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# A FOOD DEMAND SYSTEM BASED ON DEMAND FOR CHARACTERISTICS: IF THERE IS "CURVATURE" IN THE SLUTSKY MATRIX, WHAT DO THE CURVES LOOK LIKE AND WHY?

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## ABSTRACT

A food demand system is proposed, based on demand for energy, variety, and tastes of foods. By specifying utility as an explicit function of these characteristics, the entire matrix of demand elasticities can be derived for $n$ foods and one nonfood from prior specification of just four elasticities, while avoiding any assumption of separability between foods.

This framework can explain why poorest groups often are most price-responsive, but also can account for highest price-responsiveness by middle income groups. The system is applied to published food consumption data for urban and rural populations in Pakistan. Elasticities are compared with those obtained in a published Pakistan study applying an almost ideal demand system (AIDS).

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# A FOOD DEMAND SYSTEM BASED ON DEMAND FOR CHARACTERISTICS: IF THERE IS "CURVATURE" IN THE SLUTSKY MATRIX, WHAT DO THE CURVES LOOK LIKE AND WHY?*

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## 1. INTRODUCTION

In his review of empirical estimates of food price and income elasticities that are disaggregated by income group, Alderman (1986) determined that elasticities, with a few exceptions, decline (in absolute value) with increasing income. This pattern occurs across a wide range of countries, for cross-section data sets collected at various frequencies and time intervals, using several estimation techniques. More recently, Behrman and Deolalikar (1989), using national-level information on food expenditures and prices for a number of countries over time, found that food indifference curves become more sharply curved as food expenditures increase, with the implication again that price response is higher at lower incomes.[1]

The prevailing policy conclusion emerging from this literature is that the nutritional status of the poor may be quite vulnerable to upward fluctuations in prices of food staples, in that calorie availability often declines as a net result of various

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[1] Pinstrup-Andersen, Ruiz de Londoño, and Hoover (1976) presented the first empirical evidence that low income groups are more price-responsive. Although it was later criticized for methodological reasons (for example, Brandt and Goodwin 1980), this article spawned an extensive literature that is only briefly discussed here. Waterfield (1985) provides a review of part of this literature.

substitutions among foods.[2] Therefore, a policy strategy of low food prices should benefit the poor.[3]

Timmer (1981), on whose article the subtitle of this paper is based, went so far as to speculate that the utility constant ("pure substitution") term in the Slutsky equation declines (in absolute value) by approximately half as much as the income elasticity declines as incomes rise. However, the underlying causes generating the apparent higher price-responsiveness of low-income households are not well understood, apart, perhaps, from the intuitive notion that persons in high-income groups eat, for the most part, what they want to eat when food prices rise, if only because food expenditures comprise a relatively low budget share.[4]

To be without an explicit behavioral model that can account for an empirical result, which is by now widely accepted, is generally unsatisfactory. A potential benefit of such an explanatory framework is that it can reinforce, or call into question,

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[2] For example, see Timmer and Alderman (1979). An almost universal assumption among economists is that calories are the primary nutrient limiting improved nutrition in developing countries. However, recent research has led nutritionists increasingly to focus on micronutrient deficiencies that tend to come from nonstaple foods in the diets (for example, see Levin et al. 1993 and Behrman 1995b).

[3] Behrman, Deolalikar, and Wolfe (1988) caution, however, that higher food staple prices (1) will raise the incomes of some in rural areas and (2) may lead to substitution toward foods of higher nutritive value, so that higher food prices may not *necessarily* result in worsening nutrition.

[4] However, Timmer (1981) explicitly states that the *compensated* elasticities will be higher for lower income groups, so that his proposition does not depend on the income effects of price changes. Neither are the more sharply curved utility functions of Behrman and Deolalikar (1989) at high income levels a function of the income effects of price changes.

opinion as to the accuracy of past empirical results, and deepen understanding of the factors that determine demand for foods.

In particular, there was once broad agreement that the calorie intakes of the poor were highly responsive to increases in income, but this consensus no longer exists (see Behrman [1995a] for a recent synopsis of this debate). Most high estimates are derived from data collected from food expenditure surveys that have been shown to lead to upwardly biased estimates both for nutrients and individual foods (Bouis and Haddad 1992; Bouis 1994). Is it possible that price elasticities have also been overestimated from these same food expenditure surveys? For example, if (positive) income elasticities for specific foods are upwardly biased and homogeneity is imposed (for example, Pinstrup-Andersen, Ruiz de Londoño, and Hoover [1976] and, possibly, Pitt [1983]), then (negative) own-price elasticities also may be upwardly biased (in absolute value) to satisfy the restriction that price and income elasticities sum to zero.[5]

While the prospect that empirical estimates of price-response of the poor may be overstated is speculative, a reasonable argument, nevertheless, can be made on a priori

[5] A second possible cause for upwardly-biased price elasticities is also related to how food expenditure data are collected. One plausible explanation for upwardly-biased income elasticity estimates is that food transfers from higher income to lower income households go significantly underrecorded in food expenditure surveys (some empirical evidence consistent with this proposition is provided in Bouis, Haddad, and Kennedy [1992]). Thus, food purchased by higher income households (net of *measured* food leakages in the form of food given to hired laborers and poorer relatives and neighbors) overstates actual consumption, while food purchased by the poor understates actual consumption. If transfers are higher during high-price months of seasonal scarcity, the *measured* food intakes of the higher income households will appear to rise (holding other factors constant) in high price seasons, while the intakes of the poor will appear to decline. Thus, price response of the poor may be exaggerated, while price response of higher income households may be understated.

grounds that energy intakes should not be responsive to changes in food prices, if indeed the energy intakes of the poor are not income-responsive. Low-energy income elasticities indicate that poor consumers give paramount consideration to avoiding hunger, and much lower priority to purchase of nonstaple foods (which are more expensive sources of calories) and nonfoods. If the poor do place such a high preference on avoiding hunger, it would then seem inconsistent that they also would not make every effort to maintain calorie intakes as food prices vary.

The primary objective of this paper is to propose a methodological approach that eventually may help to resolve the disagreements and seeming inconsistencies described above with regard to food demand behavior of the poor, which have arisen from what are inevitably subjective interpretations of underlying behavior based on reduced-form estimates.[6] The general approach, that of motivating food demand behavior out of a desire for food characteristics, has been known for some time (Gorman 1956, 1980; Lancaster 1971), but the particular specification proposed is new. This paper seeks to capitalize on a useful property of hedonic demand systems,

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[6] For example, Timmer and Alderman (1979, 987) state that "The income elasticities for rice show that Indonesians do have *strongly held food preferences* and will exercise them as income permits." Pitt (1983, 113) asserts that "At higher levels of expenditure, households substitute foods desired on *taste* grounds even though this may mean obtaining nutrients at higher average cost." Behrman and Deolalikar (1989, 666) conclude that "Estimates suggest increasing *taste for variety* as food budgets increase." They go on to state (p. 667) that "Underlying changing food variety, of course, are different quantities of different attributes related to nutrition, taste, appearance, status value, texture, etc. As income increases, people may choose greater food variety in order to obtain different combinations of such attributes." This paper will specify mathematical expressions that provide separate, explicit measures of *variety* and of *taste* in diets, from which estimates of shadow prices for these characteristics may be derived.

that of providing an empirical link between shadow prices for explicitly defined characteristics and elasticity magnitudes for specific goods.[7]

A second valuable property of hedonic demand systems that this paper seeks to exploit is that such systems economize on the number of estimated parameters that are required to compute a complete demand matrix of own-price, cross-price, and income elasticities.[8] A new, cost-effective methodology for food-demand-parameter estimation is introduced, which is derived from the proposed behavioral structure.

Use of expenditure systems to reduce the number of parameters required for estimation has been criticized when applied at the level of food group disaggregation typically required for policy analysis (Deaton 1975; Blundell and Ray 1984). In particular, all expenditure systems assume some form of separability between utility derived from various foods, assumptions that are difficult to accept a priori. The food demand system proposed here makes the opposite assumption, that marginal utility derived from consumption of any food depends on the level of consumption of *all* other foods. Moreover, data requirements are even less stringent for this new methodology as compared with data requirements for expenditure systems.

The paper is organized as follows. In section 2, a "demand for characteristics" model is specified in which food acquisition behavior is motivated by (1) demand for *energy* to alleviate hunger, (2) demand for *variety* in the diet, and (3) demand for

[7] For example, see Gorman (1980, 851).

[8] For example, see Pudney (1981, 430).

*tastes* inherent in particular foods.  Analysis of properties of the model in section 3 will show that, in general, demand for variety in the diet increases price response. Demand for energy, described by two parameters, may increase or decrease price response.  The relative importance of shadow prices for these characteristics varies by income group, and specific foods contain all three characteristics in varying intensities.  Thus, it cannot be determined a priori that a specific income group will be more price-responsive than other income groups, with respect to changes in the prices of *all* foods.

Section 4 implements the characteristic demand framework to derive food demand matrices for urban and rural populations in Pakistan, using published data from a nationwide food expenditure survey.  These matrices are compared with food demand matrices estimated for Pakistan by Alderman (1988), using an almost ideal demand system (AIDS) framework.  In order to demonstrate empirically the link between the underlying demand for food characteristics and price-responsiveness, section 5 presents a wide range of own-price elasticities generated by this system, using the Pakistan data, under alternative assumptions as to the relative weights assigned to individual food characteristics in the utility function.  Section 6 of the paper draws final conclusions and indicates directions for future research.

## 2. A FOOD-CHARACTERISTIC DEMAND SYSTEM (FCDS)

### AN INTUITIVE INTRODUCTION

In low-income households in poor countries, families spend a high proportion of total income on food, and a high proportion of total food expenditures on a low-calorie-cost staple, to avoid going hungry. How will such low-income households react if the price of this low-calorie-cost staple (say wheat) falls? The household could afford to *substitute* some preferred staple (say rice) without going hungry (a Giffen-good outcome). A drawback of such a decision, however, is that the diet would still consist almost entirely of bland cereals. The household may prefer instead to continue eating nearly the same amount of wheat as before to meet its energy requirements, and to supplement an essentially monotonous diet with some relatively inexpensive meat. If the latter situation is the case, if nonstaple consumption is more important to the household than the superior taste of rice, then the uncompensated own-price elasticity for wheat may be (negative but) very low in absolute value.

Now suppose that the lower price of wheat in the above example prevails but that the income of the household has gone up on a permanent basis. The family can afford substantial variety in the diet represented (say) by some meat at every meal, and even can afford the relative luxury of some rice consumption. Suppose that the price of wheat rises (although still remaining below the rice price). The household is wealthy enough now not to have to worry about the specter of hunger (a low energy intake), despite the wheat price increase. The household may be willing to substitute substantial amounts of rice for wheat. Because the household pays more for cereals now, both total cereal consumption and meat consumption may be reduced marginally. However, although total utility goes down, the marginal utilities of "energy" (calorie intake) and "variety" (nonstaple consumption) have declined enough that the least utility is lost by giving up some calorie intake and nonstaple consumption, but recouping some utility from the superior "taste" of rice. A higher price response is observed for the middle-income household than for the low-income household.

This example has introduced concepts to be developed more formally below. It raises the possibility that low-income households may be *constrained* in responding to price changes by the need to consume large amounts of a low-calorie-cost staple.[9]

## MODEL SPECIFICATION

Utility is a function of energy, variety, and tastes (characteristics of quantities of food consumed) and of nonfood purchases. Total utility derived from these three characteristics and from nonfoods is the weighted sum of their individual utilities:

$$U = w_e U_e(E) + w_v U_v(V) + \sum_{i=1}^{n} w_{ti} U_{ti}(q_i) + w_{nf} U_{nf}(q_{nf}), \qquad (1)$$

[9] Behrman, Deolalikar, and Wolfe (1988) describe an analogous hypothetical situation depicted in their Figure 1, although in the context of explaining why calorie income elasticities might be zero at very low incomes and then may become positive at higher income levels.

where:

| | | |
|---|---|---|
| U | = | total utility from all food and nonfood goods, |
| q | = | quantity of a good, |
| I | = | 1,...,*n* are the *n* foods consumed, |
| E | = | a measure of energy in the diet, |
| V | = | a measure of variety in the diet, |
| $U_e$ | = | utility derived from energy, |
| $U_v$ | = | utility derived from variety, |
| $U_{ti}(q_i)$ | = | utility derived from the taste of *q* units of good *i*, |
| $U_{nf}(q_{nf})$ | = | utility derived from *q* units of the nonfood good, |
| $w_e$ | = | weight placed on utility from energy, |
| $w_v$ | = | weight placed on utility from variety, |
| $w_{ti}$ | = | weight placed on taste from individual food *i*, |
| $w_{nf}$ | = | weight placed on utility from the nonfood good. |

Utility from Energy

$$E = \sum_{i=1}^{n} z_i q_i , \tag{2}$$

where $z_i$ = a factor converting quantity of the *i*th food into calories. *E* is total calories consumed per adult equivalent.[10]

$$U_e(E) = e_2 E + e_3 E^2 , \tag{3}$$

where $e_2 > 0$ and $e_3 < 0$.

[10] The demand system being presented appeals to some notion of an *individual's* preference structure. An interesting and important, but difficult, extension of the model would be to incorporate intrahousehold distribution of food into the utility function. For now, what is being implicitly assumed is that foods are being distributed in an egalitarian fashion. Nevertheless, some account is taken of the age/gender structure of the household by expressing E in per adult equivalent terms, rather than per capita terms. What this means mathematically is that when derivatives are taken (the change in per capita $q_i$), the $z_i$ terms in equations (4) and (5) need to be corrected for the ratio of the number of household members divided by the number of household adult equivalents.

At low levels of total energy, each additional unit of energy increases utility, but at a decreasing rate. The functional form chosen, however, allows for marginal decreases in utility from additional units of energy at sufficiently high intakes of energy.

$$E_i = w_e(e_2 z_i + 2e_3 E z_i) > 0 \text{ for low-income groups}, \tag{4}$$

where

$$E_i = \frac{\partial U}{\partial U_e(e)} \frac{\partial U_e(E)}{\partial q_i}$$

and

$$E_{ij} = 2 w_e e_3 z_i z_j < 0, \tag{5}$$

where

$$E_{ij} = \frac{\partial E_i}{\partial q_j}.$$

Analogous notation is used below for $V_i$, $V_{ij}$, $T_i$, and $T_{ij}$.

Utility from Taste

$$U_{ti}(q_i) = \log(q_i), \tag{6}$$

$$T_i = w_{ti}\left(\frac{1}{q_i}\right) > 0, \tag{7}$$

$$T_{ii} = -w_{ti}\left(\frac{1}{q_i}\right)^2 < 0, \tag{8}$$

Each additional unit of taste of good *i*, no matter what the quantity, adds additional utility, but at a decreasing rate. The first derivative is positive and the

$$T_{ij} = 0. \tag{9}$$

second derivative negative, the same signs as for utility from energy for low-income groups. However, for taste, the "across food" second derivative is zero.

Utility from Variety

$$U_v(V) = \frac{M}{T}, \tag{10}$$

where M = nonstaple kilograms of food consumed per adult equivalent, and T = total kilograms of food consumed per adult equivalent.

$$\begin{aligned} V_i &= \frac{-w_v M}{T^2} < 0 \qquad \text{for } i \le s \\ V_i &= w_v \left[ \frac{(T - M)}{T^2} \right] > 0 \qquad \text{for } s < i \le n, \end{aligned} \tag{11}$$

where *i* = 1,..., *s* are staple foods.

Each additional unit of a staple good reduces utility from variety and each additional unit of a nonstaple good increases utility from variety:

$$V_{ij} = \frac{2w_v M}{T^3} > 0 \qquad \text{for } i,j \leq s$$

$$V_{ij} = \left(\frac{w_v}{T^3}\right)[2M - T] \qquad \text{for } i \leq s \text{ and } s < j \leq n. \tag{12}$$

$$V_{ij} = \left(\frac{2w_v}{T^3}\right)[M - T] < 0 \qquad \text{for } s < i,j \leq n$$

For all three sets of *i* and *j*, $V_{ij} = V_{ji}$.

Utility from Nonfoods

Utility from consumption of any food and nonfoods is assumed to be want independent. In contrast with foods, no explicit functional form is specified for utility from nonfoods. Following Frisch (1959), and in order to solve the model for the ($n$ + 1) by ($n$ + 2) matrix of food demand elasticities, it is necessary, with respect to utility from nonfoods, only to specify the following relationship:

$$\partial\left(\frac{\partial U}{\partial q_{nf}}\right)/\partial q_{nf} = \lambda \frac{P_{nf}}{q_{nf}} \left[\frac{\phi}{\eta_{nf}}\right], \tag{13}$$

where: $\phi$ = money flexibility, $\eta_{nf}$ = the nonfood income elasticity, $p_{nf}$ = price of nonfoods, and $\lambda$ = Lagrangian multiplier associated with the constrained maximization; the marginal utility of income.

Solving the Model

For any food *i*, *i* = 1, ..., *n*, from the first-order conditions:[11]

$$p_i = \frac{\frac{\partial U}{\partial U_e}}{\lambda}\left[\frac{\partial U_e}{\partial E}\;\frac{\partial E}{\partial q_i}\right] + \frac{\frac{\partial U}{\partial U_v}}{\lambda}\left[\frac{\partial U_v}{\partial V}\;\frac{\partial V}{\partial q_i}\right] + \frac{\frac{\partial U}{\partial U_T}}{\lambda}\left[\frac{\partial U_T}{\partial T}\;\frac{\partial T}{\partial q_i}\right]. \qquad (14)$$

There are *n* equations associated with (14), which, for the first food, a staple, gives:

$$p_1 = \frac{w_e}{\lambda}\left[e_2 z_1 + 2e_3 z_1 E\right] + \frac{w_v}{\lambda}\left[\frac{-M}{T^2}\right] + \frac{w_{t1}}{\lambda}\left[\frac{1}{q_1}\right]. \qquad (15)$$

Shadow prices for energy and variety are given by the product of the coefficient outside the brackets times the first partial derivatives inside the brackets, for the first and second terms in equation (15), respectively. Generally, the marginal utilities for all three characteristics will decrease (at a different rate for each characteristic) with increased food consumption at higher income levels. However, the marginal utility of income ($\lambda$) declines with income, which raises each shadow price by a constant factor. Shadow prices sum to the retail price for each food at all income levels.

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[11] Equation (14) is very similar to Gorman (1980), equation (7). His term $r_i$ and the last term on the right-hand side of equation (14), the shadow price for the intrinsic characteristics of a specific good, are similar concepts. As Boyle, Gorman, and Pudney (1977) point out, without this characteristic of "uniqueness," a good would not be consumed that was not the cheapest source of at least one characteristic (characteristics obtainable from two or more goods) in the utility function (see, also, Figure 1 in Gorman [1980]).

Consequently, the proportion of the retail price for each food accounted for by the shadow price of each characteristic will vary by income group.

Given data on food prices and food quantities (say from household surveys) and values for $w_e e_2$, $w_e e_3$, and $w_v$, it is possible to solve the *n* equations represented by equation (14) for the *n* $w_{ti}$'s.[12] Given, in addition, a value for $\phi/\eta_{nf}$ and data on nonfood expenditures, it is possible to obtain values for the entire (n+1) by (n+1) matrix of second partial derivatives of the utility function with respect to *n* foods and the nonfood good. These values, in turn, may be used to calculate the full matrix of ($n$ + 1) by ($n$ + 2) demand elasticities (for example, Henderson and Quandt 1980, 25-35). Thus, four parameters (in addition to data on *average* prices and quantities) are required a priori to solve the model for the entire matrix of demand elasticities. Prior specification of any four elasticities in the ($n$ + 1) by ($n$ + 2) demand matrix may be used to identify these four parameters.

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[12] This is not precisely correct, since $\lambda$ is unknown. However, the resulting expressions for the $w_{ti}$'s are to be used in calculating food demand elasticities that do not depend on $\lambda$.

## 3. SOME EXTENSIONS, PROPERTIES, AND METHODOLOGICAL APPLICATIONS OF THE FCDS

### EXCLUDED CHARACTERISTICS

Food consumption decisions, of course, are based on factors in addition to considerations of energy, variety, and tastes. Examples might be appearance, odor, status value, and nutrients other than calories. To the extent that such characteristics are unique to a particular food (for example, a specific odor), the above framework takes this into account. The shadow price for the "taste" of a specific food may be interpreted as a residual calculation, that is, the retail price less the shadow prices for energy and variety. This residual shadow price measures the premium that the consumer is willing to pay for all characteristics that are intrinsic (in the subjective view of the consumer) to a specific food.

Various quality grades and/or levels of preparation of a specific food, where these are important considerations, may be dealt with simply by defining a separate food category for each grade or level of preparation.[13] The premiums paid for higher quality or more preparation are presumed to be subcomponents of the "taste" shadow price.

If demands for other specific nutrients (for example, protein) or nonnutrient characteristics are important determinants of food expenditure decisions, an additional

[13] This assumes, of course, that the data are sufficiently disaggregated to follow such a strategy.

term is easily added to equation (1) for each additional characteristic. Depending on the functional form chosen for utility derived from each additional nutrient, prior knowledge of at least one additional elasticity would be required for identification of the entire demand matrix (for example, a quadratic functional form as used for energy would require prior knowledge of two additional elasticities). In initial empirical applications of this framework, however, it is informative to explore the extent to which consumption behavior can be *explained*, using only the three characteristics defined in the previous section.[14]

## ALTERNATIVE DEFINITIONS OF VARIETY

The particular index of variety presented above is based on a simple dichotomy between staples and nonstaples. Other, more complex, indices might have been constructed as functions of foods consumed in several generic food categories, for example, staples, vegetables, fruits, meats, fish, oils, and dairy. Alternatively, separate terms (indices) for these generic food categories might have been included in

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[14] In this spirit, a further extension of the model has been developed and applied in which a predesignated subgroup of the $n$ foods (for example, beef, pork, chicken, fish, and other meats) share a common additional characteristic shared by each of these foods (for example, the flavor of meat), but not by any food outside of the subgroup. Any number of subgroups may be identified (for example, fruits, vegetables), but (1) each subgroup must contain at least two foods, and (2) one additional utility function parameter (or a demand elasticity for a food from that subgroup) for each subgroup must be specified a priori in order to solve for the entire matrix of demand elasticities. For applications of this particular extension, see Bouis (1991a).

the utility function in equation (1).[15] It may well prove useful to specify more complex functions in further applications of this framework. Again, however, it is informative to see how well a simple formulation describes consumption behavior, before moving on to more complex functions.

## DIRECT ESTIMATION OF UTILITY FUNCTION PARAMETERS

Equation (15) may be rewritten as:

$$\begin{aligned} p_1 &= \frac{w_e}{\lambda}(e_2 z_1 + 2e_3 z_1 E) + \frac{w_v}{\lambda}\left(\frac{-M}{T^2}\right) + \frac{w_{t1}}{\lambda}\left(\frac{-1}{q_1}\right) \\ &= a_1 + a_2(E) + a_3\left(\frac{M}{T^2}\right) + a_4\left(\frac{1}{q_1}\right) \end{aligned} \quad . \qquad (16)$$

Equation (16) may be estimated to obtain values of the utility function parameters. In a system with *n* foods, there would be *n* such equations to estimate with identical parameters associated with energy and variety in each of the *n* equations.[16] No attempt is made here to explore rigorously the econometric problems associated with estimation of equation (16), nor to develop statistical tests to examine

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[15] As discussed in the previous section, this would require that more than four elasticities be specified a priori to solve for the entire matrix of food demand elasticities.

[16] Equation (16) is specified for a staple food. Intuitively, for observations with below average prices for that staple, one would expect (1) higher calorie consumption (a negative coefficient on $a_2$ corresponding to an expectation that $w_e$ and $\lambda > 0$, and $e_3 < 0$), (2) higher nonstaple consumption (a negative coefficient on $a_3$ corresponding to an expectation that $w_v$ and $\lambda > 0$), and (3) higher consumption of quantities of that specific staple (a negative coefficient on $a_4$ corresponding to an expectation that $w_{ti}$ and $\lambda > 0$). A positive intercept, $a_1$, is suggested by the expectation that $w_e$, $\lambda$, and $e_2 > 0$.

the assumptions made as to specific characteristics entering the utility function and the explicit functional forms used.[17]

Developing and applying such estimators and tests to data from several countries is obviously important for perhaps modifying and eventually establishing (or rejecting) the FCDS framework as a convincing model for food policy analysis. The narrower objective of this paper, however, is to show that (1) while the FCDS price and income elasticity estimates (calculated using a small number of plausible assumptions) are broadly consistent with those generated by more standard methodologies (for example, direct econometric estimation using AIDS), (2) these assumptions do not at all suggest that price-responsiveness of lowest income groups will always be highest for all foods. This task is pursued in the following sections of the paper.

## SUBSTITUTABILITY, SEPARABILITY, AND CHANGING PRICE-RESPONSIVENESS AS HOUSEHOLD INCOME INCREASES

Figure 1 depicts a plausible pattern of changes in the relative magnitudes of the marginal utilities of energy, variety, and taste as income increases. The changing slopes of the curves in Figure 1 as income increases suggest differences

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[17] For a comprehensive discussion of the considerable econometric problems involved in estimation and testing of assumptions of a characteristics model of demand applied to food, using regional time series data, see Gorman (1956; 1980), Boyle, Gorman, and Pudney (1977), but, particularly, Pudney (1981). There are important differences between the assumptions made in those articles and those made in this paper, so that the estimating equations and tests described in those articles are not applicable here.

Figure 1—Relative magnitudes of marginal utilities of food characteristics as income increases

in the rates of decline in the marginal utilities of the individual characteristics as food consumption increases. How does the changing trade-off between energy requirements and other characteristics as income increases (as reflected by changes in shadow prices for energy, variety, and tastes) affect price-responsiveness?

In evaluating the substitutability of two goods, intuitively one can consider the rate at which total utility declines as (say) one dollar is spent on good j instead of good i. When two goods are perfectly substitutable, such a reallocation leaves the consumer at exactly the same level of utility. Under the FCDS framework, *at specific levels* of income, two foods with very different characteristics may "substitute" for one another in the sense that the marginal utility losses/gains are about equal from the very different characteristics contained in a dollar's worth of either good. At other levels of income, because the marginal utilities of the three characteristics change, two foods may lose their substitutability. This is quite different conceptually from the "margarine and butter" (two foods that have very similar characteristics) example presented in some textbooks to explain a high cross-price elasticity.

It is the slopes of the curves in Figure 1 (the second derivatives given in equations [5], [8], and [12]) that are crucial to understanding substitutability of foods, and, therefore, price-responsiveness. At very low incomes, where the slope of the marginal utility curve for energy is nearly vertical, the substitutability between foods that are inexpensive sources of calories and other foods can be expected to be quite low. At a higher income, the identical substitution may appear much more attractive.[18]

To explore more rigorously the effect of the energy and other utility function parameters on price-responsiveness, it is useful to define $E_{umx}$ as the level of calorie consumption where the marginal utility from energy intake is zero, which mathematically can be expressed as $-e_2/2e_3$. From equation (15), then, the following expression may be derived:

$$w_{ti}\left(\frac{1}{q_i}\right) = p_i - 2z_i[-w_e e_3(E_{umx} - E)] + w_v\left[\frac{M}{T^2}\right](1-d) - w_v\left[\left(\frac{(T-M)}{T^2}\right]d\,, \qquad (17)$$

where

$$d = 0 \text{ for } i \le s \quad (i \text{ is a staple food});$$

and

$$d = 1 \text{ for } s < i \le n \quad (i \text{ is a nonstaple food}).$$

[18] Behrman and Deolalikar (1989) take the following view with respect to price-responsive of low-income groups (p. 667): "If concern for low-cost calories dominates food choices at very low incomes, we expect the food indifference curves to be relatively flat (so substitution among foods is considerable if relative food prices change)...."

The FCDS argument that the need for energy can reduce price-responsiveness, rests on the assumption that the *rank order* of food staples by their calorie costs remains unchanged, even as prices for these foods vary seasonally and between years. Certainly, the FCDS also predicts that price-responsiveness will be extremely high for low-income groups if, to use the intuitive example of section 2, wheat is cheaper than rice in one season, and rice is cheaper than wheat in another season (or to use a less extreme example, if the two prices nearly converge in one season, but diverge in another season).

It then becomes an empirical question whether or not calorie costs of staple foods "change rank," as it were, or nearly converge. Where markets operate reasonably efficiently and storage is possible, only under unusual circumstances would the relative prices of staple foods vary so much that the rank ordering would change.

Equation (17) may be used to evaluate relative values for $T_{ii}$ (see equation [8]) as income increases. Empirically, the region of interest will be where $E_{umx} > E$, or where the shadow price of energy is positive. Thus, all values inside the square "[ ]" brackets are positive.

Table 1 considers the effects on the second derivative of the utility function ($E_{ii}+V_{ii}+T_{ii}$) of changes in $E_{umx}$-E, $w_e e_3$, $M/T^2$, and $(T-M)/T^2$ as income increases. Food consumption data are provided in Table 2 for a set of Philippine farm households to provide some guidance and intuition for the assumptions and hypotheses/conclusions stated in Table 1.[19] More specifically with respect to the data shown in Table 2, for subsequent discussion of Table 1, it is useful to note the following:

1. Calorie intakes increase monotonically across income quintiles.
2. At the margin as incomes increase, nearly all extra calories come from nonstaple foods that are expensive sources of calories.
3. As income increases, rice consumption increases and substitutes for corn calorie for calorie; rice is a more expensive source of energy than corn.
4. On a per kilo basis, consumption of vegetables is a relatively inexpensive means of adding variety to the diet.

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[19] These Philippine data were collected using a 24-hour recall methodology. As stated earlier, data collected using this methodology generates a much weaker demand for calories as income increases than do data collected using expenditure surveys.

Table 1—Hypothesized price-responsiveness for selected foods as income increases

| Which Effect the Absolute Values of the Following Second Derivatives: | As Household Income Increases, the Following Variables/Parameters (Absolute Values) Are Observed/Hypothesized to Increase/Decrease: (1) Decrease in $E_{umx}$ - E | (2) Decrease in $w_e e_3$ | (3) Increase in $-M/T^2$ ([a]) | (4) Decrease in $(T-M)/T^2$ ([a]) |
|---|---|---|---|---|
| For a staple food: | | | | |
| $E_{ii}$ | Unchanged | Decrease | Unchanged | ... |
| $V_{ii}$ | Unchanged | Unchanged | Decrease[b] | ... |
| $T_{ii}$ (& $w_{ti}/q_i$ [c]) | Increase | Increase | Increase | ... |
| Implied price-responsiveness: | | | | |
| (A) Energy-intensive staple (e.g., corn) | Unambiguous decrease | If $E_{ii}$ dominates, then increase | If $V_{ii}$ dominates, then increase | ... |
| (B) Taste-intensive staple (e.g., rice) | Unambiguous decrease | If $T_{ii}$ dominates, then decrease | If $T_{ii}$ dominates, then decrease | ... |
| For a nonstaple food: | | | | |
| $E_{ii}$ | Unchanged | Decrease | ... | Unchanged |
| $V_{ii}$ | Unchanged | Unchanged | ... | Decrease |
| $T_{ii}$ (& $w_{ti}/q_i$ [c]) | Increase | Increase | ... | Increase |
| Implied price-responsiveness: | | | | |
| (C) Variety intensive non-staple (e.g., vegetable) | Unambiguous decrease | If $T_{ii}$ dominates, then decrease | ... | If $V_{ii}$ dominates, then increase |
| (D) Taste intensive nonstaple (e.g., meat) | Unambiguous decrease | If $T_{ii}$ dominates, then decrease | ... | If $T_{ii}$ dominates, then decrease |

[a] For staple foods, the hypothesized direction will tend to hold only where M < ½T; both numerator and denominator increase. For nonstaple foods, the numerator should remain more or less constant while the denominator increases, so that the empirical trend is much stronger than for staple foods.

[b] For staple foods, $V_{ii}$ is positive; an increasing value for $V_{ii}$ results in the overall second derivative ($E_{ii} + V_{ii} + T_{ii} < 0$) approaching zero.

[c] The hypothesized direction holds only if the taste shadow price ($w_{ti}/q_i$) increases faster with income than the consumption of $q_i$.

Table 2—Food expenditures, food prices, kilograms consumed, calorie intakes, and calories purchased per peso by expenditure quintile and food group

| Food Group | Expenditure Quintile 1 | 2 | 3 | 4 | 5 | Quintile 5 Minus Quintile 1 |
|---|---|---|---|---|---|---|
| Food expenditures (pesos per capita per week) | | | | | | |
| Rice | 2.32 | 3.77 | 4.76 | 4.51 | 10.12 | +7.80 |
| Corn | 9.64 | 9.73 | 9.19 | 8.79 | 4.40 | -5.24 |
| Other staples | 1.46 | 1.65 | 1.59 | 2.47 | 3.74 | +2.28 |
| Meat, fish | 7.25 | 9.09 | 10.77 | 15.68 | 24.09 | +16.84 |
| Vegetables | 2.71 | 2.86 | 3.58 | 3.77 | 3.85 | +1.14 |
| Fruits, snacks | 0.87 | 2.59 | 5.34 | 7.58 | 10.62 | +9.75 |
| Cooking ingredients | 2.13 | 3.22 | 3.46 | 4.77 | 4.83 | +2.70 |
| All | 26.37 | 32.91 | 38.67 | 47.59 | 61.65 | +35.28 |
| Food prices (pesos per kilogram) | | | | | | |
| Rice | 5.74 | 5.98 | 5.76 | 5.67 | 5.59 | |
| Corn | 4.36 | 4.52 | 4.50 | 4.46 | 4.46 | |
| Other staples | 2.79 | 3.39 | 2.34 | 3.72 | 5.35 | |
| Meat, fish | 19.58 | 18.82 | 20.79 | 20.63 | 23.40 | |
| Vegetables | 6.36 | 5.54 | 7.13 | 5.97 | 5.90 | |
| Fruits, snacks | 2.83 | 5.42 | 11.45 | 15.22 | 15.69 | |
| Cooking ingredients | 17.21 | 21.93 | 19.69 | 21.52 | 20.80 | |
| All | 6.04 | 6.72 | 7.42 | 8.59 | 10.15 | |
| Kilograms (per capita per week) | | | | | | |
| Rice | 0.40 | 0.63 | 0.83 | 0.80 | 1.81 | +1.41 |
| Corn | 2.21 | 2.15 | 2.04 | 1.97 | 0.99 | -1.22 |
| Other staples | 0.52 | 0.49 | 0.68 | 0.66 | 0.70 | +0.18 |
| Meat, fish | 0.37 | 0.48 | 0.52 | 0.76 | 1.03 | +0.66 |
| Vegetables | 0.43 | 0.52 | 0.50 | 0.63 | 0.65 | +0.22 |
| Fruits, snacks | 0.31 | 0.48 | 0.47 | 0.50 | 0.67 | +0.36 |
| Cooking ingredients | 0.12 | 0.15 | 0.18 | 0.22 | 0.23 | +0.11 |
| Rice and corn | 2.61 | 2.78 | 2.87 | 2.77 | 2.80 | +0.19 |
| All others | 1.75 | 2.12 | 2.34 | 2.77 | 3.28 | +1.53 |
| All | 4.36 | 4.90 | 5.21 | 5.54 | 6.08 | +1.72 |
| Calorie intakes (per adult equivalent per day) | | | | | | |
| Rice | 251 | 388 | 511 | 488 | 1,111 | +860 |
| Corn | 1,501 | 1,469 | 1,372 | 1,317 | 659 | -842 |
| Other staples | 116 | 114 | 147 | 159 | 200 | +84 |
| Meat, fish | 88 | 118 | 134 | 178 | 283 | +195 |
| Vegetables | 30 | 35 | 35 | 42 | 39 | +9 |
| Fruits, snacks | 41 | 67 | 64 | 71 | 91 | +50 |
| Cooking ingredients | 61 | 81 | 97 | 143 | 178 | +117 |
| Rice and corn | 1,753 | 1,857 | 1,884 | 1,805 | 1,770 | +17 |
| All others | 336 | 415 | 477 | 594 | 791 | +455 |
| All | 2,089 | 2,272 | 2,361 | 2,398 | 2,561 | +472 |
| Calories purchased per peso | | | | | | |
| Rice | 570 | 563 | 582 | 570 | 604 | |
| Corn | 872 | 846 | 858 | 858 | 847 | |
| Other staples | 623 | 526 | 584 | 470 | 396 | |
| Meat, fish | 87 | 79 | 84 | 72 | 69 | |
| Vegetables | 79 | 89 | 72 | 75 | 67 | |
| Fruits, snacks | 407 | 363 | 351 | 278 | 193 | |
| Cooking ingredients | 145 | 171 | 180 | 214 | 268 | |
| All | 492 | 440 | 414 | 344 | 286 | |

Source: International Food Policy Research Institute-Research Institute for Mindanao Culture survey, 1984/85.

Note: Quantity information derived from 24-hour recall survey and price information from food expenditure survey.

5. On a per kilo basis, consumption of meat is a relatively expensive means of adding variety to the diet; meat consumption increases more rapidly with income than does vegetable consumption.

With these patterns of food consumption as background, Table 1 makes the following assumptions:

1. The difference between $E_{umx}$ and E will decline as income increases; not only does E increase with income, but $E_{umx}$ may decline with income as activity levels decline, for example, as richer farmers hire labor to undertake more physically-demanding tasks.
2. $(T-M)/T^2$ will decline relatively strongly with income; the numerator is total food staple consumption, which remains constant as income increases, while the denominator increases with income.
3. $M/T^2$ will increase relatively weakly with income; both numerator and denominator increase with income; an increasing pattern will tend to hold only for certain income ranges, usually where $M < ½T$.
4. $w_e e_3$ will decline with income; note from equation (5) that the second derivative of utility from energy is constant, regardless of income level or energy consumption. This is inconsistent with the marginal utility curve drawn in Figure 1, which has a declining slope (approaching zero) as energy intakes increase. Equation (3) may be interpreted as an approximation, representing utility from energy consumption within a specific range of calorie intakes. The

fact that there is virtually no difference in calorie intakes from staple foods between the lowest and highest income groups suggests that the shape of marginal utility curve for energy may be sharply "kinked" at quite low levels of income.

As indicated in Table 1, change in price-responsiveness as income increases for any particular food depends on (1) the relative magnitudes of the changes in $E_{umx}$-E, $w_e e_3$, $M/T^2$, and $(T-M)/T^2$ as income increases *and* (2) whether that food is relatively energy-intensive, variety-intensive, or taste-intensive.[20] Because these several factors influence price-responsiveness in conflicting directions, it is impossible to specify unambiguously whether price-responsiveness will increase or decline with income for any type of food. Put differently, the FCDS structure is flexible enough to accommodate declining price-responsiveness with income, increasing price-responsiveness with income, or fluctuating increases and declines (in no particular order) in price-responsiveness as income increases.

However, it is possible to conclude from Table 1 that increasing price-responsiveness with income is most likely (1) for energy-intensive foods, to the extent that $w_e e_3$ declines with income and (2) for variety-intensive foods, to the extent that $(T-M)/T^2$ declines with income. This will be demonstrated empirically in section 5.

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[20] Thus, the structure of demand presented here violates (weak or strong) separability. The marginal rate of substitution between two staples, for example, is very much dependent on the level of nonstaple consumption, both because of the calories and variety in the diet that nonstaple foods provide.

## 4. AN APPLICATION OF THE FOOD-CHARACTERISTIC DEMAND SYSTEM

In section 2, a framework was specified that may be used for estimating food demand parameters and that is relatively easy to implement with data from published sources. To demonstrate that application of such a methodology is practical, food expenditure survey data from Pakistan are used to derive food demand parameter matrices for a seven-food-group aggregation.

To demonstrate the plausibility of the estimates generated by the FCDS framework, these estimates are compared with food demand elasticities estimated for urban and rural populations for Pakistan by Alderman (1988), using an AIDS framework for a nearly identical aggregation of seven foods. Both the FCDS and AIDS estimates utilize food expenditure data collected by the same Pakistan government agency for a nationally representative sample of households.

### DATA REQUIREMENTS

Data requirements are (1) per capita quantities consumed for each of the $n$ (in this case seven) food groups, (2) prices paid per kilogram for each food group, (3) calorie conversion rates per kilogram for each food group, (4) total nonfood expenditures, and (5) the ratio of adult equivalents over total household members. Data for calorie conversion rates and the age and gender structure of an average household typically are not available from published summaries of household expenditure surveys, but these may be accurately estimated from other sources.[21]

The food consumption and price data for Pakistan shown in Table 3 are taken from the Household Income and Expenditure Survey 1984-85 conducted by the Federal Bureau of Statistics.[22] Wheat is a much cheaper source of calories than rice and is overwhelmingly the predominant staple; its consumption shows some tendency to decline with income in urban areas. Milk, which has a high food budget share, is a very expensive source of calories relative to wheat and rice, but (in the food-characteristic demand framework) it is a relatively inexpensive source of variety. Vegetables are the cheapest source of variety, but a more expensive source of calories than milk. Meats are both an expensive source of calories and an expensive source of variety. Total food budget shares range

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[21] Estimates of per kilogram (as purchased) calorie conversion rates for disaggregate food groups may be obtained from country-specific food composition tables and dietary surveys. For specific foods (for example, wheat and rice), these conversion rates will not vary greatly across countries, so that, as is the case here, estimates available from other countries may be used. Using these assumed calorie conversion rates, per adult equivalent consumption of calories per day ranged from a low of 1,935 for the lowest income quartile in urban areas to a high of 2,910 for the highest income quartile in rural areas.

Data on household size were available for the Pakistan expenditure survey used here. However, it was necessary to make assumptions as to the ratio of adult equivalents to total household members. For the third expenditure quartile, this ratio was assumed to be 0.75. Typically, this type of demographic information is available from population censuses.

[22] Estimates were derived for four expenditure quartiles for urban and rural populations. For the sake of brevity, only the estimates for the third expenditure quartile are presented and discussed here, which may be compared with the Alderman results. See the appendices for a detailed discussion of (1) how the FCDS framework was applied to the Pakistan data and (2) a number of other computational and conceptual issues related to application of the FCDS.

Table 3—Per capita consumption, price, and calorie conversion rates for seven aggregate food groups, third income quartile, by urban and rural populations for Pakistan, 1984/85

| Urban/ Rural | Income Quartile | Food | Per Capita Consumption[a] | Market Price[b] | Calories per Kilogram[c] | Calorie Price[d] | Calorie Share | Food Budget Share | Staple? |
|---|---|---|---|---|---|---|---|---|---|
| Urban | 3 | Wheat | 1.99 | 1.06 | 3.40 | 1.06 | 0.65 | 0.17 | Yes |
| Urban | 3 | Rice | 0.24 | 2.25 | 3.50 | 2.19 | 0.08 | 0.04 | Yes |
| Urban | 3 | Milk | 1.16 | 2.06 | 0.60 | 11.68 | 0.07 | 0.19 | No |
| Urban | 3 | Meat | 0.21 | 7.38 | 1.50 | 16.73 | 0.03 | 0.12 | No |
| Urban | 3 | Vegetables | 0.76 | 1.51 | 0.30 | 17.12 | 0.02 | 0.09 | No |
| Urban | 3 | Fruits | 0.19 | 2.48 | 0.30 | 28.12 | 0.01 | 0.04 | No |
| Urban | 3 | Others | 0.60 | 7.57 | 2.60 | 9.90 | 0.15 | 0.36 | No |
| Rural | 3 | Wheat | 2.84 | 1.04 | 3.40 | 1.04 | 0.70 | 0.22 | Yes |
| Rural | 3 | Rice | 0.30 | 1.96 | 3.50 | 1.90 | 0.08 | 0.05 | Yes |
| Rural | 3 | Milk | 1.72 | 1.54 | 0.60 | 8.73 | 0.07 | 0.20 | No |
| Rural | 3 | Meat | 0.15 | 7.40 | 1.50 | 16.78 | 0.02 | 0.09 | No |
| Rural | 3 | Vegetables | 0.66 | 1.48 | 0.30 | 16.78 | 0.01 | 0.07 | No |
| Rural | 3 | Fruits | 0.12 | 2.55 | 0.30 | 28.91 | 0.00 | 0.02 | No |
| Rural | 3 | Others | 0.69 | 6.59 | 2.40 | 9.34 | 0.12 | 0.35 | No |

Source: Federal Bureau of Statistics, *Household Income and Expenditure Survey 84-85*.

[a] Kilograms per capita per week.

[b] Relative to price of cheapest grain calorie source.

[c] '000 calories per kilogram.

[d] Relative to price of cheapest grain calorie source.

from 70 percent for the lowest expenditure quartile in rural areas to 37 percent for the highest expenditure quartile in urban areas.

## PRIOR ASSUMPTIONS

One further requirement for implementing the methodology is prior knowledge of any combination of four parameters, either from the food demand elasticity matrix itself and/or parameters associated within the utility function, specifically $E_{umx}$, $w_e e_3$, $w_v$, and $\phi/\eta_{nf}$. Below are the assumptions made to fulfill this requirement for the third income quartile:

| Urban/ Rural | Observed Calorie Consumption | Four Assumptions | | | |
|---|---|---|---|---|---|
| | | Food Income Elasticity | $E_{umx}$ | $w_e e_3$ | $w_v$ |
| Urban | 2,012 | 0.45 | 2,900 | -0.100 | 0.70 |
| Rural | 2,664 | 0.50 | 3,000 | -0.094 | 0.70 |

Making a prior assumption as to the food income elasticity is equivalent to making an assumption as to the nonfood income elasticity that can be solved for using the Engel aggregation condition and the observed data on budget shares. Values for the remaining three utility function parameters are suggested by applications of the FCDS to data from seven other countries (Bouis 1989, 1990, 1991a, 1991b, 1992). While specific levels for $E_{umx}$ can be understood intuitively, specific values for

$w_e e_3$ and $w_v$ are not so easily interpretable. It turns out that the solution of the model for the urban third expenditure quartile gives income elasticities (for example) of -0.04 and 0.40 for wheat and rice, respectively. These two income elasticities might have been assumed a priori (in place of $w_e e_3$ and $w_v$) to give an identical solution.

## A COMPARISON OF THE FCDS AND AIDS ESTIMATES

Alderman (1988) has derived food demand elasticity estimates for urban and rural populations for Pakistan, applying an AIDS framework to expenditure survey data collected in 1979 and 1982. He reports elasticities evaluated at the means of the price and quantity data. For purposes of comparison with the Alderman estimates, elasticities (compensated and uncompensated) for the third income quartile are reported in Table 4 (FCDS-I), which were derived using the prior assumptions outlined in the chart above.[23] Because the Alderman nonfood income elasticity (which necessarily affects the magnitude of the food income elasticities due to the budget constraint) was substantially lower than that assumed in the above chart, a second set of estimates was derived (FCDS-II) that assumed the same nonfood income elasticity as estimated by Alderman.

Table 4, which compares the own-price and income elasticities, shows that for urban populations, there is little difference in the food demand estimates

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[23] Alderman reports estimates at mean total expenditure levels. Because the highest expenditure group earns a disproportionate share of income, mean expenditures fall within the range defined by the third expenditure quartile.

Table 4—A comparison of own-price and income elasticities for selected foods using the almost ideal demand system (AIDS) and the food-characteristic demand system (FCDS), urban and rural populations, Pakistan

| Urban/ Rural Food | Own-Price | | | | | Income | | |
|---|---|---|---|---|---|---|---|---|
| | Compensated | | | Uncompensated | | | | |
| | AIDS | FCDS-I | FCDS-II | FCDS-I | FCDS-II | AIDS | FCDS-I | FCDS-II |
| Urban | | | | | | | | |
| Wheat | -0.31 | -0.36 | -0.36 | -0.35 | -0.35 | 0.35 | -0.04 | -0.07 |
| Rice | -0.93 | -0.96 | -0.96 | -0.97 | -0.97 | 0.83 | 0.40 | 0.71 |
| Dairy (milk) | -0.76 | -0.93 | -0.92 | -0.97 | -0.99 | 1.05 | 0.41 | 0.71 |
| Meat | -1.01 | -1.03 | -1.00 | -1.07 | -1.07 | 1.30 | 0.69 | 1.21 |
| Other food | -1.02 | -0.97 | -0.90 | -1.09 | -1.11 | 0.84 | 0.66 | 1.15 |
| Nonfoods | -0.89 | -0.43 | -0.33 | -1.20 | -0.94 | 1.21 | 1.55 | 1.21 |
| Rural | | | | | | | | |
| Wheat | -0.91 | -0.25 | -0.25 | -0.24 | -0.24 | 0.36 | -0.06 | -0.10 |
| Rice | -1.91 | -0.89 | -0.88 | -0.90 | -0.90 | 0.96 | 0.39 | 0.64 |
| Dairy (milk) | -1.06 | -0.83 | -0.81 | -0.88 | -0.89 | 1.37 | 0.44 | 0.72 |
| Meat | -0.29 | -1.01 | -0.99 | -1.05 | -1.05 | 1.51 | 0.87 | 1.43 |
| Other food | -1.04 | -0.91 | -0.83 | -1.07 | -1.09 | 0.80 | 0.81 | 1.33 |
| Nonfoods | -1.07 | -0.44 | -0.33 | -1.16 | -0.88 | 1.22 | 1.61 | 1.22 |

Notes: (1) AIDS estimates are taken from Alderman (1988), Tables 4 (urban) and 1 (rural); (2) for FCDS-II, the nonfood expenditure elasticity is exogenously assumed to be equal to the AIDS nonfood expenditure elasticity; for FCDS-I, identical assumptions as to utility function parameters are used as for FCDS-II, but a higher nonfood elasticity is assumed; (3) For AIDS, the dairy group includes products other than milk; the other food group excludes pulses (not shown) in addition to wheat, rice, dairy, and meat; for FCDS, the "other food" group includes pulses and nonmilk dairy products, excludes vegetables and fruits, and excludes wheat, rice, milk, and meat.

generated by the two demand systems, with the glaring exception of the income elasticity for wheat.[24] One way of evaluating the relative plausibility of the three sets of estimates shown for urban populations in Table 4 is to compute a calorie-income elasticity, which is roughly the average of the income elasticities shown in Table 4 weighted by calorie shares. The 1984/85 data indicate that at mean income levels, wheat provides about two-thirds of calorie availability. For the AIDS estimates, this indicates a calorie-income elasticity certainly in excess of 0.5. Bouis (1994) argues that such a calorie-income elasticity is implausibly high since this would indicate weight differences of perhaps 100 percent or more per person, on average, across income groups, under a reasonable set of assumptions about activity levels, adaptation to energy stress, and weight losses/gains over time.

By contrast, the calorie-income elasticity estimate implied by the FCDS-I vector of income elasticities is on the order of 0.15. This estimate is perhaps still somewhat high, but certainly much closer to the plausible range.[25] A "low" calorie-income elasticity is an implicit "constraint" associated with the particular combination of utility function parameters specified a priori. In this regard, it is instructive to note that for the FCDS-II estimates, lowering the exogenously assumed nonfood income

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[24] There were some differences in how the food groups were defined and aggregated (see the notes at the bottom of Table 2). In particular, for the AIDS estimates dairy included products other than milk whose income elasticities are higher than for milk, while for the FCDS estimates, these other dairy products were included in the "other foods" category. This may account for some of the discrepancy between the income elasticity estimates for these two groups between techniques.

[25] The FCDS-II estimates imply a calorie-income elasticity in excess of 0.2. For this reason, it seems unlikely that the nonfood income elasticity is as low as the AIDS estimate.

elasticity (increasing the food income elasticity) has little effect on the wheat income elasticity; it is the income elasticities of other foods that increase.

The FCDS rural estimates do not differ greatly from the FCDS urban estimates; consumption patterns and prices are not dramatically different between urban and rural populations after income is controlled, and assumed utility function parameters are similar. For the AIDS estimates, however, the own-price elasticity is substantially higher for wheat and rice in rural areas as compared with urban areas, and is lower for meats. Other than supply-side income effects of price changes, it is unclear why rural consumers (who do not demonstrate dramatically different consumption patterns) would react so differently to price changes than urban consumers. This comparison serves to emphasize that the FCDS estimates, other than energy expended in earning income, are based on demand-side assumptions. They take no account of the effects of changes in prices on income or interactions between semi-subsistence production and consumption decisions. Policy simulations using the FCDS estimates must treat these supply-side income effects separately and explicitly.

## 5. PRICE-RESPONSIVENESS AND LEVEL OF INCOME

As stated in the introduction, an objective of this paper is to investigate the circumstances under which low-income groups are more price-responsive than high-income groups. This question may now be addressed empirically by combining the earlier discussion of Table 1 in section 3 with recalculations (under alternative assumptions) of the food demand matrices for Pakistan discussed in section 4.

In Table 5, several FCDS estimates of own-price elasticities for individual foods for the Pakistan urban *lowest* income quartile are generated by varying $E_{umx}$, $w_v$, and $w_e e_3$ over a wide range, holding observed food consumption levels and prices constant. This serves two purposes. First, it affords a concrete demonstration of the empirical links in the FCDS framework between the underlying demand for specific food characteristics and the degree of substitutability and price-responsiveness.[26] In particular, it provides an empirical check of the conclusions reached in Table 1. Second, general patterns of price-responsiveness with income (the "curvature" in the Slutsky matrix referred to by Timmer 1981) for particular categories of foods (staples and nonstaples; energy-intensive, variety-intensive, and taste-intensive) may be deduced.

[26] Note from Table 4 that the distinction between compensated and uncompensated price elasticities is not important empirically for individual food groups.

Table 5—FCDS own-price elasticities for Pakistan, urban lowest expenditure quartile, utility function parameters for energy and variety varied as shown, nonfood income elasticity held constant

| | Utility from Energy Parameters | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Utility from | $w_e e_3$ = -0.03 | | | $w_e e_3$ = -0.06 | | | $w_e e_3$ = -0.09 | | |
| Variety | $E_{UMX}$ | | | $E_{UMX}$ | | | $E_{UMX}$ | | |
| Parameter | 2,000 | 3,000 | 4,000 | 2,000 | 3,000 | 4,000 | 2,000 | 3,000 | 4,000 |
| $w_v$ = 0.30 | | | | | | | | | |
| Wheat | -.34 | -.38 | -.43 | -.19 | -.22 | -.28 | -.13 | -.17 | -.32 |
| Rice | -.92 | -1.05 | -1.24 | -.92 | -1.25 | -1.97 | -.93 | -1.55 | -4.92 |
| Milk | -.97 | -.99 | -1.02 | -.97 | -1.03 | -1.08 | -.97 | -1.06 | -1.15 |
| Meat | -1.03 | -1.04 | -1.06 | -1.03 | -1.07 | -1.11 | -1.03 | -1.09 | -1.16 |
| Vegetables | -1.00 | -1.02 | -1.03 | -1.00 | -1.04 | -1.08 | -1.00 | -1.06 | -1.12 |
| Fruits | -1.06 | -1.08 | -1.09 | -1.06 | -1.09 | -1.12 | -1.06 | -1.10 | -1.14 |
| Others | -1.03 | -1.05 | -1.07 | -1.03 | -1.08 | -1.12 | -1.04 | -1.10 | -1.18 |
| Nonfoods | -1.33 | -1.34 | -1.36 | -1.29 | -1.32 | -1.35 | -1.27 | -1.32 | -1.41 |
| $w_v$ = 0.70 | | | | | | | | | |
| Wheat | -.45 | -.51 | -.59 | -.22 | -.25 | -.31 | -.14 | -.18 | -.27 |
| Rice | -.84 | -.95 | -1.10 | -.84 | -1.10 | -1.62 | -.84 | -1.33 | -3.18 |
| Milk | -.98 | -1.01 | -1.03 | -.98 | -1.04 | -1.10 | -.98 | -1.07 | -1.18 |
| Meat | -1.06 | -1.08 | -1.10 | -1.06 | -1.10 | -1.15 | -1.06 | -1.13 | -1.20 |
| Vegetables | -1.05 | -1.07 | -1.09 | -1.05 | -1.09 | -1.14 | -1.05 | -1.12 | -1.20 |
| Fruits | -1.17 | -1.18 | -1.20 | -1.17 | -1.20 | -1.23 | -1.17 | -1.22 | -1.27 |
| Others | -1.07 | -1.09 | -1.12 | -1.07 | -1.12 | -1.17 | -1.07 | -1.14 | -1.23 |
| Nonfoods | -1.25 | -1.26 | -1.28 | -1.20 | -1.22 | -1.25 | -1.18 | -1.22 | -1.28 |
| $w_v$=1.10 | | | | | | | | | |
| Wheat | -.65 | -.79 | -.99 | -.25 | -.30 | -.37 | -.16 | -.19 | -.27 |
| Rice | -.79 | -.88 | -1.00 | -.77 | -.99 | -1.38 | -.77 | -1.16 | -2.35 |
| Milk | -1.04 | -1.07 | -1.11 | -1.05 | -1.12 | -1.20 | -1.05 | -1.16 | -1.30 |
| Meat | -1.10 | -1.12 | -1.14 | -1.10 | -1.14 | -1.19 | -1.10 | -1.17 | -1.25 |
| Vegetables | -1.16 | -1.19 | -1.22 | -1.16 | -1.22 | -1.29 | -1.16 | -1.26 | -1.37 |
| Fruits | -1.30 | -1.32 | -1.34 | -1.30 | -1.34 | -1.38 | -1.30 | -1.36 | -1.43 |
| Others | -1.12 | -1.15 | -1.19 | -1.11 | -1.17 | -1.23 | -1.12 | -1.19 | -1.29 |
| Nonfoods | -1.22 | -1.24 | -1.26 | -1.15 | -1.17 | -1.19 | -1.13 | -1.16 | -1.21 |

Note: Calorie consumption for this group is 1,935 per day per adult equivalent.

In Table 5, comparing estimates of declining levels of $E_{umx}$, holding $w_v$ and $w_e e_3$ constant, simulates a reduction in $E_{umx}$-E (column [1] in Table 1). Comparing estimates of declining levels of $w_e e_3$, holding $E_{umx}$ and $w_v$ constant, provides a check of the conclusions cited in column (2) in Table 1. Comparing estimates of increasing (declining) levels of $w_v$ (see columns (3) and (4) in Table 1), holding $E_{umx}$ and $w_e e_3$ constant, simulates an increase in $M/T^2$ (a reduction in $(T\text{-}M)/T^2$).[27] These comparisons are summarized in Table 6. Almost all of the conclusions reached in Table 1 are substantiated empirically.

Will low-income groups *always* be more price-responsive than high-income groups? If not, under what circumstances will low-income groups be more price-responsive? A review of Tables 1 and 6 suggests the following conclusions:

1. Analysis of differing levels of price-response at increasing levels of income should differentiate between staple and nonstaple foods and, within these two broad categories, should differentiate between foods that are energy-intensive, variety-intensive, and taste-intensive.
2. The primal desire to avoid hunger, to meet certain minimum calorie intakes, could constrain price response for staple foods at very low income levels, particularly in the range where there is a positive response to income of intakes of calories from food staples (in the aggregate). In such cases, price

[27] It should not be inferred that $w_v$ actually declines with income. Rather, a reduction in $w_v$ has the same effect *mathematically* as a decline in $(T\text{-}M)/T^2$.

Table 6—Summary of simulation results of price-responsiveness from Table 5

| Four Categories of Foods in the Diet as Characterized by FCDS | Direction of Price-Responsiveness Due to the Following Factors, Which Simulate Effects of Increase in Income: | | | |
|---|---|---|---|---|
| | Reducing $E_{UMX}$ (see column [1] in Table 1) | Reducing $w_e e_3$ (see column [2] in Table 1) | Increasing $w_v$ (see column [3] in Table 1) | Reducing $w_v$ (see column [4] in Table 1) |
| Wheat (Energy-intensive staple food; see row (A) in Table 1) | Declining | Increasing | Increasing | ... |
| Rice (Taste-intensive staple food; see row (B) in Table 1) | Declining | Declining[a] | Declining | ... |
| Vegetables (Variety-intensive nonstaple food; see row (C) in Table 1) | Declining | Declining | ... | Declining[b] |
| Meat (Taste-intensive nonstaple food; see row (D) in Table 1) | Declining | Declining | ... | Declining |

[a] This holds for $w_v$ = 0.30, but price-responsiveness is increasing for $w_v$ = 1.10; this change in direction is discussed in footnote 23.

[b] The increase in Tii (in absolute value) apparently dominates the decline in Vii for this data set; the hypothesis stated in Table 1 is that the opposite would hold true.

elasticities of cheaper food staples eaten heavily by poor may increase at higher income levels as calorie intakes from food staples (in the aggregate) increase. Within the FCDS framework, *high shadow prices for energy are inconsistent with high price-responsiveness for heavily consumed food staples.*[28]

3. A desire for variety in the diet increases the substitutability of many foods, foods with very different characteristics, particularly at low income levels. As income and variety in the diet increase and marginal utility for more variety falls, ceteris paribus price elasticities for these foods will decline (in absolute value).

4. Price elasticities of nonstaple foods, which tend to be taste-intensive, usually will decline with income. The intrinsic characteristics of individual foods (tastes) are inherently nonsubstitutable. Exceptions (increasing price elasticities over particular income ranges) may occur for inexpensive, nonstaple foods that are purchased by low-income households, primarily to provide something to eat with bland staple foods.

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[28] The two exceptions would be (1) the case described in footnote 18 wherein the rankings of staple foods by calorie cost changes due to seasonal price variability, and (2) the case where a higher value for $E_{umx}$-E for low-income groups empirically dominated the effect of a higher value for $w_e e_3$.

## 6. CONCLUSIONS AND DIRECTIONS FOR FUTURE RESEARCH

A demand for characteristics framework has been used to explain food acquisition behavior, in particular, demand for energy to alleviate hunger, demand for variety in the diet, and demand for tastes inherent in particular foods. This framework has been used to identify underlying factors that can account for the observation that, in most empirical studies in the literature, low-income groups in poor countries have demonstrated a greater responsiveness to changes in food prices than high-income groups. This framework can also explain the less frequently observed phenomenon of highest price elasticities for middle-income groups for food staples.

Apart from providing a plausible explanation for variation in price-responsiveness across income groups, development of this framework is potentially beneficial for future food policy analysis in two ways. First, the FCDS provides a methodology for computing food demand matrices with data that are often available in published form, and which therefore has the potential for substantially lowering the costs of food policy analysis. By specifying an explicit functional form for food characteristics in the utility function, it turns out that the entire matrix of price and income elasticities can be derived for a system of $n$ foods and one nonfood good from prior knowledge of just four elasticities in the $(n + 1)$ by $(n + 2)$ matrix of price and income elasticities.

Energy and variety enter the utility function in such a way that utility from consumption of any one food depends on the level of consumption of all other foods.

This avoids assumptions of strong or weak separability among food groups, which underlie existing expenditure systems and which is inappropriate for estimating a highly disaggregate food demand matrix necessary for many types of food policy analysis. A wide range of price and income elasticities can be accommodated by the FCDS, depending on relative weights assigned to the three food characteristics specified in the model.

The demand system was applied to published aggregate Pakistan data. The model was found to generate food demand estimates that are broadly similar to estimates obtained employing a widely-used, but more data-intensive and labor-intensive technique.

Second, the FCDS provides a structure for investigating and measuring a small set of underlying behavioral factors that are hypothesized to determine the observed magnitudes of price and income elasticities for specific foods. Standard econometric estimates provide little guidance for interpretation of elasticity magnitudes, apart from familiar, but hardly illuminating, labels such as "inferior," "necessities," "luxuries," and "substitutes."

Specifically, a link is provided between changes in activity patterns (energy expenditures) and changes in price and income elasticities through shifts in $E_{umx}$ and $w_e e_3$. The food-characteristic framework also provides insights into how demand elasticities might change if the relative price of a food were to fall outside of the observed range, or if a new food is introduced. Specifying utility as a function of food characteristics defines "roles" (for example, inexpensive sources of energy or variety) that various foods play in satisfying basic needs.[29]

These two properties of hedonic demand systems, (1) that they provide an empirical connection between underlying factors that drive demand behavior and elasticity magnitudes and (2) that they economize on the number of parameters that need to be estimated in the complete demand matrix, have been known for some time. From this perspective, the FCDS is a restricted form (a special case) of a more general model, which is already well-known. Its particular restrictions have yet to be formally tested. Nevertheless, the search for particular specifications that would take advantage of these two properties so as to elucidate food consumption behavior and to reduce the costs of policy analysis, has not been pursued vigorously in the literature.[30]

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[29] For example, a study of demand for potatoes and sweet potatoes in Asia, using the FCDS framework, showed that these foods are often more expensive sources of calories than more widely consumed staples, even though these roots and tubers are often perceived as "inferior staples." Higher-calorie-cost foods might be expected to have higher income elasticities than primary staples. However, if potatoes and sweet potatoes presently provide an inexpensive source of *variety* in diets, this could best explain the low (but positive) income elasticities sometimes observed for these foods. If production costs could be significantly reduced so that these roots and tubers could compete with the main staples as a source of inexpensive calories (which may or may not be technically feasible), demand could increase substantially (Bouis 1991b).

[30] While there is a large literature on estimating shadow prices for various characteristics, typically, these shadow price estimates are not then linked to magnitudes of price elasticities for specific goods.

In the broader, nonhedonic demand literature, the tendency has been toward less and less restrictive specifications, involving direct estimation of demand elasticities for specific goods. While this approach is no doubt useful in terms of the level of rigor it provides, its usefulness does depend to some extent on the quality of the data being used (and the cost of its collection and analysis). Less restrictive specifications applied to data that have systematic errors may generate even more biased estimates (for example, see Bouis 1994). Empirical tests that do not confirm restrictive, but seemingly well-conceived, hypotheses can raise questions about the quality of the data, not only the usefulness of the theories.

In search of a such workable specification, then, the following key concepts and procedures have been proposed: (1) energy provides positive utility in some ranges of consumption, but negative utility in other ranges; utility from this characteristic is defined by the magnitudes of two parameters; (2) variety in the diet is specified as a characteristic that is directly measured across foods; it is not subsumed as a food-specific characteristic motivating consumption of individual foods; increased consumption of food staples reduces variety in the diet; (3) substitutability between foods may depend on trade-offs in utility between quite different characteristics; the desirability of these trade-offs will vary by income level; and (4) price and income elasticities for goods that are estimated using conventional econometric methods may be used to identify values for utility function parameters. This paper has endeavored to demonstrate the practicality of making very specific restrictive assumptions, using a characteristics demand framework, so as to indicate a direction in which further research and more rigorous testing could provide a high return.

# APPENDICES

## APPENDIX 1

## SOLVING THE UTILITY FUNCTION TO DERIVE A COMPLETE SET OF OWN-PRICE, CROSS-PRICE, AND INCOME ELASTICITIES

### METHOD USED FOR SOLVING FOR DEMAND ELASTICITIES

The mathematical method and notation used for solving for the demand elasticities is that outlined in Henderson and Quandt (1980, 25-35). The basic strategy is to obtain values for each element in the bordered Hessian matrix of second derivatives of the utility function, and then to compute demand elasticities based on formulas that involve determinants of various subcomponents of this bordered Hessian matrix (see equations 2-30, 2-31, and 2-37 in Henderson and Quandt [1980]).

The demand system defined in equations (1) through (13) in the text is a hybrid of the Frisch (1959) technique in the limited sense that (1) utility derived from foods and nonfoods is treated as strongly separable and (2) no explicit functional form is specified for utility derived from nonfoods. However, any similarities between the food characteristic demand system (FCDS) and the Frisch methodology end there.

The FCDS differs substantially from the Frisch methodology in that explicit functional forms are proposed for utility derived from characteristics of individual foods and these utilities derived from individual foods are very much interdependent. In order (1) to illustrate the mathematical approach to be used and (2) to contrast the

FCDS methodology with the Frisch methodology, the basic results of the Frisch methodology are derived below for a four food (rows 1 through 4) and one nonfood (row 5) demand system.

To derive the Frisch methodology results, begin with the bordered Hessian matrix of second derivatives of the utility function. Except for the price borders, all off-diagonal elements are zero because of an assumption of want-independence:

$$\begin{bmatrix} f_{11} & 0 & 0 & 0 & 0 & -p_1 \\ 0 & f_{22} & 0 & 0 & 0 & -p_2 \\ 0 & 0 & f_{33} & 0 & 0 & -p_3 \\ 0 & 0 & 0 & f_{44} & 0 & -p_4 \\ 0 & 0 & 0 & 0 & f_{55} & -p_5 \\ -p_1 & -p_2 & -p_3 & -p_4 & -p_5 & 0 \end{bmatrix}.$$

To develop an expression for $f_{55}$, equations (18) and (19) below give expressions for the income elasticity for the nonfood good and for $\lambda$, which can be derived from the bordered Hessian matrix of second derivatives as shown below (see equations 2-31 and 2-36 in Henderson and Quandt [1980]):

$$\frac{Y}{q_5}\,\frac{\partial q_5}{\partial Y} = \frac{Y}{q_5}\,\frac{-D_{65}}{D}, \tag{18}$$

$$\frac{Y}{\lambda}\,\frac{\partial \lambda}{\partial Y} = \frac{Y}{\lambda}\,\frac{-D_{66}}{D}, \tag{19}$$

where:

$D_{65}$ = the cofactor of the element in the sixth row and fifth column of the bordered Hessian (the negative of the determinant of this element),

$D_{66}$ = the cofactor of the element in the sixth row and sixth column of the bordered Hessian (the determinant of this element), and

D = the determinant of the entire bordered Hessian.

Let *$D_{44}$* designate the determinant of the matrix formed by the first four rows and four columns in the bordered Hessian. Dividing equation (18) by (19), expanding $D_{65}$ by the elements in the fifth row, and expanding $D_{66}$ by the elements in the fifth column gives equation (20):

$$\frac{\eta_5}{\phi} = \frac{\lambda}{q_5} \frac{-(-p_5|D_{44}|)}{f_{55}|D_{44}|}, \qquad (20)$$

where:

$\eta_5$ = the income elasticity for nonfoods,

$\phi$ = Frisch's flexibility of the marginal utility of income, or money flexibility,

$D_{65}$ = -(-$P_5$*$D_{44}$*), and

$D_{66}$ = $f_{55}$*$D_{44}$* = $\frac{Y}{\lambda} \frac{\partial\lambda}{\partial Y}$.

Solving equation (20) for $f_{55}$ gives equation (21):

$$f_{55} = \lambda \frac{p_5}{q_5} \frac{\phi}{\eta_5}. \qquad (21)$$

Given data for $p_5$ and $q_5$ and values for $\lambda$, $\phi$, and $\eta_5$, a value for $f_{55}$

may be computed. Furthermore, given additional values for the income elasticities of the four foods and data on their prices and quantities, values can be computed for all of the elements in the bordered Hessian matrix above. It follows that own-price and cross-price elasticities may be computed, using equations 2-30 and 2-37 in Henderson and Quandt (1980). Elasticity values do not depend on $\lambda$, so that the entire matrix of own-price and cross-price elasticities may be computed, given (1) prior knowledge of all income elasticities and $\phi$ and (2) data for prices and quantities.

The reader may now skip to the next section to see how this methodology is applied to the FCDS formulation. However, to continue the illustration of this specific mathematical approach, the familiar Frisch expressions for the own-price and cross-price elasticities are developed below.

First, the determinant of the entire bordered-Hessian matrix above may be written as

$$D = f_{11}f_{22}f_{33}f_{44}f_{55}\left[\frac{p_1p_1}{f_{11}} + \frac{p_2p_2}{f_{22}} + \frac{p_3p_3}{f_{33}} + \frac{p_4p_4}{f_{44}} + \frac{p_5p_5}{f_{55}}\right].$$

But, from equation (21), $p_ip_i/f_{ii} = \theta_i\eta_i/\lambda\phi$ ($\theta$=the budget share), so that the expression for D above may be rewritten as

$$\begin{aligned} D &= \frac{f_{11}f_{22}f_{33}f_{44}f_{55}}{\lambda\phi}\left[\theta_1\eta_1 + \theta_2\eta_2 + \theta_3\eta_3 + \theta_4\eta_4 + \theta_5\eta_5\right] \\ &= \frac{f_{11}f_{22}f_{33}f_{44}f_{55}}{\lambda\phi} \end{aligned} \qquad (22)$$

Let $D_{55}$ be the cofactor of $f_{55}$:

$$D_{55} = f_{11}f_{22}f_{33}f_{44}\left[\frac{p_1p_1}{f_{11}} + \frac{p_2p_2}{f_{22}} + \frac{p_3p_3}{f_{33}} + \frac{p_4p_4}{f_{44}}\right]. \quad (23)$$
$$= \frac{f_{11}f_{22}f_{33}f_{44}}{\lambda\phi}\left[1 - \theta_5\eta_5\right]$$

Let $D_{65}$ be the cofactor of the element in the sixth row and fifth column:

$$D_{65} = -p_5f_{11}f_{22}f_{33}f_{44}. \quad (24)$$

Using equation 2-30 from Henderson and Quandt (1980),

$$\epsilon_{55} = \frac{p_5}{q_5}\left[\frac{D_{55}\lambda}{D} + \frac{q_5D_{65}}{D}\right]. \quad (25)$$

Substituting equations (22), (23), and (24) into equation (25) gives

$$\epsilon_{55} = \frac{p_5}{q_5}\left[\frac{(1 - \theta_5\eta_5)\lambda}{f_{55}} - \frac{p_5q_5}{\left(\frac{f_{55}}{\lambda\phi}\right)}\right]. \quad (26)$$

Substituting for $f_{55}$ from equation (21) into equation (26) gives

$$\epsilon_{55} = \frac{\eta_5(1 - \theta_5\eta_5)}{\phi - \theta_5\eta_5}. \quad (27)$$

Equation (27) gives the expression for the own-price elasticity. To derive the expression for the cross-price elasticity, let $D_{15}$ be the cofactor for the element in the first row and fifth column:

$$D_{15} = -p_1p_5f_{22}f_{33}f_{44}. \quad (28)$$

Using equation 2-37 from Henderson and Quandt (1980),

$$\epsilon_{51} = \frac{p_1}{q_5}\left[\frac{D_{15}\lambda}{D} + \frac{q_1 D_{65}}{D}\right]. \tag{29}$$

Substituting equations (22), (23), and (28) into equation (29) gives

$$\epsilon_{51} = \frac{p_1}{q_5}\left[\frac{(-p_1 p_5 \phi\lambda\lambda)}{(f_{11} f_{55})} - \frac{p_5 q_1}{\left(\frac{f_{55}}{\lambda\phi}\right)}\right]. \tag{30}$$

Substituting for $f_{11}$ and $f_{55}$ from equation (21) into equation (30) gives

$$\epsilon_{51} = \frac{-\theta_1 \eta_1 \eta_5}{\phi} - \theta_1 \eta_5. \tag{31}$$

It is possible to derive these relatively simple expressions, equations (27) and (31), for the own-price and cross-price elasticities, because the off-diagonal elements (except for the price borders) in the bordered Hessian matrix above are zeros. Despite considerable effort, because all of the off-diagonal elements in the bordered Hessian matrix are nonzero, no relatively simple expressions for the own-price, cross-price, and income elasticities have been developed for the FCDS. As described in the following section, however, the same basic strategy is followed of finding values for each of the elements in the bordered Hessian matrix of second derivatives and then computing the relevant determinants as given in equations 2-30, 2-31, and 2-37 in Henderson and Quandt (1980).

# THE BORDERED HESSIAN MATRIX FOR A FOUR-FOOD-AND-ONE-NONFOOD-GOOD APPLICATION OF THE FOOD-CHARACTERISTIC DEMAND SYSTEM (FCDS)

The demand system consists of four foods and one nonfood. While all of the analysis undertaken in this appendix is for a four-food-and-one-nonfood-good system, the results hold more generally for a system of *n* foods and one nonfood good.

$$\begin{bmatrix} E_{11}+V_{11}+T_{11} & E_{12}+V_{12} & E_{13}+V_{13} & E_{14}+V_{14} & 0 & -p_1 \\ E_{21}+V_{21} & E_{22}+V_{22}+T_{22} & E_{23}+V_{23} & E_{24}+V_{24} & 0 & -p_2 \\ E_{31}+V_{31} & E_{32}+V_{32} & E_{33}+V_{33}+T_{33} & E_{34}+V_{34} & 0 & -p_3 \\ E_{41}+V_{41} & E_{42}+V_{42} & E_{43}+V_{43} & E_{44}+V_{44}+T_{44} & 0 & -p_4 \\ 0 & 0 & 0 & 0 & f_{55} & -p_5 \\ -p_1 & -p_2 & -p_3 & -p_4 & -p_5 & 0 \end{bmatrix}$$

Rows and columns five and six in the above bordered Hessian matrix are identical to those in the matrix presented in the previous section. However, the remaining elements in the first four rows and columns now contain the expressions for second derivatives (the notation is identical to that presented in the text) of the FCDS utility function, which are reproduced below for convenience (including the first-order conditions; equation numbers are identical to those used in the text):

<u>Energy</u>

$$E_{ij} = 2w_e e_3 z_i z_j < 0, \tag{5}$$

<u>Taste</u>

$$T_{ii} = -w_{ti}(1/q_i)^2 < 0, \tag{8}$$

<u>Variety (a Food Staple)</u>

$$V_{ij} = \frac{2w_v M}{T^3} > 0 \qquad \text{for } i,j \leq s, \tag{12}$$

<u>First-Order Condition (a Food Staple)</u>

$$p_1 = \frac{w_e}{\lambda}\left[e_2 z_1 + 2e_3 z_1 E\right] + \frac{w_v}{\lambda}\left[\frac{-M}{T^2}\right] + \frac{w_{t1}}{\lambda}\left[\frac{1}{q_1}\right]. \tag{15}$$

The objective is to derive specific values for all of the elements in the bordered Hessian matrix above so that the complete matrix of demand elasticities then may be computed (using equations 2-30, 2-31, and 2-37 from Henderson and Quandt [1980]). Prices and quantities (and so, M and T) are observed (say from household survey data), as are the calorie conversion rates (the $z_i$'s), and total calorie consumption (E). Given prior knowledge of $w_e e_3$, all the expressions for $E_{ij}$ may be computed. Given prior knowledge of $w_v$, all the expressions for $V_{ij}$ may be computed. Given prior knowledge of $w_e e_2$ (and $w_e e_3$ and $w_v$), all the expressions for $T_{ii}$ may be computed by solving for the $w_{ti}$'s from the first-order conditions (elasticities do not depend on $\lambda$). As outlined in section 3 in the text, $E_{umx}$ and $w_e e_3$ may be used to solve for $w_e e_2$. Given prior knowledge of $\phi/\eta_{nf}$ (where $\eta_{nf}$ is the nonfood income elasticity), a value may be computed for $f_{55}$ from equation (21). Therefore, given prior values for only four parameters, values may be obtained for all of the elements in the bordered Hessian matrix.

There are two general methods for identifying values for these four parameters: (1) obtaining direct estimates of $w_e e_3$, $w_e e_2$ (or $E_{umx}$), $w_v$, and $\phi/\eta_{nf}$, *or* (2) obtaining estimates of $w_e e_3$, $w_e e_2$, $w_v$, and $\phi/\eta_{nf}$ indirectly by prior specification of four elasticities in the complete demand matrix. Use of some combination of approaches (1) and (2) is also possible.

Before proceeding with the second approach, some brief comments concerning the first approach are useful. In future research, equation (16) in the main text may provide a structure for direct estimation of $w_e e_3$, $w_e e_2$, and $w_v$, although several econometric problems would need to be resolved. There are several examples in the literature of attempts to estimate $\phi$ and $\eta_{nf}$ separately. From the nutrition literature (for example, Bliss and Stern 1978), it can be said with some certainty that $2{,}000 < E_{umx} < 3{,}500$, where calories are expressed on an adult equivalent per day basis and intakes are understood as an average for a population group (the calorie range will be wider for specific individuals with different metabolisms, activity patterns, and weights). Thus, of the four pieces of prior information required, some knowledge of what are "reasonable" magnitudes for two of these pieces of information ($E_{umx}$ and $\phi/\eta_{nf}$) are already available in the literature.

## AN ALGORITHM FOR SOLVING THE MODEL

The second approach is now discussed, that of obtaining estimates of $w_e e_3$, $w_e e_2$ (or $E_{umx}$), $w_v$, and $\phi/\eta_{nf}$ *indirectly* by prior specification of four elasticities in the complete demand matrix. Because, despite some considerable effort, attempts at deriving relatively simple expressions (such as equations [27] or [31]) from either the first-order conditions or the bordered Hessian matrix have not been successful, it has been necessary to develop a FORTRAN program that searches for a (hopefully unique) combination of $w_e e_3$, $w_e e_2$ (or $E_{umx}$), $w_v$, and $\phi/\eta_{nf}$ that generates the four elasticities in the demand matrix that are specified a priori.

The basic structure of that FORTRAN program is (1) to read in initial, "seed" values for $E_{umx}$, $w_e e_3$, $w_v$, and $\phi/\eta_{nf}$, (2) to compute values for all of the elements in the bordered Hessian matrix using these seed values, from which (3) an initial, complete demand elasticity matrix can be computed. Next, the FORTRAN program (4) compares the four elasticities in the initial matrix with the four "target" elasticities (the four elasticities specified a priori) and, based on this comparison, (5) makes appropriate adjustments in the "seed" values. Steps (2) through (5) are iterated until the four elasticities in the matrix computed from the (several times revised) "seed" values match with/converge to the four "target" elasticities—at which point, the solution for the "target" complete demand matrix has been found.

There is no guarantee, mathematically speaking, that some combination of $E_{umx}$, $w_e e_3$, $w_v$, and $\phi/\eta_{nf}$ can be found that generates a specific set of target elasticities. It is useful to mention two sets of constraints with respect to the feasibility of the solution. First, in order to ensure that the final solution is consistent with utility maximization, a requirement is that various bordered Hessian determinants must alternate in sign (Henderson and Quandt 1980, 33). Second, a more restrictive constraint is that a solution can be found within feasible ranges for $E_{umx}$ and $\phi/\eta_{nf}$.

The feasible range for $E_{umx}$ (between 2,000 and 3,500 calories per adult equivalent per day) has already been discussed above. $\eta_{nf}$ will be greater than one, but rarely greater than two. $\phi$ may range from (say) -1 to -3. De Janvry, Bieri, and Nunez (1972) derive values ranging from -2.2 to -2.6 from a regression using estimates from a number of studies in the literature; Pinstrup-Andersen, Londoño, and Hoover (1976) derive an estimate of about -1.0), so that $\phi/\eta_{nf}$ may range from -0.5 to -3.0, with a distribution of values centering around (say) -2.5/1.5 = -1.67. In all applications of the FCDS thus far undertaken (using various data sets from seven countries), solutions have been found for $E_{umx}$ and $\phi/\eta_{nf}$ that are well within these bounds.

# APPENDIX 2

## FURTHER DISCUSSION AND DETAILED RESULTS FROM PAKISTAN DEMAND ESTIMATIONS

This appendix contains a more extended discussion of the demand estimates by income group for Pakistan than that presented in section 3. Parts of section 3 are imbedded in the text below so as to provide a logical sequence of thought.

### THE DATA

Apart from prior knowledge of any four elasticities in the $(n + 1)$ by $(n + 2)$ demand matrix for a system of $n$ foods, data requirements are (1) per capita quantities consumed for each of the $n$ (in this case, seven) food groups, (2) prices paid per kilogram for each food group, (3) calorie conversion rates per kilogram for each food group, (4) total nonfood expenditures, and (5) the ratio of adult equivalents over total household members. Data for calorie conversion rates and the age and gender structure of an average household typically are not available from published summaries of expenditure survey data.

Estimates of per kilogram (as purchased) calorie conversion rates for disaggregate food groups may be obtained from country-specific food composition tables and dietary surveys. For specific foods (for example, wheat and rice), these conversion rates will not vary greatly across countries, so that, as is the case here, estimates available from other countries may be used. The assumed food-group-specific calorie conversion rates are presented in Table 7 (along with quantities consumed and prices paid, which are discussed below). Using these assumed calorie conversion rates, per adult equivalent consumption of calories per day ranged from a low of 1,935 for the lowest income quartile in urban areas to a high of 2,910 for the highest income quartile in rural areas.

Food expenditure surveys will tend to exaggerate the increase in calorie consumption as incomes increase, so that data for food quantities consumed collected using a food recall technique are to be preferred over information from a food expenditure survey (Bouis and Haddad 1992; Bouis, Haddad, and Kennedy 1992). Unfortunately, use of food recall techniques is rare for a national sample, and only food expenditure information is available for Pakistan.

Bliss and Stern (1978) argue quite persuasively, using a priori reasoning based on results found in the nutrition literature, that for moderately active populations and assuming wide bounds to account for various sources of measurement error, one would not expect to observe populations consuming below 2,000 calories per day per adult equivalent (which were not losing weight, on average) or above 3,000 calories per day per adult equivalent (which were not gaining weight, on average). Using the assumed calorie conversion rates for the

Table 7—Per capita consumption, price, and calorie conversion rates for seven aggregate food groups, by income quartile, by urban and rural populations for Pakistan, 1984/85

| Urban/ Rural | Income Quartile | Food | Per Capita Consumption[a] | Market Price[b] | Calories per Kilogram[c] | Calorie Price[d] | Calorie Share | Food Budget Share | Staple? |
|---|---|---|---|---|---|---|---|---|---|
| Urban | 1 | Wheat | 2.11 | 1.00 | 3.40 | 1.00 | 0.74 | 0.21 | Yes |
| Urban | 1 | Rice | 0.15 | 2.13 | 3.50 | 2.07 | 0.05 | 0.03 | Yes |
| Urban | 1 | Milk | 0.97 | 1.86 | 0.60 | 10.54 | 0.06 | 0.18 | No |
| Urban | 1 | Meat | 0.13 | 6.85 | 1.50 | 15.53 | 0.02 | 0.09 | No |
| Urban | 1 | Vegetables | 0.67 | 1.45 | 0.30 | 16.44 | 0.02 | 0.10 | No |
| Urban | 1 | Fruits | 0.11 | 2.31 | 0.30 | 26.19 | 0.00 | 0.03 | No |
| Urban | 1 | Others | 0.54 | 6.97 | 1.80 | 13.17 | 0.10 | 0.37 | No |
| Urban | 2 | Wheat | 2.10 | 1.03 | 3.40 | 1.03 | 0.70 | 0.19 | Yes |
| Urban | 2 | Rice | 0.20 | 2.12 | 3.50 | 2.06 | 0.07 | 0.04 | Yes |
| Urban | 2 | Milk | 1.06 | 1.91 | 0.60 | 10.83 | 0.06 | 0.18 | No |
| Urban | 2 | Meat | 0.15 | 6.99 | 1.50 | 15.85 | 0.02 | 0.09 | No |
| Urban | 2 | Vegetables | 0.69 | 1.50 | 0.30 | 17.01 | 0.02 | 0.09 | No |
| Urban | 2 | Fruits | 0.14 | 2.31 | 0.30 | 26.19 | 0.00 | 0.03 | No |
| Urban | 2 | Others | 0.58 | 7.16 | 2.20 | 11.07 | 0.12 | 0.37 | No |
| Urban | 3 | Wheat | 1.99 | 1.06 | 3.40 | 1.06 | 0.65 | 0.17 | Yes |
| Urban | 3 | Rice | 0.24 | 2.25 | 3.50 | 2.19 | 0.08 | 0.04 | Yes |
| Urban | 3 | Milk | 1.16 | 2.06 | 0.60 | 11.68 | 0.07 | 0.19 | No |
| Urban | 3 | Meat | 0.21 | 7.38 | 1.50 | 16.73 | 0.03 | 0.12 | No |
| Urban | 3 | Vegetables | 0.76 | 1.51 | 0.30 | 17.12 | 0.02 | 0.09 | No |
| Urban | 3 | Fruits | 0.19 | 2.48 | 0.30 | 28.12 | 0.01 | 0.04 | No |
| Urban | 3 | Others | 0.60 | 7.57 | 2.60 | 9.90 | 0.15 | 0.36 | No |
| Urban | 4 | Wheat | 1.96 | 1.11 | 3.40 | 1.11 | 0.57 | 0.12 | Yes |
| Urban | 4 | Rice | 0.29 | 2.54 | 3.50 | 2.47 | 0.09 | 0.04 | Yes |
| Urban | 4 | Milk | 1.57 | 2.20 | 0.60 | 12.47 | 0.08 | 0.18 | No |
| Urban | 4 | Meat | 0.38 | 8.60 | 1.50 | 19.50 | 0.05 | 0.17 | No |
| Urban | 4 | Vegetables | 0.94 | 1.56 | 0.30 | 17.69 | 0.02 | 0.08 | No |
| Urban | 4 | Fruits | 0.32 | 3.07 | 0.30 | 34.81 | 0.01 | 0.05 | No |
| Urban | 4 | Others | 0.71 | 9.72 | 3.00 | 11.02 | 0.18 | 0.36 | No |
| Rural | 1 | Wheat | 2.48 | 1.01 | 3.40 | 1.01 | 0.72 | 0.25 | Yes |
| Rural | 1 | Rice | 0.22 | 2.13 | 3.50 | 2.07 | 0.07 | 0.05 | Yes |
| Rural | 1 | Milk | 1.16 | 1.48 | 0.60 | 8.39 | 0.06 | 0.17 | No |
| Rural | 1 | Meat | 0.10 | 7.12 | 1.50 | 16.15 | 0.01 | 0.07 | No |
| Rural | 1 | Vegetables | 0.63 | 1.45 | 0.30 | 16.44 | 0.02 | 0.09 | No |
| Rural | 1 | Fruits | 0.08 | 2.35 | 0.30 | 26.64 | 0.00 | 0.02 | No |
| Rural | 1 | Others | 0.59 | 5.72 | 2.40 | 8.11 | 0.12 | 0.34 | No |

(continued)

Table 7 (continued)

| Urban/ Rural | Income Quartile | Food | Per Capita Consumption[a] | Market Price[b] | Calories per Kilogram[c] | Calorie Price[d] | Calorie Share | Food Budget Share | Staple? |
|---|---|---|---|---|---|---|---|---|---|
| Rural | 2 | Wheat | 2.53 | 1.05 | 3.40 | 1.05 | 0.68 | 0.23 | Yes |
| Rural | 2 | Rice | 0.33 | 1.76 | 3.50 | 1.71 | 0.09 | 0.05 | Yes |
| Rural | 2 | Milk | 1.47 | 1.51 | 0.60 | 8.56 | 0.07 | 0.20 | No |
| Rural | 2 | Meat | 0.12 | 6.75 | 1.50 | 15.31 | 0.01 | 0.07 | No |
| Rural | 2 | Vegetables | 0.74 | 1.28 | 0.30 | 14.51 | 0.02 | 0.08 | No |
| Rural | 2 | Fruits | 0.10 | 2.40 | 0.30 | 27.21 | 0.00 | 0.02 | No |
| Rural | 2 | Others | 0.63 | 6.17 | 2.40 | 8.74 | 0.12 | 0.34 | No |
| Rural | 3 | Wheat | 2.84 | 1.04 | 3.40 | 1.04 | 0.70 | 0.22 | Yes |
| Rural | 3 | Rice | 0.30 | 1.96 | 3.50 | 1.90 | 0.08 | 0.05 | Yes |
| Rural | 3 | Milk | 1.72 | 1.54 | 0.60 | 8.73 | 0.07 | 0.20 | No |
| Rural | 3 | Meat | 0.15 | 7.40 | 1.50 | 16.78 | 0.02 | 0.09 | No |
| Rural | 3 | Vegetables | 0.66 | 1.48 | 0.30 | 16.78 | 0.01 | 0.07 | No |
| Rural | 3 | Fruits | 0.12 | 2.55 | 0.30 | 28.91 | 0.00 | 0.02 | No |
| Rural | 3 | Others | 0.69 | 6.59 | 2.40 | 9.34 | 0.12 | 0.35 | No |
| Rural | 4 | Wheat | 3.06 | 1.04 | 3.40 | 1.04 | 0.66 | 0.18 | Yes |
| Rural | 4 | Rice | 0.37 | 2.21 | 3.50 | 2.15 | 0.08 | 0.05 | Yes |
| Rural | 4 | Milk | 2.14 | 1.59 | 0.60 | 9.01 | 0.08 | 0.19 | No |
| Rural | 4 | Meat | 0.25 | 7.97 | 1.50 | 18.07 | 0.02 | 0.11 | No |
| Rural | 4 | Vegetables | 0.78 | 1.53 | 0.30 | 17.35 | 0.01 | 0.07 | No |
| Rural | 4 | Fruits | 0.20 | 2.86 | 0.30 | 32.43 | 0.00 | 0.03 | No |
| Rural | 4 | Others | 0.86 | 7.57 | 2.40 | 10.73 | 0.13 | 0.37 | No |

Source: Federal Bureau of Statistics, *Household Income and Expenditure Survey 1984-85*.

[a] Kilograms per capita per week.

[b] Relative to price of cheapest grain calorie source.

[c] '000 calories per kilogram.

[d] Relative to price of cheapest grain calorie source.

seven-food aggregate food groups, then, the estimates of total calorie consumption are in the plausible range.

For each expenditure quartile, calorie availability is higher for rural populations than for urban populations and increases more rapidly for rural populations moving from low to high expenditure quartiles. These patterns are typical of other countries in Asia (Bouis 1989) and may be due both to a real phenomenon, greater energy expenditures in rural areas, and the fact that significant amounts of food purchased by high-income rural households is eaten in the form of in-kind wages or as guest meals by low-income rural households (Bouis and Haddad 1992). However, the divergence between calorie availability and calorie intakes for rural areas would appear to be far less of a problem for the Pakistan data than for food expenditure data for five other countries in Asia (Bouis 1989).

Data on household size were available for the Pakistan expenditure survey used here; however, it was necessary to make assumptions as to the ratio of adult equivalents to total household members. Age and gender structure for countries with similar average household sizes and incomes do not vary a great deal, so that, where this information is otherwise unavailable, rough assumptions may be made for these data inputs as well. The assumptions made for Pakistan are given in the final column of the chart presented later in this section. As suggested by Philippine data (Bouis 1990), the age structure was assumed to increase marginally with increased incomes.

Typically, per capita incomes of households increase during later stages of the life cycle.

The food consumption and price data for Pakistan shown in Table 7 are taken from the *Household Income and Expenditure Survey 1984-85* conducted by the Federal Bureau of Statistics. Prices given in Table 7 are initialized on the cost of wheat for low-income consumers in urban areas. Wheat is a much cheaper source of calories than rice and is overwhelmingly the predominant staple; consumption shows some tendency to decline with income in urban areas. Milk, which has a high food budget share, is a very expensive source of calories relative to wheat and rice, but (in the food-characteristic demand framework) is a relatively inexpensive source of variety. Vegetables are the cheapest source of variety, but a more expensive source of calories than milk. Meats are both an expensive source of calories and an expensive source of variety. Total food budget shares (not shown) range from 70 percent for the lowest expenditure quartile in rural areas to 37 percent for the highest expenditure quartile in urban areas.

## PRIOR ASSUMPTIONS

One further requirement for implementing the methodology is prior knowledge of any combination of four food-demand elasticities and/or four parameters in the utility function, from which shadow prices can be derived for the characteristics of energy, variety, and taste. Below are the assumptions made to fulfill this requirement:

| Urban/ Rural | Income Quartile | Food Income Elasticity | $E_{UMX}$ | $w_e e_3$ | $w_v$ | Ratio of Adult Equivalents Over Household Members |
|---|---|---|---|---|---|---|
| Urban | 1 | 0.60 | 3,100 | -0.088 | 0.70 | 0.71 |
| | 2 | 0.53 | 3,000 | -0.094 | 0.70 | 0.73 |
| | 3 | 0.45 | 2,900 | -0.100 | 0.70 | 0.75 |
| | 4 | 0.35 | 2,800 | -0.108 | 0.70 | 0.77 |
| Rural | 1 | 0.65 | 3,200 | -0.082 | 0.70 | 0.71 |
| | 2 | 0.58 | 3,100 | -0.088 | 0.70 | 0.73 |
| | 3 | 0.50 | 3,000 | -0.094 | 0.70 | 0.75 |
| | 4 | 0.35 | 2,900 | -0.100 | 0.70 | 0.77 |

Food income elasticities assumed for the various expenditure quartiles were selected arbitrarily, although the pattern of selected values adheres to Engel's Law. The sensitivity of the elasticity estimates to these assumptions is addressed in section 3 of the paper (compare the FCDS-I and FCDS-II estimates in Table 4) and is also further discussed below.

The column labeled $E_{umx}$ indicates those levels of calorie consumption (per adult equivalent) at which the marginal utilities to further calorie consumption are zero. These levels are reduced marginally for successive expenditure quartiles under the assumption that activity levels are lower at higher income levels. For the same reason, they are increased marginally for rural populations as compared with urban populations in the same expenditure quartile. Again, although the selected pattern would appear to be reasonable, the levels are selected arbitrarily in the sense that they have not been estimated econometrically.

For the urban third expenditure quartile, two additional assumptions were made—that the income elasticities for wheat and for rice are -0.04 and 0.40, respectively, as suggested by the arc income elasticities for those foods (presented below in Table 9). These prior assumptions along with the data presented in Table 7 permit solution of the model for the urban third income quartile.

Solution of the model, in turn, identifies values for $w_e e_3$ (-0.100) and for $w_v$ (0.70) for the urban third quartile, which can then be used to derive values for these same parameters for the remaining seven population groups. Given $E_{umx}$ for two groups (see the chart above) and $w_e e_3$ for one group (for example, the urban third quartile), gives $w_e e_3$ for the second group. This is derived in Appendix 5 (in particular, see equation [43]). $w_v$ is simply held constant across all population groups, which is a restrictive assumption. In view of the apparent sensitivity of the own-price elasticity of wheat to assumptions about $w_v$ (see Table 5 in the text), this last assumption restricts the range of price response observed across income groups in Table 8 below.

An alternative strategy for deriving a solution for the urban third quartile would have been to choose four elasticities from the Alderman results, which, in turn, would have identified specific values for $E_{umx}$, $w_e e_3$, $w_v$, and $\phi/\eta_{nf}$. If this alternative had been used, the reader might have conjectured that the results for the two systems matched up so well (see the discussion below) because estimates for the AIDS system were used to derive estimates for the FCDS system. That (1) the resulting income elasticity estimates correspond so closely to the arc income elasticities for the data set, that (2) the own-price elasticities estimates (with the exceptions of wheat, rice, and meat for rural areas) are quite similar to the Alderman results, and that (3) the utility function parameters identified for Pakistan are similar to those derived for the Philippines (Bouis 1990), suggest the possibility that some common utility function parameters may underlie demand in developing countries, even though elasticities for specific foods may differ widely between countries.

## THE ELASTICITY ESTIMATES DERIVED USING THE FCDS

Table 8 presents the full matrices of demand elasticity estimates by expenditure quartile and urban and rural populations, derived using the FCDS and the data and assumptions discussed above. Table 9 compares the income elasticity estimates with the arc income elasticities computed from the survey data summarized in Table 7.

Table 9 shows that the estimated income elasticities all decline across income quartiles. In part, this pattern results from (observed) declining food budget shares and assumptions made with respect to food income elasticities. In general, the levels and patterns of declines in income elasticity estimates for individual foods

Table 8—Estimated (uncompensated) food demand elasticities, by expenditure quartile, by urban and rural populations for Pakistan

| | Wheat | Rice | Milk | Meat | Vegetables | Fruits | Others | Nonfoods | Income |
|---|---|---|---|---|---|---|---|---|---|
| Urban first quartile (low income) | | | | | | | | | |
| Wheat | -.35 | .05 | .06 | .03 | -.00 | .00 | .15 | .01 | .05 |
| Rice | .28 | -1.04 | -.01 | .01 | -.03 | -.00 | .04 | .13 | .62 |
| Milk | .01 | .00 | -1.02 | .03 | .22 | .03 | .13 | .11 | .51 |
| Meat | -.05 | -.00 | .01 | -1.09 | .01 | .00 | -.01 | .20 | .93 |
| Vegetables | -.05 | -.00 | .42 | .05 | -1.08 | .04 | .19 | .08 | .36 |
| Fruits | -.09 | -.01 | .20 | .02 | .15 | -1.19 | .09 | .15 | .69 |
| Others | -.03 | -.00 | .01 | -.00 | .01 | .00 | -1.11 | .19 | .92 |
| Nonfoods | -.23 | -.02 | -.11 | -.01 | -.07 | -.01 | -.07 | -1.23 | 1.75 |
| Urban second quartile | | | | | | | | | |
| Wheat | -.34 | .07 | .06 | .03 | -.01 | -.00 | .19 | -.00 | -.00 |
| Rice | .30 | -1.00 | .00 | .01 | -.02 | -.00 | .07 | .14 | .50 |
| Milk | .01 | .00 | -.99 | .03 | .20 | .04 | .13 | .13 | .46 |
| Meat | -.03 | -.00 | .03 | -1.08 | .02 | .00 | -.00 | .24 | .83 |
| Vegetables | -.05 | -.00 | .40 | .05 | -1.07 | .05 | .19 | .10 | .34 |
| Fruits | -.08 | -.01 | .21 | .02 | .14 | -1.17 | .09 | .18 | .61 |
| Others | .01 | .00 | .02 | .00 | .02 | .00 | -1.10 | .23 | .81 |
| Nonfoods | -.19 | -.02 | -.09 | -.01 | -.06 | -.01 | -.06 | -1.21 | 1.65 |
| Urban third quartile | | | | | | | | | |
| Wheat | -.35 | .08 | .05 | .04 | -.01 | -.00 | .24 | -.02 | -.04 |
| Rice | .29 | -.97 | .01 | .02 | -.02 | -.00 | .10 | .18 | .40 |
| Milk | .01 | .00 | -.97 | .04 | .17 | .04 | .12 | .18 | .41 |
| Meat | -.00 | .00 | .04 | -1.07 | .03 | .01 | .00 | .31 | .69 |
| Vegetables | -.05 | -.01 | .38 | .06 | -1.04 | .06 | .19 | .13 | .28 |
| Fruits | -.06 | -.01 | .19 | .03 | .13 | -1.13 | .09 | .24 | .52 |
| Others | .05 | .01 | .04 | .00 | .03 | .01 | -1.09 | .30 | .66 |
| Nonfoods | -.13 | -.02 | -.07 | -.01 | -.05 | -.01 | -.05 | -1.20 | 1.55 |
| Urban fourth quartile | | | | | | | | | |
| Wheat | -.34 | .09 | .06 | .06 | -.02 | -.00 | .29 | -.07 | -.08 |
| Rice | .27 | -.93 | .02 | .03 | -.01 | -.00 | .12 | .24 | .27 |
| Milk | .02 | .00 | -.93 | .05 | .13 | .04 | .11 | .27 | .30 |
| Meat | .02 | .00 | .04 | -1.03 | .03 | .01 | .01 | .43 | .49 |
| Vegetables | -.03 | -.00 | .32 | .08 | -1.00 | .06 | .17 | .20 | .22 |
| Fruits | -.03 | -.00 | .14 | .03 | .08 | -1.06 | .07 | .36 | .41 |
| Others | .07 | .01 | .04 | .01 | .03 | .01 | -1.04 | .41 | .47 |
| Nonfoods | -.07 | -.01 | -.04 | -.04 | -.01 | -.02 | -.04 | -1.18 | 1.38 |

(continued)

Table 8 (continued)

| | Wheat | Rice | Milk | Meat | Vegetables | Fruits | Others | Nonfoods | Income |
|---|---|---|---|---|---|---|---|---|---|
| Rural first quartile (low income) | | | | | | | | | |
| Wheat | -.30 | .07 | .06 | .02 | -.00 | .00 | .18 | -.00 | -.02 |
| Rice | .23 | -.99 | -.02 | .00 | -.03 | -.00 | .05 | .07 | .69 |
| Milk | -.01 | -.00 | -.98 | .03 | .21 | .02 | .14 | .05 | .53 |
| Meat | -.15 | -.01 | -.02 | -1.08 | -.00 | .00 | -.03 | .12 | 1.18 |
| Vegetables | -.12 | -.01 | .39 | .03 | -1.13 | .02 | .15 | .06 | .61 |
| Fruits | -.17 | -.02 | .17 | .01 | .09 | -1.18 | .05 | .10 | .95 |
| Others | -.06 | -.01 | .01 | -.00 | .01 | .00 | -1.14 | .11 | 1.08 |
| Nonfoods | -.33 | -.03 | -.14 | -.01 | -.07 | -.01 | -.09 | -1.15 | 1.82 |
| Rural second quartile | | | | | | | | | |
| Wheat | -.30 | .09 | .06 | .02 | -.00 | .00 | .17 | -.01 | -.03 |
| Rice | .35 | -.93 | .01 | .01 | -.02 | -.00 | .08 | .07 | .43 |
| Milk | -.01 | -.00 | -.94 | .02 | .18 | .02 | .12 | .08 | .52 |
| Meat | -.10 | -.01 | -.00 | -1.07 | .00 | .00 | -.02 | .16 | 1.04 |
| Vegetables | -.08 | -.01 | .44 | .03 | -1.10 | .03 | .15 | .07 | .47 |
| Fruits | -.14 | -.02 | .16 | .01 | .08 | -1.14 | .04 | .13 | .87 |
| Others | -.03 | -.01 | .01 | -.00 | .01 | .00 | -1.10 | .15 | .97 |
| Nonfoods | -.26 | -.04 | -.12 | -.01 | -.06 | -.01 | -.07 | -1.15 | 1.72 |
| Rural third quartile | | | | | | | | | |
| Wheat | -.24 | .07 | .07 | .02 | .00 | .00 | .16 | -.02 | -.06 |
| Rice | .32 | -.90 | .01 | .01 | -.01 | -.00 | .07 | .11 | .39 |
| Milk | .01 | .00 | -.88 | .03 | .14 | .02 | .12 | .12 | .44 |
| Meat | -.06 | -.01 | .01 | -1.05 | .01 | .00 | -.01 | .24 | .87 |
| Vegetables | -.07 | -.01 | .37 | .03 | -1.09 | .02 | .12 | .13 | .49 |
| Fruits | -.10 | -.01 | .17 | .01 | .07 | -1.11 | .05 | .20 | .73 |
| Others | -.01 | .00 | .03 | .00 | .01 | .00 | -1.07 | .22 | .81 |
| Nonfoods | -.21 | -.02 | -.10 | -.01 | -.03 | -.01 | -.06 | -1.16 | 1.61 |
| Rural fourth quartile | | | | | | | | | |
| Wheat | -.22 | .08 | .07 | .03 | .00 | .00 | .17 | -.04 | -.08 |
| Rice | .29 | -.85 | .02 | .01 | -.00 | .00 | .08 | .16 | .30 |
| Milk | .03 | .00 | -.84 | .03 | .12 | .03 | .12 | .18 | .33 |
| Meat | -.01 | -.00 | .03 | -1.03 | .01 | .00 | .00 | .35 | .64 |
| Vegetables | -.03 | -.00 | .33 | .03 | -1.05 | .03 | .12 | .20 | .38 |
| Fruits | -.05 | -.01 | .15 | .01 | .05 | -1.07 | .05 | .30 | .56 |
| Others | .03 | .00 | .04 | .00 | .02 | .00 | -1.02 | .32 | .60 |
| Nonfoods | -.12 | -.02 | -.06 | -.01 | -.02 | -.00 | -.04 | -1.16 | 1.43 |

Table 9—Comparison of arc income elasticities between expenditure quartiles and income elasticity estimates, by expenditure quartiles, by food, and by urban and rural populations, Pakistan

| | | Urban | | | | Rural | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | Arc Income Elasticity Between Expenditure Quartile | | | Estimated Income Elasticity | Arc Income Elasticity Between Expenditure Quartile | | | Estimated Income Elasticity |
| Food | Expenditure Quartile | 2 | 3 | 4 | | 2 | 3 | 4 | |
| Wheat | 1 | -0.02 | -0.09 | -0.03 | 0.05 | 0.07 | 0.21 | 0.12 | -0.02 |
| | 2 | | -0.16 | -0.04 | -0.00 | | 0.38 | 0.16 | -0.03 |
| | 3 | | | -0.01 | -0.04 | | | 0.10 | -0.06 |
| | 4 | | | | -0.08 | | | | -0.08 |
| Rice | 1 | 1.40 | 0.94 | 0.41 | 0.62 | 1.81 | 0.53 | 0.34 | 0.69 |
| | 2 | | 0.62 | 0.27 | 0.50 | | -0.28 | 0.09 | 0.43 |
| | 3 | | | 0.21 | 0.40 | | | 0.31 | 0.39 |
| | | | | | 0.27 | | | | 0.30 |
| Milk | 1 | 0.39 | 0.31 | 0.27 | 0.51 | 0.97 | 0.70 | 0.43 | 0.53 |
| | 2 | | 0.29 | 0.29 | 0.46 | | 0.52 | 0.34 | 0.52 |
| | 3 | | | 0.35 | 0.41 | | | 0.32 | 0.44 |
| | 4 | | | | 0.30 | | | | 0.33 |
| Meat | 1 | 0.65 | 0.96 | 0.84 | 0.93 | 0.73 | 0.72 | 0.76 | 1.18 |
| | 2 | | 1.24 | 0.92 | 0.83 | | 0.77 | 0.81 | 1.04 |
| | 3 | | | 0.80 | 0.69 | | | 0.87 | 0.87 |
| | 4 | | | | 0.49 | | | | 0.64 |
| Vegetables | 1 | 0.13 | 0.21 | 0.18 | 0.36 | 0.63 | 0.07 | 0.12 | 0.61 |
| | 2 | | 0.31 | 0.22 | 0.34 | | -0.33 | 0.04 | 0.47 |
| | 3 | | | 0.23 | 0.28 | | | 0.87 | 0.73 |
| | 4 | | | | 0.22 | | | | 0.38 |
| Fruits | 1 | 1.15 | 1.14 | 0.83 | 0.69 | 0.91 | 0.72 | 0.76 | 0.95 |
| | 2 | | 1.10 | 0.77 | 0.61 | | 0.61 | 0.75 | 0.87 |
| | 3 | | | 0.68 | 0.52 | | | 0.87 | 0.73 |
| | 4 | | | | 0.41 | | | | 0.56 |
| Others | 1 | 0.31 | 0.17 | 0.14 | 0.92 | 0.25 | 0.25 | 0.23 | 1.08 |
| | 2 | | 0.11 | 0.13 | 0.81 | | 0.29 | 0.27 | 0.97 |
| | 3 | | | 0.18 | 0.66 | | | 0.32 | 0.81 |
| | 4 | | | | 0.47 | | | | 0.60 |

match up quite well with the arc income elasticity estimates, with the exception of the catchall "other foods" category. This is remarkable in the sense that the elasticity estimates for any one expenditure quartile are derived independently of any food consumption, price, or income information for any other expenditure quartile.

The income elasticity for wheat, the least expensive calorie source, is nearly zero for low-income groups and falls below zero for high-income groups. Income elasticities for vegetables, the least expensive source of variety, are positive, but lowest among the nonstaple foods. Meats are among the most expensive sources of calories and variety, and have among the highest income elasticities. Rice, which costs about twice as much as wheat per kilogram, but is an inexpensive calorie source relative to nonstaple foods, has a moderately high income elasticity.

The tendency for the estimated own-price elasticities to decline across expenditure quartiles is very weak, much weaker than the tendency for income elasticities to decline with income. Given the zero homogeneity restriction, this means that as income elasticities decline, some cross-price elasticities increase, in particular, the cross-price elasticities with respect to nonfoods. At higher incomes, as the marginal utilities of energy and variety fall substantially, equation (1) tends to approach a system that is strongly separable, although food consumption is constrained by the disutility of high levels of calorie consumption.

Note that the cross-price elasticity for wheat in demand for rice (two foods that play the same "role" as staples in the diet) is relatively high. This cross-price elasticity is highest for middle income levels for both urban and rural populations. The FCDS estimates presented in Table 8 include the "income effect" term in the Slutsky equation. The Timmer (1981) proposition that price elasticities decline with income refers only to the compensated (utility constant) term in the Slutsky equation. Price elasticity estimates presented by Alderman (1988) are compensated elasticities. The budget shares and income elasticities for individual foods are sufficiently low that the compensated and uncompensated FCDS estimates are not substantially different; the compensated cross-price elasticities of wheat in demand for rice are also highest for the second expenditure quartile (see, also, Table 4 in the text and Table 10).

## A COMPARISON WITH THE AIDS ESTIMATES

Alderman (1988) has derived food demand elasticity estimates for urban and rural populations for Pakistan, applying an AIDS framework to expenditure survey data collected in 1979 and 1982. He reports elasticities evaluated at the means of the price and quantity data. For purposes of comparison with the Alderman estimates, elasticities (compensated and uncompensated) for the third expenditure quartile in Table 8 are reported in Table 4 (in the text) and Table 10 (FCDS-I). Alderman reports estimates at mean total expenditure levels. Because the highest

Table 10—A comparison of cross-price elasticities for selected foods, using the almost ideal demand system (AIDS) and the food characteristic demand system, urban and rural populations, Pakistan

| Food Price / Food Quantity | Urban | | | | | Rural | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Compensated | | | Uncompensated | | Compensated | | | Uncompensated | |
| | AIDS | FCDS-I | FCDS-II | FCDS-I | FCDS-II | AIDS | FCDS-I | FCDS-II | FCDS-I | FCDS-II |
| Wheat | | | | | | | | | | |
| Wheat | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... |
| Rice | 0.42 | 0.32 | 0.32 | 0.29 | 0.26 | 1.73 | 0.37 | 0.36 | 0.32 | 0.28 |
| Dairy (milk) | -0.42 | 0.04 | 0.04 | 0.01 | -0.01 | -0.12 | 0.07 | 0.06 | 0.01 | -0.02 |
| Meat | -0.50 | 0.05 | 0.05 | 0.00 | -0.05 | -0.16 | 0.04 | 0.04 | -0.06 | -0.14 |
| Other food | -0.08 | 0.11 | 0.11 | 0.05 | 0.01 | 0.02 | 0.09 | 0.09 | -0.01 | -0.07 |
| Nonfoods | -0.05 | -0.00 | -0.00 | -0.13 | -0.11 | -0.07 | -0.01 | -0.01 | -0.21 | -0.16 |
| Dairy (milk) | | | | | | | | | | |
| Wheat | -0.37 | 0.05 | 0.05 | 0.05 | 0.06 | -0.13 | 0.06 | 0.06 | 0.07 | 0.07 |
| Rice | 0.42 | 0.04 | 0.06 | 0.01 | -0.01 | 0.65 | 0.05 | 0.07 | 0.01 | -0.00 |
| Dairy (milk) | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... |
| Meat | 0.00 | 0.10 | 0.12 | 0.04 | 0.01 | -0.58 | 0.11 | 0.14 | 0.01 | -0.02 |
| Other food | 0.01 | 0.10 | 0.12 | 0.04 | 0.01 | 0.11 | 0.12 | 0.14 | 0.03 | -0.01 |
| Nonfoods | 0.00 | 0.07 | 0.06 | -0.07 | -0.06 | -0.03 | 0.08 | 0.06 | -0.10 | -0.08 |
| Other foods | | | | | | | | | | |
| Wheat | -0.15 | 0.23 | 0.23 | 0.24 | 0.24 | 0.02 | 0.15 | 0.14 | 0.16 | 0.16 |
| Rice | 0.32 | 0.17 | 0.21 | 0.10 | 0.09 | -0.55 | 0.15 | 0.19 | 0.07 | 0.06 |
| Dairy (milk) | 0.02 | 0.19 | 0.23 | 0.12 | 0.11 | 0.13 | 0.20 | 0.24 | 0.12 | 0.11 |
| Meat | 0.14 | 0.13 | 0.20 | 0.00 | -0.02 | -0.35 | 0.16 | 0.24 | -0.01 | -0.03 |
| Other food | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... |
| Nonfoods | 0.01 | 0.22 | 0.17 | -0.05 | -0.04 | 0.06 | 0.25 | 0.19 | -0.06 | -0.05 |
| Nonfoods | | | | | | | | | | |
| Wheat | -0.34 | -0.04 | -0.03 | -0.02 | 0.00 | -0.20 | -0.04 | -0.03 | -0.02 | 0.01 |
| Rice | -0.75 | 0.38 | 0.30 | 0.18 | -0.05 | -0.20 | 0.28 | 0.21 | 0.11 | -0.08 |
| Dairy (milk) | 0.04 | 0.39 | 0.30 | 0.18 | -0.05 | -0.08 | 0.32 | 0.24 | 0.12 | -0.08 |
| Meat | 0.49 | 0.65 | 0.51 | 0.31 | -0.09 | 0.76 | 0.63 | 0.47 | 0.24 | -0.17 |
| Other food | 0.03 | 0.62 | 0.49 | 0.30 | -0.09 | 0.11 | 0.58 | 0.44 | 0.22 | -0.16 |
| Nonfoods | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... |

Note: Own-price elasticity comparisons are reported in Table 4 in the text.

expenditure group earns a disproportionate share of income, mean expenditures fall within the range defined by the third expenditure quartile.

Because the Alderman nonfood income elasticity was substantially lower than that assumed in the derivation of the elasticities shown in Table 4 (which necessarily affects the level of food income elasticities due to the budget constraint), a second set of estimates was derived (FCDS-II) that assumed the same nonfood income elasticity as estimated by Alderman.

Cross-price elasticities are compared in Table 10 for four goods with high budget shares: wheat, dairy, other foods, and nonfoods. Because the own-price and income elasticities between the two demand systems match up so well (with the exceptions noted above), the sum of the cross-price effects for a specific food between the two systems are similar in view of the zero homogeneity constraint. However, individual AIDS and FCDS cross-price effects (that is, the price of a specific food in demand for a specific food) tend to be quite different. For example, the urban AIDS wheat price *compensated* cross elasticities are mostly negative, indicating complementarity between wheat and these other foods, while the corresponding FCDS estimates are nearly all positive, indicating substitutability.

The remaining urban dairy and other food cross-price elasticities are very similar between the two demand systems, except for the high AIDS cross-price effect of dairy on demand for rice. The rural cross-price effects between the two demand systems are substantially different. This is perhaps because the AIDS estimates are influenced by supply-side price effects, while the FCDS estimates are not. Policy simulations using the FCDS estimates must treat these supply-side income effects separately and explicitly.

## APPENDIX 3

## ADDITIONAL RESULTS FOR SECTION 5 OF THE PAPER: VARYING CHARACTERISTIC PARAMETER VALUES TO INVESTIGATE PROPERTIES OF THE FCDS

This appendix discusses the results of recomputing demand elasticity matrices under alternative assumptions from those used to generate Table 5. The purpose of presenting various examples is to provide a better intuitive understanding of the properties and implicit assumptions of the FCDS.

### GRAPHICAL REPRESENTATION OF THE FIRST-ORDER CONDITIONS

As background for this discussion, Figure 2 facilitates an understanding of the factors that determine the relative magnitudes of income elasticities for various foods. The total height of each rectangle, measured against the vertical axis, represents the retail price for a specific food. As equations (14) and (15) in the text show, the (say per kilo) price paid for each food is the sum of the shadow prices paid for energy, variety, and tastes of individual foods.

A simplification used in constructing Figure 2 is an assumption that the calorie conversion rate per kilo is constant across the five foods depicted (say

Figure 2—The retail price for each food is the sum of the shadow prices for energy, variety, and tastes of individual foods

2,000 calories per kilo).[31] The particular individual pictured is willing to pay $1.00 at the margin (given his/her level of calorie consumption) for 2,000 calories. If income were to increase, and, thus, calorie consumption, the shadow price for energy would fall below $1.00 per 2,000 calories (even though the retail prices for all foods remain constant).

Each kilo of vegetables and meat provides an identical amount of variety in the diet (this is a simplifying assumption explicitly included in the model, and is not specific to Figure 2). At the margin, the individual pictured is willing to pay $1.50 for each extra kilo of variety. Note that staple consumption (wheat and rice) reduces variety in the diet; the model assumes that this shadow price is negative for staples, which is difficult to show graphically.[32]

Thus, the difference between the retail price and the sum of the shadow prices for energy and variety is the premium that the consumer is willing to pay, at the margin, for the specific intrinsic characteristics ("tastes") contained in an extra kilo of any food. That premium is relatively small for inferior staples such as wheat, and relatively large for expensive meats. For food staples, as the shadow price of variety

[31] For example, vegetables tend to have about a tenth as many calories per kilo as staples, although these fractions vary widely for individual foods. Thus, the shaded portions for "energy" for vegetables in Figure 2 should be smaller than depicted.

[32] Shadow prices for energy may also be negative at sufficiently high levels of calorie consumption (see equation [4]).

increases, the proportion of the retail price accounted for by "taste" *increases*. For nonstaples, the taste shadow price declines, ceteris paribus, as $w_v$ increases.

Using this framework, foods tend to fall into four categories: (1) inexpensive, nonpreferred staples with negative income elasticities, (2) preferred staples with positive income elasticities, (3) inexpensive sources of variety (nonstaple foods) with income elasticities below 0.5 and sometimes negative, and (4) expensive nonstaple foods for which the taste shadow price predominates (income elasticities above 0.5 and sometimes above 1.0). As values for $E_{umx}$, $w_e e_3$, and $w_v$ change, and so the shadow prices for energy and variety change, the relative proportions accounted for by taste change at differential rates across individual foods. *The FCDS generates the highest income elasticities for foods for which the shadow price of taste is a high proportion of the total retail price*. These might be called "taste-intensive" foods.

## THE FCDS AND REVEALED PREFERENCES

### Effect of the Shadow Price of Variety

The income elasticities, corresponding to the own-price elasticities presented in Table 5 in the text, are presented in Table 11. In comparing these income elasticities across parameter values, it is instructive to decipher what they "reveal" about food consumption behavior. For example, note that vegetable income elasticities (vegetables are the cheapest source of variety in the diet for this

Table 11—FCDS income elasticities for Pakistan, urban lowest expenditure quartile, utility function parameters for energy and variety varied as shown, nonfood income elasticity held constant

| Utility from Variety Parameter | Utility from Energy Parameters | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | $w_e e_3$ = -0.03 | | | $w_e e_3$ = -0.06 | | | $w_e e_3$ = -0.09 | | |
| | $E_{UMX}$ | | | $E_{UMX}$ | | | $E_{UMX}$ | | |
| | 2,000 | 3,000 | 4,000 | 2,000 | 3,000 | 4,000 | 2,000 | 3,000 | 4,000 |
| $w_v$ = 0.30 | | | | | | | | | |
| Wheat | .09 | .09 | .09 | -.06 | -.09 | -.13 | -.12 | -.17 | -.32 |
| Rice | .50 | .55 | .62 | .46 | .61 | .94 | .44 | .75 | 2.41 |
| Milk | .62 | .62 | .62 | .66 | .66 | .66 | .67 | .68 | .67 |
| Meat | .83 | .82 | .82 | .88 | .87 | .87 | .90 | .89 | .85 |
| Vegetables | .58 | .58 | .57 | .62 | .60 | .58 | .63 | .60 | .55 |
| Fruits | .73 | .72 | .71 | .77 | .75 | .73 | .79 | .75 | .68 |
| Others | .81 | .81 | .81 | .87 | .87 | .87 | .88 | .89 | .87 |
| Nonfoods | 1.75 | 1.75 | 1.75 | 1.75 | 1.75 | 1.75 | 1.75 | 1.75 | 1.75 |
| $w_v$ = 0.70 | | | | | | | | | |
| Wheat | .20 | .22 | .24 | -.03 | -.05 | -.09 | -.11 | -.15 | -.26 |
| Rice | .58 | .62 | .68 | .50 | .63 | .89 | .47 | .74 | 1.77 |
| Milk | .48 | .47 | .47 | .53 | .53 | .52 | .54 | .54 | .54 |
| Meat | .88 | .87 | .86 | .96 | .96 | .96 | .99 | .99 | .97 |
| Vegetables | .36 | .35 | .34 | .39 | .36 | .34 | .40 | .36 | .31 |
| Fruits | .67 | .66 | .65 | .73 | .71 | .68 | .75 | .71 | .65 |
| Others | .87 | .86 | .85 | .95 | .96 | .96 | .98 | .99 | .99 |
| Nonfoods | 1.75 | 1.75 | 1.75 | 1.75 | 1.75 | 1.75 | 1.75 | 1.75 | 1.75 |
| $w_v$ = 1.10 | | | | | | | | | |
| Wheat | .37 | .43 | .52 | .00 | -.02 | -.05 | -.10 | -.14 | -.23 |
| Rice | .66 | .72 | .77 | .51 | .63 | .84 | .48 | .70 | 1.41 |
| Milk | .36 | .34 | .31 | .42 | .42 | .42 | .44 | .44 | .44 |
| Meat | .89 | .87 | .83 | 1.03 | 1.04 | 1.04 | 1.07 | 1.08 | 1.07 |
| Vegetables | .17 | .17 | .16 | .17 | .14 | .11 | .17 | .12 | .05 |
| Fruits | .64 | .63 | .61 | .71 | .69 | .67 | .73 | .69 | .64 |
| Others | .87 | .85 | .81 | 1.02 | 1.03 | 1.04 | 1.06 | 1.08 | 1.09 |
| Nonfoods | 1.75 | 1.75 | 1.75 | 1.75 | 1.75 | 1.75 | 1.75 | 1.75 | 1.75 |

particular disaggregation of these Pakistan data) are much higher for *low* values of $w_v$, which may seem counterintuitive—a positive relationship might have been expected between $w_v$ and income elasticities for variety-intensive foods.

To interpret this result, consider that each solution in Table 11 is derived using the same input data for food quantities and prices (shown in Table 7 for the urban lowest income quintile). Consumers A and B are observed to eat identical amounts of vegetables (and other foods) facing identical prices and having identical incomes. However, suppose that consumers A and B have different preference functions, specifically $w_v$=0.30 for consumer A (the top portion of Table 11) and $w_v$=1.10 for consumer B (the bottom portion of Table 11). The vegetables that consumer B eats are valued (relative to consumer A) for the variety they provide. This "reveals" that as income goes up, consumer B (who, along with consumer A, will be less concerned about variety as income increases and more concerned with tastes of individual foods; see Figure 1) will exhibit a (relatively) weak demand for vegetables.

Consumer A (a true vegetable lover), despite a low value for $w_v$, is eating the same amount of vegetables as consumer B. This "reveals" a (relatively) strong preference for the "taste" of vegetables on the part of consumer A. Consumer A's demand for vegetables will remain (relatively) strong as income increases.

Put differently, if the prior information had been that the income elasticity for vegetables for consumer A was (say) 0.60 and for consumer B was (say) 0.15

(again, both consumers having identical diets), this would "reveal" a relatively low value for $w_v$ for consumer A and a higher value for consumer B.

Turning to wheat and rice, despite the (relatively) high *disutility* of food staple consumption in terms of reducing variety for consumer B (because $w_v$ is relatively high), consumer B eats the same amount of food staples as consumer A. For consumer B, this reveals a relatively strong preference for the "taste" of wheat and rice and, therefore, higher income elasticities for food staples.

Effect of the Shadow Price of Energy

Restricting the discussion to consumers like B (that is, holding $w_v$ constant at 1.10), the different income elasticities for wheat between $w_e e_3 = -0.03$ and $w_e e_3 = -0.09$ provide an interesting contrast. For $w_e e_3 = -0.03$, the shadow price for energy is relatively low; the shadow price for the taste for wheat is correspondingly high. The taste shadow price for wheat is high enough (in particular because of the high negative effect of the variety shadow price; refer to Figure 2) that it accounts for as high a proportion of the total retail price as most nonstaple foods. Thus, the wheat income elasticity is positive and relatively high.

Again for $w_e e_3 = -0.03$, consider two consumers, B1 and B2, consuming the same diets, but with $E_{umx} = 2,000$ for consumer B1 and $E_{umx} = 4,000$ for consumer B2 (B2 may weigh considerably more than B1 and/or may have a more strenuous activity pattern than B1, and so requires more calories to reach the same level of "hunger satiation"). That B2 eats the same diet as B1 "reveals" that B2 is willing to accept more hunger (has a lower shadow price of energy than B1) in return for more taste in the diet. Income elasticities for wheat and rice are higher and income elasticities for nonstaples are marginally lower for B2 as compared with B1.

A different (perhaps more realistic) pattern of behavior is depicted for $w_e e_3$ = -0.09. The shadow price for energy is relatively high; the shadow price for taste for wheat is correspondingly low. The taste shadow price for wheat is low enough, in fact (in particular because of the high positive effect of the energy shadow price; refer to Figure 2) that it accounts for a much lower proportion of the total retail price as compared with nonstaple foods. Thus, the wheat income elasticity is negative.

Comparing B1 and B2 ($E_{umx}$ = 2,000 and 4,000, respectively), again B2 reveals a lower shadow price for energy and so a relative preference for the "tastes" of foods. However, in contrast with the case where $w_e e_3$ = -0.03, wheat is now ranked quite low relative to other foods in terms of the "taste" it provides. That B2 gives relatively more weight to "taste-intensive" foods means that the income elasticity for wheat will be even more negative than for B1.

Finally, holding $E_{umx}$ constant, consider the case where $w_e e_3$ = -0.03 for consumer B3 and $w_e e_3$ = -0.09 for consumer B4. B4's shadow price for energy is relatively high, "revealing" that wheat, in particular, is consumed for its energy content and not for its taste. Consequently, B4's demand for wheat will be weak as income increases, relative to B3's demand for wheat.

The Shadow Price of Energy and Calorie-Income Elasticities

The calorie-income elasticity can be computed as the weighted sum of the income elasticities for individual foods, weighted by their calorie shares. For these Pakistan data, wheat receives a weight of 0.74 (see Table 7). Focusing on the bottom of Table 11 ($w_v$ = 1.10), calorie-income elasticities are high for $w_e e_3$ = -0.03 (a minimum of 0.4) and considerably lower for $w_e e_3 = -0.09$. *It is not an underlying strong demand for energy, but for tastes of staple foods, which is "driving" the high calorie-income elasticities in the lower left-hand portion of Table 11 and the high own-price elasticities for wheat in the lower left-hand portion of Table 5 in the text*.[33]

Under the FCDS framework, a high shadow price for energy (for example, $w_e e_3$ = -0.09), ceteris paribus, is associated with *low* price-response and reveals *weak* demand for "energy-intensive" foods as income increases. A high shadow price for energy *does not* explain the often observed phenomenon of highest price response for staple foods for the lowest income groups, but rather the opposite conclusion, that price response should be low.

## AGGREGATING WHEAT AND RICE INTO ONE FOOD

In Table 5, the own-price elasticity for wheat is always lower (in absolute value) than the own-price elasticity of rice. In Table 12, where wheat and rice are

[33] Such magnitudes are implausible because of the increases in body weights that they imply (Bouis 1994).

aggregated into a single food, the own-price elasticity for wheat and rice combined is always *lower* (in absolute value) than for wheat in Table 5, because there is no possibility of substitution between wheat and rice. The own-price elasticities of the remaining foods remain largely unaffected.

In Table 5, the income elasticity for wheat is always lower than that of rice. In Table 13, the income elasticity for wheat and rice combined is always within the range between the income elasticities wheat and rice in Table 5, and is a weighted average with wheat receiving the higher weight. The income elasticities of the remaining foods remain largely unaffected.

## LOWERING THE PRICE OF RICE

The price of rice (which is indexed on the price of wheat) was lowered to 1.50 from 2.13; the resulting own-price and income elasticities are presented in Tables 14 and 15, respectively. Lowering the retail price of rice reduces the shadow price of taste for rice, which is calculated as a residual of the retail price minus the shadow prices for energy and variety (all food quantities and other food prices remain constant in this "experiment"). As discussed earlier in this appendix, a reduced taste component is associated with lower income elasticities

Table 12—FCDS own-price elasticities for Pakistan, urban lowest expenditure quartile, utility function parameters for energy and variety varied as shown, nonfood income elasticity held constant, combining wheat and rice into one food

| Utility from Variety Parameter | Utility from Energy Parameters | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | $w_e e_3$ = -0.03 | | | $w_e e_3$ = -0.06 | | | $w_e e_3$ = -0.09 | | |
| | $E_{UMX}$ | | | $E_{UMX}$ | | | $E_{UMX}$ | | |
| | 2,000 | 3,000 | 4,000 | 2,000 | 3,000 | 4,000 | 2,000 | 3,000 | 4,000 |
| $w_v$ = 0.30 | | | | | | | | | |
| Wheat/rice | -.34 | -.37 | -.41 | -.18 | -.19 | -.22 | -.11 | -.13 | -.14 |
| Milk | -.97 | -.99 | -1.02 | -.97 | -1.03 | -1.09 | -.98 | -1.06 | -1.16 |
| Meat | -1.03 | -1.04 | -1.06 | -1.03 | -1.07 | -1.11 | -1.03 | -1.09 | -1.16 |
| Vegetables | -1.00 | -1.02 | -1.03 | -1.00 | -1.04 | -1.08 | -1.00 | -1.06 | -1.12 |
| Fruits | -1.06 | -1.08 | -1.09 | -1.06 | -1.09 | -1.12 | -1.06 | -1.10 | -1.14 |
| Others | -1.03 | -1.05 | -1.07 | -1.03 | -1.08 | -1.13 | -1.04 | -1.10 | -1.18 |
| Nonfoods | -1.32 | -1.34 | -1.35 | -1.28 | -1.30 | -1.33 | -1.26 | -1.30 | -1.34 |
| $w_v$ = 0.70 | | | | | | | | | |
| Wheat/rice | -.45 | -.51 | -.58 | -.21 | -.23 | -.26 | -.13 | -.14 | -.17 |
| Milk | -.98 | -1.01 | -1.03 | -.98 | -1.04 | -1.10 | -.98 | -1.07 | -1.18 |
| Meat | -1.06 | -1.08 | -1.10 | -1.06 | -1.10 | -1.15 | -1.06 | -1.13 | -1.20 |
| Vegetables | -1.05 | -1.07 | -1.09 | -1.05 | -1.09 | -1.14 | -1.05 | -1.12 | -1.20 |
| Fruits | -1.17 | -1.18 | -1.20 | -1.17 | -1.20 | -1.23 | -1.17 | -1.22 | -1.27 |
| Others | -1.07 | -1.09 | -1.12 | -1.07 | -1.12 | -1.17 | -1.07 | -1.14 | -1.23 |
| Nonfoods | -1.25 | -1.26 | -1.27 | -1.19 | -1.21 | -1.23 | -1.18 | -1.21 | -1.24 |
| $w_v$ = 1.10 | | | | | | | | | |
| Wheat/rice | -.67 | -.80 | -.99 | -.25 | -.29 | -.33 | -.15 | -.17 | -.19 |
| Milk | -1.04 | -1.07 | -1.11 | -1.05 | -1.12 | -1.20 | -1.05 | -1.16 | -1.30 |
| Meat | -1.10 | -1.12 | -1.14 | -1.10 | -1.14 | -1.19 | -1.10 | -1.17 | -1.25 |
| Vegetables | -1.16 | -1.19 | -1.22 | -1.16 | -1.22 | -1.29 | -1.16 | -1.26 | -1.38 |
| Fruits | -1.30 | -1.32 | -1.34 | -1.30 | -1.34 | -1.38 | -1.30 | -1.36 | -1.43 |
| Others | -1.12 | -1.15 | -1.19 | -1.12 | -1.17 | -1.23 | -1.12 | -1.20 | -1.29 |
| Nonfoods | -1.22 | -1.24 | -1.26 | -1.14 | -1.16 | -1.18 | -1.13 | -1.15 | -1.18 |

Table 13—FCDS income elasticities for Pakistan, urban lowest expenditure quartile, utility function parameters for energy and variety varied as shown, nonfood income elasticity held constant, combining wheat and rice into one food

| Utility from Variety Parameter | Utility from Energy Parameters | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | $w_e e_3$ = -0.03 | | | $w_e e_3$ = -0.06 | | | $w_e e_3$ = -0.09 | | |
| | $E_{UMX}$ | | | $E_{UMX}$ | | | $E_{UMX}$ | | |
| | 2,000 | 3,000 | 4,000 | 2,000 | 3,000 | 4,000 | 2,000 | 3,000 | 4,000 |
| $w_v$ = 0.30 | | | | | | | | | |
| Wheat/rice | .14 | .14 | .14 | -.02 | -.03 | -.04 | -.08 | -.10 | -.12 |
| Milk | .63 | .62 | .62 | .66 | .67 | .68 | .68 | .69 | .71 |
| Meat | .83 | .83 | .83 | .89 | .89 | .89 | .91 | .91 | .91 |
| Vegetables | .59 | .58 | .57 | .62 | .61 | .60 | .64 | .62 | .60 |
| Fruits | .73 | .72 | .71 | .78 | .76 | .75 | .80 | .77 | .74 |
| Others | .82 | .82 | .82 | .87 | .88 | .89 | .90 | .91 | .92 |
| Nonfoods | 1.75 | 1.75 | 1.75 | 1.75 | 1.75 | 1.75 | 1.75 | 1.75 | 1.75 |
| $w_v$ = 0.70 | | | | | | | | | |
| Wheat/rice | .24 | .26 | .28 | .01 | .01 | -.00 | -.06 | -.08 | -.10 |
| Milk | .48 | .48 | .47 | .53 | .53 | .53 | .55 | .55 | .56 |
| Meat | .88 | .88 | .87 | .97 | .98 | .98 | 1.00 | 1.01 | 1.02 |
| Vegetables | .36 | .35 | .34 | .39 | .37 | .35 | .40 | .37 | .33 |
| Fruits | .68 | .67 | .65 | .74 | .72 | .70 | .76 | .72 | .69 |
| Others | .87 | .86 | .86 | .96 | .97 | .98 | .99 | 1.01 | 1.03 |
| Nonfoods | 1.75 | 1.75 | 1.75 | 1.75 | 1.75 | 1.75 | 1.75 | 1.75 | 1.75 |
| $w_v$ = 1.10 | | | | | | | | | |
| Wheat/rice | .41 | .47 | .56 | .05 | .04 | .03 | -.05 | -.07 | -.10 |
| Milk | .36 | .34 | .31 | .43 | .42 | .42 | .44 | .45 | .45 |
| Meat | .89 | .87 | .83 | 1.04 | 1.05 | 1.08 | 1.08 | 1.10 | 1.11 |
| Vegetables | .18 | .17 | .16 | .17 | .15 | .11 | .17 | .12 | .06 |
| Fruits | .64 | .63 | .61 | .72 | .70 | .69 | .74 | .71 | .67 |
| Others | .87 | .85 | .81 | 1.03 | 1.04 | 1.06 | 1.07 | 1.10 | 1.13 |
| Nonfoods | 1.75 | 1.75 | 1.75 | 1.75 | 1.75 | 1.75 | 1.75 | 1.75 | 1.75 |

Table 14—FCDS own-price elasticities for Pakistan, urban lowest expenditure quartile, utility function parameters for energy and variety varied as shown, nonfood income elasticity held constant, rice price lowered from 2.13 to 1.50

| Utility from Variety Parameter | Utility from Energy Parameters $w_e e_3 = -0.03$ $E_{UMX}$ 2,000 | 3,000 | 4,000 | $w_e e_3 = -0.06$ $E_{UMX}$ 2,000 | 3,000 | 4,000 | $w_e e_3 = -0.09$ $E_{UMX}$ 2,000 | 3,000 | 4,000 |
|---|---|---|---|---|---|---|---|---|---|
| $w_v = 0.30$ | | | | | | | | | |
| Wheat | -.35 | -.39 | -.45 | -.20 | -.25 | -.38 | -.15 | -.22 | ... |
| Rice | -.88 | -1.08 | -1.38 | -.89 | -1.40 | -3.32 | -.89 | -2.01 | ... |
| Milk | -.97 | -.99 | -1.02 | -.97 | -1.03 | -1.09 | -.97 | -1.06 | ... |
| Meat | -1.03 | -1.04 | -1.06 | -1.03 | -1.07 | -1.11 | -1.03 | -1.09 | ... |
| Vegetables | -1.00 | -1.02 | -1.03 | -1.00 | -1.04 | -1.08 | -1.00 | -1.06 | ... |
| Fruits | -1.06 | -1.08 | -1.09 | -1.06 | -1.09 | -1.12 | -1.06 | -1.10 | ... |
| Others | -1.03 | -1.05 | -1.07 | -1.03 | -1.08 | -1.12 | -1.04 | -1.10 | ... |
| Nonfoods | -1.33 | -1.34 | -1.36 | -1.29 | -1.31 | -1.34 | -1.27 | -1.32 | ... |
| $w_v = 0.70$ | | | | | | | | | |
| Wheat | -.45 | -.51 | -.60 | -.22 | -.27 | -.36 | -.15 | -.21 | -2.33 |
| Rice | -.79 | -.93 | -1.14 | -.78 | -1.15 | -2.18 | -.79 | -1.53 | -39.99 |
| Milk | -.98 | -1.01 | -1.04 | -.98 | -1.04 | -1.10 | -.98 | -1.07 | -1.18 |
| Meat | -1.06 | -1.08 | -1.10 | -1.06 | -1.10 | -1.15 | -1.06 | -1.13 | -1.20 |
| Vegetables | -1.05 | -1.07 | -1.09 | -1.05 | -1.09 | -1.14 | -1.05 | -1.12 | -1.20 |
| Fruits | -1.17 | -1.18 | -1.20 | -1.17 | -1.20 | -1.23 | -1.17 | -1.22 | -1.27 |
| Others | -1.07 | -1.09 | -1.12 | -1.07 | -1.12 | -1.17 | -1.07 | -1.14 | -1.23 |
| Nonfoods | -1.25 | -1.26 | -1.28 | -1.20 | -1.22 | -1.24 | -1.18 | -1.22 | -1.38 |
| $w_v = 1.10$ | | | | | | | | | |
| Wheat | -.65 | -.79 | -.99 | -.26 | -.31 | -.39 | -.16 | -.21 | -.47 |
| Rice | -.72 | -.83 | -.99 | -.70 | -.98 | -1.63 | -.70 | -1.24 | -5.44 |
| Milk | -1.04 | -1.08 | -1.11 | -1.05 | -1.12 | -1.20 | -1.05 | -1.16 | -1.30 |
| Meat | -1.10 | -1.12 | -1.14 | -1.10 | -1.14 | -1.19 | -1.10 | -1.17 | -1.25 |
| Vegetables | -1.16 | -1.19 | -1.22 | -1.16 | -1.22 | -1.29 | -1.16 | -1.26 | -1.37 |
| Fruits | -1.30 | -1.32 | -1.34 | -1.30 | -1.34 | -1.38 | -1.30 | -1.36 | -1.43 |
| Others | -1.12 | -1.15 | -1.19 | -1.12 | -1.17 | -1.23 | -1.12 | -1.19 | -1.29 |
| Nonfoods | -1.22 | -1.24 | -1.26 | -1.15 | -1.17 | -1.19 | -1.13 | -1.16 | -1.21 |

Note: ... indicates that solution is not consistent with utility maximization.

Table 15—FCDS income elasticities for Pakistan, urban lowest expenditure quartile, utility function parameters for energy and variety varied as shown, nonfood income elasticity held constant, rice price lowered from 2.13 to 1.50

| Utility from Variety Parameter | Utility from Energy Parameters | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | $w_e e_3$ = -0.03 | | | $w_e e_3$ = -0.06 | | | $w_e e_3$ = -0.09 | | |
| | $E_{UMX}$ | | | $E_{UMX}$ | | | $E_{UMX}$ | | |
| | 2,000 | 3,000 | 4,000 | 2,000 | 3,000 | 4,000 | 2,000 | 3,000 | 4,000 |
| $w_v$ = 0.30 | | | | | | | | | |
| Wheat | .10 | .10 | .10 | -.05 | -.07 | -.13 | -.11 | -.15 | ... |
| Rice | .34 | .39 | .46 | .26 | .40 | .97 | .22 | .57 | ... |
| Milk | .62 | .62 | .62 | .66 | .66 | .67 | .67 | .68 | ... |
| Meat | .83 | .82 | .82 | .88 | .88 | .87 | .90 | .89 | ... |
| Vegetables | .58 | .58 | .57 | .62 | .60 | .59 | .63 | .61 | ... |
| Fruits | .73 | .72 | .71 | .77 | .75 | .73 | .79 | .76 | ... |
| Others | .82 | .82 | .82 | .87 | .87 | .88 | .88 | .89 | ... |
| Nonfoods | 1.75 | 1.75 | 1.75 | 1.75 | 1.75 | 1.75 | 1.75 | 1.75 | ... |
| $w_v$ = 0.70 | | | | | | | | | |
| Wheat | .21 | .22 | .24 | -.02 | -.04 | -.08 | -.09 | -.13 | -1.14 |
| Rice | .42 | .46 | .52 | .28 | .39 | .73 | .24 | .49 | 13.09 |
| Milk | .48 | .48 | .47 | .53 | .53 | .53 | .54 | .54 | .50 |
| Meat | .88 | .88 | .87 | .96 | .97 | .97 | .99 | .99 | .87 |
| Vegetables | .36 | .35 | .34 | .39 | .37 | .34 | .40 | .36 | .26 |
| Fruits | .67 | .66 | .65 | .73 | .71 | .68 | .75 | .71 | .56 |
| Others | .87 | .86 | .86 | .95 | .96 | .96 | .98 | .99 | .89 |
| Nonfoods | 1.75 | 1.75 | 1.75 | 1.75 | 1.75 | 1.75 | 1.75 | 1.75 | 1.75 |
| $w_v$ = 1.10 | | | | | | | | | |
| Wheat | .38 | .44 | .53 | .01 | -.00 | -.03 | -.08 | -.12 | -.28 |
| Rice | .53 | .59 | .67 | .30 | .39 | .61 | .24 | .43 | 2.06 |
| Milk | .36 | .34 | .32 | .43 | .42 | .42 | .44 | .44 | .44 |
| Meat | .89 | .87 | .84 | 1.04 | 1.04 | 1.05 | 1.07 | 1.08 | 1.08 |
| Vegetables | .17 | .17 | .16 | .17 | .14 | .11 | .17 | .12 | .05 |
| Fruits | .64 | .63 | .61 | .71 | .69 | .67 | .73 | .69 | .64 |
| Others | .87 | .85 | .82 | 1.02 | 1.03 | 1.05 | 1.06 | 1.08 | 1.10 |
| Nonfoods | 1.75 | 1.75 | 1.75 | 1.75 | 1.75 | 1.75 | 1.75 | 1.75 | 1.75 |

Note: ... indicates that solution is not consistent with utility maximization.

(compare Tables 11 and 15) in that it "reveals" that rice is being consumed primarily for the energy it provides (relative to the higher rice price scenario of Table 11).

Analysis of changed price-responsiveness is more complex. On the one hand, as the price of rice declines, it becomes a more attractive substitute for wheat as an inexpensive staple. On the other hand, in that the taste shadow price for rice has declined, spending a dollar on rice (high calories, positive taste, negative variety) instead of nonstaple foods (low calories, positive taste, positive variety) is now less attractive. Whether the former factor (increased price-responsiveness) or the latter factor (reduced price-responsiveness) predominates depends on the relative magnitudes of the shadow prices for variety ($w_v$) and energy ($E_{UMX}$, $w_e e_3$). Own-price elasticities for rice are sometimes higher and sometimes lower in Table 5 (a high rice price) as compared with Table 14 (a low rice price). High values for $E_{UMX}$ and $w_e e_3$ and low values for $w_v$ (a relatively high energy shadow price) are associated with higher own-price elasticities for rice in Table 14 as compared with Table 5.

## APPENDIX 4

## AN EXTENSION OF THE FCDS: FOOD GROUPS

Footnote 14 refers to an extension of the FCDS in which a predesignated subgroup of the *n* foods (for example, beef, pork, chicken, fish, and other meats) share a common additional characteristic shared by each of these foods, but not by any food outside of the subgroup. This extension is developed below. Equations (6) through (9) from the text, which refer to utility from tastes of individual foods, are reproduced for convenience.

$$U_{ti}(q_i) = \log(q_i), \tag{6}$$

$$T_i = w_{ti}\left(\frac{1}{q_i}\right) > 0, \tag{7}$$

$$T_{ii} = -w_{ti}\left(\frac{1}{q_i}\right)^2 < 0, \tag{8}$$

$$T_{ij} = 0. \tag{9}$$

In equations (6) through (9), each additional unit of taste of good i, no matter what the quantity, adds additional utility, but at a decreasing rate. The first derivative is positive and the second derivative negative. The "across food" second derivative is zero, which may be a reasonable assumption for broad food groups.

However, for estimating a highly disaggregate food matrix in which several individual foods for a broad food group are specified (for example, if in place of an aggregate "meat" category, the food demand matrix includes individual meats such as pork, chicken, beef, fish, and other meats), the assumption implicit in equation (9) may be unrealistically strong. This may be remedied by specifying which individual foods fall into K broad food groups and respecifying that portion of equation (1) in the text that refers to the utility from taste of foods as follows[34]:

$$w_t U(T) = \sum_{i=1}^{n} w_{ti} \log(q_i) + \sum_{k=1}^{K} w_k \log(G_k), \tag{32}$$

where

$$G_k = \sum_{i=1}^{n} d_{ki} \cdot q_i \tag{33}$$

and $d_{ki} = 1$ for all i belonging to food group k; $d_{ki} = 0$ otherwise.

In the first and second derivatives and first order conditions below, *it is assumed that only one food group (say, meats) has been specified* and that food numbers 5 through 8 belong to the meat group (for a system of say ten foods and one one-food). For foods belonging to the meat group, $G_k$ is observed and $d_{ki}$ and $d_{kj}$ are 1. Thus, $w_k$ must also be specified a priori before it is possible to solve for the $w_{ti}$'s from the first-order conditions:

---

[34] All *n* individual foods need not be included in one of the K food groups. It is conceivable, though it would not be a frequent occurrence, that an individual food may be included in two or more of the K food groups. A food group could include both staple and nonstaple foods, or only staple foods, or only nonstaple foods.

First Derivative for Taste:

$$T_i = w_{ti}\left(\frac{1}{q_i}\right) + w_k \cdot \left(\frac{1}{G_k}\right). \qquad (34)$$

First-Order Condition for Food 5 (a Meat):

$$P_5 = \frac{w_e}{\lambda}\left[e_2 z_5 + 2e_3 z_5 E\right] + \frac{w_v}{\lambda}\left[\frac{(T-M)}{T^2}\right] + \frac{w_{t5}}{\lambda}\left[\frac{1}{q_5}\right] + \frac{w_k}{\lambda}\left[\frac{1}{G_k}\right]. \qquad (35)$$

Second Derivatives for Taste:

$$T_{ii} = -w_{ti}\left(\frac{1}{q_i}\right)^2 - w_k \cdot d_{ki} \cdot \left(\frac{1}{G_k}\right)^2. \qquad (36)$$

$$T_{ij} = T_{ji} = -w_k \cdot d_{ki} \cdot d_{kj} \cdot \left(\frac{1}{G_k}\right)^2. \qquad (37)$$

For each food group specified, prior specification of one additional demand elasticity is required (in addition to the original four elasticities/utility function parameters) in order to be able to solve the first-order conditions for the $w_{ti}$'s for that food group.

Compare Figure 3 with Figure 2 to see that specification of these "group-specific" characteristics reduces the proportion of the retail price accounted for by the shadow price of "taste," the residual that accounts for whatever

Figure 3—The retail price for each food is the sum of the shadow prices for energy, variety, and the tastes of individual foods; for meats, this sum also includes a shadow price for a common characteristic that is inherent in meats

characteristic(s) is(are) inherent in a specific food and not shared by any other food.

Addition of these food-group-specific characteristics adds a "degree of freedom" for each food group, which (1) allows for a greater "stratification" of the income elasticities within a food group (that is, increases the range between low and high elasticities for foods in the group) and (2) increases the magnitudes of the cross-price elasticities between foods in that group.

These properties are shown in Tables 16, 17, and 18 taken from Bouis (1991a). For these Philippine data (Table 16), animal and fish products were specified as a group containing seven individual foods. Note in Table 17 the range of income elasticities between (say) fresh fish (an inexpensive meat) and beef (an expensive meat). The relatively high cross-price elasticities between animal and fish products are shown in Table 18.

The Philippines data also provide an interesting contrast with the Pakistan results in that, because of import policies, wheat is a more expensive source of calories than rice. Consequently, income elasticities are substantially higher for wheat than rice.

Table 16—Input data for calculating food demand matrix, urban Philippines, lowest income quartile, 1978

| Food Group | Price per Kilogram[a] | Quantity Consumed[b] | '000 Calories per Kilogram | Share of Food Budget | Share of Total Expenditures | Price per '000 Calories[c] | Nonstaples? | Meat? |
|---|---|---|---|---|---|---|---|---|
| Corn | 1.218 | .112 | 3.500 | .01 | .01 | .35 | .0 | .0 |
| Rice | 1.368 | 1.827 | 3.510 | .25 | .17 | .39 | .0 | .0 |
| Wheat/bread | 2.597 | .140 | 3.650 | .04 | .03 | .71 | 1.0 | .0 |
| Vegetable/fruit | .873 | 1.302 | .296 | .11 | .08 | 2.95 | 1.0 | .0 |
| Other foods | 2.333 | .721 | 2.068 | .17 | .12 | 1.13 | 1.0 | .0 |
| Fresh fish | 3.197 | .455 | .554 | .14 | .10 | 5.77 | 1.0 | 1.0 |
| Other meat | 3.247 | .280 | .725 | .09 | .06 | 4.48 | 1.0 | 1.0 |
| Pork | 6.494 | .063 | 3.444 | .04 | .03 | 1.89 | 1.0 | 1.0 |
| Beef | 8.656 | .021 | 1.667 | .02 | .01 | 5.19 | 1.0 | 1.0 |
| Poultry | 9.740 | .014 | 1.500 | .01 | .01 | 6.49 | 1.0 | 1.0 |
| Eggs | 6.494 | .042 | 1.500 | .03 | .02 | 4.33 | 1.0 | 1.0 |
| Milk/product | 4.468 | .224 | 1.063 | .10 | .07 | 4.20 | 1.0 | 1.0 |
| Nonfoods | 4.364 | 1.000 | .000 | .00 | .30 | .00 | .0 | .0 |
| Household size = | | | | 6.9 | | | | |
| Adult equivalents = | | | | 5.7 | | | | |
| Food expenditures = | | | | 10.2 | | | | |
| Nonfood expenditures = | | | 4.4 | | | | | |
| Income = | | | | 14.5 | | | | |

Source: Bouis (1991a).

[a] Price per kilogram is indexed on the price paid for a kilogram of corn grits by the lowest income quartile in rural areas (index = 1.00 = price paid for one kilogram of corn grits).

[b] Quantity consumed is kilograms per capita per week.

[c] Price per '000 calories uses the index for the price of corn (= indexed expenditures for a food/(kilograms for a food x calories per kilogram).

Table 17—Comparison of arc income elasticities between income groups and income elasticity estimates, by income group by food, urban Philippines

| Food | Income Group | Arc Income Elasticity Between Income Group 2 | 3 | 4 | Estimated Income Elasticity |
|---|---|---|---|---|---|
| Rice | 1 | -.01 | .00 | -.00 | 0.14 |
| | 2 | | -.01 | .02 | 0.12 |
| | 3 | | | -.03 | -0.03 |
| | 4 | | | | -0.16 |
| Corn | 1 | .68 | .05 | -.04 | -0.33 |
| | 2 | | -.15 | -.41 | -0.83 |
| | 3 | | | -.30 | -0.57 |
| | 4 | | | | -0.02 |
| Wheat/bread | 1 | .07 | .10 | .07 | 0.47 |
| | 2 | | .16 | .28 | 0.52 |
| | 3 | | | .20 | 0.34 |
| | 4 | | | | 0.07 |
| Vegetables/fruits | 1 | .08 | .09 | .05 | 0.44 |
| | 2 | | .10 | .20 | 0.58 |
| | 3 | | | .13 | 0.56 |
| | 4 | | | | 0.55 |
| Other foods | 1 | -.03 | .05 | .05 | 1.28 |
| | 2 | | .16 | .28 | 0.80 |
| | 3 | | | .20 | 0.75 |
| | 4 | | | | 0.62 |
| Fresh fish | 1 | .08 | .02 | .01 | 0.34 |
| | 2 | | .01 | -.06 | 0.09 |
| | 3 | | | .06 | 0.07 |
| | 4 | | | | -0.03 |
| Other fish/meat/poultry | 1 | .02 | .08 | .06 | 0.28 |
| | 2 | | .15 | .30 | 0.30 |
| | 3 | | | .16 | 0.24 |
| | 4 | | | | 0.27 |
| Pork | 1 | .08 | .31 | .22 | 1.41 |
| | 2 | | .53 | 1.02 | 1.18 |
| | 3 | | | .41 | 0.76 |
| | 4 | | | | 0.35 |
| Beef | 1 | -.23 | .09 | .17 | 1.99 |
| | 2 | | .82 | 1.02 | 1.35 |
| | 3 | | | .77 | 1.10 |
| | 4 | | | | 0.68 |
| Poultry | 1 | .68 | .89 | .67 | 2.08 |
| | 2 | | .82 | 1.27 | 1.35 |
| | 3 | | | .62 | 0.96 |
| | 4 | | | | 0.69 |
| Eggs | 1 | .11 | .26 | .21 | 1.75 |
| | 2 | | .47 | .73 | 1.18 |
| | 3 | | | .47 | 0.80 |
| | 4 | | | | 0.30 |
| Milk/milk products | 1 | .11 | .18 | .10 | 1.24 |
| | 2 | | .21 | .49 | 1.46 |
| | 3 | | | .18 | 0.96 |
| | 4 | | | | 0.66 |

Source: Bouis (1991a).

Table 18—Food demand elasticity estimates, urban Philippines, lowest income quartile

| Food Group | Corn | Rice | Wheat/ Bread | Vegetables Fruits | Other Foods | Fresh Fish | Other Meat | Pork | Beef | Poultry | Eggs | Milk/ Products | Nonfoods | Income |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Corn | -1.59 | 1.15 | .27 | -.20 | .52 | -.03 | .01 | .09 | .01 | .00 | .02 | .05 | .03 | -.33 |
| Rice | .06 | -.66 | .22 | -.20 | .41 | -.06 | -.01 | .07 | .01 | .00 | .01 | .03 | -.01 | .14 |
| Wheat/bread | .09 | 1.44 | -2.39 | .73 | -.22 | .05 | .00 | -.08 | -.01 | .00 | -.01 | -.03 | -.04 | .47 |
| Vegetables/fruits | -.03 | -.48 | .23 | -.96 | .71 | .02 | .03 | .04 | .00 | .00 | .01 | .03 | -.04 | .44 |
| Other foods | .03 | .41 | -.07 | .41 | -1.73 | -.06 | -.05 | -.04 | -.01 | .00 | -.01 | -.05 | -.12 | 1.28 |
| Fresh fish | -.01 | -.14 | .02 | .02 | .04 | -1.41 | .60 | .10 | .03 | .02 | .06 | .36 | -.03 | .34 |
| Other meat | .00 | -.05 | .01 | .04 | .02 | .97 | -1.80 | .10 | .03 | .02 | .06 | .36 | -.03 | .28 |
| Pork | .01 | .19 | -.09 | .03 | -.19 | .25 | .15 | -1.72 | .00 | .00 | .01 | .08 | -.13 | 1.41 |
| Beef | -.02 | -.24 | -.05 | -.10 | -.13 | .04 | .02 | -.01 | -1.34 | .00 | .00 | .01 | -.18 | 1.99 |
| Poultry | -.02 | -.28 | -.05 | -.11 | -.13 | .00 | .00 | -.01 | .00 | -1.28 | .00 | -.01 | -.19 | 2.08 |
| Eggs | -.01 | -.18 | -.05 | -.07 | -.12 | .16 | .10 | .00 | .00 | .00 | -1.48 | .05 | -.16 | 1.75 |
| Milk/products | -.01 | -.12 | -.03 | -.03 | -.07 | .44 | .27 | .04 | .01 | .01 | .02 | -1.65 | -.11 | 1.24 |
| Nonfoods | -.02 | -.29 | -.04 | -.11 | -.10 | -.15 | -.10 | -.02 | .00 | .00 | -.01 | -.06 | -.86 | 1.77 |

Source: Bouis (1991a).

## APPENDIX 5

## DERIVING AN EXPRESSION FOR THE RELATIONSHIP BETWEEN UTILITY FUNCTION PARAMETERS FOR ENERGY ACROSS SOCIOECONOMIC GROUPS

How are the utility function parameters for energy between low-income and high-income groups related? Equation (43) is derived here to provide one possible mathematical relationship. Equation (43) is used to solve for estimates of $w_e e_3$ for various Pakistan urban/rural income groups, as described in Appendix 2.

In general, different socioeconomic groups will have different activity levels, so that the level of energy consumption at which $\partial U_e(E)/\partial E=0$ will vary for each group; call this level $E_{LMX}$ for a low-income group and $E_{HMX}$ for a high-income group. Intuitively, it can be expected that a low-income group will be more active than a high-income group, so that $E_{LMX} > E_{HMX}$. However, the derivations that follow do not depend on such an assumption.

To begin, a reasonable assumption is that

$$U_{e1}(E_{LMX}) = U_{eh}(E_{HMX}), \qquad (38)$$

where

$$U_{e1}(E_{LMX}) = e_{2L}E_{LMX} + e_{3L}E_{LMX}^2 \qquad (39)$$

and

$$U_{eh}(E_{HMX}) = e_{2H}E_{HMX} + e_{3H}E_{HMX}^2. \tag{40}$$

One way of interpreting the assumption made in equation (38) is to consider an individual who shifts to a more active occupation, requiring an increment in calorie consumption equal to $E_{LMX}$ - $E_{HMX}$ to maintain the same weight in the new occupation as in the previous one. Equation (38) implies that utility from energy consumption is equal between the two occupations at consumption levels of $E_{LMX}$ (new occupation) and $E_{HMX}$ (old occupation).

Next, by definition,

$$-E_{LMX} = \frac{e_{2L}}{2e_{3L}} \quad \Rightarrow \quad -2E_{LMX}e_{3L} = e_{2L} \tag{41}$$

and

$$-E_{HMX} = \frac{e_{2H}}{2e_{3H}} \quad \Rightarrow \quad -2E_{HMX}e_{3H} = e_{2H}. \tag{42}$$

Substituting equations (41) and (42) into equations (39) and (40), respectively, before substituting equation (39) and (40) into (38), gives

$$\begin{aligned} -2e_{3L}E_{LMX}^2 + e_{3L}E_{LMX}^2 &= -2e_{3H}E_{HMX}^2 + e_{3H}E_{HMX}^2 \\ e_{3L} &= e_{3H}\frac{E_{HMX}^2}{E_{LMX}^2} \end{aligned} \tag{43}$$

At a specific level of energy intake, E (below $E_{LMX}$ and $E_{HMX}$), intuitively, one would expect that the marginal utility from energy intake for the more active group would be greater than the marginal utility from energy intake for the less active group.

To determine the conditions under which this is the case, letting $MU_L = \partial U_{le}(E)/\partial E$ and $MU_H = \partial U_{he}(E)/\partial E$,

$$MU_L = e_{2L} + 2e_{3L}E = 2e_{3H}\frac{E_{HMX}^2}{E_{LMX}^2}(-E_{LMX} + E), \tag{44}$$

$$MU_H = e_{2H} + 2e_{3H}E = 2e_{3H}(-E_{HMX} + E). \tag{45}$$

$MU_L$ is greater than $MU_H$ if

$$\frac{MU_L}{MU_H} = \frac{E_{HMX}^2(-E_{LMX} + E)}{E_{LMX}^2(-E_{HMX} + E)} > 1$$

or

$$E > \frac{\frac{E_{HMX}}{E_{LMX}}(E_{LMX} - E_{HMX})}{1 - \frac{E_{HMX}^2}{E_{LMX}^2}}. \tag{46}$$

For the range of values of $E_{LMX}$ and $E_{HMX}$ chosen for Pakistan reported in the text (3,200 to 2,800 calories per day per adult equivalent), equation (46) holds for observed calorie intakes above 1,575, which is the case for all income groups.

## APPENDIX 6

## MISCELLANEOUS RELATIONSHIPS/DERIVATIONS

The following derivations are by-products of unsuccessful efforts to solve the first-order conditions for a relatively simple expression for $q_i = f(p_1,...p_n, p_{nf}, Y)$. They may be useful for future work with the FCDS.

### RELATING MONEY FLEXIBILITY AND THE CALORIE-INCOME ELASTICITY

The first-order conditions are given below for two foods (a staple and a nonstaple, food #1 and #2, respectively) and nonfoods.

$$p_1 = \frac{w_e}{\lambda}\left[e_2 z_1 + 2e_3 z_1 E\right] + \frac{w_v}{\lambda}\left[\frac{-M}{T^2}\right] + \frac{w_{t1}}{\lambda}\left[\frac{1}{q_1}\right], \tag{47}$$

$$p_2 = \frac{w_e}{\lambda}\left[e_2 z_2 + 2e_3 z_2 E\right] + \frac{w_v}{\lambda}\left[\frac{T-M}{T^2}\right] + \frac{w_{t2}}{\lambda}\left[\frac{1}{q_2}\right], \tag{48}$$

$$p_{nf} = \frac{\frac{\partial U}{\partial U_{nf}}}{\lambda}\left[\frac{\partial U_{nf}}{\partial q_{nf}}\right] = \frac{w_{nf}}{\lambda}\left[\frac{\partial U_{nf}}{\partial q_{nf}}\right]. \tag{49}$$

Also,

$$Y = p_1q_1 + p_2q_2 + p_{nf}q_{nf}. \tag{50}$$

In this simple case,

$$E = z_1q_z + z_2q_2, \tag{51}$$

$$M = q_2 \; ; \quad T = q_1 + q_2 \; ; \quad T-M = q_2. \tag{52}$$

This is *not* done in the text, but to simplify the discussion, assign a specific functional form for nonfoods:

$$U_{nf}(q_{nf}) = \log\ q_{nf}. \tag{53}$$

Substituting for M, T, E, and $\partial U_{nf}/\partial q_{nf}$, and multiplying equation (47) by $q_1$, equation (48) by $q_2$, and equation (49) by $q_{nf}$ gives

$$p_1q_1 = \frac{w_e z_1 q_1}{\lambda}\left[e_2 + 2e_3(z_1q_1 + z_2q_2)\right] + \frac{w_v}{\lambda}\left[\frac{-q_1q_2}{(q_1 + q_2)^2}\right] + \frac{w_{t1}}{\lambda}, \tag{54}$$

$$p_2q_2 = \frac{w_e z_2 q_2}{\lambda}\left[e_2 + 2e_3(z_1q_1 + z_2q_2)\right] + \frac{w_v}{\lambda}\left[\frac{q_1q_2}{(q_1 + q_2)^2}\right] + \frac{w_{t2}}{\lambda}, \tag{55}$$

$$p_{nf}q_{nf} = \frac{w_{nf}}{\lambda}. \tag{56}$$

Multiply equations (54), (55), and (56) by $\lambda$, sum these three equations, and substitute for Y from equation (50). This gives (also using equation [51])[35]:

$$\lambda Y = w_e \left[ e_2 E + 2e_3 E^2 \right] + w_{t1} + w_{t2} + w_{nf}. \tag{57}$$

For convenience, let Z denote the RHS of equation (57). Taking the partial derivative of equation (57) with respect to income,

$$\frac{\partial \log\lambda}{\partial Y} + \frac{\partial \log Y}{\partial Y} = \frac{\partial \log Z}{\partial Y},$$

$$\frac{1}{\lambda}\frac{\partial\lambda}{\partial Y} + \frac{1}{Y} = \frac{1}{Z}\frac{\partial Z}{\partial Y},$$

$$\frac{Y}{\lambda}\frac{\partial\lambda}{\partial Y} = \left[\frac{E}{Z}\frac{\partial Z}{\partial E}\right]\left[\frac{Y}{E}\frac{\partial E}{\partial Y}\right] - 1. \tag{58}$$

The LHS of equation (58) is the money flexibility. The second bracketed term on the RHS of equation (58) is the calorie-income elasticity, which is usually found to be positive. To look more closely at the first bracketed term on the RHS of equation (58), take the partial derivative of the RHS of equation (57) with respect to E:

$$\frac{\partial Z}{\partial E} = w_e(e_2 + 4e_3 E) \tag{59}$$

or

[35] Equation (57) turns out to be quite general; the variety terms always disappear from the summation. For the case of *n* foods, and any disaggregation of these *n* foods between staples and nonstaples (at least one staple and one nonstaple), the list of $w_{ti}$'s would have *n* terms instead of just two terms.

$$\left(\frac{E}{Z}\right)\frac{\partial Z}{\partial E} = \frac{w_eE(e_2 + 2e_3E) + w_eE(2e_3E)}{w_eE(e_2 + 2e_3E) + w_{t1} + w_{t2} + w_{nf}}. \quad (60)$$

Equation (59) is less than zero for $E > E_{umx}/2$, which is the relevant empirical range, so that the numerator of equation (60) is negative. Where the marginal utility of calories is positive (that is, $e_2 + 2e_3E > 0$), the denominator of equation (60) is positive, so that the entire expression is positive. Therefore, the first bracketed term on the RHS of equation (58) is negative, while the second bracketed term is positive. Therefore, where the marginal utility of calories is positive, the money flexibility will be less than -1. Where the marginal utility of calories is zero, equation (60) will remain negative. However, at sufficiently high intakes of calories, the denominator of equation (60) may become negative, at which point the money flexibility falls between -1 and 0.

If the simplifying assumption of equation (53) is dropped, it is no longer possible simply to write $\partial Z/\partial Y = (\partial Z/\partial E)(\partial E/\partial Y)$, so that these results do not necessarily hold. Nevertheless, controlling the effects of $\partial(w_{nf}\, q_{nf}\, [\partial U_{nf}\,(q_{nf})/\partial q_{nf}]/\partial Y$ on Z, there is tendency for the money flexibility to take on values less than -1.

THE RATIO OF TWO FOOD INCOME ELASTICITIES

A relatively simple expression can be developed for the ratio of two food income elasticities as described below. The determinant of the entire bordered Hessian in Appendix 1 may be expressed as

$$D = f_{55}|D_{55}| - p_5 p_5 |D_{44}|, \tag{61}$$

where

$$D_{55} = \begin{vmatrix} x_{11} & x_{12} & x_{13} & x_{14} & -p_1 \\ x_{21} & x_{22} & x_{23} & x_{24} & -p_2 \\ x_{31} & x_{32} & x_{33} & x_{34} & -p_3 \\ x_{41} & x_{42} & x_{43} & x_{44} & -p_4 \\ -p_1 & -p_2 & -p_3 & -p_4 & 0 \end{vmatrix}$$

and

$$D_{44} = \begin{vmatrix} x_{11} & x_{12} & x_{13} & x_{14} \\ x_{21} & x_{22} & x_{23} & x_{24} \\ x_{31} & x_{32} & x_{33} & x_{34} \\ x_{41} & x_{42} & x_{43} & x_{44} \end{vmatrix}$$

and

$$x_{11} = E_{11} + V_{11} + T_{11} \; ; \; x_{12} = E_{12} + V_{12} \; . \; . \; . \; ;$$

and so forth. D is required in the computation of the income elasticity of any food, say the first food (see equation [2-31] in Henderson and Quandt [1980]):

$$\eta_1 = -\left(\frac{Y}{q_1}\right)\left(\frac{D_{61}}{D}\right).$$

$D_{61}$ is cofactor for the last element in the first row of the bordered Hessian:

$$D_{61} = \begin{vmatrix} x_{12} & x_{13} & x_{14} & 0 & -p_1 \\ x_{22} & x_{23} & x_{24} & 0 & -p_2 \\ x_{32} & x_{33} & x_{34} & 0 & -p_3 \\ x_{42} & x_{43} & x_{44} & 0 & -p_4 \\ 0 & 0 & 0 & f_{55} & 0 \end{vmatrix} \tag{62}$$

$$= f_{55} \begin{vmatrix} x_{12} & x_{13} & x_{14} & -p_1 \\ x_{22} & x_{23} & x_{24} & -p_2 \\ x_{32} & x_{33} & x_{34} & -p_3 \\ x_{42} & x_{43} & x_{44} & -p_4 \end{vmatrix} = f_{55}|D_{14}| .$$

By analogy, for the second food, equation (63) may be written

$$D_{62} = -f_{55}|D_{24}| , \tag{63}$$

so that

$$\frac{\eta_1}{\eta_2} = \frac{-q_2}{q_1} \frac{|D_{14}|}{|D_{24}|} . \tag{64}$$

Note that the ratio of two food income elasticities is independent of any prior information related to nonfoods; it does not involve $\phi/\eta_{nf}$.

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