	0.1425	0.0246	0.0004	0.0246	0.0206	0.0174	0.0005	0.0177	0.0114
1967	0.0647	0.0083	0.0608	0.0242	0.1651	0.0265	0.1386	0.1385	0.0124	0.0006	0.0093	0.0301	0.0145	0.0006	0.0145	0.0102
1968	0.0585	0.0086	0.0534	0.0241	0.1609	0.0263	0.1342	0.1373	0.0099	0.0012	0.0060	0.0283	0.0147	0.0007	0.0147	0.0091
1969	0.0511	0.0078	0.0461	0.0229	0.1524	0.0239	0.1274	0.1367	0.0104	0.0011	0.0075	0.0225	0.0143	0.0007	0.0143	0.0083
1970	0.0424	0.0071	0.0367	0.0245	0.1376	0.0209	0.1149	0.1292	0.0144	0.0007	0.0115	0.0303	0.0160	0.0006	0.0164	0.0081
1971	0.0418	0.0068	0.0364	0.0248	0.1247	0.0191	0.1026	0.1281	0.0139	0.0011	0.0096	0.0364	0.0176	0.0004	0.0183	0.0084
1972	0.0395	0.0065	0.0348	0.0201	0.1165	0.0178	0.0952	0.1240	0.0141	0.0014	0.0116	0.0210	0.0153	0.0005	0.0159	0.0073
1973	0.0362	0.0063	0.0315	0.0180	0.1055	0.0166	0.0854	0.1149	0.0106	0.0017	0.0077	0.0179	0.0140	0.0006	0.0142	0.0071
1974	0.0330	0.0048	0.0296	0.0178	0.1007	0.0151	0.0833	0.1028	0.0102	0.0004	0.0092	0.0140	0.0130	0.0006	0.0132	0.0068
1975	0.0310	0.0047	0.0276	0.0173	0.0949	0.0142	0.0783	0.0986	0.0135	0.0000	0.0131	0.0159	0.0143	0.0003	0.0150	0.0065
1976	0.0317	0.0050	0.0275	0.0206	0.0847	0.0121	0.0697	0.0932	0.0104	0.0005	0.0080	0.0235	0.0113	0.0006	0.0110	0.0084
1977	0.0327	0.0057	0.0278	0.0209	0.0784	0.0110	0.0643	0.0904	0.0122	0.0014	0.0087	0.0257	0.0101	0.0007	0.0095	0.0086
1978	0.0340	0.0065	0.0280	0.0233	0.0728	0.0095	0.0597	0.0896	0.0159	0.0034	0.0107	0.0255	0.0087	0.0012	0.0074	0.0083
1979	0.0321	0.0070	0.0249	0.0274	0.0664	0.0082	0.0540	0.0885	0.0173	0.0045	0.0102	0.0325	0.0085	0.0016	0.0066	0.0096

Table 1. Continued

Year	Income Inequality				Government Inequality				Investment Inequality				Industrial Inequality			
	J	J_R	$J_{G\text{-}7}$	$J_{Other\text{-}7}$	J	J_R	$J_{G\text{-}7}$	$J_{Other\text{-}7}$	J	J_R	$J_{G\text{-}7}$	$J_{Other\text{-}7}$	J	J_R	$J_{G\text{-}7}$	$J_{Other\text{-}7}$
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1980	0.0286	0.0065	0.0211	0.0297	0.0623	0.0072	0.0507	0.0867	0.0213	0.0030	0.0159	0.0363	0.0097	0.0019	0.0074	0.0106
1981	0.0298	0.0072	0.0214	0.0308	0.0583	0.0067	0.0463	0.0902	0.0300	0.0066	0.0215	0.0375	0.0105	0.0026	0.0073	0.0119
1982	0.0254	0.0068	0.0169	0.0309	0.0566	0.0063	0.0456	0.0849	0.0216	0.0039	0.0152	0.0359	0.0132	0.0026	0.0103	0.0130
1983	0.0275	0.0075	0.0183	0.0331	0.0537	0.0060	0.0432	0.0809	0.0191	0.0062	0.0091	0.0411	0.0140	0.0029	0.0107	0.0138
1984	0.0319	0.0085	0.0216	0.0369	0.0544	0.0065	0.0441	0.0760	0.0297	0.0093	0.0141	0.0665	0.0139	0.0038	0.0091	0.0164
1985	0.0326	0.0087	0.0219	0.0392	0.0596	0.0070	0.0500	0.0717	0.0276	0.0089	0.0122	0.0667	0.0152	0.0040	0.0100	0.0202
1986	0.0322	0.0088	0.0215	0.0379	0.0574	0.0071	0.0483	0.0657	0.0246	0.0067	0.0117	0.0641	0.0149	0.0037	0.0101	0.0198
1987	0.0313	0.0088	0.0212	0.0331	0.0580	0.0063	0.0508	0.0586	0.0220	0.0069	0.0113	0.0433	0.0135	0.0029	0.0099	0.0157
1988	0.0304	0.0088	0.0203	0.0314	0.0547	0.0061	0.0480	0.0536	0.0203	0.0067	0.0105	0.0369	0.0141	0.0024	0.0114	0.0143

Notes: G-7 (United States, Canada, Japan, U.K., Germany, France, Italy); G-O (Austria, Belgium, Denmark, The Netherlands, Norway, Ireland, and Spain); J is total inequality, J_R is regional inequality, and J_g is within inequality.

Table 2. Initial and Final Investment Per Capita and Growth Rate for 14 OECD Countries, 1950–1988

Country	1950	1988	Growth Rate*
Canada	1,562	4,661	3
United States	1,640	3,513	2
Japan	185	3,878	21
U.K.	516	2,465	4.8
West Germany	752	3,050	4
France	845	3,094	3.7
Italy	569	2,921	5
Austria	544	3,534	6.5
Belgium	812	2,598	3.2
Denmark	1,120	2,473	2.2
Netherlands	1,013	2,275	2.3
Norway	1,394	4,404	3.2
Ireland	488	1,287	2.6
Spain	341	1,921	5.6

* Growth is calculated by dividing the final investment per capita value by the initial value.

Cointegration

Several studies have combined inequality measures with multiple-regression techniques (Amos; Braun; McGillivray; Ram 1989, 1992). This study, however, attempts to establish the co-movement of these inequality indices over time in order to analyze the long-run equilibrium relationships between income inequality and factors that influence growth. To accomplish this, cointegration is used in this analysis to determine the long-run relationships among the four inequality indices for the 14 OECD countries.

Testing for cointegration involves analyzing the residuals from a cointegrating regression for stationarity. If the cointegrating equation is stationary, then the variables are cointegrated (Maddala; Moss). A prerequisite of cointegration analysis is that the variables under consideration are integrated in the same order. A time-series variable is integrated in order d if the dth difference of x_t is stationary and is denoted by $I(d)$. Maddala suggests examining graphs and using unit-root tests to determine if a time series is stationary.

The graphs of the inequality of income, government and investment expenditures, and

Table 3. OECD Unit Root Tests[a]

Tests	Income	Government	Investment	Industry
ADF[a,b]	10.38	8.30	11.21	10.08
Phillips[c]	10.68	8.54	11.53	10.37

Note: Only the second differenced results are reported.
[a] The reported values are for zero lagged difference terms.
[b] The critical values for the Augmented Dicky-Fuller test and Phillips test are 3.56, and 2.94 for the 0.01, and 0.05 confidence levels, respectively.
[c] The reported values are for 1 autocorrelation term.

industrial employment for the OECD countries were examined and are confirmed to be $I(2)$ (not shown). Unit-root tests were then used to determine the order of integration of the time series. The results of the augmented Dickey-Fuller (ADF) and the Phillips tests (Fuller; Evans and Savin; Engle and Granger 1991; Phillips; Maddala) are presented in Table 3 and indicate that the inequalities in income, investment expenditure, industrial employment, and government expenditure with a 99% confidence level are $I(2)$, which supports the interpretation of the graphs.[8] Given that all of the variables are $I(2)$, pairwise-cointegration tests are conducted. Engle and Granger (1987) show that two $I(d)$ variables, x_t and y_t, are cointegrated in order (d, b) if there exists a constant $B \neq 0$ such that $u_t = y_t - \alpha - Bx_t$ is integrated in order $(d - b)$, $b > 0$. If these restrictions are satisfied then x_t and y_t are cointegrated, which is written as $CI(d, b)$. In this example, α is a constant and u_t is the residual vector.

[8] Increasing the number of lags (autocovariance terms for Phillips tests) in the model has no effect on the significance level for the $I(2)$ series.

Durbin Watson and the Augmented Dickey-Fuller (CADF) tests (Maddala) are used on the following sets of inequality measures: income − investment, income − government expenditures, and income − industrial employment. The results from the two pairwise tests are somewhat inconsistent. Table 4 shows that the Durbin Watson statistic is significantly different from zero at the 5% level for the following pairs: income inequality and investment-expenditure inequality, and income inequality and the inequality in the number of people employed in industry. The Durbin Watson statistic is not significantly different from zero for government inequality and income inequality. The Durbin Watson tests indicate that nonpairwise cointegration is rejected for investment and the number of people in industry, but cannot be rejected for government expenditure. The CADF tests tell a different story from that of the Durbin Watson test results. Government expenditures and the number of people employed in industry regressions reject noncointegration at the 1% level. The investment regression does not reject noncointegration.

Conflicting results from different time-series tests are common (Maddala). The distribution of the Durbin Watson has not yet been fully investigated. The general rule is that the smaller the statistic, the greater the chance that the null hypothesis of noncointegration is not rejected. Engle and Yoo (1987, 1991) conclude that the Durbin Watson is not useful for testing cointegration. In general, the test results indicate that income inequality is cointegrated with the inequalities of the other three variables. This suggests that there exists a long-run equilibrium among income inequality

Table 4. OECD Pairwise Tests for Cointegration

Tests	Government	Investment	Industry
Durbin-Watson Regression[a]	0.296	0.429	0.471
Augmented Dickey-Fuller Regression[b,c]	6.07	2.76	4.27

[a] The critical values for the Durbin-Watson regression are 0.511, 0.386, 0.322 for the 0.01, 0.05, and 0.10 confidence levels, respectively (Engle and Granger 1987).
[b] The critical values for the Augmented Dickey-Fuller regression are 4.02, 3.4, and 3.09 for the 0.01, 0.05, and 0.10 confidence levels, respectively.
[c] The reported values are for zero lags.

Table 5. Estimated Eigenvalues and Eigenvectors from Johansen's Multiple Cointegration Test[a]

Eigenvalue (ρ):			
0.688	0.504	0260	0.143
Eigenvectors (B):			
−35.718	−225.140	133.669	215.392
21.627	23.452	−33.151	−111.575
22.905	154.150	−41.214	−107.683
188.173	179.047	−358.899	31.076

and the inequality in government and investment expenditure, and the number of people employed in industry for the 14 OECD countries. However, the results only illustrate that two variables are cointegrated at a time, not all four simultaneously, and the hypothesis of all four variables being cointegrated is what needs to be addressed. As a final step, this study analyzes whether multiple cointegration exists among the four variables.

Multiple Cointegration

The results from the pairwise-cointegration tests are somewhat inconclusive. Those results only suggest that certain pairs of the inequalities appear to be cointegrated. As a final step, this study analyzes whether multiple cointegration exists among the four variables. Due to the complication of having four $I(2)$ variables, Johansen's $I(2)$ procedure was used because of its maximum-likelihood properties (consult Johansen 1992a, 1992b for details on the procedure).

The first step of Johansen's test is to solve the eigenvalue problem. The solution to this problem provides eigenvalues and their associated eigenvectors, which are presented in Table 5. Let p be the number of variables, and r the number of significant eigenvalues. The value of r can be determined by reading from top to bottom of the Q_r column (Table 6, column 4) and comparing the observed values with the 95% critical value (C_{p-r}) for $p - r$ degrees of freedom found in Table 6. Conditional on r, the value of s (the number of common $I(1)$ trends) can be chosen by reading the row associated with the selected r value in the $Q_{r,s}$ rows and comparing the observed values with the critical values at the bottom of the table (C_{p-r-s}). The results from the 14 OECD country analysis are similar to the results from just analyzing the G-7. Both appear to have two cointegrating vectors. This is not surprising considering that the G-7 dominates the 14 OECD countries.

The trace statistic Q_r clearly rejects $r = 0$, since the test statistic is 83.55 and the 95% calculated quantile is 49.09. The hypothesis H_1 of $r \leq 1$ is also rejected with the statistic 41.65 and the quantile 31.62. The hypothesis H_2 of $r \leq 2$ is a borderline case and cannot be rejected since the statistic 16.40 corresponds approximately to the 95% quantile (17.65) in the asymptotic distribution. Based on $r = 2$, the two estimated cointegrating vectors (B) are

Table 6. Johansen Test Statistics

$p - r$	r			$Q_{r,s}$		Q_r	C_{p-r} (95%)
4	0	99.633	46.278	16.756	2.807	83.555	49.097
		$s = 0$	$s = 1$	$s = 2$	$s = 3$		
3	1		73.589	27.506	4.782	41.646	31.618
			$s = 0$	$s = 1$	$s = 2$		
2	2			38.685	3.930	16.398	17.652
				$s = 0$	$s = 1$		
1	3				27.406	5.557	8.106
					$s = 0$		
$p - r - s$	4	3	2	1			
C_{p-r-s}(95%)	49.097	31.618	17.652	8.106			

[a] These results were calculated using a RATS program written and provided by Dr. Johansen. $P = 4$ represents the number of variables, r the number of significant eigenvalues, and s the number of common $I(1)$ trends.

given by the first two columns of the eigenvectors in Table 5.

To determine the value of s, the row equal to $r = 2$ in Table 6 is read. The hypothesis $H_{2,0}$, that $r = 2$ and $s = 0$, is rejected based on the test statistic 38.69 and the quantile 17.65. The next test $H_{2,1}$, that $r = 2$ and $s \leq 1$, cannot be rejected. This is determined by comparing $Q_{2,1} = 3.9$ with the quantile of 8.11. Therefore, the number of common $I(2)$ trends in the data series is $p - r - s = 1$, and the number of common $I(1)$ trends is $s = 1$.

There is one common $I(2)$ trend that drives all of the variables. The vector $B'X_t$ in this case is just one linear combination, and it is $I(1)$ (not stationary). However, this representation is made stationary by including the differences, that is $B'X_t + \kappa B_{\perp}^{2\prime}\Delta X_t$ (Johansen 1991c, 1991d), where the κ coefficient is equal to $(\alpha'\alpha)^{-1}\alpha'\Gamma B_{\perp}^{2}(B_{\perp}^{2\prime}B_{\perp}^{2})^{-1} = \alpha_{\perp}^{1\prime}\Gamma B_{\perp}^{2}(B_{\perp}^{2\prime}B_{\perp}^{2})^{-1}$, $\alpha = \alpha(\alpha'\alpha)^{-1}$, and $B_{\perp}^{2} = B_{\perp}\eta_{\perp}$. The two normalized stationary relationships are:

$$(1)\quad \text{INC} - .61\text{GOV}_t - .64\text{INV}_t - 5.27\text{IND}_t + 128.4\Delta\text{INC} + 48.83\Delta\text{GOV}_t + 184.39\Delta\text{INV}_t - 3.68\Delta\text{IND}_t$$

and

$$(2)\quad \text{INC} - .1\text{GOV}_t - .69\text{INV}_t - .80\text{IND}_t - 135.12\Delta\text{INC} - 51.4\Delta\text{GOV}_t - 194.1\Delta\text{INV}_t + 3.88\Delta\text{IND}_t$$

The B vectors used to determine Equations (1) and (2) are the first two columns of the eigenvector in Table 5, with the exception of being normalized by the income coefficient. These two equations represent the long-run equilibrium among the four inequality indices for the 14 OECD countries. Given that there are two stationary relationships, the equilibrium can be thought of as a plane instead of a line in hyperspace.[9]

[9] Hyperspace in this case refers to a four-dimensional space with two stationary relationships forming an equilibrium within this space. Since there are two relationships, the equilibrium is a plane.

Table 7. Cointegrating Adjustment Coefficients from the G-7

$B_{\perp}^{2}$	$\alpha_{\perp}^{2}$
11.397	0.006
4.334	0.010
16.366	−0.006
−0.327	0.056

The next step is to use all the information from this estimation to determine which variable or variables are determining this equilibrium. Both eigenvectors appear to have strong relationships. The first eigenvector may have the following cointegrating relationship (1, −1, −1, *) and the second cointegrating relationship may be (1, *, −1, −1). The first vector indicates that income inequality, government–expenditure inequality, and investment–expenditure inequality form a stationary equilibrium. The second-vector case is where income inequality, investment–expenditure inequality, and the inequality in the number of people employed by industry form a stationary equilibrium. The interesting development here is that government expenditure plays a more important role in the 14 OECD countries than in the G-7. The inequality in government expenditure has no effect on the G-7 equilibrium when analyzed without the Other 7.

The coefficient $B_{\perp}^{2}$ in Table 7 shows which variables are actually $I(2)$. The variable that has a coefficient closest to one or negative one is the common $I(2)$ trend. $\alpha_{\perp}^{2}$ represents the average speed of adjustment toward the estimated equilibrium and is interpreted as the linear combination that describes the common $I(2)$ trend. A small coefficient indicates a slow adjustment and a large coefficient represents a fast adjustment.

The $B_{\perp}^{2}$ vector indicates that the inequality in the number of people employed in industry is the $I(2)$ variable. Hence, the $I(2)$-ness of the model is ascribed to the inequality of industrial employment. This means that when an innovation occurs causing the inequalities to be out of equilibrium, the inequality in industrial employment adjusts the most and the quickest to restore the equilibrium. This is the same for the G-7 case when it is run separately.

The result that industrial employment is the main force is confirmed by α_1^2 in Table 7. As in the G-7 case, the heaviest weight is given to the inequality in industrial employment. Therefore, the stabilizing force in this model is the inequality in industrial employment. However, the second largest value for α_1^2 is the inequality of government expenditure. Hence, when the economy is out of equilibrium, the inequality of government expenditure helps to return the economy to the long-run equilibrium. This result is contrary to the G-7 results where government expenditure is not important to the equilibrium process.

In summary, the stationary equilibrium is dependent on two stationary relationships. The first stationary relationship for the OECD is described as the inequality in income, government expenditure, and investment expenditure. The second relationship is the inequality in income inequality, investment-expenditure inequality, and the inequality in the number of people employed in industry. These two stationary relationships form a long-run equilibrium and can be described as a plane in four-dimensional space, which acts as an attractor every time the four inequalities deviate from this equilibrium. It is also determined that the inequality in government expenditure has no effect on the G-7 equilibrium but it does for the 14 OECD countries when analyzed as a whole. The inequality in industrial employment is determined to be the common $I(2)$ trend. Whenever an innovation occurs in one of the inequalities and a deviation from the long-run equilibrium exists, industrial employment adjusts first to return the economy to the long-run equilibrium followed by a slower adjustment in government expenditures.

Summary and Conclusion

Theil's inequality index lends itself to further analysis using regression or cointegration models. The fact that it is symmetrical and additive assists in determining driving forces of convergence or nonconvergence. In this study, 14 OECD countries are shown to have converged in terms of per capita income, per capita government expenditures, per capita investment expenditures, and industrial employment over the time period 1950–1988. Theil's index reveals that Japan's rapid growth and the slow growth of the United States and Canada strongly influences convergence in the G-7. Since the G-7 has a heavier weight in the model due to population, it strongly influences the overall convergence of the 14 OECD countries. Theil's index is then extended to address the issue of long-run relationships among the various inequality indices.

The four total inequality indices are tested for long-run relationships using cointegration analysis. The pairwise-cointegration tests, although inconclusive, suggest that income inequality is cointegrated with the other three inequalities. However, our major goal is to determine if all four inequality indices are cointegrated, hence, Johansen's multiple cointegration test is used. Results from Johansen's multiple cointegration $I(2)$ test supports the hypothesis that a long-run equilibrium exists among the inequalities in income, investment expenditure, and industrial employment for the 14 OECD countries. Industrial employment appears to be the driving force in returning these economies to their long-run equilibrium when some external factor shocks the economies out of equilibrium. Government expenditure is shown to also adjust at a slower rate to help return the economies to equilibrium. Interestingly, when the G-7 model is run independent of the Other 7, government expenditure has no role in returning the economy to the equilibrium. The G-7 model emphasizes the key factors for economic convergence to be investment expenditure and industrial employment. Government expenditure does not appear to influence convergence or growth in the G-7 countries.

The implications of these results are important when considering the economic growth in middle- to high-income countries. Countries can mimic either the G-7 model, which relies on investment and industrial employment for growth and convergence, or it can follow the 14 OECD countries' approach, which adds government expenditures to the list. This study may assist policy makers in reevaluating the reliance on public expenditures to improve their economic growth. These interesting issues should be pursued as data become available, or when econometric methods are better able to handle small samples.

Appendix. Theil's Inequality Index

Theil's income inequality index (Theil 1979, 1989) when applied to n countries can be written as

$$(\text{A.1}) \quad J = \sum p_i \log \frac{p_i}{y_i}$$

where p_i is the population share of country i relative to total population, and y_i is its income share relative to total income.[10]

This measure can be decomposed additively to measure inter- and within inequality. Let $R_1, \ldots, R_g$ represent regions such that each country is in only one region. Let P_g and Y_g be the population and income shares of region R_g, $P_g = \Sigma_i p_i$, and $Y_g = \Sigma_i y_i$ where the summations are over $i \in R_g$ ($g = 1, \ldots, G$). The extension of (A.1) to regions is:

$$(\text{A.2}) \quad J_R = \sum_{g=1}^{G} P_g \log \frac{P_g}{Y_g}$$

which measures inter-regional inequality among G regions, while

$$(\text{A.3}) \quad J_g = \sum_{i \in R_g} \left(\frac{p_i}{P_g}\right) \log \left[\left(\frac{p_i}{P_g}\right) \Big/ \left(\frac{y_i}{Y_g}\right)\right]$$

measures the inequality among the countries of region R_g. It is then easily verified that

$$(\text{A.4}) \quad \text{J} = \text{J}_\text{R} + \text{J}^* \text{ where } J^* = \sum_{g=1}^{G} P_g Y_g$$

which is an additive decomposition expressing total inequality J among the n countries as the sum of regional inequality J_R, and the average within-region inequality J^*. This average is a weighted average with weights proportional to the populations.

[10] All logarithms in this paper are natural logarithms.

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