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Estimation of General and Commodity-Specific Inflation Rates Using Linear Time-Varying Constraints

D.S. Prasada Rao, H.E. Doran, and E.A. Selvanathan

In this paper, we consider the problem of estimating general and commodity-specific inflation rates by the stochastic approach considered in Clements and Izan (1987) and Selvanathan (1989). In order to achieve identification of commodity-specific rates, a linear constraint usually is imposed, and to make it operational, the constraint is generally imposed at an average over the time periods in the series. This paper uses recently developed methodology for estimation of econometric models with time-varying constraints (O'Donnel, Rambaldi, and Doran) to relax the constraint imposed at average shares and to derive commodity-specific inflation rates.

Key Words: generalized inverses, index number, inflation, stochastic approach, time-varying constraints

In this paper, we investigate the estimation of general inflation rates as well as commodityspecific growth rates in prices using the "new stochastic approach" discussed in Clements and Izan (1981, 1987), Selvanathan (1989), and Selvanathan and Prasada Rao. The approach essentially derives estimates of overall inflation rates as weighted averages of itemwise price changes modeled using a regression framework. Since the initial exposition in Clements and Izan (1981), several papers have extended the basic model. Selvanathan (1991, 1993) examined an application leading to the standard errors of widely used Laspeyres and Paasche index numbers. Selvanathan and Prasada Rao extended the standard model, resulting in generalized Tornqvist index numbers for multilateral comparisons.

Clements and Izan (1987) modified the ba-

sic model in Clements and Izan (1981) to allow for systematic changes in relative prices. In order to facilitate identification of the parameters in the model, the commodity-specific price changes were constrained to satisfy a linear restriction that the budget share weighted sum of the relative price changes is equal to zero. Selvanathan (1989) also focused on the same model. Because budget shares of different commodities vary over time, the approach followed in these papers was one of imposing the identifying restriction at budget shares that are averages over time. In order to accommodate the restriction at average shares, the specification of the model had to be changed. A major implication of this approach is that the estimates of the general inflation rates for each time period are no longer equal to the Tornqvist indices in which the original stochastic model was supposed to result.

Some of these issues and some other aspects of the stochastic approach were raised by Diewert. In a recent paper, Crompton addressed questions relating to the variance

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specifications of the stochastic model, but the issue of using average budget shares still remains as a major source of inconsistency of the twin aims of deriving Tornqvist indices using the stochastic approach and of identifying commodity-specific inflation rates.

Our main objective in this paper is to relax the identifying restriction at the average budget shares and propose a model that allows the relative price changes for different commodities to vary over time, satisfying a different identifying restriction for each period. The paper is organized as follows. First, we describe the specification and estimation of the new model that allows for time-varying constraints on commodity-specific inflation rates, together with the general rate of inflation. Later, we provide empirical results derived using Australian data and contrast the results of the estimation obtained under the conventional approach.

The Measurement of Inflation and Relative Prices Using Time-Varying Constraints

The stochastic approach to index numbers considers the price change of a commodity to have three components: (1) the overall rate of inflation or the common trend in all prices, (2) the systematic change in its relative price, and (3) a random component.

In contrast to the models used in Clements and Izan (1987) and Selvanathan (1989), a more general model, allowing for a time-varying β , is specified in this paper. The model is given by

(1)
$$Dp_{ii} = \alpha_t + \beta_{ii} + \varepsilon_{ii}$$
$$i = 1, 2, \ldots, n, \qquad t = 1, 2, \ldots, T$$

where

 $Dp_{it} = \log(p_{it}/p_{it-1}) = \text{price log change over}$ periods t and t - 1 of commodity i;

 α_t = the overall rate of inflation or the common trend in all prices in period t;

 β_{it} = systematic change in the relative price of *i* in period *t*; and

 ε_{it} = a zero mean random term.

The random terms, ε_{ii} , are assumed to be independent over commodities and time,

(2)
$$cov(\varepsilon_{it}, \varepsilon_{is}) = 0$$
 $i \neq j, t \neq s$,

with the variance structure

$$V(\varepsilon_{it}) = \xi_t^2.$$

In order to make the model identifiable, the following constraints are imposed. For each t,

(3)
$$\sum_{i=1}^{n} \bar{w}_{it} \beta_{it} = 0 \qquad t = 1, 2, 3, \ldots, T,$$

where the \bar{w}_{it} are averages of exogenous, known commodity consumption shares. In effect, we want to decompose the observed commodity inflation rate, Dp_{it} (i = 1, 2, ..., n; t = 1, 2, ..., T), into the general inflation rate, α_{it} , and a commodity-specific rate, β_{it} , such that all β_{it} satisfy the identifying restriction in Equation (3).

The model can be written in the form of a standard linear regression model as in Equation (4),

$$(4) y_{ii} = \alpha_i x_{ii} + \beta_{ii} x_{ii} + \varepsilon_{ii},$$

where $y_{ii} = Dp_{ii}$ and $x_{ii} = 1$.

For each t, Equation (4) can be written as

$$\begin{pmatrix} \mathbf{y}_{1t} \\ \mathbf{y}_{2t} \\ \vdots \\ \mathbf{y}_{nt} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & \mathbf{x}_{1t} & 0 & \cdots & 0 \\ 0 & 0 & 0 & \mathbf{x}_{2t} & \vdots & & \vdots \\ \vdots & \vdots & \vdots & \vdots & 0 & \cdots & 0 \\ 0 & 0 & 0 & \mathbf{x}_{nt} & 0 & \cdots & 0 \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_T \end{pmatrix}$$

$$+ \begin{pmatrix} \mathbf{x}_{1t} & 0 & \cdots & 0 \\ 0 & \mathbf{x}_{2t} & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & \mathbf{x}_{nt} \end{pmatrix} \begin{bmatrix} \beta_{1t} \\ \beta_{2t} \\ \vdots \\ \beta_{nt} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \vdots \\ \varepsilon_{nt} \end{bmatrix}$$

$$\mathbf{y}_t = \mathbf{Z}_{1t} \boldsymbol{\alpha} + \mathbf{Z}_{2t} \boldsymbol{\beta}_t + \mathbf{E}_t$$

$$= [\mathbf{Z}_{1t} & \mathbf{Z}_{2t}] \begin{bmatrix} \boldsymbol{\alpha} \\ \boldsymbol{\beta}_t \end{bmatrix} + \mathbf{E}_t.$$

Therefore,

$$\mathbf{y}_{t} = \mathbf{Z}_{t} \mathbf{\delta}_{t} + \mathbf{E}_{t}$$

where

$$\mathbf{Z}_{t} = [\mathbf{Z}_{1t} \quad \mathbf{Z}_{2t}]_{n \times (T+n)} \quad \text{and} \quad \boldsymbol{\delta}_{t} = \begin{bmatrix} \boldsymbol{\alpha} \\ \boldsymbol{\beta}_{t} \end{bmatrix}_{(T+n) \times 1}. \quad (9) \quad M = \sum_{t=1}^{T} (\mathbf{y}_{t}^{*} - \mathbf{Z}_{t}^{*}\boldsymbol{\gamma})'(\mathbf{y}_{t}^{*} - \mathbf{Z}_{t}^{*}\boldsymbol{\gamma})$$

We need to estimate the model in Equation (5) using T time-series observations imposing the constraint in Equation (3) for $t = 1, 2, 3, \ldots$, T, which can be written in the form

(6)
$$\mathbf{R}_{i}\delta_{i}=0$$
,

where
$$\mathbf{R}_{t} = [0 \dots 0 | \vec{w}_{1t} \ \vec{w}_{2t} \dots \vec{w}_{nt}].$$

In contrast to the standard approach used in the stochastic approach to index numbers, where disturbances are assumed to be heteroscedastic with variances inversely related to the expenditure shares, the approach we use in this paper estimates α_i and β_{ii} by minimizing the criterion function,

(7)
$$M = \sum_{t=1}^{T} (\mathbf{y}_t - \mathbf{Z}_t \mathbf{\delta}_t)' \mathbf{W}_t (\mathbf{y}_t - \mathbf{Z}_t \mathbf{\delta}_t),$$

subject to the time-varying restriction

(8)
$$\mathbf{R}_{i}\mathbf{\delta}_{i}=0.$$

In the above criterion function, $\mathbf{W}_t = \operatorname{diag}(\bar{w}_{1t})$ $\bar{w}_{2r}, \ldots, \bar{w}_{nt}$). Thus, in minimizing the sums of squares, importance is directly related to the budget shares of different commodities. For more details on the use of the criterion function in the derivation of Torngvist index numbers (Kloek and Theil; Theil), see Prasada Rao and Doran.

The general solution of Equation (7) subject to Equation (8) is of the form

$$\delta_t = (\mathbf{I}_{T+n} - \mathbf{R}_t^+ \mathbf{R}_t) \gamma_{tt}$$

where \mathbf{R}_{t}^{+} is the Moore Penrose generalized inverse of \mathbf{R}_n and $\mathbf{\gamma}_t$ is an arbitrary vector.

We now make the identifying assumption $\gamma_t = \gamma$, a constant vector. Then we minimize

(9)
$$M = \sum_{i=1}^{T} (\mathbf{y}_{i}^{*} - \mathbf{Z}_{i}^{*} \mathbf{\gamma})' (\mathbf{y}_{i}^{*} - \mathbf{Z}_{i}^{*} \mathbf{\gamma})$$

with respect to y, where

$$\mathbf{y}_{t}^{*} = \mathbf{W}_{t}^{1/2} \mathbf{y}_{t};$$

$$\mathbf{Z}_{t}^{*} = \mathbf{W}_{t}^{1/2} \mathbf{Z}_{t} (\mathbf{I}_{(T+n)} - \mathbf{R}_{t}^{+} \mathbf{R}_{t}).$$

An estimate of $\hat{\gamma}$ is obtained by minimizing Equation (9), that is,

(10)
$$\hat{\mathbf{\gamma}} = (\mathbf{Z}^* \mathbf{Z}^*)^{-1} \mathbf{Z}^* \mathbf{y}^*,$$

where $Z^* = [Z_1^* \ Z_2^* \ \dots \ Z_T^*]'$ and $y^* =$ $y_1^* y_2^* \dots y_T^*$]. Then the time-varying vector of parameters satisfying the restrictions given in Equation (3) is estimated using

(11)
$$\hat{\mathbf{\delta}}_t = (\mathbf{I}_{T+n} - \mathbf{R}_t^+ \mathbf{R}_t) \hat{\mathbf{\gamma}}.$$

The estimator of δ , in Equation (11) has many desirable properties. First, it satisfies the timevarying constraints stipulated in Equation (3). Second, the estimator in Equation (11) collapses to the restricted least squares estimator if the constraints present are invariant over time. Finally, the performance of the estimator, δ_{r} is closely related to the performance of the least squares estimator in the general regression model. Assumption of constancy of γ and the implied explanatory power can be assessed using the usual R^2 measure, and the estimator $\hat{\gamma}$ is easy to compute. Proofs of these properties can be found in O'Donnell, Rambaldi, and Doran.

Structure of the Estimator δ_t

In this section, we derive the exact form for the estimator of the parameters that satisfy the time-varying parameter restrictions. The estimator, $\hat{\delta}_n$, in Equation (11) is examined using the specific structure of the matrices Z, and the vector **R**, defining the time-varying restrictions. Based on Equations (11) and (10), the structure depends entirely on the form of the Moore Penrose inverse \mathbf{R}_{t}^{+} involved.

Because \mathbf{R}_t is of the form $[\mathbf{0}_{1\times T} \ \mathbf{w}'_{t(1\times n)}]$, it can be seen that \mathbf{R}_{+}^{+} is given by

$$\mathbf{R}_{t}^{+} = \begin{bmatrix} \mathbf{0}_{T \times 1} \\ \frac{\bar{w}_{t}}{\bar{w}_{t}' \bar{w}_{t}} \end{bmatrix}.$$

Now,

$$(\mathbf{I} - \mathbf{R}_t^+ \mathbf{R}_t) = \begin{bmatrix} \mathbf{I}_t & 0 \\ 0 & \mathbf{P}_t \end{bmatrix}$$

where

$$\mathbf{P}_t = \mathbf{I}_n - \frac{\bar{w}_t \bar{w}_t'}{\bar{w}_t' \bar{w}_t}$$

is the projection matrix with the property

$$\mathbf{P}_{t}\bar{w}_{t}=0.$$

In order to derive the expression for $\hat{\gamma}$, we first partition matrix \mathbf{Z}_t as

$$\mathbf{Z}_t = [\mathbf{Z}_{1t} \ \mathbf{Z}_{2t}].$$

Then, we have

$$\mathbf{Z}_{t}^{*} = \mathbf{W}_{t}^{1/2} \mathbf{Z}_{t} (\mathbf{I} - \mathbf{R}_{t}^{*} \mathbf{R}_{t})$$

$$= \mathbf{W}_{t}^{1/2} [\mathbf{Z}_{1t} \mathbf{Z}_{2t}] \begin{pmatrix} \mathbf{I}_{T} & 0\\ 0 & \mathbf{P}_{t} \end{pmatrix}$$

$$= [\mathbf{W}_{t}^{1/2} \mathbf{Z}_{1t} \mathbf{W}_{t}^{1/2} \mathbf{Z}_{2t} \mathbf{P}_{t}]$$

and

$$\hat{\mathbf{y}} = (\mathbf{Z}^*' \ \mathbf{Z}^*)^{-1} \mathbf{Z}^*' \mathbf{y}^*$$

where $\mathbf{Z}^* = [\mathbf{Z}_1^* \ \mathbf{Z}_2^* \ \dots \ \mathbf{Z}_T^*]$. Given these expressions, it is easy to show that

$$\mathbf{Z}^{*'}\mathbf{Z}^{*}$$

$$= \begin{bmatrix} \sum_{t} (\mathbf{Z}'_{1t}\mathbf{W}_{t}\mathbf{Z}_{1t}) & \sum_{t} (\mathbf{Z}'_{1t}\mathbf{W}_{t}\mathbf{Z}_{2t}\mathbf{P}_{t}) \\ \sum_{t} (\mathbf{P}_{t}\mathbf{Z}'_{2t}\mathbf{W}_{t}\mathbf{Z}_{1t}) & \sum_{t} (\mathbf{P}_{t}\mathbf{Z}'_{2t}\mathbf{W}_{t}\mathbf{Z}_{2t}\mathbf{P}_{t}) \end{bmatrix}.$$

We can use the structure of \mathbf{Z}_{1t} and \mathbf{Z}_{2t} in Equation (5) and show that

$$\mathbf{Z}^{*'}\mathbf{Z}^{*} = \begin{bmatrix} \mathbf{I}_{T} & 0 \\ 0 & \sum_{t} \mathbf{P}_{t}\mathbf{Z}_{2t}'\mathbf{W}_{t}\mathbf{Z}_{2t}\mathbf{P}_{t} \end{bmatrix}$$
$$= \begin{bmatrix} \mathbf{I}_{T} & 0 \\ 0 & \sum_{t} \mathbf{P}_{t}\mathbf{W}_{t}\mathbf{P}_{t} \end{bmatrix}.$$

Based on the expressions derived and substituting for y_i , and Z_i , from Equation (5), it can be seen that

$$\hat{\mathbf{\gamma}} = \begin{bmatrix} \sum_{t} \mathbf{Z}_{1t}^{t} \mathbf{W}_{t} \mathbf{y}_{t} \\ \left(\sum_{t} \mathbf{P}_{t} \mathbf{W}_{t} \mathbf{P}_{t}\right)^{-1} \sum_{t} \mathbf{P}_{t} \mathbf{Z}_{2t}^{\prime} \mathbf{W}_{t} \mathbf{y}_{t} \end{bmatrix}$$

$$= \begin{bmatrix} \sum_{t} w_{t} D p_{t} \\ \left(\sum_{t} \mathbf{P}_{t} \mathbf{W}_{t} \mathbf{P}_{t}\right)^{-1} \sum_{t} \mathbf{P}_{t} \mathbf{Z}_{2t}^{\prime} \mathbf{W}_{t} D p_{t} \end{bmatrix}.$$

From this expression, we have the estimator for the parameters α and β_{i} , which are respectively given by

(12)
$$\hat{\boldsymbol{\alpha}} = \sum_{t} \bar{\mathbf{w}}_{t} D p_{t}$$

and

(13)
$$\hat{\boldsymbol{\beta}}_t = \mathbf{P}_t \left(\sum_t \mathbf{P}_t \mathbf{W}_t \mathbf{P}_t \right)^{-1} \left(\sum_t \mathbf{P}_t \mathbf{W}_t D p_t \right).$$

Equation (12) shows that the estimator of α does not vary with the time-varying restrictions and is equal to the Tornqvist index, which is commonly used in index number literature. The time-varying commodity effects are estimated using Equation (13).

Empirical Results

Now we present an application of the model in Equation (1) to estimate annual inflation rates and commodity-specific relative price changes incorporating the time-varying restriction of Equation (3). We use Australian private final consumption expenditure data (at current and constant prices) for n = 7 commodity groups; namely, food (food, beverages, and tobacco), clothing (clothing and foot-

wear), housing (gross rent, fuel, and power), furniture (furniture, furnishings, and household equipment and operation), health (medical care and health expenses), transport (transport and communication), and miscellaneous for the period 1976–1995.

The empirical estimation of inflation and commodity-specific effects imposing timevarying restrictions is undertaken with a multistage procedure. In the first stage, we estimate the parameters with Equation (11),

$$\hat{\mathbf{\delta}}_t = (\mathbf{I}_{T+n} - \mathbf{R}_t^* \mathbf{R}_t) \hat{\mathbf{\gamma}},$$

where $\hat{\gamma}$ is estimated using Equation (10).

In the second stage, we account for possible heteroscedasticity using the residuals derived from the model in Equation (4). The residuals derived were used in estimating variances ξ_t^2 with Equation (10).

$$\hat{\xi}_{t}^{2} = \frac{1}{n-1} \sum_{i=1}^{n} \bar{w}_{it} (Dp_{it} - \hat{\alpha}_{t} - \hat{\beta}_{it})^{2}.$$

The resulting estimates, $\hat{\xi}_{i}^{2}$, were used in transforming Equation (4), leading to

$$\frac{y_{it}}{\hat{\xi}_t} = \alpha_t \frac{x_{it}}{\hat{\xi}_t} + \beta_{it} \frac{x_{it}}{\hat{\xi}_t} + v_{it}^*.$$

This model is re-estimated to obtain estimates in Equations (12) and (13).

The following discussion highlights the salient features of the results derived. Only results from the second stage are discussed here.

- (1) The reduced model in Equation (10) resulting from reparameterization fits the Australian price data quite well with an R^2 of .94. This indicates the adequacy of the constancy of the $\hat{\gamma}$ assumption used in the study.
- (2) The estimated relative price movements, measured by $\hat{\beta}_{ir}$, satisfy the time-varying constraint restriction,

$$\sum_{i=1}^n \bar{w}_{ii}\hat{\beta}_{ii} = 0,$$

for each of the time periods.

Table 1. Estimates of Inflation Based on Time-Varying Constraint Model

	Estimate of	
	Inflation	Standard Error
Year	$(\hat{\boldsymbol{\alpha}}_i \times 100)$	(×100)
1977	9.24	.43
1978	8.69	.71
1979	10.02	.49
1980	9.44	.52
1981	9.11	.57
1982	10.21	.46
1983	7.20	.30
1984	4.52	1.07
1985	7.70	.26
1986	8.23	.45
1987	6.91	.52
1988	7,29	.40
1989	6.27	.34
1990	5.71	.58
1991	2.43	.40
1992	1.65	.38
1993	1.63	.37
1994	1.75	.37
1995	2.56	.41

- (3) The inflation parameter vector, $\alpha = [\alpha_1, \alpha_2, \dots, \alpha_T]'$, is not subject to any restrictions.
- (4) Estimates of α were to be the same in all estimates, δ_n , and were invariant over time

Estimates of α and the associated standard errors are presented below. Another feature of the inflation rates below is that they correspond to estimates resulting from the application of the Tornqvist index. Simple algebra can be used to show that $\hat{\alpha}_i$ is equal to the Tornqvist index.

The estimates of α_t and β_{it} are calculated by Equations (12) and (13). We present the estimates for all α_t and their standard errors in Table 1 and the β_{it} in Figures 1 to 7. As can be seen from Table 1, for example, the rate of inflation for 1995 has been estimated to be 2.5% with a standard error of 0.4%.

Because commodity-specific effects, as measured by beta coefficients, were modeled to vary over time and were restricted to satisfy Equation (3) for each period, we have 19 es-

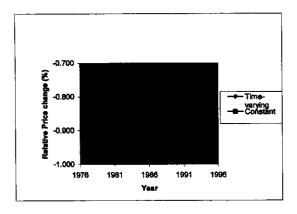


Figure 1. Estimates for Relative Price Changes for Clothing

timates for each coefficient. These estimates are presented in Figures 1 to 7. Each graph corresponds to a particular commodity with the horizontal line representing relative price changes for different commodities estimated under the assumption of time-invariant β_i , and the \bar{w}_{ii} are replaced by the average budget share, \bar{w}_{ii} averaged over time. A number of interesting features can be observed from the profiles of the time-dependent relative price changes.

The first major feature is that the profiles of each time-dependent coefficient deviates significantly from its respective time-invariant relative price change. These profiles indicate a considerable loss of information regarding relative price changes.

The second significant feature is that the profiles of the relative price changes are quite

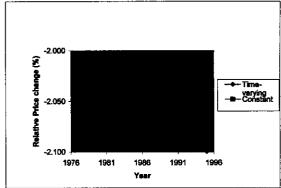


Figure 3. Estimates for Relative Price Changes for Furniture

different for different commodities. For commodities like clothing, food, and furniture (Figures 1 to 3), relative price changes exhibit a secular decline over time. In contrast, housing relative prices (Figure 4) have an inverted U shape showing an increase in relative price changes up to 1986 and then a sharp decline until 1995. The relative price changes for health (Figure 5) provide an altogether different picture. Although the general trend appears to move upward, these movements appear to be cyclical. A similar picture in the opposition direction, a general downward trend with short-term cyclical movements, is seen in the case for transport (Figure 6). For the miscellaneous group (Figure 7), the relative price changes generally are moving upward. In all these cases, results where a single constraint (constant over all the time periods)

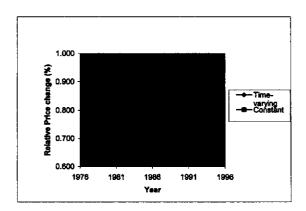


Figure 2. Estimates for Relative Price Changes for Food

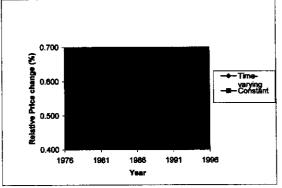


Figure 4. Estimates for Relative Price Changes for Housing

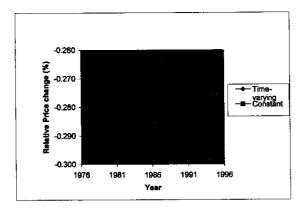


Figure 5. Estimates for Relative Price Changes for Health

is imposed in order to achieve identifiability provides a fairly different picture and, often, masks temporal movements in relative prices of different commodity groups. The overall conclusion to be drawn is that useful information emerges from the model that allows for time-varying parameters satisfying time-varying constraints.

Concluding Remarks

The main purpose of the paper is to demonstrate the feasibility of extending the basic stochastic model underlying the estimation of general and commodity-specific inflation rates to include time-varying inflation rates. This was achieved through a more general specification that allows the parameters of the stochastic model to vary over time and to then

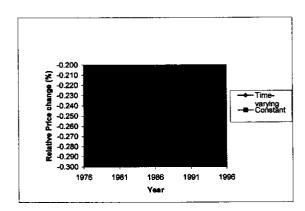


Figure 6. Estimates for Relative Price Changes for Transport

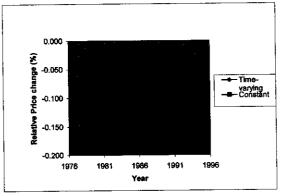


Figure 7. Estimates for Relative Price Changes for Miscellaneous

impose the identifying restrictions. Empirical results from the extended model provide extremely useful insights into the time-varying nature of inflation rates associated with different commodity groups. In summary, the paper provides a variation of the stochastic approach that allows the possibility of deriving Tornqvist indexes for the overall rate of inflation and at the same time of providing estimates of relative price changes for different commodities for each time period.

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