Year 1982 *Paper* 187*R*

An Open-Access Fishery with Rational Expectations

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187 rev.

Working Paper No. 187 Rev.

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Giannini Foundation of Agricultural Economics
December, 1982

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I. INTRODUCTION

Since the stock of fish changes slowly, a model of an open-access fishery must explain the evolution of the fish stock and the boat fleet as well as determine their equilibrium levels.[2] While the changes in the fish stock are described by a biological rule, entry and exit from the fishing industry are determined by how expectations are formed. We examine a general model in which entry and exit are proportional to the present value of expected profits and compare two special cases of adaptive (myopic) expectations and rational expectations.

Although both expectations mechanisms lead to the same equilibrium values, they lead to different adjustment paths. Where expectations solely depend on current values (myopic expectations), it is well known that boats enter the fishery only when the fish stock is greater than its steady state stock.[3] With rational expectations, however, boats may enter when the fish stock is much lower than its steady state value provided the boat fleet is sufficiently small. This difference in adjustment is observable and, in principle, leads to an obvious test on how agents form expectations.

Section 2 develops a general dynamic model for studying fisheries. In Section 3, we show that the model standardly used in the literature is equivalent to a myopic, adaptive expectations model. A rational expectations model is analyzed in Section 4. The standard and rational expections models are compared in Section 5. Section 6 illustrates this comparison using a simulation based on a Schaefer model of the Pacific halibut fishery. Section 7 presents the conclusions.

2. AN OPEN-ACCESS FISHERY

In this section, we develop a general dynamic version of Gordon's [12] open-access fishery model which describes the evolution of the fish stock, x, and the boat fleet, s, over time. In the next two sections, we show that the standard model and the rational expectations model are special cases of this general model.

The change in the fish population, $\dot{x}$, is its natural rate of growth, $f(x)$, less the fish catch.[4] The natural growth function, $f(\cdot)$, is assumed to be positive on the open interval $(0,K)$, where K is the carrying capacity of the fishing grounds (the largest possible fish population) and zero at $x = 0$ and K.[5] The function is assumed to be analytic and $f'' < 0$. A version widely used in empirical studies is the logistic function popularized by Schaefer [16]: $gx(1 - x/K)$, where g, a positive constant, is the ("intrinsic") growth rate for small levels of x.

It is commonly assumed that each boat catches an amount of fish per unit time which is proportional to the stock of fish.[6] By appropriate choice of the units of measurement of boats, the proportionality constant may be set equal to one so that the total catch per unit time is just sx.[7] Given these assumptions, the growth rate of the population is

(1) $$\dot{x} = f(x) - sx.$$

The rate of change of the boat fleet is assumed to be proportional to the present value of expected quasi rents, y. Letting δ be the constant of proportionality,[8]

(2) $\dot{s} = \delta y.$

The present value of expected quasi rents,

(3) $$y = \int_t^\infty e^{-r(z - t)} \pi^e(z)dz,$$

depends on the real rate of interest, r, and the expected quasi rents, $\pi^e(z)$, at each instant, z. Since the catch per boat is x, the revenue per boat is px, given a fixed price, p. Taking per boat costs, c, as fixed per unit of time, actual quasi rents are $\pi(z) = px - c$ at time z.

Equations (1) to (3) describe a very simple fishery model for general expectations. In the next two sections, special cases of myopic and rational expectations are examined.

3. THE STANDARD MODEL

Following Smith [17, 18], virtually the entire dynamic fisheries literature has assumed that entry is proportional to current profits. Beddington, Watts, and Wright [2], Berck [3], Leung and Wang [14], and others use this model to examine extinction and other issues while Clark [7], Mohring [15], Smith [17, 18], and Southey [19], among others, compare this model to an optimal exploitation model.

Since both fisheries and boats exist for long periods of time, a more natural approach is to assume that potential entrants base their entry decision on the present value of profits (quasi rents) where they use current profits as a myopic (adaptive) estimate of future profits. That is, expected profits, $\pi^e(z)$, are equal to current profits, $\pi(t)$, for all $z > t$. Making this substitution, equation (3) becomes

(4) $$y(t) = \frac{\pi(t)}{r} = \frac{px - c}{r}.$$

Substituting equation (4) into equation (2), we obtain

(5) $$\dot{s}(t) = \frac{\delta}{r}\,(px - c).$$

Equation (5) is formally equivalent to the models of Smith [17] and others who assume that entry is proportional to instantaneous profits where their constant of proportionality is δ/r, in our terms. Equations (1) and (5) constitute the standard model.

It has been shown (cf., Clark [7]) that the standard model has an equilibrium at $\dot{x} = 0$ and $\dot{s} = 0$ described by $x^* = c/p$ and $s^* = f(x)/x$, which for $f(\cdot)$ logistic is $s^* = g(1 - c/pK)$. The nature of the equilibrium is determined by finding the eigenvalues of a linearized version of the system (1) and (5) about the equilibrium, (x^*, s^*):

(6) $$\begin{pmatrix} \Delta\dot{x} \\ \Delta\dot{s} \end{pmatrix} = \begin{pmatrix} f'(x^*) - s^* & -x^* \\ \frac{\delta p}{r} & 0 \end{pmatrix} \begin{pmatrix} \Delta x \\ \Delta s \end{pmatrix},$$

where $\Delta x = x - x^*$ and $\Delta s = s - s^*$. This differential system is subject to the initial conditions that x_0 and s_0 are given.

The eigenvalues of this linearized system are

(7) $$\mu = \frac{a \pm \sqrt{a^2 + 4m/r}}{2},$$

where $a \equiv f'(x^*) - s^*$ and $m \equiv -x^* p\delta$. Limiting the model to those in which a is negative (such as the logistic, where $a = -gx^*/K$), the equilibrium is a stable node if $a^2 + 4m/r \geq 0$ and a stable vortex (stable focus) if $a^2 + 4m/r < 0$.[9]

4. THE RATIONAL EXPECTATIONS MODEL

Entrepreneurs in the rational expectations or perfect foresight fishery model base their entry decision on a correct estimate of all future quasi rents. Since their decision depends not just on the current catch but also on future ones, the rational expectations model is mathematically different from the myopic model in a fundamental manner. In this section, we show that the rational expectations model can be described by a three-equation, autonomous differential equation system. By finding the eigenvalues of the Jacobian matrix of the system linearized about its equilibrium, we show that the solution paths near the equilibrium are restricted to lie on a two-dimensional manifold in the three-dimensional phase space. A phase space is then used to show the location of the manifold relative to the standard (x,s) plane. Finally, we use standard methods for plane autonomous systems to characterize the equilibrium point.

In the rational expectations or perfect foresight model, $\pi^e(z) = \pi(z)$. Thus, the present value of expected profits, y, equals the present value of realized profits, or

$$(8) \qquad y(t) = \int_t^\infty e^{-r(z - t)} [px(z) - c]dz,$$

which, on taking the derivative with respect to time, gives

$$(9) \qquad \dot{y} = ry - (px - c).$$

Of course, some information is lost in moving from equation (8) to (9): only (8) includes a value for y(0).

As before, boats enter in proportion to the expected present value of quasi rents,

(10) $\dot{s} = \delta y,$

and the equation for the evolution of the fish stock is, as before,

(11) $\dot{x} = f(x) - sx.$

The rational expectations model is the three-equation system--equations (9), (10), and (11)--together with the initial conditions, x_0 and s_0 given, and $y(0)$ chosen to satisfy the integral equation (8).

The first step in analyzing this model is to linearize equations (9), (10), and (11) about their equilibrium. As in the standard model, the single non-zero equilibrium, $(\dot{x} = \dot{s} = \dot{y} = 0)$, is given by $x^* = c/p$, $s^* = f(x^*)/x^*$, and, additionally, $y^* = 0$. The linearized model is

(12) $$\begin{pmatrix} \Delta\dot{x} \\ \Delta\dot{s} \\ \Delta\dot{y} \end{pmatrix} = \begin{pmatrix} f'(x^*) - s^* & -x^* & 0 \\ 0 & 0 & \delta \\ -p & 0 & r \end{pmatrix} \begin{pmatrix} \Delta x \\ \Delta s \\ \Delta y \end{pmatrix} .$$

The Jacobian matrix in equation (12) is J, and its characteristic polynominal is,

(13) $H(\lambda) = \lambda^3 - (a + r)\lambda^2 + ar\lambda + m,$

where again $a = f'(x^*) - s^*$ and $m = -x^*p\delta$. The signs of the real parts and the existence of imaginary parts of the eigenvalues of J [which are the roots of $H(\lambda)$] determine the nature of the equilibrium of the rational expectations model.

PROPOSITION 1: There are two eigenvalues (possibly real) with negative real parts and one positive eigenvalue.

The proof is shown in four steps. First, either all the roots are real or there is one real root and a complex conjugate pair. This result is a simple consequence of the Fundamental Theorem of Algebra (Albert [1], p. 148) and can also be shown from Tartaglia's formula for the solution of a cubic (Beyer [5], p. 9). Let λ_1 be the real root; the other two roots may be real as well.

Second, there are either: (a) three positive roots, (b) one positive and two negative roots, or (c) one positive root and a complex conjugate pair. The product of the roots equals the negative of the determinant of J (Beyer [5], pp. 9-11) which is positive: $\lambda_1\lambda_2\lambda_3 = -m > 0$. Therefore, none of the roots can be zero. If all three roots are real, then (a) and (b) are the only possibilities. If two of the roots, λ_2 and λ_3, form a complex conjugate pair, then the remaining root must be positive because the product of a complex conjugate pair is always positive. Since there is always one real positive root, let it be λ_1.

Third, there is exactly one positive real root; possibility (a), that there are three positive roots, can be rejected. Descartes' rule of signs states that the number of positive roots of $H(\lambda)$ cannot exceed the number of variations in sign of the terms of $H(\lambda)$ when λ is taken to be a positive number (Davis [11], p. 228). The coefficients of the λ^3, λ, and constant terms of $H(\lambda)$ [1, $ar(< 0)$, and $m(< 0)$, respectively] all have determinate signs, but the sign of the coefficient on the λ^2 term [$-(a + r)$] is indeterminate. If $a + r < 0$, the sign pattern is (+,+,-,-); while, if $a + r > 0$, the pattern is (+,-,-,-). With either sign of $a + r$, there is exactly one change in the sign pattern so that there is at most one positive root. From the second point above, there is at least one positive root, so there is exactly one positive root.

Finally, if two of the roots are a complex conjugate pair, their real parts are negative. The sum of the pairwise products of the roots of H is equal to the coefficient of the linear term of H(•) (Beyer [5], p. 35):

$$(14) \qquad \lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_2\lambda_3 = ar < 0.$$

Let z_1 be the real part and z_2 be the imaginary part of the complex conjugate roots. Substituting into equation (14) and rearranging gives

$$z_1 < -\frac{z_1^2 + z_2^2}{2\lambda_1}$$

which shows that z_1, the real part of λ_2 and λ_3, is negative since z_1^2, z_2^2, and λ_1 are all positive. This step completes the proof.

Much is known about dynamical systems whose linear approximations about the equilibrium have eigenvalues as described in Propostion 1. The next proposition summarizes the relevant properties.

PROPOSITION 2: For any large t_0, there exists in (x,s,y) space (1) a real analytic two-dimensional manifold, S, containing the equilibrium, $(x^*,s^*,0)$, such that any solution (x,s,y) which is on S at t_0 satisfies $(x,s,y) \to (x^*,s^*,0)$ as $t \to \infty$, and (2) a one-dimensional analytic manifold, U, also containing the equilibrium such that $(x,s,y) \to (x^*,s^*,0)$ as $t \to -\infty$. Further, the eigenvector associated with λ_1 is tangent to U at $(x^*,s^*,0)$.

REMARK: Heuristically, U is an unstable manifold in the sense that solutions that start on U other than at the equilibrium diverge from the equilibrium as $t \to \infty$.

Since the system (9)-(11) is analytic and one eigenvalue is positive while the other two have negative real parts, this proposition follows from Theorems 4.1 and 5.1 and a corollary to Theorem 5.1 in Coddington and Levinson [9] and Theorem 6.17 in Irwin [13].

The next task is to locate the stable manifold, S, in (x,s,y) space. Figure 1 is the phase diagram in (x,s,y) space.[10] The figure measures y vertically so that the horizontal plane is (x,s,0). The three separatrices are described by: (1) $\dot{x} = 0$, the plane $(x,f'(x),y)$; (2) $\dot{s} = 0$, the plane $(x,s,0)$; and (3) $\dot{y} = 0$, the plane $(x,s,(px-c)/r)$. The intersection of these three planes is the unique, nonzero equilibrium point $E = (x^*,s^*,y^*)$ which, in the Schaefer model, is $(c/p,g[1 - c/pk],0)$. The three separatrices divide the phase space into eight sectors (numbered from 1 to 8 in Figure 1). Table I gives the direction of travel of a solution path in each of these sectors.

Since S is analytic, on a small enough neighborhood of E, a plane tangent to S at E can approximate S arbitrarily closely. In this neighborhood of E, the tangent plane enters only six of the sectors. It does not intersect sectors 3 and 8 because these are terminal isosectors (paths that enter these sectors remain in them and cannot reach E). For instance, sector 3 is terminal because a trajectory cannot exit through its boundary which consists of parts of the $\dot{y} = 0$, $\dot{s} = 0$, and $\dot{x} = 0$ planes. Since sector 3 is below (has smaller y than) the $\dot{s} = 0$ plane and $\dot{y}$ is negative in sector 3, a trajectory cannot exit through this plane. The same type of argument shows the trajectory cannot escape through the other two planes on the boundary and shows that sector 8

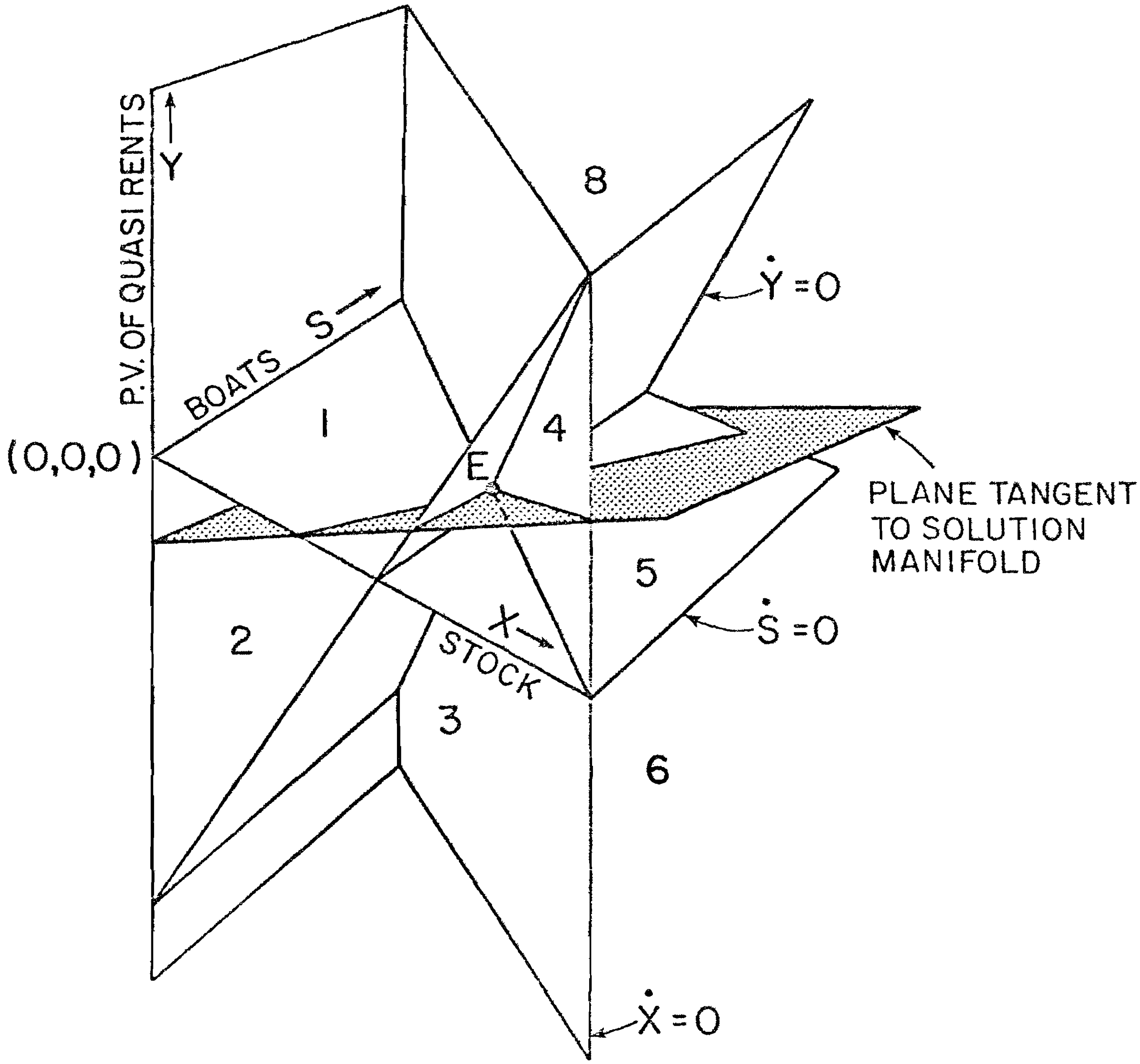

Figure 1. Rational Expectations Model Phase Diagram (Schaefer Example)

Note: $E = (x^*, s^*, y^*) = \left(\frac{c}{p}, g\left[1-\frac{c}{pK}\right], 0\right)$

Region 7 is not shown. From this angle, region 7 lies below $\dot{s}=0$, to the left of $\dot{y}=0$, and to the right of $\dot{x}=0$.

TABLE I

Phases

Sector	x	s	y	Heuristic description
1	+	+	+	Goes to 4 or to 8 terminal
2	+	-	+	May directly approach E or goes to 1 or 3
3	+	-	-	Trapped in this region
4	+	+	-	Goes to 3 (terminal) or to 5
5	-	+	-	May directly approach E or goes to 6 or 8 (terminal)
6	-	-	-	Goes to 3 (terminal) or to 7
7	-	-	+	Goes to 2 or to 8 (terminal)
8	-	+	+	Trapped in this region

is also terminal. The tangent plane must intersect sectors 2 or 5 since, as Table I shows, these are the only sectors from which paths can actually reach E and the $\lim_{t\to\infty} (x,s,y) = E$.

Exclusion of the tangent plane from two sectors and inclusion in two others almost places the plane in the phase space. It remains to show that the plane does not include the line $(c/p,s,0)$, the locus of zero present value, and instantaneous profits. A trajectory that begins on that line (other than at E) at time t_0 must enter sector 4 and then return to the line through sector 5 at time t_1. By definition,

$$y(t_0) = y(t_1) + \int_{t_1}^{\infty} e^{-r(z - t_1)} (px - c)\, dz.$$

But $y(t_1) = y(t_0)$ and the integral is everywhere positive $(px > c)$ on sectors 4 and 5 which is a contradiction. Thus, the tangent plane cannot include the zero profit line.

Since the tangent plane cannot intersect sectors 3 and 8, it cannot be parallel to the $\dot{y} = 0$ plane so it must intersect the $\dot{y} = 0$ plane. To avoid entering the terminal sector 3, at least one point on the tangent plane must be interior to sector 4. The equilibrium, E, gives us a second point. A third point must be in sector 2 or 5 by the earlier argument. These points fix the location of the tangent plane as shown in Figure 1. Since S is arbitrarily close to the tangent plane on a small enough neighborhood of E, S enters the same sectors as the tangent plane on that neighborhood.

Thus, S is included in all sectors except 3 and 8. In contrast, U lies only in sectors 3 and 8. We first show that the tanget to U at E lies in these sectors. Since λ_1 is the only eigenvalue of J with a positive real part, $L(t) = b_1 e^{\lambda_1 t} v_1$ (where b_1 is nonzero and v_1 is the eigenvector corresponding to the positive eigenvalue, λ_1) is the solution to equation (12) tangent to U. Since v_1 is an eigenvector, $\dot{L}(t) = \lambda_1 L(t) = \lambda_1 e^{\lambda_1 t} b_1 v_1$. Thus, if $b_1 > 0$, each element of the vector L(t) has the same sign as the corresponding element of v_1. If $b_1 < 0$, L(t) has the opposite sign.

By a process of elimination, it is possible to show that L(t) must be in sectors 3 or 8. For example, a vector in sector 8 is displaced from the equilibrium $(\Delta x, \Delta s, \Delta y)$ negatively in the x direction ($\Delta x < 0$), positively in the s direction ($\Delta s > 0$), and positively in the y direction ($\Delta y > 0$); so the sign pattern of v_1 if it were in sector 8 would be (-,+,+). As shown in Table I, the sign pattern of L(t) in sector 8 is also (-,+,+), so v_1 can be in sector 8. Repeating this type of argument for the other seven sectors shows that v_1 can only be in sectors 3 and 8. Since L is tangent to U at E, U is in sectors 3 and 8 near E. Furthermore, since sectors 3 and 8 are terminal sectors, U is only in sectors 3 and 8.

PROPOSITION 3: No solution to equations (9)-(11) except E itself lies on U or in sectors 3 or 8.

In sector 8, $-(px-c) > 0$, so y always grows at least as fast as ry; that is, y grows at least exponentially. But no solution can have this property because the largest possible value for y is $(pK-c)/r$. A similar argument (y cannot be smaller than $-c/r$) can be used to eliminate sector 3.

Although the solution space for system (12) appears to be three-dimensional, the choice of $y(0)$ condition for the original problem forces the solution to lie on a two-dimensional manifold approximated by a plane near the equilibrium point. The initial conditions, x_0 and s_0, by themselves do not completely determine whether the starting point is on S or U or on neither manifold. The position of $L(0)$ in $\mathbb{R}^3$ is also dependent on $y(0)$ which is determined by the integral condition (8). The following two propositions limit $L(0)$ to be (1) on S in a neighborhood of E and (2) globally, not to be on $U - \{E\}$.

PROPOSITION 4 (Local Asymptotic Stability Theorem): All solutions to equations (8)-(11) in a small neighborhood of E lie on S.

The same theorems that guarantee the existence of S and U guarantee that there is a measure of distance on $\mathbb{R}^3$ and a ball of radius, ξ, about E, $B_\xi(E)$, such that, if L solves equations (9)-(11), $L(t_0) \in B_\xi(E)$, and $L(t_0) \notin S$, then (1) the distance between L and S grows exponentially over time on $B_\xi(E)$ and (2) the distance between L and U decreases. Let 2δ be the minimum distance between the points at which paths beginning on U exit $B_\xi(E)$ and the boundaries of sectors 3 and 8. Clearly, $2\delta \leq \xi$. Let W be $\{w \in \mathbb{R}^3 \mid \text{dist}(U,w) < \delta\}$. Since $E \in U$, W is an open set containing E. $L(t_0) \in W$ and $L(t_0) \notin S$ implies that, for some $T > t_0$, $L(T)$ exits $B_\xi(E)$ and, when it does, it is closer to U than the boundary of either sector 3 or 8 is to U. Thus, $L(T)$ is in sector 3 or 8. Since no solution that enters sector 3 or 8 can meet the integral condition, $y(0)$ must be chosen so that $L(t_0) \in S$. The stability of S completes the proof.

PROPOSITION 5: The equilibrium is a stable vortex point if λ_2 and λ_3 are a complex conjugate pair and a stable node if λ_2 and λ_3 are negative and real (the stable node is degenerate if λ_2 and λ_3 are equal).[11]

Using Tartaglia's solution of the cubic, the conditions when each possibility occurs can be described. Let

$$w_1 = \frac{a^2 - ar + r^2}{9},$$

$$w_2 = \frac{27m - 2a^3 + 3a^2r + 3ar^2 - 2r^3}{54},$$

and

$$w_3 = (w_1)^3 + (w_2)^2$$

(15)

$$= \frac{-a^2r^4 + 2a^3r^3 - a^4r^2 - 4mr^3 + 6amr^2 + 6a^2mr + 27m^2 - 4a^3m}{108}.$$

All the roots are real (stable node) if $w_3 \leq 0$ (the two negative roots are equal if $w_3 = 0$), and there is one real root and a complex conjugate pair (stable vortex) if $w_3 > 0$. By taking the limits of w_3, it is easy to show that, as $r \to 0$ or $a \to 0$, the equilibrium is a stable vortex; while, as $r \to \infty$, $a \to 0$, or $\delta \to 0$ (i.e., $m \to 0$), the equilibrium is a stable node.

A fishery is said to be extinct if the fish stock is zero from some time T forward. A fishery is asymptotically extinct if, for every ξ greater than zero, there is a time T such that the fish stock is less than ξ at all times greater than T. Extinction implies asymptotic extinction.

PROPOSITION 6: Asymptotic extinction is impossible.

The proof is by contradiction. Let L(t) be within ξ of the $x = 0$ plane for all $t > T$. Since $y = \int e^{r(t-z)} (px - c)dz$ and $x < \xi$, y(t) is at most $(p\xi - c)/r$ which is negative for small ξ. In turn, $\dot{s} = \delta y \leq \delta (p\varepsilon - c)/r$, so $s(t) \xrightarrow{\text{uniformly}} 0$ as $t \to \infty$. Thus, if L(t) leads to asymptotic extinction, it must occur at $x = 0$ and $s = 0$. Linearizing (9)-(11) at $x = 0$ and $s = 0$ [replace x^* and s^* with zero in equation (12)] shows that $\dot{x} > 0$; so x cannot remain within ξ of zero which is a contradiction.

5. COMPARISON

Both models have the same equilibrium and neither model admits extinction.[12] The rational expectations model differs from the standard (myopic) model both in the character of its equilibrium and in the regions of the (x,s) space where entry occurs. The two differences are summarized in Propositions 6 and 7.

PROPOSITION 7: For a given choice of parameters p, c, r, and δ, the rational expections model may have a stable node while the myopic model has a stable vortex point or vice versa.

To show these possibilities exist, it is sufficient to compare the conditions determining the character of the equilibria for the two models. The characteristics of the standard model's equilibrium depends on the sign of the discriminant in equation (7):

$$(16) \qquad n \equiv a^2 + \frac{4m}{r}.$$

If $n \geq 0$, the standard model has a stable node; otherwise, it has a stable vortex.

The characteristic of the rational expectations model depends on the sign of w_3 [see equation (15)]. If $w_3 \leq 0$, the rational expectations model has a stable node; while, if $w_3 > 0$, it has a stable vortex. From inspection, it is obvious that equations (15) and (16) are not identical.

To show that the two approaches to equilibrium in the two models can differ, consider the following exercise. Suppose for each interest rate, r, we choose δ (and, hence, $m = -xp\delta$) so that n in equation (16) remains constant. Since the equilibrium in both the models are identical and independent of both r and δ [$x^* = c/p$, $s^* = f(x)/x$, and $y^* = 0$], we can set these parameters without affecting the equilibrium.

We could, for example, set δ so that, for each r, $n < 0$, so that the standard model has a stable vortex. Solving equation (16) for m and substituting into equation (16) gives

$$w_3 = \Big\{-16a^2r^4 + 32a^3r^3 - 16(n - a^2)r^2$$

$$(17) \qquad - 16a^4r^2 + 24a(n - a^2)\, r - \frac{16a^3(n - a^2)}{r}$$

$$+ 27\,\frac{(n - a^2)^2}{r^2} + 24a^2(n - a^2)\Big\}/1728.$$

As $r \to \infty$, w_3 becomes negative, and the rational expectations model has a stable node. By construction, as $r \to \infty$, n remains constant so that the standard model's equilibrium is a stable vortex. If we let $r \to 0$, then n remains constant while w_3 becomes positive. Here, both models have a stable vortex.[13]

PROPOSITION 8: The regions of the (x,s) plane in which entry occurs differ between the rational expectations and standard models. In the standard model, entry will occur only when $x \geq x^*$; while, in the rational expectations model, there are $x < x^*$ associated with $s < s^*$ for which entry occurs.

As Clark [7] and others have demonstrated, entry occurs in the standard model to the right of the $\dot{s} = 0$ line in (x,s) space. Thus, entry occurs only if $x > x^*$ (where instantaneous profits are positive).

Entry occurs in the rational expectations model whenever the stable manifold is above the $\dot{s} = 0$ plane (see Figure 1). By projecting the part of the stable manifold above the $\dot{s} = 0$ plane (x,s,0) onto the $\dot{s} = 0$ plane, we obtain the set of (x,s) pairs for which quasi rents (y) are positive and, hence, for which entry occurs in the rational expectations model. Since the stable manifold is above $\dot{s} = 0$ for some points in sector 1, there are points in s at which $\dot{s} > 0$ in the rational expectations model and not in the standard model. An example is shown in the next section.

6. SIMULATIONS

The basic points raised by Propositions 7 and 8 can be illustrated using a Schaefer model of the Pacific halibut fishery (area 2) and simulating its behavior near the equilibrium, as shown in Figure 2. Mohring [15] has estimated the parameters of such a model.[14] The growth rate at zero stock, g, is 0.001925 per day; the natural carrying capacity, K, is 17.63 tens of millions of pounds; and the catchability coefficient, α, is 0.001007 per thousand skate soaks days.[15]

Prior to 1930, there was minimal control of this fishery, so the stock at the end of the 1929 season approximates the open-access equilibrium stock:

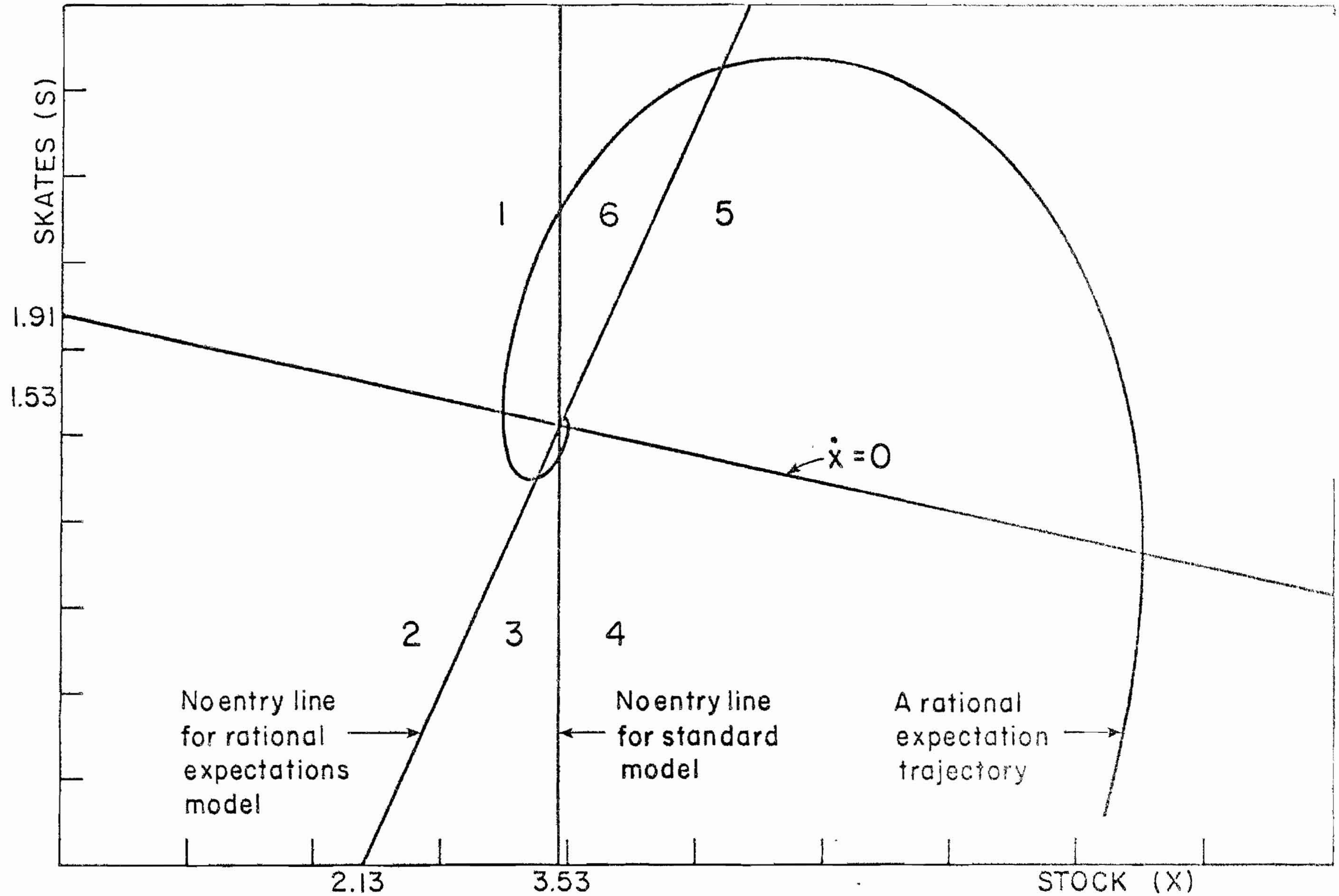

Figure 2. Schaefer Model of Pacific Halibut

$x^* = 3.53$ tens of millions of pounds.[16] These figures imply that the equilibrium effort, s^*, is 1.52886 thousand skates per day. For illustration, we assume the real annual rate of interest is 3.65 percent (so that the daily rate is 0.0001). We roughly estimate that the actual δ is 0.004598465.[17]

In order to determine whether the adaptive and rational expectations models have stable vortex or stable focus points, it is necessary to solve for the eigenvalues under both models. The eigenvalues of the rational expectations ($\lambda_1,\lambda_2,\lambda_3$) and standard ($\mu_1,\mu_2$) models are shown in Table II. For low rates of adjustment ($\delta \leq 0.000000479$), both models have real eigenvalues and, hence, stable nodes. When $0.000000479 < \delta \leq 0.00000147$, the rational expectations model has a stable node (the roots are real), while the standard model has a stable vortex (the roots are complex). For a larger δ, both models have a pair of complex roots and stable vortex points.

Thus, for slow rates of adjustments, both models predict a direct approach to equilibrium; for moderate rates of adjustment, the rational expectations model predicts a direct approach while the standard model predicts a spiraling approach (alternately over- and undershooting); and for fast rates of adjustment, both models predict a spiraling approach. In the Pacific halibut example, the rational expectations model has a stable node over a larger range of values of δ. This result need not be true in general.[18]

It is also possible to calculate when entry occurs under both models. The eigenvectors which correspond to the negative eigenvalues (λ_2 and λ_3) for this δ are

$$(18)\qquad v_2 = \begin{pmatrix} 1 + i \\ -.42235 + 1.1985i \\ .91298 - .19543i \end{pmatrix} \quad \text{and} \quad {}_3v = \begin{pmatrix} 1 - i \\ -.42235 - 1.1985i \\ .91298 + .19543i \end{pmatrix},$$

TABLE II

Pacific Halibut Fishery Area 2

δ	Rational expectations model			Standard model	
	λ_1	λ_2	λ_3	μ_1	μ_2
.004598465	.003246	-.001766-.00288i	-.001766+.00288i	-.000193-.01924i	-.000193+.01924i
.00001	.000378	-.000332-.00032i	-.000332+.00032i	-.000193-.00088i	-.000193+.00088i
.00000147	.000201	-.000251	-.000235	-.000193-.00029i	-.000193+.00029i
.000000479	.000149	-.0000716	-.000362	-.000193	-.000193
.0000001	.000114	-.0000185	-.000381	-.000022	-.000363

The general solution to equation (12) is

$$\text{(19)} \qquad L(t) = \begin{pmatrix} x(t) - x^* \\ s(t) - s^* \\ y(t) \end{pmatrix} = \sum_{i=1}^{3} b_i e^{\lambda_i t} v_i$$

where v_i is the eigenvector corresponding to the i^{th} eigenvalue of J and b_i is determined from the initial conditions. Since, as was shown above, the only solutions lie in the stable manifold, b_1 must be zero. Thus, at $t = 0$,

$$\text{(20)} \qquad b_2 v_2 + b_3 v_3 = \begin{pmatrix} x_0 - x^* \\ s_0 - s^* \\ y_0 \end{pmatrix} = \begin{pmatrix} x_0 - 3.53 \\ s_0 - 1.52886 \\ y_0 \end{pmatrix}.$$

Expanding (20), we find that

$$\text{(21)} \qquad (1 + i)b_2 + (1 - i)b_3 = x_0 - 3.53,$$

and

$$\text{(22)} \qquad (-.42235 + 1.1985i)b_2 - (.42235 - 1.1985i)b_3 = s_0 - 1.52886.$$

Solving equations (21) and (22) simultaneously for b_2 and b_3 and substituting into equation (20) gives

$$\text{(23)} \qquad y_0 = 1.45966 - .68383x_0 + .624175s_0.$$

Equation (23) determines pairs of values of x_0 and s_0 which are consistent with an initial present value of expected quasi rents of y_0. That is, equation (23) is the equation for the plane which is tangent to the stable manifold, S. This equation can be used to determine when entry occurs in the rational expectations model near the equilibrium.

Figure 2 illustrates the Pacific halibut case. The rational expectations model's zero entry line is the upward-sloping line which hits the x axis at 2.13 which was determined by setting $y_0 = 0$ in equation (24). This line represents the intersection of the plane tangent to the stable manifold and the (x,s,0) plane (see Figure 1). Along this line near the equilibrium, there is neither entry nor exit. Since this line is not coincident with the standard model's vertical $\dot{s} = 0$ line, the two models make different predictions about entry and exit when the number of boats and the stock of fish are both low or both high.

Table III shows the direction of the flows in the rational expectations and standard models for the six sectors in Figure 2 which are determined by the $\dot{x} = 0$ line and $\dot{s} = 0$ lines for the two models. Entry occurs (s is positive) in sectors 3, 4, and 5 in the rational expectations model and sectors 4, 5, and 6 in the standard model.

In the standard model, entry occurs to the right of its vertical $\dot{s} = 0$ line (where instantaneous profits are positive). Entry occurs if $x > x^*$, and exit occurs if $x < x^*$. Thus, in the standard model, entry does not depend on the number of boats (skates) currently in the industry.

In contrast, in the rational expectations model, entry and exit depend on both the stock of fish and the number of boats. All else the same, entry is more likely to occur the smaller is s and the greater is x. It should be noted, however, that entry may occur even if $x > x^*$ if s is sufficiently smaller than s^*.

TABLE III

The (x,s) Phase Space

Sector	Rational expectations $\dot{x}$	Rational expectations $\dot{s}$	Adaptive expectations $\dot{x}$	Adaptive expectations $\dot{s}$
1	-	-	-	-
2	+	-	+	-
3	+	+	+	-
4	+	+	+	+
5	-	+	-	+
6	-	-	-	+

Figure 2 also shows a computer-generated simulation of the nonlinear model. The adjustment path spirals as it approaches the equilibrium. Although the path for the adaptive expectations model is qualitatively the same, it approaches the equilibrium value slower.

The (x,s,0) points in the simulation are close to the ones generated by the linear approximation, equation (23), as shown in Table IV. Until the stock differs from the equilibrium value by 46 percent ($x = 5.160$), the simulated s(t) and the linear approximation to s(t) are within 3 percent of each other. Even when $x = 5.160$, the linear approximation of s(t) is within 18 percent of s(t). These differences are caused by deviations between the true, nonlinear model and its linear approximation and by errors in simulating the nonlinear system.

7. CONCLUSIONS

A model of an open-access fishing industry must explain the evolution of the stocks of fish and fishing vessels as well as determine their equilibrium levels. As Spence [20] and others have shown, in some cases, stock adjustments may take years.

In a general model of the rate of change of boats, entry is a function of the present value of expected profits. Traditionally, entry has been modeled as a function of instantaneous profits. Within the context of our general model, the standard model is equivalent to assuming that potential entrants have myopic, adaptive expectations.

In contrast, the literature has assumed that an optimal social policy should look at the entire stream of profits over time; that is, perfect foresight is used. It is peculiar, therefore, that these two models are so often

TABLE IV

A Comparison of the (x,s,0) Values from the Linear Approximation and from a Simulation of the Nonlinear System

x	s simulation	s linear approximation
3.5299	1.5288	1.5287
3.5306	1.5295	1.5295
3.5254	1.5247	1.5238
3.5610	1.5571	1.5628
3.3210	1.3343	1.2999
5.1600	2.8132	3.3147

compared and contrasted since they differ in both objective functions and in the form of expectations which are used. It is more enlightening to hold constant the type of expectations used while comparing these two market organizations.

Thus, we examined the properties of an open-access, rational expectations (perfect foresight) model. Both the myopic, standard model and the rational expectations model have the same equilibrium but differ in how they adjust.

One somewhat surprising result of our analysis is that, in both models, the equilibrium may not be approached directly: overshooting and undershooting of the equilibrium level of ships may alternately occur (i.e., the equilibrium point is a stable vortex). An important difference, however, is that entry can occur at lower levels of fish stock and exit at higher levels in the rational expectations model than in the standard model.

Simple simulations based on Schaefer models of the Pacific halibut suggest that entry behavior under these two models will differ. Because the standard model can be nested in a more general model, it should be possible to test for the type of expectations used in ways analogous to those used in the macroeconomics literature. A simple, indirect test between myopic and rational expectations consists of examining entry behavior at fish and boat stocks below the equilibrium levels.

Though the rational expectations, open-access model differs from the standard model in its approach to equilibrium while having the same equilibrium, it differs from the optimal solution on both grounds. For noninfinite interest rates, the optimal fish stock will be larger than in the open-access, rational expectations model; and the fishing fleet is optimally adjusted so that the equilibrium is directly approached (instead of possibly spiraling).[19] In the limit as interest rates become infinite, the standard, the rational expectations, and the optimal model all (degenerately) approach the same solution.

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FOOTNOTES

[1]Giannini Foundation Paper No. 672 (reprint identification only). We wish to thank C. Cartwright for drawing the three-dimensional phase diagram. We are also grateful to C. Clark, F. Fisher, R. Rosenman, V. Smith, P. Varaiya, and an anonymous referee for helpful comments.

[2]For example, Spence [20] provides empirical evidence that adjustment in the whaling industry takes years.

[3]We restrict our model to fishing industries without crowding of boats: entry hurts existing boat owners only to the degree that the fish stock is reduced; boats do not physically interfere with each other.

[4]The assumption that $f(\cdot)$ is solely a function of x is made for expositional simplicity (cf., Schaefer [16]). In contrast, the Beverton-Holt [4] model assumes that natural growth is a function of the age classes of the stock and not just the total stock. In such a model, gear can be selective (e.g., one can harvest just one age class), a concept which is meaningless in the simpler model used here.

[5]This assumption is made for expositional simplicity and is not necessary. An alternative model is one with "critical depensation" (see Clark [7], pp. 16 and 17), where $f(\cdot)$ is negative on $(0,\underline{x})$, equals zero at $\underline{x}$, and is positive on $(\underline{x},K)$.

[6]See, for example, Clark [7], pp. 14-17. Many empirical studies based on the Schaefer [16] model, such as Mohring [15], have tested and used this specification.

[7]The catchability (scaling) coefficient is superfluous in the theoretical exposition; but, in the discussion of the simulations in Section 6, it is

convenient to use a catchability coefficient, α, which is different than one. Thus, in that section, the catch per unit time is αsx.

[8]If the cost of building or converting boats increases with the speed of entry (or exit), according to the quadratic cost function $(1/2\delta)s^2 + F$ (F is fixed cost), then the marginal cost of a new entrant is s/δ. Entry will occur until the marginal cost equals the present value of expected quasi rents, $s/\delta = y$, which gives us equation (2).

To the degree that entry in a fishery is the result of conversion (e.g., the gear is changed so that a new type of fish can be caught), it may be reasonable to assume that the rate of entry equals that of exit as shown in equation (2). Smith [18] discusses this symmetry assumption; Clark, Clarke, and Munro [8] analyze an optimally managed fishery without exit; and Cremer [10] examines irreversible investment in a nonrenewable resource model. We follow the otherwise universal practice of assuming symmetry.

[9]See Clark [7], pp. 183-190 and 203-204 for a detailed explanation of dynamic systems. A stable node implies that the equilibrium is directly approached (there is no overshooting). In a stable vortex (or stable focus, spiral, or focal point) the characteristics approach the equilibrium as $t \to \infty$, but they do so by spiraling about it.

[10]For the purposes of the graph, a Schaefer model is assumed which implies that the intersection of the $\dot{x} = 0$ and $\dot{s} = 0$ planes is a straight line with a negative slope in the $(x,s,0)$ plane.

[11]In a rational expectations macromodel with inventory, Blinder and Fischer [6] have shown that there is a "business cycle" which is a similar phenomenon to the spiraling approach paths (stable vortex) demonstrated here.

[12]Berck [3] shows that, given the very strong assumptions made in this paper, extinction does not occur in the standard model.

[13]By setting $w_3 = 0$ (for $n = 0$) and solving for a as a function of r, we learn that there are at most 3 real negative roots. Thus, as r ranges from 0 to ∞, there can be at most 2 intervals of r where $w_3 < 0$ and 2 where $w_3 > 0$. Since we have limit results as r goes to 0 or ∞, we know there are either 1 or 3 real negative roots. We conjecture (but have not yet proved) that there is exactly one. Thus, where $n = 0$, the standard model has a stable node, while the rational expectations model may have a stable node or stable focus depending on values of r.

[14]Note that Mohring [15] estimated a standard growth equation and treated effort (boats or fishing lines) as exogenous. The fishery was regulated during the estimation period. Unlike our simple example, Mohring [15] (reasonably) allows p to vary with the catch.

[15]Instead of using boats, Mohring [15] measured the number of skate soaks (fishing lines) which is related to the number of boats: skates are increased by increasing the number of boats. In the discussion of the Pacific halibut fishery, s refers to skates.

[16]This approach is suggested by Clark [7], p. 48. It assumes that no technological change took place during the relevant time period.

[17]Using equation (2) and $x^* = 3.53$ tens of millions of pounds, we calculate the cost per skate per year, c, to be $7,998. Since, in 1965, x was 10.6; α was 0.001007; p was 22.5 cents per pound; the change in thousands of skate soaks, s, per (average) day was 0.2018 (from 1964 to 1965); and the fishing grounds were open roughly 100 days out of the year, we can use equation (2) (given our assumed daily interest rate of 0.0001) to obtain $\delta = 0.0000008995$. We suspect that the true δ is higher since boats can be quickly converted from catching one type of fish to another.

[18] In contrast, consider a Schaefer model of the antarctic fin whale. Clark ([7], pp. 49-51) estimates that, for the antarctic fin whale, $g = 0.08$, $K = 400{,}000$, and $x^* = 40{,}000$. Here, the rational expectations model has multiple negative roots at $\delta = 0.001996$ and 0.0005528, while the standard model has multiple roots at 0.0024.

Thus, for $0.001996 > \delta > 0.0005528$, the rational expectations model has a stable vortex; otherwise, it has a stable node. The standard model has a stable vortex for $\delta > 0.0024$; while, for a smaller δ, it has a stable node. Thus, the standard model has a stable node where the rational expectations model spirals $(0.001996 > \delta > 0.0024)$.

[19] Clark, Clarke, and Munro [8] show that, if capital is irreversible, then the optimal path will cycle (once) about the equilibrium.

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