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A Watershed-Based Economic Model of Alternative Management Practices in Southern Agricultural Systems

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We investigated the environmental impacts of alternative cultural practices within a watershed under different water quality standards. We used experimental data on nutrient runoff to determine the optimal amount of broiler litter application in hay production in Louisiana. To compensate for the lack of experimental data, we used biophysical simulation models to find the optimal combination of agricultural best-management practices in a watershed in Mississippi. The results indicated that stricter environmental standards lower total profit potential and litter utilization.

Key Words: broiler litter, cropping systems, optimization, watershed level modeling, water quality

JEL Classifications: C6, Q1, Q25, D21

There are increased concerns about the negative impact of row-crop production and broiler litter on water quality in the southeastern United States. Row-crop agriculture and the broiler industry contribute significantly to agricultural revenue in most of the states in the region. The dominant use of broiler litter in the region has been its application as a source of crop nutri-

ents. Runoff from cropland on which broiler litter has been applied can impair water quality. Given this possibility, an optimal combination of broiler litter, crop choices, crop acreage, and best-management practices (BMPs) can help minimize nutrient and sediment runoff and improve water quality. We examine the economic and environmental impacts of optimal combinations of crops, broiler litter application, and optimal BMPs under a set of alternate water quality standards.

To accomplish the objective of finding an optimal combination of crops and broiler litter application rates in a given watershed, we used both physical data available from experimental plots and biophysical simulation. We examined the potential effects of BMPs on both water quality and on profitability, which can be used to establish policies that promote BMP adoption. Although producers with a sense of environmental stewardship will adopt

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certain levels of BMPs, acceptance and optimal implementation of BMPs will ultimately depend on the effect of alternative cultural and structural practices on farm profitability. Weather conditions and other sources of uncertainty can make onsite experimentation using BMPs costly for individual farmers. Furthermore, a full assessment of the effects on environmental quality and profitability in terms of input costs and yields requires knowledge of how systems will perform in the long run.

To evaluate the full scope of the economic effects of BMPs, impacts on agriculture at both the farm and watershed level must be addressed. At the farm level, BMPs allow farmers to reduce soil loss with accompanying nutrient and chemical losses, possibly providing benefits to farmers in the form of increased soil productivity. Farmers may perceive, however, that cultural and structural practices that prevent soil erosion also result in reduced crop yields and/or increased costs. Thus, benefits from avoided soil loss might be countered by potential profit reductions, rendering BMPs unattractive to individual farmers.

At the watershed level, societal benefits may accrue from reductions in runoff that can degrade offsite water quality and ecosystems. Furthermore, it must be recognized that BMP implementation in a watershed will require a cooperative effort among farmers, because the method will be effective only if the majority of area in the watershed stays under BMPs. As with any economic activity that requires a coordinated effort to be successful, proper incentives for participation—such as maintenance of profit—must be considered.

Watershed- and farm-level economic effects must be evaluated to understand the magnitude of gains and losses to individual farmers through use of BMPs. In the present article, we develop a bioeconomic model to demonstrate novel ways in which farmers can use crop-management practices to optimize profits as well as contribute to improvements in environmental quality. Because actual experience with BMP implementation will be correlated with exogenous factors, such as weather, several years of experience would be

needed to demonstrate the expected outcome on farm profits and environmental quality. By using simulation models, we are able to generate a number of expected economic and environmental outcomes under various assumptions about BMP implementation.

Background

We used two watersheds in describing a typical watershed in southern agricultural systems comprising of crop/broiler mixes. These two watersheds are representative of crop/broiler producing watersheds in Louisiana and Mississippi. In Louisiana, we chose the Bayou D'Arbonne watershed. The watershed is approximately 1,480 square miles, with the majority of the land in use for agriculture and forestry. The Bayou D'Arbonne watershed is composed of portions of three parishes: Union, Claiborne, and Lincoln. These parishes have the highest numbers of broilers in Louisiana. Broiler litter use in agriculture is considered an important source of nonpoint pollution in the Bayou D'Arbonne watershed. In 1993, a Louisiana nonpoint source pollution assessment report identified broiler production facilities and on-site waste water systems as suspected sources of water quality impairment in the watershed.

The Deep Hollow Lake watershed, located in LeFlore County, Mississippi, is the second watershed chosen. The watershed is approximately 400 acres, surrounding a 20-acre lake, with nearly 250 acres under row-crop cultivation. Both structural and cultural BMPs, as well as conventional practices, have been used in the farming system, and the primary crops in this watershed are cotton and soybeans. It should be noted that the entire watershed is under the management of only one producer, so the individual farm model coincides with the watershed model.

Our model is based on experimental data. However, when experimental data were not available, we used a bioeconomic small watershed model to calculate the effects of alternative management systems. The model merges physical and biological data to analyze various management decisions and to simul-

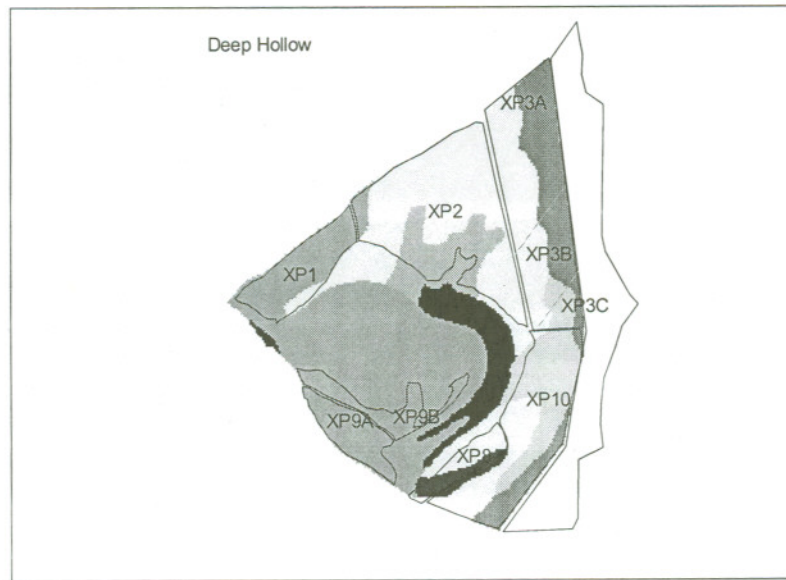


Figure 1. Soil Map and Fields—Deep Hollow Watershed

taneously determine optimal management in terms of profit and environmental quality. The bioeconomic model uses the agricultural policy environmental extender (APEX) (Blackland Research Center; Williams, Arnold, and Srinivasan), which was developed as an extension of the erosion productivity–impact calculator (EPIC) model at the small watershed level by the US Department of Agriculture’s Agricultural Research Service, Soil Conservation Service, and Economic Research Service in the early 1980s (Sharpley and Williams 1990a, 1990b). APEX is a relatively recent extension of EPIC. APEX is designed to simulate biophysical processes and the interaction of cropping systems with management practices, soils, and climates over long time periods. APEX captures timing of planting and harvesting and the use of cultural BMPs and produces environmental parameters in which water flows through small watersheds as surface, channel, and subsurface flow. APEX has flexibility in allowing for model calibration with existing data. For our study, we calibrated our model to correspond with onsite empirical measures of environmental parameters. Although few studies exist that have used the APEX program, there is a wide body of literature that has used EPIC to measure the en-

vironmental and economic impacts of cropping practices. Cautions need to be taken, because the economic model is extremely sensitive to the results of the APEX model. Therefore, it is essential to obtain good quality on-site data to calibrate the biophysical model.

Model

Our analytical approach uses two steps. When available, we used the available experimental data to derive the optimum pollution level and crop enterprise mix. When experimental data were limited, we developed the biophysical model in which we used APEX to estimate runoff and yields under a number of scenarios, including various combinations of crops and tillage practices. The outputs of interest from this model were expected crop yields and expected runoff of nitrogen, phosphorous, and sediment. In the second stage of analysis, we used an optimization model along with information on yields, crop prices, production costs, and environmental parameters derived from APEX to estimate optimal cropping systems with and without environmental constraints. The following two economic optimization models were used:

I. The profit maximization model with a restriction on a pollutant runoff:

$$\max(PY - C) \times X_{f,t,c}$$

subject to

$$i) \quad X \leq L$$

$$ii) \quad \eta_K \cdot X \leq \text{Base } K_i \text{ Runoff} \cdot (1 - 0.20)$$

Here, P represents a vector of crop prices, Y represents a vector of output bushels per acre, C is total cost per acre, X is the planted acreage, and L is total available land. The subscripts on X represent subfield, tillage practice, and crop practice. Model I maximizes the profit, subject to the constraint on land and the amount of runoff allowed. ηK_i represents K_i pollutant runoff obtained from APEX associated with each field activity. When we are not concerned about restricting pollutant runoff, the second constraint need not be considered. Base K_i runoff indicates the amount of runoff obtained from each field under conventional tillage. Under the desired solution, the runoff is decreased by 20% from the base level of pollutants runoff.

II. The dynamic goal programming model with carryover effect:

$$\min \sum_{t=1}^T \sum_{i=1}^I W_{it} S_{it}$$

subject to

Goal constraint:

$$\beta_{it} X_{it} + S_{it} + S_{it-1} + \dots + S_{it-k} = G_{it}$$

Nongoal constraint:

$$\gamma_{it} X_{it} \leq N_{it} \quad i = 1, \dots, I \quad t = 1, \dots, T$$

$$I = I + 1, \dots, I + L$$

W is the weight associated with a given goal, S is the deviation from the targeted goal, X is the activities, β and γ are the technical coefficients, i denotes the i th constraint, and G_{it} is the desired goal for the i th constraint variable in time period t . N is the factor constraint such as land and labor availability in time period t . The subscript associated with t in S denotes the carryover of pollutant above the goal to the next period. Land constraint in the opti-

mization model coincided with the actual hay acres during the experimental period from which the data were obtained for the study. Furthermore, we used land as an equality constraint. For our purpose, the ranking of goals followed, from the highest weight to the lowest weight, reduction of phosphorus runoff, phosphorus leaching, potassium runoff, and potassium leaching. The weights were selected on the basis of the seriousness of the problems in the given watershed. Here, we consider the severity of the phosphorus runoff problem to be twice as high as the potassium runoff problem.

Data

For the Louisiana study, we obtained data on P and K runoff and leachates from Eichhorn. That study was conducted to measure the water quality impact of broiler litter and commercial fertilizer application when applied on hay field at the Hill Farm Research Station at Homer, Louisiana, from 1998 to 2000. Different rates of stacked broiler litter and commercial fertilizer were applied on Coastal Bermuda Grass hay. The descriptions of alternative treatments and resulting pollutant runoff are presented in Table 1.

Nitrogen, phosphorus, and potassium runoff and leachates were reported on an annual basis over the experimental period. Data on hay production under alternate broiler and commercial fertilizer rates were also reported on an annual basis. Data on labor and capital used per acre of hay land were based on the cost and return estimates of Bermuda Grass hay developed by Boucher and Gillespie. It is assumed that 40 hours of labor per week are available for an average 400-acre size farm. The cost of capital used is calculated at the rate of 8% per year. Bermuda Grass hay acreage and the number of broilers in the given watershed were gathered from the *Louisiana Agricultural Statistics Bulletin*. An estimate of litter production within the counties in the watershed was based on broiler numbers published by the Louisiana Agricultural Statistics Service.

For the case of the Deep Hollow Lake wa-

Table 1. Commercial Fertilizer and Stacked Broiler Litter (SBL) Rates Used for Different Treatments and Resulting Runoff of Phosphorus (P) and Potash (K) from a Hay Field

	Tons/acre			Total P Runoff (pounds/acre)	Total K Runoff (pounds/acre)
	First Year	Second Year	Third Year		
Control	0	0	0	<1	46
Commercial Fertilizer ^a	0.59	1.18	1.18	8	65
SBL-2	2	4	4	14	58
SBL-4	4	8	8	34	83
SBL-8	8	16	16	53	88

^a Chemical fertilizer is applied, on average, as 18-5-19 NPK in two increments during the first year and four increments during the second and the third years. SBL is applied in one increment during the first year and two increments during the second and third years.

tershed in Mississippi, we ran the APEX model for 22 subfields of unique soil. The model was calibrated on the basis of the similar experimental results. We simulated scenarios under a number of alternative assumptions, to find out how cultural practices and BMPs can affect yields and environmental outputs over a 25-year time period. The specific scenarios included crop-tillage combinations under conventional tillage, conservation tillage, and no-till. The crops considered were continuous cotton, continuous soybeans, and a cotton/soybean rotation. We generated these outputs from the APEX model to use them as inputs in the economic optimization model.

Results and Discussion

Louisiana

The level of phosphorus concentration in Lake D'Arbonne varies from 0.01 ppm to 0.12 ppm, with an average level of 0.05 ppm (Eichhorn). A dynamic goal programming model was used to find the optimal hay acreage and broiler litter application at the current average level of phosphorus concentration (0.05 ppm) in Lake D'Arbonne. The model was run under the assumption of a 3-year time horizon in which phosphorus carryover from one year to another was considered. The optimal solutions described here show the optimal hay acreage and the choice of nutrient treatments. The selection of optimal treatment of commercial fertilizer and stacked broiler litter are presented in Table 2.

As shown in Table 2, the optimization model indicated that, out of 4,667 acres of land, chemical fertilizer should be applied on 3,687 acres of hay land at the rate of 0.59 tons per acre, and the remaining 981 acres should be applied with stacked broiler litter at the rate of 8 tons per acre in the first year. In the second year, the optimal allocation was found similar to the previous year, but commercial fertilizer application expanded along with hay acreage. In the third year, only a fraction of total land was cultivated with commercial fertilizer and the other left for control (no application of commercial fertilizer or broiler litter). Thus, it can be concluded that, to maintain the phosphorus concentration level of Lake D'Arbonne at 0.05 ppm, most of the available land should be allocated to Coastal Bermuda Grass using commercial fertilizer.

A sensitivity analysis was conducted with varying levels of phosphorus concentration as goals. The results are presented in Table 2. When the phosphorus concentration desired in the lake was increased to 0.12 ppm, a significant portion of land was cultivated using stacked broiler litter. This is justifiable, because stacked broiler litter would produce more runoff but cost less than commercial fertilizer. As we relaxed the phosphorus concentration level, the number of acre of land allocated to control diminished, whereas the number of acres of land allocated for commercial fertilizer increased. When the phosphorus concentration was set at 0.01 ppm, most of the hay land was cultivated without using any fertilizer or broiler litter.

Table 2. Optimal Combination of Nutrient Treatments and Hay Acres under Various Levels of Phosphorus Concentration in Bayou D'Arbonne Watershed, Louisiana

Year, Treatment	Desired Level of Phosphorus Concentration			
	0.01 ppm	0.05 ppm	0.08 ppm	0.12 ppm
Optimal Hay Acreage				
First				
Control	3,764.7	0.0	0.0	0.0
Commercial Fertilizer	902.0	3,686.5	2,271.9	202.8
SBL-2	0.0	0.0	0.0	0.0
SBL-4	0.0	0.0	0.0	685.8
SBL-8	0.0	980.5	2,395.0	3,778.3
Second				
Control	4,746.0	0.0	0.0	0.0
Commercial Fertilizer	392.7	5,447.2	4,433.3	3,668.0
SBL-4	598.3	0.0	0.0	0.0
SBL-8	0.0	0.0	0.0	0.0
SBL-16	0.0	289.8	1,303.7	2,068.7
Third				
Control	4,769.0	247.6	0.0	0.0
Commercial Fertilizer	501.3	5,419.4	4,341.0	1,625.6
SBL-4	0.0	0.0	1,325.9	4,041.4
SBL-8	0.0	0.0	0.0	0.0
SBL-16	0.0	0.0	0.0	0.0
Three-year Cumulative Net Return (\$)	393,167	1,541,296	1,649,419	1,776,400

Note: SBL is stacked broiler litter.

For the phosphorus level of 0.05 ppm, stacked broiler litter should not be applied in the third year. This is because allowing the use of stacked broiler litter would result in an increased phosphorus level beyond the 0.05 ppm level. Therefore, the desired solution was not to continuously use stacked broiler litter, to keep the phosphorus level at 0.05 ppm. However, if we allowed the phosphorus level to increase beyond 0.05 ppm (0.08 or 0.12 ppm), the number of hay acres under commercial fertilizer decreased, whereas the acres cultivated with stacked broiler litter increased. The sensitivity result further showed that, if stacked broiler litter was used as a plant nutrient source, the cumulative impact of the 3 years' application would suggest that the appropriate level of stacked broiler litter would be 4 tons per acre. This conclusion is drawn from the optimal nutrient treatment chosen in the third year under a phosphorus level of 0.08 and 0.12

ppm.¹ These results are consistent with other studies in which 4 tons of stacked broiler litter application was recommended level of litter to apply in hay production.

Mississippi

The APEX model was run for a 25-year period on the basis of each type of crop grown on each subfield. We observed that the runoff parameters associated with sediment loss decreased with decreased tillage intensity, whereas nitrogen runoff tended to increase with decreased tillage because of reduced topsoil permeability. Obviously, in terms of environmental outcomes, there are tradeoffs among the various tillage practices. On the

¹ The use of stacked broiler litter will cause the accumulation of phosphorus. Of course, expanding the time horizon will further reduce the use of broiler litter in crop production because of the carryover effect.

other hand, soybean cultivation provides a way to mitigate nitrogen runoff, compared with cotton. Cotton/soybean rotations also result in a lowering of nitrogen runoff in most cases.

To investigate the impact of various practices in the watershed on profit and environmental quality, we also developed a series of mathematical models in which we found the maximum watershed profit under a number of constraints. We used budget and operation data to derive input and output prices, labor and machinery costs, and so on. The model was run using a number of different constraints, including acreage, labor and, in some cases, environmental standards.

The outputs of the economic optimization model included total expected watershed profit, optimal cropping (i.e., crop acreage and practices to be used in each subfield), and gross expected nitrogen, phosphorus, and sediment runoff in the watershed. Thus, the model can tell us, for a given scenario, which crops should be planted in which field under which practice to obtain the maximum profit while still achieving a certain environmental goal.

Results from the baseline model consists of cultivation of cotton using conventional tillage are shown in Table 3. The model output included the total watershed profits, expected total watershed level nutrient and runoff, and optimal level of land under cotton on each subfield. After running the baseline model, which constrains operations to only cotton cultivation with conventional tillage, we allowed the model to choose optimal combinations of both tillage and crops but incorporated constraints on the amount of runoff allowed. There were two assumptions for environmental constraints, the first requiring a 20% reduction in nitrogen runoff, and the second a 20% reduction in sediment runoff. The results of the models (Table 4) showed that a policy of nitrogen reduction to 80% of its baseline level was significantly less costly to producers than a policy that restricted sediment to 80% of the baseline. That is, producer profits were greater by 6% if sediments rather than nitrogen were constrained. It is notable that the sediment-constrained model profit was 40%

Table 3. Optimization Model Base Case Scenarios, With Conventional Tillage and Without Environmental Constraint

Field	Planted Acreage, Continuous Cotton	Returns per Field (\$)
XP3A	21.1	2,119
XP3B	9.4	918
XP3C	3.3	330
XP10	33.3	130
XP1	23.9	2,193
XP2W	36.0	3,287
XP2E	37.0	3,357
XP8	4.6	456
XP9A	15.8	1,656
XP9B	10.1	1,084
Total Watershed Profit		\$18,529
Environmental Outcome		
Nitrogen Runoff (pounds/acre)		1,090
Phosphorus Runoff (pounds/acre)		39
Sediment Loss (net tons)		63.811

higher than the actual practices model, yet sediment levels were only 86% of the actual practices model and nitrogen levels in the constrained model were 37% higher. Because sediment is a more serious problem than nitrogen runoff in the Mississippi Delta, our results suggest that a tradeoff of more nitrogen runoff for less sediment runoff is optimal. In the case of our nitrogen-constrained model, the shadow price per pound of nitrogen runoff reduction was \$3.25. That is, if the producer were to be charged this amount for every pound of nitrogen running into the watershed, then his optimal profit would occur at the point at which nitrogen runoff is reduced by 20% (i.e., 872 pounds for the watershed in 1 year). Likewise, in the sediment-constrained model, the shadow price was found to be \$126 per ton. Thus, a farmer would maximize profits at the point at which total annual sediments for the watershed are 51.049 tons.

Conclusions

A dynamic-goal programming model was used to analyze the optimal use of commercial fertilizer and stacked broiler litter in hay production under alternative phosphorus concen-

Table 4. Optimization Model with Environmental Constraints, N-Standard, Deep Hollow Lake Watershed, Mississippi

20% N-Reduction Regulation					
Field	Planted Acreage		Conventional Tillage Acreage	No-Till Acreage	Returns per Field (\$)
	Continuous Cotton	Cotton/ Soybean			
XP3A	21.4		15.1	3.0	2,093
XP3B	9.4		4.4	2.5	903
XP3C	3.3			1.7	311
XP10	27.2	6.1	3.5	14.9	2,808
XP1	23.9			11.9	2,165
XP2W	36.0		33.6	1.2	3,275
XP2E	37.0		18.6	9.2	3,189
XP8	4.6		4.6		456
XP9A	15.8		15.8		1,656
XP9B	10.1		10.1		1,084
Total Watershed Profit		\$17,940			
Environmental Outcomes					
Nitrogen Runoff (pounds)			872		
Phosphorous Runoff (pounds)			31		
Sediment Loss (net tons)			63.811		
20% S-Reduction Regulation					
Field	Planted Acreage		Conventional Tillage Acreage	No-Till Acreage	Returns per Field (\$)
	Continuous Cotton	Cotton/ Soybean			
XP3A	21.1		21.1		2,119
XP3B	6.4	3.0	6.4	1.5	835
XP3C		3.3	3.3		236
XP10	33.3		33.3		3,130
XP1	3.7	20.2	3.7	10.1	1,618
XP2W	33.6	2.4	36.0		3,220
XP2E	8.8	28.2	37.0		2,577
XP8	4.6		4.6		456
XP9A	15.8		15.8		1,656
XP9B	10.1		10.1		1,084
Total Watershed Profit		\$16,931			
Environmental Outcomes					
Nitrogen Runoff (pounds/acre)			1,044		
Phosphorous Runoff (pounds/acre)			29		
Sediment Loss (net tons)			51.049		

tration levels in Louisiana. Using this model, the allocation of land, the level of commercial fertilizer use, and the level of stacked broiler litter use were determined for each year. The dynamic-goal programming model provided

the optimal solution under different scenarios. In general, the results showed that, as the phosphorus concentration levels were relaxed, the available land was mostly allocated for stacked broiler litter. If the goal was to set a

low phosphorus concentration level (less than or equal to current average phosphorus level), then more land should be allocated for commercial fertilizer application or even no fertilizer. On the other hand, if we are not concerned about an increased level of phosphorus concentration, then more land can be cultivated with the use of stacked broiler litter as a nutrient source. The results also indicated that 4 tons of stacked broiler litter was the optimal rate for use in Coastal Bermuda Grass hay production. This is consistent with previous studies on broiler litter application rates. This level allowed the use of broiler litter as plant nutrient source while minimizing the level of non-point source pollution and optimizing returns.

When nitrogen runoff and sedimentation are of concern, as in the case of Mississippi watershed, we found that a nitrogen standard would be a much cheaper policy to institute both in terms of profit and taxes (as measured by the shadow price). Given a sediment standard (20% reduction from baseline), total watershed profits would be \$16,931, but the total tax on sediment would be \$6,422. The profits for the nitrogen standard would be \$17,939 for the watershed, whereas total taxes would be just \$207.

Another policy implication that arises from the constrained models is that nitrogen con-

straints can lead to certain acreage being taken out of production entirely. Thus, the model indicated that marginal land could be put into the conservation reservation program or similar other programs. Such programs can ensure environmental goals are met, while still allowing positive producer income.

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