Portfolio Theory and the Demand for Futures: theory and the case of California cotton

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I. Introduction

Although farmers and storers can both make use of the futures markets to reduce risk, surveys such as those by the CFTC show that very few farmers (5 percent) participate directly in the futures markets and most speculate rather than hedge. Asked why they did not participate, about one-third of the farmers said they did not know how the markets worked and about one-third said it was not worth it to them to participate. Ten percent said it was not worth the effort to find out whether it was worth it to them. From this survey, the Commodity Futures Trade Commission (CFTC) concluded it should provide more education to farmers so that more farmers would hedge.

The information in the survey--basically the findings that one-third of the farmers are ignorant--is not sufficient for either the finding that it is ignorance that prevents farmers from using futures or for the finding that education will increase their use of the futures market. Indeed, it may well be that sober introspection and careful economic analysis lead to the conclusion that farmers should either not participate in these markets or that they should speculate. The purpose of this paper is to lay out the factors that influence farmer hedgery and empirically estimate their importance.

The basic framework is portfolio analysis, which we empiricize by the assumption that utility is a function of mean and variance, and is, of course, the same assumption as that made by Peck, Rutledge, or Rolfo. The model differs from Peck or Rutledge in that they assume hedging is costless, and we settle the cost of hedging as an empirical matter. The costs of hedging are the risk premium which is empirically important and a tying up of credit which does not influence the empirical results in this paper.

The model differs from all three previous authors in the simultaneous choice of both crops and futures. This approach consolidates the choice of crops approach of Freund and others with the choice of the futures approach of Rutledge, Peck, Rolfo, and others. Empirically, the simultaneous choice of crops and futures makes a large difference in the optimal hedge and in the location of the mean variance frontier.

The paper is in four sections. The theory section (II) lays out the mean variance framework and discusses its limitations. It provides a brief example to show the importance of simultaneous choice of crops and futures; it explicates the role of costs of futures holding and of credit or debt; and it explains the role of forecasting. The next two sections implement the theory.

The estimation section (III) uses regression techniques to predict the mean and variance of return for the crops and futures. It discusses the choice between time series methods and a prediction model. It presents estimates of the means and variances by generalized least squares on data largely derived from the Kern County Agricultural Comissioner's reports. As other authors (Lin, Dean, and Moore) who investigated this cropping system have found, alfalfa and barley have the lowest means and variances of the sampled crops, cotton has higher mean and variance, and sugar beets and potatoes have the highest. As Cootner found, selling cotton futures involves a substantial expected loss--the risk premium. The means and variances of Section III are combined with a quadratic program to get the results of Section IV.

The empirical results section (IV) substantiates the role of cost of hedging and of simultaneous choice of crops and futures. The naively calculated hedge is 3.4 percent. It accounts for neither the costs of futures trading nor for other ways to diversify risk. The true optimal hedge is a much larger, though still not very large 11 percent.

II. Theory

Factors Affecting Mean and Variance

Constrained by their landholdings and their credit lines, farmers choose futures holdings and cropping patterns to maximize the expected utility of their income. As an approximation, expected utility can be taken as a function of mean and variance of income where futures holdings affect both the mean and variance.

This section discusses a portfolio model and points out the limitations of a one-period mean variance model. Then the section discusses the effects of futures and crops in creating a diversified portfolio, with special attention to the costs of futures and the effects of fixed debt. The last part of the section describes the mathematical portfolio model that is empiricized later in this paper.

One-period models, such as those of Tobin, Markowitz, and Sharpe, achieve great simplicity at the expense of not modeling the savings decision or the behavior of adaptive control. Even though Samuelson has solved the more general lifetime portfolio problem, the limitations his model places on the distribution of returns and on the utility functions, as well as the more basic matter of computability, have led all the recent workers (Rutledge, Peck, and Rolfo) on futures demand as portfolio choice to use the one-period framework. Although the omission of the savings decision is not serious in this context, the omission of adaptive control may substantially alter the calculation of the optimal hedge. As information on the progress of the crop becomes available, yield risk is reduced and the percent hedged should rise. The model presented here follows the earlier workers in this area and ignores the effects of adaptive control.

Having settled for a one-period model, the next important choice for the analyst is the family of utility functions. Generalized mean variance functions--U (M, V), increasing in mean M and decreasing in variance V--are known to approximate the other utility functions common in finance (Levy and Markowitz) and have the advantage of taking their maximum on the computable efficient set (or mean-variance frontier). This locus of minimum variance for a given mean makes all the possible portfolios that maximize U (M, V) quite independent of which U is actually chosen. Thus, the analyst is freed from having to choose a specific utility function, and he can analyze the portfolios in the whole U (M, V) maximal class at once. The major drawback to the use of mean variance analysis in agriculture is that yields are known to have skewed distributions (Day) which might distort the ability of variance to represent downside risk. By use of a Chebychev inequality on semivariance, it is possible to check on the effect of nonsymmetric distributions on portfolio choice. This project is left for the empirical results section.

What to include in the menu of assets is at least as important as the choice of time frame or utility function. The literature emphasizes either the role of diversifying crops to reduce variance in income (Freund; Carter and Dean; and Lin, Dean, and Moore) or the role of futures to diversify variance (Rutledge; Peck; and Rolfo). Both schools of thought are important, and there is no reason why they cannot be considered simultaneously: Indeed, as a brief and extreme example will make clear, both the crop and futures approach to risk diversification must be considered simultaneously. There are two states of nature: good (g) and bad (b); two crops, A_g and A_b; and a future, F. In the good state of nature, the payoffs to a unit holding of A_g, A_b, and F are 1, -1, -1, while, in the bad state of nature, the payoffs are

-1, 1, 1. The payoffs of these assets are perfectly correlated. Perhaps the crops grow with certainty, and the uncertain prices of the crops are perfectly negatively correlated.

A zero variance portfolio can be achieved in one of three ways: (1) either grow the crops in equal proportion, (2) grow the good crop and go long in the futures, or (3) grow the bad crop and short the futures. If the futures are free, then all three ways of getting zero variance have the same mean and belong to the efficient set. (Note that adding futures to the $1/2\ A_g$ + $1/2\ A_b$ portfolio <u>increase</u> the variance.) Any other price for futures (without transaction costs) leads to the futures being the only way to get a mean variance efficient point, and the introduction of a transaction cost (of, say, 3) makes the use of crops alone the only mean variance efficient point. As this example shows, mean-variance efficient portfolios require the simultaneous choice of crops and futures.

In a mean variance framework, the mean is as important as the variance. Crops are grown (presumably) because, on the average, the value of the crops exceeds the costs of growing the crops. From this expected crop income, the payments for the land and equipment that made it possible must be subtracted. These debt payments, which shift the mean variance frontier to the left, make any crop futures plan have a lower mean for a given variance or, in common parlance, be riskier. After netting debt payments from income, the issue of futures losses or gains still remains. Except for Rolfo, no other writers have considered the effect of futures on mean income.

Futures are costly because they tie up credit and because of brokerage fees. Neither of these costs need be very great with a sympathetic banker (who will lend money at close to prime so the cost of the money is the prime less the T-bill rate and who will not impinge on the farmers other credit lines) and with a discount broker (who will charge about $20 for a "round turn"). A greater cost is most likely to be the expected loss on the short sale of the futures. Without predictive ability on the part of farmers, the Keynes-Hicks-Cootner theory of speculative markets holds that storers and producers of commodities will pay speculators to take the price risk of holding commodities. Indeed, this cost which can run several cents per pound for cotton (and is assumed away in Peck's work on eggs),[1] provides a major reason for farmers not to hedge. On the contrary, if farmers have good predictive ability, then they may enter the futures market for the same reason as speculators to make an (expected) profit on their futures position.

In short, (1) farmers make simultaneous decisions on crops and futures, (2) they may evaluate losses differently than gains (skewness), (3) futures holdings tie up their credit, (4) mortgages leave them leveraged, (5) hedging is expected to result in a loss, and (6) forecasting may allow a gain on futures.

The Model

More formally, a farmer has fixed acreage, L, debt, D, and allocatable credit (and wealth), W. The total landholding of L acres is split among crops with acreage, A_i, so

$$\Sigma A_i = L$$

In a more compact notation, let $A = (A_1 \dots A_n)$ and $a = (1, \dots, 1)$ so

$$a'A = L \tag{1}$$

Allocatable credit, W, is split between futures, F_i, that tie up f_i dollars each or is unused, B. Each futures impinges on the credit constraint by the maximum amount the farmer is prepared to lose on that contract (worst possible variation margin) and the initial margin. Again, utilizing vector notation,

$$(2) \qquad W = f'F + B.$$

More generally, wealth could have been allocated to financial assets other than the implicit bond, B, or futures. Stocks, options, city real estate, or any other asset could be included.

The constraints on wealth (or credit) and land define the farmer's choice set. After he has chosen acreage, A, and futures, F, the state of nature—which is the profitability per acre of his crops, x, and the gains or losses from his futures positions, z, including the interest paid on the margin posted as T bills—become known, and he receives his income, Y. Composed of profits from crops, losses (or profits) from futures, interest cost on credit, interest paid on deposits, and interest paid on fixed debt (farm income), this income is stochastic.

$$(3) \qquad Y = x'A + z'F - rW + rB - rD.$$

From this expression and the definitions of $\tilde{x} = x - \bar{x}$, $\tilde{y} = y - \bar{y}$, and $\tilde{z} = z - \bar{z}$ where the bars denote the means and E is the expectation operator, one can calculate the mean and variance of income for given futures and cropping plans.

$$(4) \qquad M = \bar{x}'A + \bar{z}'F + r\,(B - W) - rD$$

$$(5) \qquad V = A'E\,(\tilde{x}\tilde{x}')\,A + F'E\,(\tilde{z}\tilde{z}')F + 2A'E\,(\tilde{x}\tilde{z}')F.$$

The three terms in the variance equation (4) are (a) the variance from crops, (b) the variance from futures, and (c) the covariance of crops and futures. The covariance term includes the covariance of each crop and all futures, so the covariance of wheat and wheat futures as well as wheat and cotton futures enter the management decision.

Letting λ shadow the land constraint and γ shadow credit, the Kuhn-Tucker conditions for an optima are written in terms of the derivatives of the Lagrangian, L,

$$L = U\ (M,\ V) + \lambda\ (L - a'A) + \gamma\ (W - f'F - B)$$

$$L'_A = U_m\ X' + U_v\ (2A'\ E\ [\tilde{x}\tilde{x}] + 4\ F'E\ [\tilde{z}\tilde{x}']) - a'\lambda \leq 0 \tag{6}$$

$$\text{and } L_{A_i} \cdot A_i = 0 \qquad \text{for every } i.$$

$$L'_F = U_m\ \bar{\bar{Z}} + U_v\ (2F'\ E\ [\tilde{z}\tilde{z}'] + 4\ A'E\ [\tilde{x}\tilde{z}']) - \gamma f' \leq 0 \tag{7}$$

$$\text{and } L_{F_i} \cdot F_i = 0 \qquad \text{for every } i.$$

$$L_B = U_m r - \gamma \leq 0 \quad \text{and} \quad L_B \cdot B = 0 \tag{8}$$

and (1) and (2) above which are just the equality constraints. In addition, there are nonnegativity constraints, but they do not help in the discussion and will not be stated. Equation (6) states that either a crop is not grown or, at an optimal portfolio, its mean return times the marginal utility of the mean plus twice its variance and its covariance with each other crop and futures times the marginal utility of variance equals the shadow price of land.

Equation (7) gives the credit cost for each future in the portfolio which equals the marginal utility of the mean times the mean return for a future plus the marginal utility of variance times twice the variance plus the covariance of the futures with each other crop and futures.

As equations (6)-(8) amply illustrate, computation of an optimal program requires estimates of mean and the variances and covariances which is the task of the next section.

III. Means and Variances

Before estimating the mean variance frontier and examining the effects of a simultaneous choice of crops and futures and the other interesting issues, it is necessary to estimate the means and variance-covariance matrix of the several crop and futures activities. These variances and means are usually constructed by time series analysis, but the Gauss-Markov theorem argues for the use of generalized least squares (Fried). The GLS prediction errors estimate the variance, while the prediction estimates the mean. Empirical difficulties encountered in the estimation include the construction of the futures variables, the poor quality of the cost data, and the less-than-heartening fit for the sugar beets equation. This section proceeds by discussing the estimator, the crops data and results, and the futures data and equations.

The Estimator

Gauss-Markov estimates provide the best linear unbiased predictor in the usual sense; any other unbiased linear predictor has a variance-covariance matrix that exceeds the Gauss-Markov predictor by a positive semidefinite matrix. Therefore, any other predictor will probably direct a mean-variance decision-maker's choice toward a higher mean, lower (true) variance plan. Since the distortion of choice in a suboptimal plan involves a real loss in welfare, the

Gauss-Markov theorem is a powerful argument for the use of a properly specified regression technique.

When, as is true here, there is no problem of simultaneous equation bias, the best linear unbiased predictors of the returns per acre and futures gains are given by generalized least squares. Durbin-Watson tests on the individual equations show that autocorrelation is not a problem, but it is a part of the hypothesis that there is contemporaneous covariance among the prediction errors of which the regression errors are a part. To account for this (possible) contemporaneous correlation in the errors, the equations are jointly estimated by a variant of the Zellner seemingly unrelated regression (SUR) technique.

Crop Activities Data

The basic data on crop yields and revenues come from the *Kern County Agricultural Crop Reports*, while the costs are from the Extension Service cost-of-production data and represent the judgment of the farm advisers. The farm advisers' cost estimates conform very closely to the 1969 Economic Research Service (Sutherland, Carlson, and Hoover) estimates for cotton costs but differ greatly from Eric Thor, Sr.'s sugar beets estimates. The quality of cost data seems to be reflected in the regression results: the cotton equations fit best, and the sugar beets fit worst. The crop data, which are county average data, do not reflect the variance among farms within a year--an additional source of risk.

All of the data have been deflated by a California consumer price index (1978 = 1.00) because the agents are presumed to be interested in real income, not money income. Table 1 gives the definition of the variables.

Table 1. Variable Definitions and Sources

Dependent variables	Revenues less costs per acre for each crop deflated by California consumer price index, except sugar beets for which revenues and costs, deflated, are used separately.
Revenues	Value of production divided by harvested acres. For cotton, includes both lint and seed. Source: Agricultural Crop Report, Kern County, annual issues.
Cost	Variable costs from harvest and preharvest plus equipment and irrigation interest and depreciation, excluding charges for land. Source: Cost of Production. University of California, Cooperative Extension Service, Kern County data, various years. Intervening years are interpolated by regression on index of prices paid by farmers.
LCOSTA	Deflated lagged cost of alfalfa per acre.
FPREDA	December 15 futures price of September wheat deflated by California consumer price index times once lagged yield per acre of alfalfa. Source of yield per acre: Agricultural Crop Report.
FPREDB	December 15 futures price of September wheat deflated by California consumer price index times once lagged yield per acre of barley. Source: Agricultural Crop Report. California CPI in Economic Report of the Governor. Sacramento, 1979.
LCOSTB	Deflated lagged cost of producing barley per acre.
LCOSTC	Deflated lagged cost of producing cotton per acre.
FPREDC	April 15 futures price of December cotton lint, deflated, times lagged yield per acre. Source: Agricultural Crop Report.
LPREDS	Lagged revenues of cottonseed. Source: Agricultural Crop Report.
FPREDP	January 15 futures price of May potatoes, deflated, times lagged yield per acre. Data for 1969-1977.
DUMP	Dummy; 1 for 1963-1968.
LCOSTP	Lagged deflated cost of producing potatoes per acre.
LPREDP	Lagged deflated revenues per acre for potatoes. Source: Agricultural Crop Report.
LCOSTSB	Lagged deflated cost of producing sugar beets per acre.
LPREDSB	Lagged revenue of sugar beets. Source: Agricultural Crop Report.

The ideal form for the equations would be estimation of the restricted profit function (or quasi rent function) as a function--perhaps a translog--of expected input prices and output price. The observations on inputs are so sparse that this sort of procedure is not feasible; instead, costs of production are simply based on lagged costs.

The form chosen for the quasi rent equation is

$$\pi = \alpha + \beta P^e \cdot Y^e + \gamma C^e$$

where P^e is expected real price and is usually proxied by an appropriate futures price, Y^e is expected yields (last year's yields), and C^e are expected real costs (last year's costs). Theory suggests that, ceteris paribus, increases in output price should induce increase in yields and costs per acre. And the Keynes-Hicks-Cootner theory of futures has the futures price underestimating the expected spot. For both these reasons, β should be greater than one. The impact of changing P^e on costs is not testable in this formulation, and the crude form of the costs data sheets would not support such estimation.

The Estimation: Crops

All but the sugar beets equation performed reasonably as can be seen in Table 2. The futures price of wheat was successful in explaining profits in both alfalfa and barley.[2] The fits in these equations were good (R^2 = .77 and .45, respectively); and the t ratios were substantial (7.6 and 4.6, respectively). The Durbin-Watson statistic from the OLSQ results (which are omitted for brevity) confirmed that autocorrelation was not a problem. The lagged cost variables should be close to and below one because of the slight

Table 2. Crop Regressions, 1963-1977

Crop	Coefficient	Standard Error	t Ratio
		Seemingly Unrelated Regression[a]	
		Profits	
Alfalfa			
CONSTANT	78.87	16.79	4.69
LCOSTA	- 0.72	0.09	7.81
FPREDA	7.68	1.00	7.61
	R^2 = .77		
Barley			
CONSTANT	5.35	38.95	0.13
FPREDB	11.52	2.52	4.56
LCOSTB	- 0.48	0.45	1.05
	R^2 = .45		
Cotton			
CONSTANT	715.75	136.68	5.23
LCOSTC	- 3.24	0.45	7.15
FPREDC	0.69	0.10	6.79
LPREDS	1.53	0.40	3.82
	R^2 = .88		
Potatoes			
CONSTANT	646.59	832.62	0.77
FPREDP	0.41	0.13	3.01
LPREDP	0.35	0.20	1.73
DUMP	209.46	343.98	0.60
LCOSTP	- 1.51	1.17	1.28
	R^2 = .23		
		Revenues	
Sugar beets[b]			
CONSTANT	178.94	66.77	2.67
LPREDSB	0.45	0.19	2.39
	R^2 = .22		
		Costs	
Sugar beets			
CONSTANT	43.79	29.02	1.50
LCOSTSB	0.79	0.13	5.91
	R^2 = .58		

[a]The seemingly unrelated regression technique was used simultaneously on all crops and futures. For clarity of exposition, the futures results are presented in Table 3, *infra*, p. 17.

[b]Sugar beets profits are computed as revenues minus costs.

Source: Computed

downward trend in real costs--a result of technical progress. For alfalfa, the coefficient was two standard deviations below one--a little too low at -.70 but of the right sign and significantly different from zero. For barley, the coefficient, though of the right sign, was not significantly different from zero; and the point estimate, -.48, was far too low. On the whole, the alfalfa and barley equations were judged to have performed satisfactorily.

The cotton equation contains a variable for both predicted seed revenue, based on a lag, and lint revenue, based on the futures price. Although both were of the right sign and significant, the expectation that the coefficient of FPREDC would be greater than one (a change in price would increase both yield and price and backwardation) was not borne out. The cost variable--also of the right sign with a high t ratio--was also too large, about -3 rather than the expected -1. Given the crude functional form (and the high R^2 and good Durbin-Watson), these results are judged to be good.

Root crops are widely believed to be risky and the equations for sugar beets and potatoes bear out this wisdom with large prediction errors, low R^2 (.23, .22), and a general lack of significance in the coefficients. The bright spots in these equations are the lack of autocorrelation in the OLSQ results, the expected signs on all the potato variables, and the surprisingly successful use of the potato futures (FREDP) in explaining California potato profits. The sugar beets equations were broken into separate cost and revenue equations because of sign problems when estimated in the same form as the other equations. Profits were then estimated as revenue minus cost which are unbiased but with larger prediction errors than the GLS estimator. The domestic sugar futures were tried as an explanatory variable, but they were insignificant.

Futures

The futures strategies considered are those most appropriate to a grower: the sale at planting time of futures that mature slightly after harvest. Provision is made for hedging or speculating and the sale or purchase of futures after prespecified price changes. Trading after a prespecified price change is a crude way of accounting for limited credit (which plays no role in the empirical results) and for a mean-variance, decision-makers' disproportionate concern with large price swings. This logic is familiar to other insurance buyers as the rationale for deductibles. For both of these reasons, futures are imagined to be purchased with a stop-loss order varying between 4 cents and $1.00 per unit (pound or bushel). For example, 50,000 pounds (one contract) of cotton sold with a 25-cent stop-loss amount (SLA) requires a $12,500 line of credit. If the futures rises 25 cents, the contract would be repurchased. These stop-loss orders allow the construction of complicated strategies that start speculative and end as hedges such as two long contracts with a 10-cent stop loss and one short contract. On a price rise, the position remains net one long; on a price fall, it reverts to one contract short (and a loss of 10 cents per pound times two contracts, or $10,000 for cotton).

The gains (or losses) from selling a particular contract in a given year were calculated as follows: The opening price, P_0 was taken as the price at the close of trading on the 15th of the month in which the crop was planted. The date the contract was liquidated, T, was the same as the end of the contract or the first time that the contract lost the stop-loss amount. For a short SLA = $P_T - P_0$ defines T, while for a long SLA = $P_0 - P_T$.

The actual amount lost for a short is $P_T - P_0$ plus interest at 10 percent on margin and variation margin plus brokerage fees. The 10 percent interest rate is a compromise; it is too high in the early years and too low today. Moreover, hedgers posting T bills pay only the difference between the T-bill rate and the prime, always less than 10 percent. Changing the interest rate makes little empirical difference.

The natural choices for commodities contracts are: cotton to hedge cotton lint (but not seed); sugar to hedge beets, but the three different contracts and two institutional structures for sugar in the last 15 years rendered this contract useless and it was omitted; Maine potatoes to hedge potatoes (used even though empirically the correlation with the California new potatoes is slight); and a grain to hedge alfalfa and barley. Corn or wheat are the natural choices. Their prices are highly correlated (.5), so only one was chosen. Corn is closer to alfalfa as an animal feed, but wheat exhibits normal backwardation. The reported results use wheat, but calculations with corn futures produce substantially the same results.

The mean and variance of holding futures were estimated by linear regression and reported in Table 3. The Keynes-Hicks theory of backwardation with the Cootner wrinkle holds that, during the period consumption comes from storage, the price of futures should rise effecting a risk premium paid by storers to speculators. The regression results show this risk premium as an expected loss to the short position. The lack of significance (low t ratios) was cited as evidence against the Keynes-Hicks-Cootner theory by Telser and others. In this formulation, high t ratios would imply almost certain profits from holding a long position, so the low t ratios is almost expected. Adding

Table 3. Returns from Futures Positions

Code	Commodity	Position	Limit	Expected Return	Standard Error	t	R^2
			1978 cents per pound or bushel				
SCOT10	Cotton	Short	10	- 2.10	0.12	0.200	0.003
SCOT25	Cotton	Short	25	7.80	0.16	0.520	0.020
SCOT50	Cotton	Short	50	9.10	0.18	0.540	0.020
LCOT10	Cotton	Long	10	10.10	0.16	0.690	0.040
LCOT25	Cotton	Long	25	3.10	0.18	0.190	0.003
SPOT4	Potatoes	Short	4	- 0.89	0.18	0.510	0.020
SPOT10	Potatoes	Short	10	- 0.41	0.02	0.290	0.006
LPOT4	Potatoes	Long	4	- 0.60	0.02	0.380	0.010
LPOT10	Potatoes	Long	10	- 0.47	0.02	0.320	0.008
SWHT25	Wheat	Short	25	-19.06	0.29	0.660	0.030
SWHT50	Wheat	Short	50	- 6.22	0.45	0.140	0.002
LWHT25	Wheat	Long	25	-30.34	0.31	0.980	0.070
LWHT50	Wheat	Long	50	0.47	0.95	0.005	0.000

Source: Computed by Zellner's seemingly unrelated regression technique.

other variables, such as planting intentions or opening price, does not improve the predictive power of the equations.

IV. Empirical Results

With the econometric estimates of mean and variance constructed in the past section and the opening futures prices for 1978, we used quadratic programming to construct the mean variance frontier, of which some values are tabled in the last column of Table 4. The cropping plans and futures holdings along this frontier are described in the next section. Following the description of the frontier, the Risk and Debt subsection gives two ways of choosing a point on the frontier and evaluates the effect of fixed debt on the optimal choice. Then succeeding sections show that the benefits of simultaneous choice of crops and futures are quite large and that the simultaneous choice has a large effect on the amount of hedging. The last sections show that the price of hedging is a significant determinant of the amount hedged and that forecasting can make a difference. Taken together, these sections show that a full portfolio approach, taking a proper account of the price of hedging, makes a significant difference in the policies a farmer should adopt and that these policies do not include a strong reliance on the futures markets.

The Frontier

The mean variance frontier starts with less risky and moves to more risk crops while going from hedge to speculative futures positions. At low mean returns (about 100,000 per acre), cotton and alfalfa are grown; about 11 percent of the cotton is hedged. Although barley is also a low-risk crop, it is omitted from the portfolio because it is projected to cause losses in 1978. Moving up the mean variance frontier entails substituting some sugar beets for the cotton and alfalfa and changing the futures position to a more speculative

Table 4. The Mean-Variance Frontier for Six Asset Choices

Mean Return	Cotton and 250 Acres of Alfalfa	Cotton and Alfalfa	All Crops	All Crops and SCOT50	All Crops and Cotton Futures	All Crops and Futures
1978 --- (dollars) ---						
97,250	1.952	1.680	1.142	1.134	1.128	.802
116,700	2.808	2.452	1.642	1.642	1.620	1.158
136,150	[a]		2.240	2.240	2.224	1.574
155,600			2.996	2.996	2.974	2.080
175,050			4.132	4.132	3.988	2.694
194,500			6.084	6.084	5.622	3.412
320,000			30.000	30.000	23.000	16.000

[a] Blanks indicate this mean cannot be achieved.

Source: Computed.

stance. In the data, going long in cotton has an expected positive return and a mean variance trade-off somewhat comparable to growing sugar beets; so, by the time mean income reaches $200,000, the futures position is to start by speculating (Texas hedging) in cotton and only revert to a true hedge position if the price moves 10 cents against the farmer. The positions advocated in the sugar beets portion of the mean variance frontier are suspect because of two pecularities in the data: (1) the cost of growing sugar beets is suspect and probably too low and (2) the sample wheat futures and sugar beet profits have the astounding covariance of -.5, leading to the cross-hedging of sugar beets in the wheat market. Although it is hard to accept the recommendation that sugar beets be grown, a number of other crop combinations currently grown in Kern County could just as easily fill the expected high-profit mildly speculative niche: to name a few, carrots, garlic, soybean-barley double crop, tomatoes, and dry beans.

The further reaches of the mean-variance frontier are characterized by a potato cropping system. Because early potatoes are a very risky perishable fresh vegetable, very few operators (34 in the Bank of America sample of 1,028, cited by Pope) grow them, although these growers are each so large that potatoes are the largest acreage vegetable in Kern County. Potatoes are not hedgable because the contracts traded are for Maine and Northwest potatoes and correlate almost not at all (-.01) with Kern potato profits. Moreover, the trade between mean and variance is so extreme at the point on the efficient set where potatoes are grown that outright speculating in cotton is part of the efficient portfolio. Agents capable of accepting the risk-variance trade-off of growing potatoes are also willing to accept the mean variance trade-off of pure speculative activity in the futures market.

Risk and Debt

The evaluation of risk is the choice of a point on the MV efficient set, either by a utility function written explicitly in mean and variance or by a safety-first criteria. An example of a mean-variance function is ln (M) - V/M^2 which approximates ln (Y). The function takes its maximum at the point on the frontier at which the mean is 450,000. When the problem is altered to include a fixed debt load of $80,000 (1,000 acres at the cheap price of $1,000 per acre (80 percent) financed by a 10 percent mortgage), the optimal point on the frontier has mean 370,000. Another way to view mean-variance frontiers is to convert the efficient set to a safety-first statement through the Chebychev inequality prob $(Y < M - k\sqrt{V}) < 1/k^2$; or if skewness is a problem, a new variant (Berck and Hihn) of the inequality in terms of semivariance, SV, prob $(Y < M - k\sqrt{V}) < SV/(k^2V)$. The Chebychev bounds are well known not to be tight, but the experience in growing cotton in this century gives results much closer to inference from Chebychev than from inference from the normal (two observations at $M - 5\sqrt{V}$ in 50 years) and the distribution of income (not yield) is so close to symmetric that use of semivariance tightens the bound to $\leq 1/(2k^2)$. The probability of meeting a fixed payment of $80,000 increases as the mean return increases until the mean return is 300,000 and probability of success is 23 percent; it decreases thereafter. To break even (zero payment), the safest mean return is about 100,000. Clearly, "safety first" is very sensitive to fixed payments; just as clearly, growing cotton and sugar beets (which gives the greatest chance of netting $80,000) is not a very safe activity.

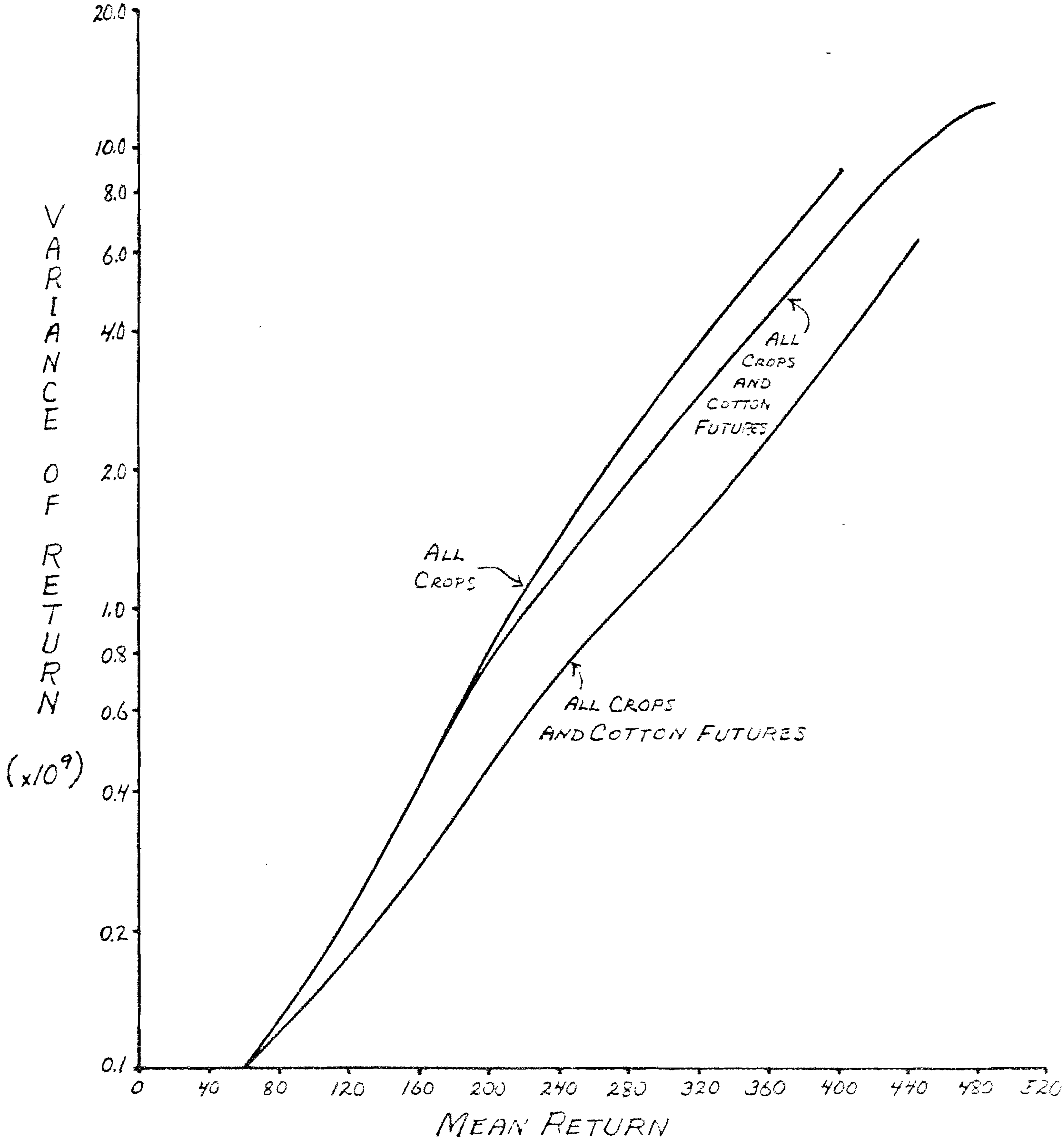
Benefits of Diversification

Crop diversification and futures positions make a large difference in the mean-variance frontier. Table 4 gives the frontier for six plans in the "cotton region," while Figure 1 shows the whole frontier for three asset choice plans. Limited to cotton and alfalfa with no futures, the greatest expected return possible is $126,000; a $116,700 return incurs a variance of $2.54\ 10^9$. When the menu of assets is expanded to include all crops, the same mean return can be achieved with two-thirds the variance ($1.64\ 10^9$) by growing a small quantity (about 100 acres) of sugar beets. Adding all cotton futures to the mix changes nothing; it lessens the variance by less than 1 percent. The addition of the other futures again makes a perceptible difference: variance is reduced to about $1.16\ 10^9$ which is about a 40 percent reduction. This variance reduction is achieved by growing less cotton, more alfalfa, and almost twice as many sugar beets as in the crops-only solution while, at the same time, adopting a complicated speculative position in cotton and "hedging" eight contracts (4,400 bushels) of wheat. The cotton position is start one contract (50,000 pounds) long. If 50 cent per pound or $25,000 is lost, sell two contracts for a new position of a one-contract hedge.

Effect of Diversification on Hedging

Crop diversification, the introduction of complex stop-loss hedge positions, and the introduction of other than cotton futures all make large differences in the percent of the cotton crop hedged. The hedge advocated by Rolfo (who added cost to Peck's hedging formula) is a simple hedge without crop diversification, that is, a hedge based on the covariance of the cotton futures with California cotton profits, incorporating yield and basis risk and not assuming the hedge to be costless. The Peck-Rolfo hedge is calculated at 3.4 percent

FIGURE 1. MEAN-VARIANCE FRONTIER



of the crop and amounts to half a contract sold on the New York Mercantile Exchange. In Table 5 the mean return is tabled against the percentage of the crop hedged. The first column, "Cotton (Naive)," is the Peck-Rolfo hedge. The second and third columns, "Cotton (Initial)" and "Cotton (Deep)," are the percentages hedged initially and the position taken if the price of cotton should drop 50 cents.

As the table shows, the effect of introducing a complex hedging strategy more than triples the open interest under conditions of a large price fall. Mean-variance utility maximizers go to much greater lengths to protect against large (> 50 cents) changes in price than they do to protect against small changes. For reasonable ($120,000-$140,000) mean income targets, the addition of alfalfa (column labeled "Cotton and Alfalfa") to the crop mix on which the hedge is based doubles the size of the simple hedge but leaves the complex hedge much the same.

Since growing alfalfa (much less sugar beets) substantially diversifies the risks of farming cotton (predicted profits from cotton and alfalfa have a correlation of -.25), one might naively expect that the demand for futures would be reduced by this crop diversification. In fact, the cotton hedge is negatively correlated with both cotton and alfalfa (this is statistically possible) so that crop diversification makes the cotton hedge more desirable.

Allowing cropping plans including sugar beets (columns labeled "All Crops") reduce the simple hedge to near zero and make little difference in the complex hedges for low mean plans. Higher on the mean-variance frontier, with the mean exceeding $140,000, the inclusion of all crops allows making higher expected returns through growing sugar beets rather than speculating in cotton. The percentage hedge figures reflect this lack of speculation. Finally,

Table 5. Cotton Hedged

Mean Return	Cotton			Cotton and Alfalfa			All Crops			All Crops and All Futures		
	Naive	Sophisticated Initial Hedge	Sophisticated Deep Hedge	Naive	Sophisticated Initial Hedge	Sophisticated Deep Hedge	Naive	Sophisticated Initial Hedge	Sophisticated Deep Hedge	Naive	Sophisticated Initial Hedge	Sophisticated Deep Hedge
1978 --(dollars)--	--(percent)--											
97,250	3.4	3.1	13.0	6.8	6.3	14.3	1.5	1.8	10.1	10.8	-16.4	10.8
116,700	3.4	3.1	13.0	5.0	4.6	11.9	1.5	1.5	9.1	10.8	-16.4	10.8
136,150	[a]	-11.9	52.4		- 8.1	46.1	0.3	0.07	6.5	10.8	-16.4	10.8
155,600		-32.6	105.7		-28.2	106.4	0.0	-1.8	4.1	8.1	-20.1	8.1
175,050		-36.6	124.2		-34.6	122.4	0.0	-6.2	0.0	5.4	-23.9	5.4
194,500		-39.1	144.3		-37.0	142.5	0.0	-8.4	0.0	2.9	-27.4	2.9

[a] Blanks indicate this mean cannot be achieved.

Source: Computed.

the "All Crops and Futures" column gives the result if simultaneous positions in wheat and potatoes are taken. The complex hedge is to start (up to a mean of $160,000) slightly speculative 16 percent long and switch to an 11 percent hedge if the price changes are large.

Price of a Hedge

The price or "risk premium" of a hedge is the expected loss per pound from selling the futures. For cotton, the short position entails a loss of 9.3 cents per pound which is a large fraction of the 17.3 cents per pound expected profit from growing cotton. The demand for hedging can be traced out by varying its price or risk premium and recording the consequent demand for hedging. In a $124,000 expected income plan in which only cotton and alfalfa are grown and the only future is the the hedge, SCOT50, a 1 cent change in the risk premium results in a 9/10 percent change in percent hedged. Although the difference in hedging between a hedge that costs 9 cents and one that is free is nearly double, the total percent of the crop hedge is still quite low. If more complicated hedging strategies are permitted, the "demand for hedge" will require a decrease in the profitability of the long position at the same time the "risk premium" of short position is decreased. If this were not done, it would be possible to make large and certain profits by taking offsetting longs and shorts. Table 6 shows the demand curves for hedging for all crops and futures except LCOT10, SCOT10, and SCOT25. At low expected returns, the hedge doubles if the risk premium is eliminated, while at high expected returns, the ratio is closer to 10 times. As these experiments show, the size of the risk premium has a large effect on the percent of the crop hedged.

Forecast

It is hard to do justice to forecasting in this model because the futures price forms the basis for the forecasts of the crop prices and because the

Table 6. Pounds Hedge and Percent Hedge with SCOT50 and LCOT25 Set at 1-Cent Intervals and Actual Hedge

Mean Return, 1978 Dollars	Unit	SCOT50 (cents per pound) 0.0 / LCOT25 (cents per pound) 0.0	-.01 / .004	-.02 / .008	-.03 / .012	-.04 / .016	-.0469 / .0164	-.05 / .020
97,250	pounds	81,936	70,171	58,080	48,568	38,812	29,174	28,897
	percent	26.6	23.4	20.0	17.1	14.1	10.8	10.8
155,600	pounds	110,762	95,091	79,357	63,781	48,567	33,183	33,893
	percent	24.4	21.3	18.2	15.0	11.7	8.1	8.4
194,500	pounds	104,763	84,139	65,862	49,991	34,779	13,317	14,335
	percent	22.5	18.1	14.1	10.6	7.4	2.3	3.1

Source: Computed.

one-shot decision rule, rather than the adaptive control formulation, does not allow forecasts to be used effectively. The table on the price of a hedge gives evidence on what happens when the expectations for the cash price diverges from that derived by a constant (greater than one) times the futures price. A decrease in the expected loss from a short is a narrowing of the difference between the observed futures price and the expected price of the settlement time. Thus, a decrease in the expected loss is analagous to a prediction of a lower cash price than is predicted in the model, and it leads—as one would expect--to a greater hedge position.

Conclusion

The empirical section shows what the theory section surmises--that the costs of hedging and the opportunity to diversify risk by growing other crops each substantially change the optimal hedge and the opportunity set for California farmers. Although commodity exchanges would like the additional hedging volume that could be generated by farmers, educating cotton farmers will not produce such additional volumes: The price of hedging is set high enough so that storers of cotton who have no yield risk find hedging worthwhile, while producers whose yield risk makes this insurance very imperfect find the price to be too high.

Since this paper considers only a planting-time hedge, it leaves unsettled the optimal path for a hedge to take between planting and harvest. At planting time, there is yield risk; but, as the crop year progresses, that risk is resolved until, at harvest, the producer has no yield risk and is just the same as any other storer of commodities. It seems very reasonable that the hedge should evolve over the crop year from the hedge appropriate to a producer to that appropriate for a storer. The mathematical tools for investigating the influence of new information on hedging are an adaptive control and are well known, but there is a twofold empirical problem: constructing a yield prediction model that is updated dependent on the progress of the crop and constructing a price prediction equation not wholly dependent upon the futures price. The rewards to such a model are that it would point the way for the profitable use of futures by farmers.

Footnotes

*I would like to thank Paul A. Samuelson, Andrew Schmitz, Gordon C. Rausser, and James N. Boles for helpful comments; also, Ms. Connie Cartwright for her able assistance with the data and estimation. The errors, however, are mine.

[1]Peck (1975) considered using regression equations for prediction of futures mean-square error in her study of eggs. She credits Fried with the idea. In fact, she assumed the mean return of egg futures was zero, and she estimated the mean-squared error from the differences of a predicted and actual series.

[2]The problem with corn, which seems the more natural choice, is that it is harvested too early in the season to be well timed with respect to the bulk of the alfalfa cutting or barley harvest. What it gains in closeness as an animal feed, it loses in timing.

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