Journal of Agricultural and Applied Economics, 35,3(December 2003):599–609

Price Sensitivities for U.S. Frozen Dairy Products

Leigh J. Maynard and Venkat N. Veeramani

Price elasticities and flexibilities for frozen dessert products were estimated from weekly scanner data, with emphasis on functional form selection, system misspecification testing, and endogeneity testing. Reciprocals of elasticities and elasticity matrix inversion were invalid means of obtaining flexibility estimates, leaving direct estimation as the only viable, albeit resource-intensive, approach.

Key Words: dairy, demand, price elasticities, price flexibilities

JEL Classifications: C32, C52, D11, D12, Q11

In the dairy sector, both private and public decision makers require contemporary demand analysis. For example, in 1998 the International Dairy Foods Association commissioned a retail demand analysis of a wide range of dairy products, which was motivated by the industry's perception that dairy demand was becoming more elastic (Maynard). The own-price elasticity of demand for frozen dairy products was a salient issue in a 1999 $6.4 million breach-of-contract case, *Fiola v. Nissen Bakery.* Recently, the U.S. General Accounting Office (GAO) sought contemporary dairy demand elasticities for use in preparing its analysis of Northeast Dairy Compact impacts.

One gap in the existing dairy demand literature is estimation of price flexibilities of demand (the percentage change in price given a 1% increase in quantity) from inverse demand systems. Although quantity is the individual consumer's choice variable, aggregate quantity of perishable dairy products at any given time may be predetermined, and price is a choice variable from the retailer's perspective. Product manufacturers, risk managers, and commodity analysts can use demand flexibilities to forecast price changes resulting from supply shocks. Demand flexibilities may be used in price transmission models and in market power studies of price distortion.

One might be tempted to substitute reciprocals of price elasticities of demand where flexibilities are needed. A reciprocal relationship would theoretically hold only for goods that had no substitutes or complements. One might next be tempted to substitute the inverse of the price elasticity matrix for the matrix of flexibilities. Inverting the elasticity matrix may be theoretically appropriate, but the stochastic nature of elasticity estimates may introduce numerical instability in the inverted elasticity matrix, rendering the results empirically inappropriate as flexibility estimates.

Huang (1994, 1996) and Eales debated about how potential simultaneity of prices and quantity should affect estimation of elasticities

Leigh J. Maynard and Venkat N. Veeramani are, respectively, assistant professor and graduate research assistant, Department of Agricultural Economics, University of Kentucky, Lexington, KY.

The authors wish to thank the International Dairy Foods Association and A.C. Nielsen for providing the data used in this study, and two anonymous reviewers for helpful comments. All errors remain the responsibility of the authors.

and flexibilities. Huang argued that flexibilities should always be estimated directly, whereas Eales countered that simultaneity tests should first determine whether ordinary or inverse demand models were appropriate. If prices were predetermined and quantities were endogenous, Eales argued that an ordinary demand system was appropriate, and that flexibilities should be obtained by inverting the elasticity matrix. Huang (1996) responded that inverting a matrix estimated from stochastic variables would produce inaccurate and possibly unstable flexibility estimates. Huang clarified that small (in absolute value) elasticity estimates do not necessarily imply large flexibility estimates.

The objective of this study was to estimate price elasticities and price flexibilities, generically referred to as price sensitivities, of retail demand for seven product types within the frozen dessert category. The analysis was performed on weekly U.S. average scanner data (i.e., national average weekly prices and total national weekly volume in pounds). Synthetic ordinary and inverse demand systems were estimated that allowed flexibility in how expenditure shares affected parameter estimates. System Durbin-Wu-Hausman tests were used to test whether quantity-dependent (ordinary) or price-dependent (inverse) systems were appropriate at the weekly retail level.

Frozen desserts are not a frequent subject of statistical demand analysis. Boehm estimated price elasticities of demand for ice cream and frozen novelties, Huang (1993) estimated price elasticities for ice cream as part of a large-scale complete demand system for food using annual U.S. data from 1953 to 1990, and Maynard and Liu estimated price elasticities for ice cream, frozen yogurt, and frozen novelties. To the authors' knowledge, this is the most comprehensive demand analysis of frozen dairy products and their primary substitutes. Although product manufacturers, retailers, and occasionally policy makers benefit from estimates at this level of product disaggregation, the results also motivate a general discussion about the role applied researchers could play in making valid demand sensitivity estimates more accessible for private and public decision making.

Methods

Ordinary Synthetic Demand System

Elasticities were estimated directly from an ordinary (i.e., quantity-dependent) conditional demand system. A synthetic model developed by Barten aided model selection by parameterizing, rather than assuming, the influence of expenditure shares on marginal expenditure shares and Slutsky terms.

Lee, Brown, and Seale and Brown, Lee, and Seale (1994) provide details of the ordinary synthetic demand system. The synthetic system nests four differential demand systems: the Rotterdam, the linear approximate almost ideal demand system (LA/AIDS), the CBS system (named after the Dutch Central Bureau of Statistics), and the NBR system (named after the National Bureau of Research). The Rotterdam model is specified as

$$w_i d \ln q_i = \theta_i d \ln Q + \sum_j \pi_{ij} d \ln p_j,$$

where w_i denotes the expenditure share of the ith good, q_i denotes the quantity demanded of the ith good, $d \ln Q$ denotes the Divisia volume index, and p_j denotes the price of the jth good. The differential form of the LA/AIDS model is specified as

$$dw_i = \beta_i d \ln Q + \sum_j \gamma_{ij} d \ln p_j.$$

Marginal budget shares and Slutsky terms are treated as constants in the Rotterdam model, but they are treated as functions of budget share levels in the LA/AIDS model. The CBS model has the LA/AIDS income coefficients and the Rotterdam price coefficients, whereas the NBR model has Rotterdam income coefficients and LA/AIDS price coefficients. One first estimates the following model to identify if any of the four nested specifications best describes the data:

$$w_i d \ln q_i = (d_i + \delta_1 w_i) d \ln Q + \sum_j [e_{ij} - \delta_2 w_i(\delta_{ij} - w_j)] d \ln p_j,$$

where δ_{ij} denotes the Kronecker δ such that $\delta_{ij} = 1$ if $i = j$, and $\delta_{ij} = 0$ if $i \neq j$. The parameter d_i is a weighted average of the expenditure parameters β_i and θ_i in the LA/AIDS and Rotterdam models, respectively. Likewise, the parameter e_{ij} is a weighted average of the compensated price parameters γ_{ij} and π_{ij} in the LA/AIDS and Rotterdam models, respectively.

$$d_i = \delta_1 \beta_i + (1 - \delta_1)\theta_i;$$

$$e_{ij} = \delta_2 \gamma_{ij} + (1 - \delta_2)\pi_{ij}.$$

Restricting the value of δ_1 and δ_2 yields the following demand systems:

Rotterdam	$\delta_1 = \delta_2 = 0$	
LA/AIDS	$\delta_1 = \delta_2 = 1$	
CBS	$\delta_1 = 1,$	$\delta_2 = 0$
NBR	$\delta_1 = 0,$	$\delta_2 = 1.$

Likelihood ratio tests evaluated with two degrees of freedom allow one to choose which set of restrictions (if any) adequately describes the data.

One may either impose restrictions on δ_1 and δ_2 and re-estimate a specific model or obtain elasticity estimates directly from the synthetic model (typically at the expenditure share means):

expenditure elasticity

$$\eta_i = (d_i + \delta_1 w_i)/w_i$$

compensated price elasticity

$$\eta_{ij} = [e_{ij} - \delta_2 w_i(\delta_{ij} - w_j)]/w_i$$

uncompensated price elasticity

$$\eta_{ij}^* = \eta_{ij} + w_j \eta_i.$$

Standard errors of the elasticity estimates can be calculated using values drawn from the parameter covariance matrix. The formula for the standard error of a given compensated price elasticity η_{ij}, calculated at sample mean budget shares, is as follows:

$$\text{S.E.}\ (\eta_{ij}) = \left[\frac{\text{var}(e_{ij})}{\bar{w}_i^2} + \text{var}(\delta_2)(\delta_{ij} - \bar{w}_j)^2 - 2\frac{(\delta_{ij} - \bar{w}_j)}{\bar{w}_i}\text{cov}(e_{ij}, \delta_2)\right]^{1/2}.$$

Theoretical demand restrictions in the synthetic model are as follows, where equations are indexed by i and price terms within an equation are indexed by j:

Adding-up

$$\sum_i d_i = 1 - \delta_1,$$

$$\sum_i e_{ij} = 0 \qquad \text{for all } j$$

Homogeneity

$$\sum_j e_{ij} = 0 \qquad \text{for all } i$$

Symmetry

$$e_{ij} = e_{ji} \qquad \text{for all } i, j.$$

Inverse Synthetic Demand System

Flexibilities were estimated directly from an inverse (i.e., price-dependent) conditional demand system. Brown, Lee, and Seale (1995) developed a synthetic inverse demand system analogous to Barten's synthetic ordinary demand system and applied it to orange varieties for which quantities were expected to be predetermined. Brown, Lee, and Seale (1995) provide details that supplement the following summary.

The synthetic inverse system nests four differential inverse demand systems: the inverse Rotterdam (RIDS), the almost ideal inverse demand system (AIIDS), the Laitinen-Theil system, and the RAIIDS system (a RIDS/AIIDS hybrid). The relationships between expenditure shares and compensated quantity and Antonelli coefficients are parameterized to relax the maintained assumptions of specific inverse demand systems. The Antonelli matrix is the generalized inverse of the Slutsky substitution matrix, with each element representing the compensated price impact of a one-unit increase in quantity (Deaton and Muellbauer, p. 57). The synthetic inverse demand system is

$$w_i d \ln \pi_i = (d_i + \delta_1 w_i) d \ln Q + \sum_j [e_{ij} - \delta_2 w_i(\delta_{ij} - w_j)] d \ln q_j,$$

where π_i denotes p_i/x, x denotes total expenditure, and all other variables are defined as in the ordinary demand system. The parameter d_i is a weighted average of the scale parameters in the RIDS and AIIDS models, respectively. Likewise, the parameter e_{ij} is a weighted average of the compensated quantity parameters in the RIDS and AIIDS models.

Restricting the value of δ_1 and δ_2 yields the following inverse demand systems:

RIDS	$\delta_1 = \delta_2 = 0$	
AIIDS	$\delta_1 = \delta_2 = 1$	
Laitinen-Theil	$\delta_1 = 1,$	$\delta_2 = 0$
RAIIDS	$\delta_1 = 0,$	$\delta_2 = 1.$

Likelihood ratio tests evaluated with two degrees of freedom allow one to choose which set of restrictions best describes the data.

As with the ordinary synthetic model, one may either impose restrictions on δ_1 and δ_2 and re-estimate a specific model, or obtain flexibility estimates directly from the synthetic model:

scale flexibility

$$f_i = (d_i + \delta_1 w_i)/w_i$$

compensated price flexibility

$$f_{ij} = [e_{ij} - \delta_2 w_i(\delta_{ij} - w_j)]/w_i$$

uncompensated price flexibility

$$f^*_{ij} = f_{ij} + w_j f_i.$$

The calculation of flexibility standard errors is analogous to that of elasticity estimates.

Theoretical demand restrictions in the synthetic inverse model are as follows, where equations are indexed by i and price terms within an equation are indexed by j:

Adding-up

$$\sum_i d_i = -1 + \delta_1,$$

$$\sum_i e_{ij} = 0 \qquad \text{for all } j$$

Homogeneity

$$\sum_j e_{ij} = 0 \qquad \text{for all } i$$

Symmetry

$$e_{ij} = e_{ji} \qquad \text{for all } i, j.$$

Misspecification Testing

The system misspecification testing procedure advocated by McGuirk et al. was used to identify econometric violations. Performing isolated tests (e.g., for autocorrelation) risks erroneous inferences if the underlying assumptions of the test are violated (e.g., adequate functional form). Likewise, single-equation tests are inappropriate in a system setting because they fail to recognize contemporaneous relationships across equations. The McGuirk et al. procedure addressed both issues by advocating estimation of joint conditional mean and joint conditional variance test statistics (Rao, p. 556) that are system analogs to a single-equation F-test.

The joint conditional mean test was implemented by regressing estimated residuals from the original equations of interest on all original right-hand-side variables and three additional sets of terms used to identify functional form, parameter instability, and serial independence violations. The functional form terms were squared predicted values from each of the original equations (i.e., a RESET test); the parameter instability terms were trend and trend-squared variables; and lagged estimated residual terms allowed testing for independence (i.e., no autocorrelation). The error covariance matrix from this unrestricted regression was compared with that from a restricted regression of estimated residuals on only the original right-hand-side variables. As detailed in McGuirk et al., the Rao F-test statistic is calculated as follows:

$$F = \left[\frac{1 - \Lambda^{1/t}}{\Lambda^{1/t}}\right]\frac{rt - 2z}{pq},$$

where Λ equals the determinant of the unrestricted error covariance matrix divided by the determinant of the restricted error covariance matrix, p denotes the number of additional regressors in the unrestricted model, q denotes the number of equations in the system, v denotes the unrestricted error degrees of freedom, $t = [(p^2q^2 - 4)/(p^2 + q^2 - 5)]^{1/2}$, $r = v - (p - q + 1)/2$, and $z = (pq - 2)/4$. The test statistic is distributed approximately $F(pq, rt - 2z)$. In this application, SAS software was used. The system testing framework was implemented by saving error covariance matrices from unrestricted and restricted regressions run in proc MODEL; the matrices were then read into proc IML modules that calculated Rao F-test statistics for any parametric restrictions of interest.

The joint conditional variance test was implemented by regressing squared estimated residuals from the original equations of interest on an intercept and three additional sets of terms comparable with those in the joint conditional means test, but used to identify static heteroscedasticity, error variance instability, and dynamic heteroscedasticity (i.e., ARCH errors). The static heteroscedasticity terms were squared predicted values from each of the original equations, the error variance instability terms were trend and trend-squared variables, and the dynamic heteroscedasticity terms were lagged squared estimated residuals. If the joint conditional mean and/or joint conditional variance tests were rejected, the cause of rejection was generally evident from the significance of individual test parameters and was confirmed with equation-by-equation F-tests and system-wide Rao F-tests of individual econometric violations.

Endogeneity Testing

Quantity-dependent demand models produce consistent elasticity estimates when prices are predetermined or exogenous. Inverse demand models are appropriate when quantities are predetermined, and they are commonly used when biological lags characterize food production. Incorrect assumptions about exogeneity produce biased and inconsistent estimates (see, e.g., Pindyck and Rubinfeld, ch. 11). If endogenous variables appear on both sides of an equation, consistent estimates may be obtained by replacing each endogenous right-hand-side variable with its corresponding predicted values obtained from a regression on exogenous instruments.

Suppose prices are predetermined in a demand system, but one needs to obtain flexibility estimates (for example, to calculate a Lerner index of market power-induced price distortion). Should one invert the consistently estimated elasticity matrix, or should one estimate an inverse system via instrumental variables?

Eales argued that simultaneity tests should first determine whether ordinary or inverse demand models were appropriate. If prices were predetermined, Eales argued that flexibilities should be obtained by inverting the elasticity matrix. Although agreeing that the flexibility matrix is theoretically equivalent to the inverted elasticity matrix, Huang (1996) used the Cauchy-Schwartz inequality to demonstrate that the inverse of a directly estimated elasticity matrix will not equal the flexibility matrix estimated from the same data. Furthermore, inverted statistical estimates may be unstable. Huang (1994, 1996) suggested direct flexibility estimation.

The debate clarified, but did not resolve, the analytical tradeoffs between simultaneity bias and parameter instability. The approach used in this study integrated both perspectives. Elasticities and flexibilities were estimated directly from quantity-dependent and price-dependent models, respectively. If system Durbin-Wu-Hausman tests rejected exogeneity of right-hand-side variables, instrumental variable (IV) estimators could be used to obtain consistent estimates. IV estimators such as 3SLS are consistent but are generally biased (Davidson and MacKinnon, p. 217). The potential bias of the IV estimator was deemed less costly than the potential instability of inverting parameter matrices biased by simultaneity.

Simultaneity tests are only valid if the underlying models are statistically adequate (i.e., free of significant econometric violations) and were therefore performed following misspecification testing and correction. The system analog to the Durbin-Wu-Hausman test was performed by regressing potentially endogenous variables on a set of exogenous and predetermined instruments and including the residuals as regressors in the original demand model (McGuirk et al.; Davidson and MacKinnon, p. 239). Rao F-test statistics, as described in the misspecification testing section, were calculated to assess the system-wide joint significance of the generated residual terms. Rejection would indicate that parameter estimates in the demand system were significantly affected by simultaneity of right-hand-side variables, in which case the affected demand system would be estimated via 3SLS instead of SUR.

In the ordinary demand system, price terms ($d \ln p_i$) were jointly tested for exogeneity, as was the Divisia volume index ($d \ln Q$). In the inverse demand system, quantity terms ($d \ln q_i$) were jointly tested, as was $d \ln Q$. Instruments in all cases consisted of current and lagged exogenous variables and lagged (potentially) endogenous variables.

Eales and Unnevehr (1993) performed Hausman tests using livestock production costs as instruments in ordinary and inverse AIDS models of meat demand. The annual data suggested that only beef quantity was predetermined; all other prices and quantities were endogenous. Brown, Behr, and Lee performed Hausman tests using current and lagged exogenous variables and lagged endogenous variables as instruments on a conditional ordinary Rotterdam system for fruit juices using weekly scanner data. Neither prices nor conditional expenditures were found to be endogenous. The exogeneity of conditional expenditures was interpreted as support for rational random behavior. Lee, Brown, and Seale also determined that $d \ln Q$ was exogenous in a complete ordinary AIDS system using annual Taiwanese data for highly aggregated goods.

Data and Estimation

Demand for frozen dessert products was estimated using weekly national average retail scanner data provided by A.C. Nielsen via the International Dairy Foods Association for the weeks ending August 3, 1996, through November 21, 1998 ($n = 121$). The raw data consisted of nominal U.S. average prices and U.S. total quantities for seven products: ice cream, frozen yogurt, sherbet, sorbet, branded frozen novelties, private label frozen novelties, and "other packaged frozen" products. Nominal prices were deflated by the Consumer Price Index and linearly interpolated to obtain a weekly series. The distinction between branded and private label sales of frozen novelties (but no other product categories) reflected the needs of the sponsor at the time the data were collected. The data represented sales at retail grocery stores with over $2 million in annual sales and were similar to the juice data used by Brown, Behr, and Lee in that they were highly aggregated across space but quite disaggregated across time and form. Table 1 provides descriptive statistics.

Given the extremely small share of frozen dairy products in the total consumer budget, conditional demand systems were estimated, with total expenditures defined as expenditures on the group of seven frozen dairy product categories. Recall the overall goal of comparing inverted elasticity matrices to directly estimated flexibility matrices. The inverse of the conditional demand elasticity matrix is equal to the inverse of the corresponding submatrix in a complete demand elasticity matrix if the cross-price elasticities of "all other goods" with respect to all frozen dairy product prices equal zero (Eales). It is hard to imagine a scenario in which a change in ice cream prices, for example, would significantly influence consumers' consumption of "all other goods." Thus the estimation of conditional versus complete demand systems appears unlikely to affect the conclusions of the analysis.

Results

Regarding choice of functional form, Table 2 shows likelihood ratio tests for the restrictions on the parameters δ_1 and δ_2 that correspond to specific demand systems (e.g., Rotterdam, LA/AIDS, CBS, and NBR in the case of the or-

Table 1. Descriptive Statistics of Weekly U.S. Scanner Data, 8/3/96–11/21/98

	M	SD	Min	Max
Quantities (000)				
Ice cream	10,599.43	1,530.14	7,939.96	14,295.07
Frozen yogurt	862.19	180.16	525.39	1,322.48
Sherbet	421.62	65.08	311.15	563.47
Sorbet	68.47	15.25	43.48	96.81
Branded novelties	7,414.34	2,572.68	3,670.27	12,793.66
Private label novelties	2,631.32	909.71	1,135.57	4,586.91
Other frozen	72.34	20.27	48.33	182.07
Nominal prices				
Ice cream	\$6.16	\$0.27	\$5.67	\$7.13
Frozen yogurt	\$7.58	\$0.24	\$7.01	\$8.25
Sherbet	\$5.54	\$0.30	\$5.01	\$6.38
Sorbet	\$20.41	\$0.73	\$18.46	\$21.79
Branded novelties	\$3.60	\$0.18	\$3.24	\$4.02
Private label novelties	\$2.03	\$0.11	\$1.77	\$2.32
Other frozen	\$25.58	\$1.63	\$20.91	\$28.90
Expenditure shares				
Ice cream	60.35%	3.25%	53.78%	66.67%
Frozen yogurt	6.07%	1.11%	4.39%	8.15%
Sherbet	2.16%	0.15%	1.92%	2.80%
Sorbet	1.28%	0.11%	1.06%	1.51%
Branded novelties	23.69%	3.08%	17.84%	29.08%
Private label novelties	4.74%	0.63%	3.39%	5.93%
Other frozen	1.72%	0.44%	1.24%	3.38%

Table 2. Functional Form, Misspecification, and Endogeneity Test Statistics

Likelihood Ratio Tests Reject Common Functional Forms			
Ordinary Demand	L.R. Statistic	Inverse Demand	L.R. Statistic
Rotterdam ($\delta_1 = 0, \delta_2 = 0$)	58.34**	RIDS ($\delta_1 = 0, \delta_2 = 0$)	262.27**
AIDS ($\delta_1 = 1, \delta_2 = 1$)	11.70**	AIIDS ($\delta_1 = 1, \delta_2 = 1$)	188.03**
CBS ($\delta_1 = 1, \delta_2 = 0$)	29.78**	Laitinen-Theil ($\delta_1 = 1, \delta_2 = 0$)	32.62**
NBR ($\delta_1 = 1, \delta_2 = 1$)	41.83**	RAIIDS ($\delta_1 = 0, \delta_2 = 1$)	471.99**
System Joint Misspecification Tests Suggest Model Adequacy			
Ordinary Demand	Rao *F*-Statistic	Inverse Demand	Rao *F*-Statistic
Joint conditional mean	0.51[a]	Joint conditional mean	0.33[a]
Joint conditional variance	0.35[b]	Joint conditional variance	0.09[b]
System Endogeneity Tests Suggest SUR is Appropriate			
Ordinary Demand	Rao *F*-Statistic[c]	Inverse Demand	Rao *F*-Statistic[c]
$d \ln p_1, \ldots, d \ln p_7$	0.36	$d \ln q_1, \ldots, d \ln q_7$	1.73
$d \ln Q$	0.49	$d \ln Q$	1.78

Note: ** denotes likelihood ratio statistic > critical χ^2 value for 2 df at the .01 level.

[a] $F^c_{.05}(70,64) = 1.50$.

[b] $F^c_{.05}(70,72) = 1.48$.

[c] $F^c_{.05}(7,108) = 2.09$.

dinary system). All four specific models were rejected at the .01 level in both the ordinary and inverse demand systems. Each estimated value of δ_1 and δ_2 was significantly different from zero at the .01 level, and the estimated values of δ_2 in each system were significantly different from one at the .01 level. Accordingly, the unrestricted synthetic models were used for subsequent estimation. The test results illustrate the strength of the synthetic model in helping to avoid inadequate functional form choices that could lead to specification bias.

Table 2 also contains system joint misspecification test statistics for each demand system. In the ordinary demand system, the joint conditional mean and joint conditional variance system misspecification tests were initially rejected, apparently due to multiple econometric violations. Eales and Unnevehr (1988, p. 522) replaced current budget shares with lagged budget shares "to avoid simultaneity problems" in constructing Stone's price index in an AIDS model. When the same approach was applied in constructing the analogous Divisia volume index, all evidence of econometric violations disappeared. The lagged budget shares were highly correlated with current budget shares (e.g., 94% for ice cream).

In the inverse system, the system joint conditional mean test was initially rejected, apparently due to a serial independence violation. The violation was confirmed by a system Rao test devoted only to serial independence. Each equation in the model was respecified as a first-order autoregressive process, after which the independence, joint conditional mean, and joint conditional variance tests were not rejected.

The data added up by construction. *F*-tests failed to reject any of the homogeneity and symmetry restrictions at the .05 level in the ordinary demand system, and all theoretical restrictions were imposed in subsequent estimation. In the inverse demand system, one homogeneity restriction and several symmetry restrictions (mostly involving the "other packaged frozen" category) were rejected and not imposed.

The lower portion of Table 2 presents the results of system Durbin-Wu-Hausman tests for exogeneity of right-hand-side variables. Neither prices in the ordinary system, nor quantities in the inverse system, were sufficiently endogenous to significantly affect the vector of contrasts between parameters estimated via SUR versus 3SLS. System Durbin-Wu-Hausman tests for exogeneity of conditional expenditures (i.e., the $d \ln Q$ variable) were also not rejected at the .05 level. Although not statistically significant, the test statistics were substantially higher (i.e., closer to rejection) in the inverse demand system than in the ordinary demand system. Eales and Unnevehr (1993) generally rejected exogeneity in annual data, whereas Brown, Behr, and Lee failed to reject exogeneity in weekly prices. The frozen dairy product data suggest that market-clearing adjustments in both prices and quantities occurred over durations exceeding 1 week. The implication is that ordinary and inverse demand systems may both be consistently estimated via SUR without resorting to an IV estimator such as 3SLS.

Table 3 contains the compensated price elasticity matrix estimated from the ordinary synthetic demand system. Adjusted R^2 statistics ranged from 0.43 in the sherbet equation to 0.93 in the ice cream equation. All product categories were own-price elastic, with frozen yogurt, branded frozen novelties, and other frozen products being the most elastic. The elasticity magnitudes were similar to those of other dairy products estimated from scanner data (Maynard and Liu), which tend to be higher in absolute value than estimates obtained from data that are more aggregated over form and time. Product storability may also influence the magnitude of elasticities estimated from scanner data. Vickner and Davies obtained extremely elastic results from a brand-level analysis of spaghetti sauces, which are more storable than frozen dairy products. The dominant roles of ice cream and branded frozen novelties in the frozen dessert category are evident in the cross-price elasticities. All but two of the significant cross-price terms indicated substitute relationships among product categories.

Table 4 contains the compensated price flexibility matrix. Explanatory power was

Table 3. Compensated Price Elasticity Matrix, Estimated from Ordinary Demand System

	Ice Cream	Frozen Yogurt	Sherbet	Sorbet	Branded Novelties	Private-label Novelties	Other Frozen
Ice cream	**−1.00****	0.12**	0.03**	0.03**	0.69**	0.15**	−0.02
	(0.09)	(0.02)	(0.01)	(0.00)	(0.07)	(0.02)	(0.02)
Frozen yogurt	1.18**	**−1.64****	0.12*	0.06**	0.21	0.03	0.03
	(0.18)	(0.10)	(0.06)	(0.02)	(0.17)	(0.09)	(0.06)
Sherbet	0.91**	0.35*	**−1.11****	−0.09	0.08	−0.06	−0.07
	(0.31)	(0.16)	(0.19)	(0.07)	(0.31)	(0.18)	(0.10)
Sorbet	1.34**	0.31**	−0.16	**−1.27****	−0.05	−0.15	−0.02
	(0.21)	(0.12)	(0.12)	(0.17)	(0.22)	(0.13)	(0.06)
Branded novelties	1.75**	0.05	0.01	−0.00	**−1.81****	−0.16**	0.16**
	(0.18)	(0.04)	(0.03)	(0.01)	(0.17)	(0.06)	(0.05)
Private-label novelties	1.94**	0.04	−0.03	−0.04	−0.79**	**−1.44****	0.32**
	(0.30)	(0.11)	(0.08)	(0.04)	(0.28)	(0.18)	(0.09)
Other frozen	−0.67	0.12	−0.09	−0.01	2.23**	0.88**	**−2.46****
	(0.72)	(0.20)	(0.12)	(0.05)	(0.67)	(0.24)	(0.32)

Notes: Standard errors are in parentheses.
** and * denote statistical significance at the .01 and .05 levels, respectively.

higher in the price-dependent system, with adjusted R^2 statistics ranging from 0.85 in the "other packaged frozen" equation to 0.99 in the ice cream equation. Significant complementary relationships were much more common in the inverse demand system results. Given that price is the choice variable in inverse demand systems, strategic behavior by manufacturers and retailers offers a plausible avenue for future investigation of complementary inverse demand relationships.

The primary finding of the analysis is the relationship between own-price elasticity estimates in Table 3 and own-price flexibility estimates in Table 4. All own-price flexibilities were negative, significant, and less than 1 in

Table 4. Compensated Price Flexibility Matrix, Estimated from Inverse Demand System

	Ice Cream	Frozen Yogurt	Sherbet	Sorbet	Branded Novelties	Private-label Novelties	Other Frozen
Ice cream	**−0.09****	0.02**	0.01**	0.00**	0.04**	0.00*	0.01**
	(0.01)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Frozen yogurt	0.22**	**−0.40****	−0.06**	0.11**	0.11**	0.03	−0.02*
	(0.02)	(0.02)	(0.02)	(0.02)	(0.03)	(0.02)	(0.01)
Sherbet	0.15**	0.02	**−0.18****	−0.00	−0.01	0.03	−0.01
	(0.04)	(0.03)	(0.03)	(0.01)	(0.04)	(0.03)	(0.01)
Sorbet	0.15**	0.01	−0.00	**−0.24****	0.07	0.02	−0.01
	(0.03)	(0.03)	(0.02)	(0.03)	(0.04)	(0.03)	(0.01)
Branded novelties	0.10**	0.03**	−0.00	0.00	**−0.15****	0.03**	−0.01**
	(0.01)	(0.01)	(0.00)	(0.00)	(0.02)	(0.01)	(0.00)
Private-label novelties	0.06*	0.13**	0.01	−0.07**	0.14**	**−0.27****	−0.00
	(0.03)	(0.02)	(0.01)	(0.02)	(0.04)	(0.03)	(0.01)
Other frozen	0.39**	−0.16*	0.22**	−0.17*	−0.09	−0.02	**−0.15****
	(0.06)	(0.07)	(0.06)	(0.08)	(0.10)	(0.08)	(0.02)

Notes: Standard errors are in parentheses.
** and * denote statistical significance at the .01 and .05 levels, respectively.

absolute value (inflexible). Although this is qualitatively consistent with the elastic values in Table 3, prices appeared to be less elastic than the reciprocal or inverse of the flexibilities would suggest. Alternatively, prices were less flexible than the reciprocal or inverse of the elasticities would suggest. The same phenomenon existed in the Eales and Unnevehr (1993) SUR estimates and in Huang (1994).

The statistical and economic significance of off-diagonal terms implies that inverting the elasticity matrix, rather than calculating own-price reciprocals, is necessary to obtain theoretically valid flexibility estimates from elasticity estimates. However, inversion of both the elasticity and flexibility matrices produced implausible values exceeding 100. The relatively large magnitudes of off-diagonal estimates with respect to the dominant ice cream category appeared to be the main culprit in rendering inversion useless as an alternative to direct estimation.

Discussion

The objective of this study was to provide public and private decision makers with valid demand elasticity and flexibility estimates for frozen dairy products and their primary substitutes, with particular emphasis on functional form selection, misspecification testing, and treatment of endogeneity. The results supported Huang's (1994, 1996) contention that inverting an elasticity matrix will produce substantially different outcomes than direct estimation of flexibilities from a price-dependent demand system.

The most interesting discussion point involves the debate between Huang (1994, 1996) and Eales regarding the propriety of obtaining flexibility estimates by direct estimation or inversion of the elasticity matrix. Inverting the elasticity matrix is theoretically appropriate, and calculating own-price flexibilities as reciprocals of elasticities would be theoretically appropriate only for products with no substitutes or complements. If weak substitute/complement relationships exist, the reciprocals may not differ significantly from their inverted counterparts (Huang's 1994 results illustrate this). The bigger threat is the difference between inverted elasticity matrices and directly estimated flexibilities. In Eales and Unnevehr (1993), inverted elasticities differ from estimated flexibilities by 15% (pork) to 128% (chicken). In Huang (1994), inverted elasticities differ from estimated flexibilities by 37% (high-quality beef) to 1,071% (manufacturing-grade beef). In the present application to less aggregated products, inversion resulted in clearly unreasonable estimates, illustrating the sensitivity to numeric structure referred to by Huang (1996).

Earlier in the manuscript, Lerner's index of price distortion was used as an example where researchers have alternately used inverted elasticities (e.g., Schroeter) or advocated direct flexibility estimation (e.g., Sexton). The flexibility discrepancies described above could easily make the difference between attributing either modest or extreme price distortions to market power, with subsequent impacts on policy recommendations.

Theoretical rationale notwithstanding, it appears empirically inappropriate in many cases to use inverted elasticity matrices as demand flexibilities. Where does this leave the agency analyst or consultant who does not have the time or data to estimate flexibilities directly? In marketing courses, we tell our students that elasticities and flexibilities are useful because they can be inserted into many economic models without requiring that a full-blown demand study accompany every economic analysis. Analysts would be well-served by a publicly accessible collection of directly estimated demand elasticities and flexibilities for food products over a wide range of temporal, spatial, and product aggregation. A coordinated effort could exploit economies of scale in methods development, data collection, and estimation procedures, and would generate outputs of value both within the discipline and among our stakeholders. By more clearly demonstrating the usefulness of applied economic research to those who make economic decisions but do not read academic journals, such efforts could help maintain long-run support for the discipline. A reviewer correctly noted that the devil would be in the details.

[Received June 2002; Accepted January 2003.]

References

Barten, A.P. "Consumer Allocation Models: Choice of Functional Form." *Empirical Economics* 18(1993):129–58.

Boehm, W.T. "The Household Demand for Major Dairy Products in the Southern Region." *Southern Journal of Agricultural Economics* 7(1975): 187–96.

Brown, M.G., R.M. Behr, and J.-Y. Lee. "Conditional Demand and Endogeneity? A Case Study of Demand for Juice Products." *Journal of Agricultural and Resource Economics* 19(1994): 129–40.

Brown, M.G., J.-Y. Lee, and J.L. Seale, Jr. "Demand Relationships Among Juice Beverages: A Differential Demand System Approach." *Journal of Agricultural and Applied Economics* 26(1994):417–29.

———. "A Family of Inverse Demand Systems and Choice of Functional Form." *Empirical Economics* 20(1995):519–30.

Davidson, R., and J.G. MacKinnon. *Estimation and Inference in Econometrics.* New York: Oxford University Press, 1993.

Deaton, A., and J. Muellbauer. *Economics and Consumer Behavior.* Cambridge: Cambridge University Press, 1980.

Eales, J. "A Further Look at Flexibilities and Elasticities: Comment." *American Journal of Agricultural Economics* 78(1996):1125–29.

Eales, J.S., and L.J. Unnevehr. "Demand for Beef and Chicken Products: Separability and Structural Change." *American Journal of Agricultural Economics* 70(1988):521–32.

———. "Simultaneity and Structural Change in U.S. Meat Demand." *American Journal of Agricultural Economics* 75(1993):259–68.

Huang, K.S. "A Complete System of U.S. Demand for Food." Technical Bulletin no. 1821, USDA, Economic Research Service, Washington, DC, September 1993.

———. "A Further Look at Flexibilities and Elasticities." *American Journal of Agricultural Economics* 76(1994):313–17.

———. "A Further Look at Flexibilities and Elasticities: Reply." *American Journal of Agricultural Economics* 78(1996):1130–31.

Lee, J.-Y., M.G. Brown, and J.L. Seale, Jr. "Model Choice in Consumer Analysis: Taiwan, 1970–89." *American Journal of Agricultural Economics* 76(1994):504–12.

Maynard, L.J. "Dairy Product Markets: Does Price Really Matter?" Panel session, 1999 Dairy Forum, International Dairy Foods Association, Naples, FL, January 17–20, 1999.

Maynard, L.J., and D. Liu. "Fragility in Dairy Product Demand Analysis." Paper presented at the annual meetings of the American Agricultural Economics Association, Nashville, TN, August 8–11, 1999.

McGuirk, A., P. Driscoll, J. Alwang, and H. Huang. "System Misspecification Testing and Structural Change in the Demand for Meats." *Journal of Agricultural and Resource Economics* 20(1995):1–21.

Pindyck, R.S., and D.L. Rubinfeld. *Econometric Models and Economic Forecasts,* 3rd ed. New York: McGraw-Hill, 1991.

Rao, C.R. *Linear Statistical Inference and Its Applications,* 2nd ed. New York: Wiley, 1973.

Schroeter, J.R. "Estimating the Degree of Market Power in the Beef Packing Industry." *Review of Economics and Statistics* 70(1988):158–62.

Sexton, R.J. "Industrialization and Consolidation in the U.S. Food Sector: Implications for Competition and Welfare." *American Journal of Agricultural Economics* 82(2000):1087–104.

United States General Accounting Office. "Dairy Industry: Estimated Economic Impacts of Dairy Compacts." GAO-01-866, Washington, DC, September 2001.

Vickner, S.J., and S.P. Davies. "Estimating Market Power and Pricing Conduct in a Product-Differentiated Oligopoly: The Case of the Domestic Spaghetti Sauce Industry." *Journal of Agricultural and Applied Economics* 31(1999):1–13.