Optimal Management of Renewable Resources with Growing Demand and Stock Externalities

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Division of Agricultural Sciences
UNIVERSITY OF CALIFORNIA

Working Paper No. 63
Rev.

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Giannini Foundation of Agricultural Economics
September 1979

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* Giannini Foundation Paper No. . I am thankful to Robert Solow, Vincent P. Crawford, Theodore Groves, and Andreu Mas-Colell for helpful comments. The remaining errors are mine.

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OPTIMAL MANAGEMENT

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OPTIMAL MANAGEMENT OF RENEWABLE RESOURCES WITH GROWING DEMAND AND STOCK EXTERNALITIES

1. INTRODUCTION

Economists often characterize optimal management of renewable resources in terms of a simple capital market equilibrium rule such as the rate of interest equals the rate of increase of population growth, a rule that leads to lower steady state resource stocks than the maximum sustainable yield prescribed by many biologists and environmentalists. Indeed, the biologists' rule which amounts to the economists' simple rule with a zero interest rate is fallacious, but the simple capital theory rules are little better because of costs, externalities, and increasing demand. Rules that treat cost properly are well known; in general, harvesting costs lead to increased steady state stocks. Externalities of the common pool or open access sort are known to lead to inefficiency, but the policy prescription is simply to restrict access so that the resource again follows a (cost adjusted) simple capital market rule. Besides the well-known open access externality, many renewable resources such as range or forest provide a positive externality such as fishing in their streams, hiking across their expanses, and drinking water which runs off them. It is not customary for the landowner to be compensated for the water, hiking, or fishing his land provides, so these services are positive external economies. These positive external economies are often thought to be related to the stock of the underlying resource: the size or number of trees influences the quality of the hiking experience and the quantity of runoff. Optimal renewable resource policy should account for these externalities. Growing demand provides the last of the modifications to the simple capital theory rules presented here. When demand grows in a particular exponential fashion, the steady state stock should be larger than it would otherwise be. To be precise, the entrepreneurs will act exactly as if they faced a stationary demand and a lower interest rate. In fact, if the numbers are fortuitously chosen, rational entrepreneurs (or a regulating agency) could end up acting as if the interest rate were zero, which is exactly the environmentalist-biologist maximum sustainable yield rule.

Although the growing demand model requires no government intervention to achieve an optimal path (assuming correct expectations), the case of positive-valued externalities does require intervention for optimality. Besides simply mandating the optimal harvest path, the government might try a number of tax or subsidy strategies. It turns out that harvest costs play an essential and surprising role in determining the efficacy of many of these policies.

Section 2.1 presents a partial equilibrium model of a renewable resource with harvest costs which draws heavily on Clark [2] and which has its roots in the work of Smith [16], Scott [14], and ultimately Lotka [9].

In Theorem 1 the model is used to characterize a market equilibrium. In a market equilibrium that starts with a large resource stock, price starts low but rises faster than the rate of interest toward a steady state price. As the price goes up over time, the resource stock is diminished toward a steady state stock. Section 2 contains Theorem 2 which describes the market allocation over time if the demand curve shifts out at an exponential rate m. In this case of exponentially increasing demand, there is no steady state for price: in the long run it increases at rate m; however, stock does approach a steady state. The simple capital theory rule relating rate of interest to rate of growth increase and rate of price increase is modified by replacing the rate of interest with the rate of interest less m. To illustrate how exponentially increasing demand might come about, there is an example of a simple economy with a good manufactured from a renewable and nonrenewable resource. Section 3 is concerned with the optimal management of a resource when the resource stock per se has consumption value, an area first researched by Lusky [10] and Vousden [18] for nonrenewable resources. Theorem 3 shows that the optimal resource policy with valued stock will have a higher steady state stock than the optimal policy without valuation of the stock; the theorem also presents the optimal pricing rule. Theorem 4 shows how to decentralize the solution of Theorem 3 using a stock subsidy. Theorems 5 and 6 are concerned with the results of actually observed resource policies that are designed to protect stocks. In Theorem 5 it is shown that taxing a product (lumber) would increase the steady state stock of trees only if unit harvest costs were not constant. Irrational conservation is examined in Theorem 6: the result is that legislated inefficiency does indeed raise steady state stock.

2. MODEL

2.1. *Partial Equilibrium Model*

Seven assumptions on the entrepeneurs and consumers define the partial equilibrium model.

Assumption 1--*The Growth Curve.* In order to elucidate the price path of a renewable resource in a competitive environment, it is necessary to obscure the details of the population's age distribution. Accordingly, let x, a scalar, be the stock of the exploited population, and let its growth be described by the usual differential equation $\dot{x} = f(x) - h$, where h is the harvest or cut, and f is concave, twice continuously differentiable, positive only on the open interval (0, K), and zero at 0 and K.

Assumption 2--*Price Takers.* There are many producers, each of whom owns his own pool of the resource (forestland, fish pond, etc.) and acts as price taker.

Assumption 3--*Certainty Prediction.* These producers know the demand for the resource and use this knowledge to predict future prices with certainty.

Assumption 4--*Present Value Maximization.* Each producer is assumed to maximize the present discounted value of resource harvest at discount rate r.

Assumption 5--*Costs.* The unit cost of extracting the resource is given by c(x), a monotone decreasing differentiable function of x.

Assumption 6--*Demand.* Consumers are represented by a downward sloping demand curve Q(p) which is continuously differentiable and is the same in every period (section 2.2 relaxes this assumption). The $\lim_{p\to\infty} Q = 0$ and $Q > 0$. Any information on the relative prices of the resource and other goods in the future is reflected by this demand curve.

Assumption 7--*Equilibrium.* The market-clearing equation states $Q(p) = h$ or harvest equals demand at every instant.

With these assumptions, it is possible to describe the resulting partial equilibrium.

Theorem 1. *If Assumptions 1 through 7 hold, then there is a price path that equates supply and demand and:*

a. Along the present value maximizing path, $\dot{p}/(p - c) = (r - f) + c'f/(p - c)$ *so when unit costs are constant* ($c' = 0$), *the rate of net price increase plus the rate of growth increase* (f') *equals the rate of interest.*

b. *There is a steady state x^*, p^*, h^* defined by $\dot{p} = 0$ and $Q(p^*) = f(x^*)$, and every equilibrium path leads to a steady state.*

c. *The steady state is unique if $c'' = 0$.*

d. *If the initial stock is greater than the steady state stock, then the initial price is lower than the steady state price; and when $c' = 0$, $\dot{p}/(p - c) > r$ when $x > x^*$.*

To show 1(a), it is necessary to solve the problem of the producer faced with a differential price path.

The producer's problem is:

$$\max_{c} \int_0^\infty e^{-rt} \, [p - c(x)] \, h \, dt \tag{1}$$

subject to

$$\dot{x} = f(x) - h$$

$$x \geq 0, \qquad h \in [0, \infty],$$

where p(t) is the expected and actual price at time t. The supply of resource is h(t) from each of the identical producers, and, for convenience, h(t) is taken as the industry supply; that is, the industry is treated as having only one price-taking firm.

The first step in solving this problem is to apply the maximum principle of Pontragin <u>et al</u>. [13] to the producer's problem. Let

$$H = e^{-rt} \, [p - c(x)] \, h + \lambda \, [f(x) - h] \tag{2}$$

be the Hamiltonian. Necessary conditions for an optimum are:

$$\dot{x} = \frac{\partial H}{\partial \lambda} = f(x) - h \tag{3}$$

$$\dot{\lambda} = -\frac{\partial H}{\partial x} = -f'(x)\ \lambda + c'h\ e^{-rt} \tag{4}$$

$$\text{transversality condition } \lim_{t\to\infty} \lambda x = 0$$

and the maximum principle: choose h to maximize H at every time t. Because H is linear in h, h is either zero or infinity or $\lambda = e^{-rt}\ (p - c)$.

The corner conditions have been avoided by judicious choice of demand functions. No demand function derived from a budget-constrained problem can admit infinite output at positive price, so the possibility that $h = \infty$ can be dismissed. On the other hand, the harvest could well be zero at a chokeoff price, and this was ruled out by Assumption 6.

On the above conditions, $\lambda = e^{-rt}$ p everywhere. Differentiate

$$[p - c(x)]\ e^{-rt} = \lambda, \tag{5}$$

and substitute it into Eq. (4) to get

$$\dot{p} = (r - f')\ (p - c) + c'f(x). \tag{6}$$

Eq. (6) shows part (a) of the theorem and provides one of two equations necessary to show part (b) of the theorem. In the case that unit costs are constant, Eq. (6)--when rearranged--states that the rate of net price $\dot{p}/(p - c)$ increase is the rate of interest less the rate of growth increase (f'). Since $f'' < 0$--rate of growth increase decreases in stock--it is true that price increases fastest when the stock is large. In particular, when f' is negative, as is the case with a resource just beginning to be exploited, price increases faster than the rate of interest. The economic sense of the situation is that entrepreneurs holding large stocks lose interest--they could harvest the resource and put the money in a bank--and they lose growth. Growth happens at rate f, and f' is the growth lost or gained from a small change in the stock. The rate of price increase--capital gains--must be enough to make up for both the lost interest and the lost growth. When costs are not constant, the matter is more difficult. Price increases slower than (r - f') (p - c) by the amount c'f which compensates for the extra costs incurred by reducing the stock.

Parts (b) and (c) of the theorem follow from examining the differential equation system described by Eq. (6), the price equation; the simultaneous solution of the growth and demand equations,

$$\dot{x} = f(x) - Q(p); \tag{7}$$

and the transformed transversality conditions in terms of p,

$$\lim_{t\to\infty} p\, e^{-rt}\, x = 0. \tag{8}$$

Eq. (7) states that the change in stock equals growth less market demand. Since Eqs. (6) and (7) describe a differential equation system that depends continuously on its variables, there is a solution to the equations at each point on R_+^2 [8]. Most of these solutions do not meet the transversality condition [Eq. (8)]. Those for which $\lim_{t\to\infty} p$ and $\lim_{t\to\infty} x$ are constant, called a stationary point, clearly do meet the transversality condition. Being a stationary point is not quite enough; it must also be possible to "get there" from somewhere else--solutions starting in other places must end at the stationary point. If the linearized differential equations system at the stationary point has at least one negative eigenvalue, it makes it possible to "get to" that stationary point. Saddle points have one negative and one positive eigenvalue.

First, take the special case--part (c) of the theorem.

Lemma 1. *When there are constant marginal costs* [$c'(x) = 0$], *this equation system has only one stationary point on the positive orthant.*

Since p is zero if $r = f'(x)$ and f' everywhere decreases in x, there is a single value x^* such that $r = f'(x^*)$. To find p^*, set $Q(p) = f(x^*)$. Again, p^* is unique because $Q' < 0$ everywhere.

Lemma 2. *When marginal costs are constant, the stationary point* (x^*, p^*) *is a saddle point.*

To verify this statement, linearize the differential equations about (x^*, p^*) and find the characteristic values. Let Δp and Δx be the state variables expressed as a deviation from the steady state values. That is, $\Delta p = p - p^*$ and $\Delta x = x - x^*$. To a first-order approximation, the problem of a renewable resource is:

$$\begin{pmatrix} \Delta \dot{p} \\ \Delta \dot{x} \end{pmatrix} = \begin{bmatrix} 0 & -f''(x^*)p^* \\ -Q'(p^*) & f'(x^*) \end{bmatrix} \begin{pmatrix} \Delta p \\ \Delta x \end{pmatrix}. \tag{9}$$

The characteristic polynomial is $b^2 - bf' - Q'f''p^*$, and the characteristic roots are

$$\begin{aligned} b_1 &= \left(f' + \sqrt{f'^2 + 4Q'f''p^*}\right)/2 \\ b_2 &= \left(f' - \sqrt{f'^2 + 4Q'f''p^*}\right)/2 \end{aligned} \tag{10}$$

which are real and of opposite sign, b_2 being negative, since the term $4Q'f''p^* > 0$ ($Q' < 0$ and $f'' < 0$). The opposition of the signs of the eigenvalue is sufficient to show that (x^*, p^*) is indeed a saddle point.

To aid in discussion, Figure 1 presents the p − x phase space. That the $\dot{p} = 0$ locus is a vertical line is plain. The U shape of the $\dot{x} = 0$ locus results from the shape of the growth curve f: for almost all values of Q(p), there are two values $\hat{x}$ such that $f(\hat{x}) = Q(p)$; by the concavity of f, it has a unique maximizer x_{max}; and by the downward slope of the demand curve, this must be associated with the least p.

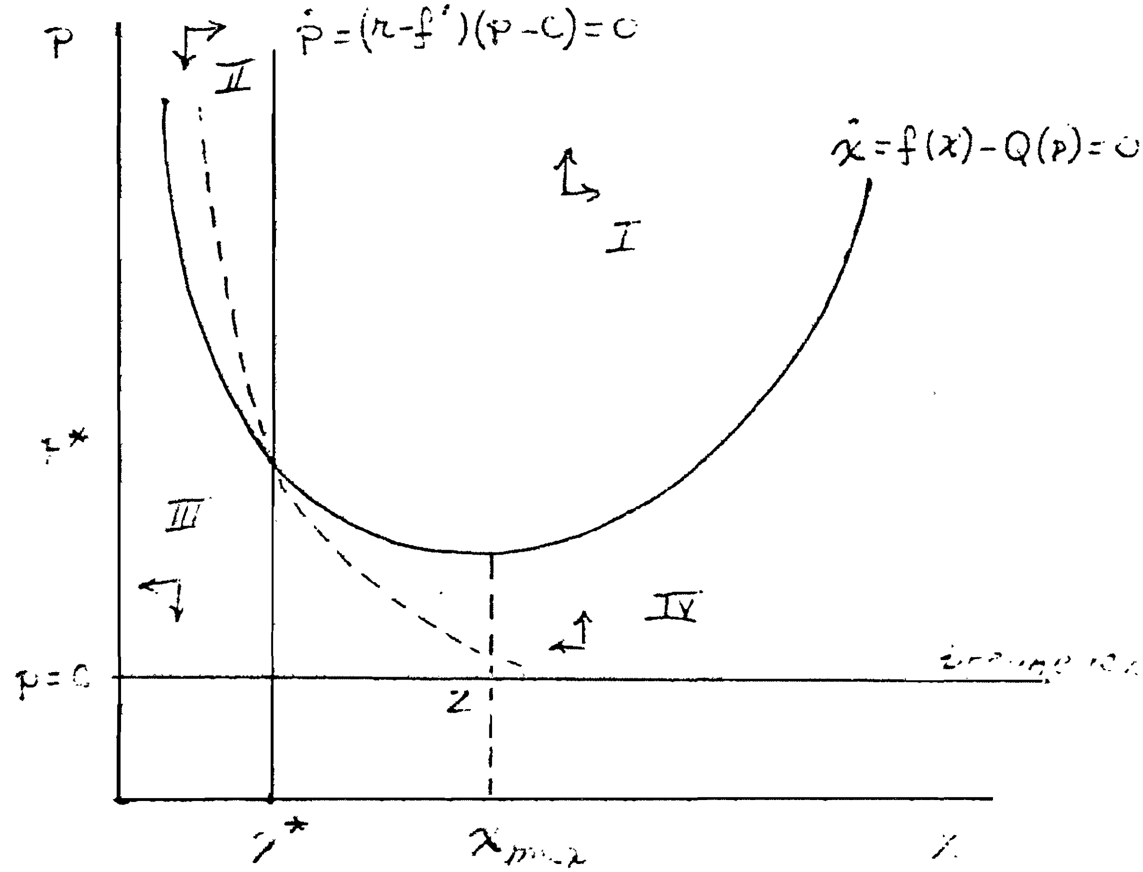
The placement of the $\dot{p} = 0$ locus to the left of x_{max} is not accidental: f' is positive only between 0 and x_{max}, so $(r - f')$ can only be zero somewhere on that interval. As $x \to 0$ or K, p will approach an infinite price. The convergent arm (labeled z) exists because (p^*, x^*) is known to be a saddle point [8]. The arrows, whose directions are easily verifiable, show that the convergent arm must reside in regions II and IV. The horizontal line in the diagram at p = c is the break-even point. In the half plane below it, nothing would be harvested ($h \equiv 0$); but given the assumptions on demand (no supply brings an infinite price), the convergent arm will always lie above the break-even line. Any trajectory other than z must eventually end in quandrants I and III, both of which lead to violations of the transversality conditions.

Turning to the more general case of part (b) of the theorem:

Lemma 3. *The* $\dot{p} = 0$ *curve crosses* $\dot{x} = 0$ *an odd number of times from above. Each such crossing is a saddle point.*

When unit costs are not constant, the $\dot{p} = 0$ locus is given by

$$p = c - \frac{c'f}{r - f'}. \tag{11}$$



Phase Space when C ≡ 0

Figure 1.

As shown in Figure 2, the slope of the $\dot{p} = 0$ curve is generally downward: it has a pole at $r - f'(x) = 0$; and examination of

$$\left.\frac{dp}{dx}\right|_{\dot{p}=0} = \frac{-f''\ (p - c) + (r - f')\ (-c') + c''f + c'f'}{(f' - r)} \qquad (12)$$

shows that all the terms of the numerator save the last are positive while the denominator is negative. The last term in the numerator is positive for $x > x_{max}$, so the $\dot{p} = 0$ curve slopes downward on $x > x_{max}$. Although it would be easy to draw cases of multiple equilibria (Colin Clark [2] has done this), assume the equilibrium is unique. Since the $\dot{p} = 0$ curve starts above the $\dot{x} = 0$ curve, it must cross it (an odd number of times) from above. This crossing is a saddle point. The linearized system is

$$\begin{pmatrix} \Delta\dot{p} \\ \Delta\dot{x} \end{pmatrix} = \begin{bmatrix} (r - f') & -f''\ (p - c) + (r - f')\ (-c) + c''f + c'f' \\ -Q' & f'(x) \end{bmatrix} \begin{pmatrix} \Delta p \\ \Delta x \end{pmatrix} \qquad (13)$$

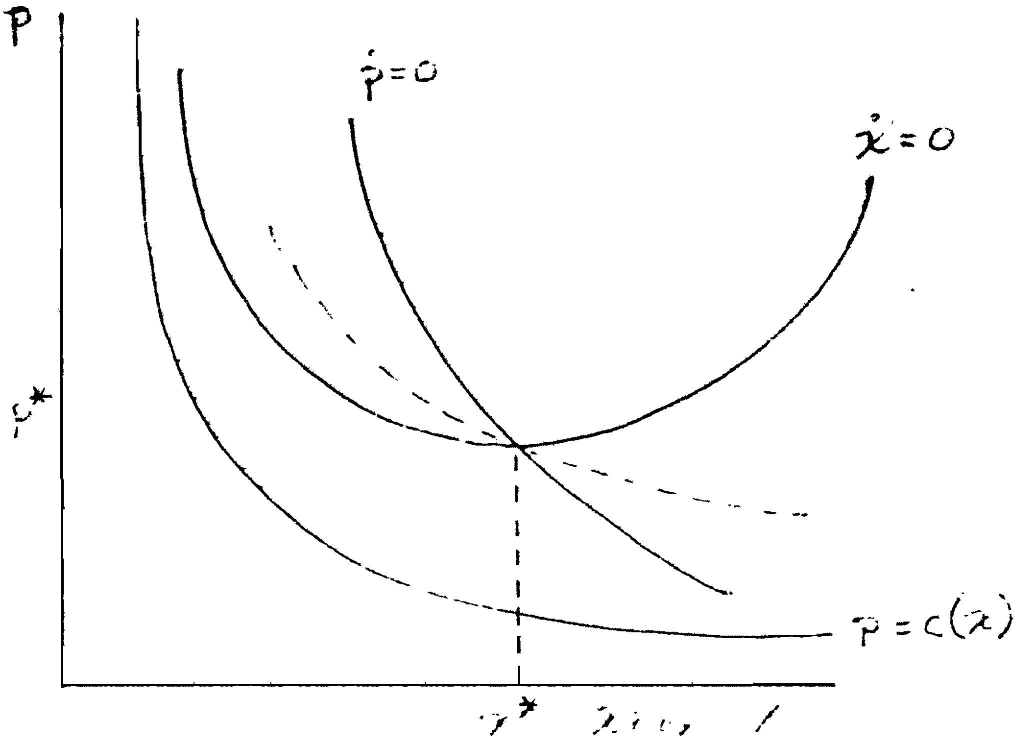
That $\dot{p} = 0$ croses $\dot{x} = 0$ from above is equivalent to its having a greater slope than $\dot{x} = 0$; in symbols,

$$\frac{(B)}{(r - f')} > \frac{f'(x)}{-Q'} \qquad (14)$$

where (B) is the upper right-hand corner term of the linearized system. Noticing that $r - f'$ and $-Q'$ are positive and rearranging yields

$$(r - f')\ (f') - Q'(B) \leq 0. \qquad (15)$$

This shows that the determinant of the system is negative, the eigenvalues are real and of opposite sign, and the equilibrium is a saddle point.



P - x Phase Space

$C' \neq 0$

Figure 2.

Figure 2 depicts this system. Again, the break-even curve [p = c(x)] has been drawn in; it slopes downward because low stock leads to high cost. As drawn, the costs of harvest if $x < x_{break-even}$ are infinite.

The existence of the harvest path is not quite enough to assure that the agents actually get on it. The individual entrepreneur must be constantly checking both the differential equation conditions (he must see that price is rising at the right rate) and the transversality conditions (he must see that the price-quantity path leads to a steady state and not some infinte price disaster from which he should profit). This problem was first studied by Hahn [6]. Stiglitz [17] finds these expectations--rational with respect to both day-to-day price change and the transversality conditions--to be a very strong assumption. For instance, suppose a resource owner observes a price lower than he expects. If he believes his calculations and his estimate of the total resource stock, he withholds his supply from the market and attempts to establish a "long" position in the commodity; these actions tend to push price up and lead to stability. To the contrary, assume the producer is unsure of the total resource stock. When price is lower than expected, the producer concludes that the true industry stock was greater than he thought. He adjusts his plan by selling more and depressing price further: clearly an unstable situation. Which of these scenarios is more realistic, is open to question. For want of a better assumption, it is assumed that the expected prices are realized and are on the convergent arm. This was Assumption 3.

2.2. *Nonautonomous Demand*

Stationary demand curves of the sort used in Theorem 1 do not account for increased demand incident on such things as growing population, increasing wealth, or changing relative prices. When these time-varying factors affect demand by moving the demand curve outward at an exponential rate, the partial equilibrium model can still be solved. Clark and Munroe [3] treat the related case of exogenously increasing price. Theorem 2 characterizes the solution. Following the proof of the theorem is an example of how changing relative prices in a competitive world led to exponentially shifting resource demand.

Theorem 2. *If Assumptions 1 through 4 hold, Assumption 5 is replaced by* $c = 0$, *and Assumption 6 is replaced by demand* $= Q\ (e^{-mt}\ p)$ *and* $r > m$, *then*

a. *There is a unique steady state* x^* *at which price increases at rate* $\dot{p}/p = m$ *and* x^* *is defined by* $f'(x^*) = r - m$.

b. *Along any profit-maximizing path,* $\dot{p}/p = r - f'$.

c. *Along an optimal path whenever* $x > x^*$, $\dot{p}/p > r - m$ *and* $q = pe^{-mt} < q^*$ *where* $f(x^*) = Q(q^*)$.

Proof. The same steps--maximizing firm profits and substituting in the demand curve--that lead to Eqs. (6), (7), and (8) give

$$\frac{\dot{p}}{p} = (r - f') \tag{16}$$

$$\dot{x} = f(x) - Q\ [e^{-mt}\ p(t)] \tag{17}$$

$$\lim_{t\to\infty} p(t)\ e^{-rt}\ x(t) = 0 \qquad \text{(transversality)} \tag{18}$$

A solution involves exponentially increasing prices at rate m. Choose a new variable $q(t) = e^{-mt}\ p(t)$ and rewrite the system in terms of this new variable:

$$\frac{\dot{q}}{q} = [r - m - f'(x)] \tag{19}$$

$$\dot{x} = f(x) - Q(q) \tag{20}$$

$$\lim_{t\to\infty} q\ e^{(m-r)t}\ x(t) = 0. \tag{21}$$

When the time preference (or interest) rate is greater than the rate of demand increase ($r > m$), there is a steady state in the $q - x$ space which corresponds to a solution where $\dot{p}/p = m$. The steady state occurs at a resource stock $x^*(m)$ that is higher than the zero increase in demand steady state stock $x^*(0)$. Because of the concavity of f, $x^*(\cdot)$ is monotonically increasing in m. One troublesome detail remains: if $m > r$, then the proposed solution no longer meets the transversality condition--the economic problem underlying the transversality condition is the finiteness of the present value of a unit of the resource sold in the future. When prices go up faster than the rate of interest forever, the present value of even a carelessly made plan would be infinite. Criteria concerning vector dominance or overtaking--not present value--are appropriate for judging plans in these unhappy circumstances; below, the discussion is limited to $m < r$.

Since the equations for the nonautonomous demand case are the same as those that lead to Theorem 1, save the replacement of p by q and r by $r - m$, the analogous theorem in the new phase space is also true.

2.3. *An Example*

One situation in which demand is not autonomous is a world dependent upon only a renewable and an exhaustible resource. From the outset, it will be clear that it is a grim world: neither manufactured capital nor technical progress nor high elasticities of substitution are allowed to preclude the Malthusian doom (Dasgupta and Heal [5] and Stiglitz [17] investigate these reasons for continued growth with exhaustible resources).

A single consumption good (g) is produced by a Cobb-Douglas production function with parameters $\bar{\alpha}$ and $\bar{\beta}$ from the renewable resource (stock x, flow h) and the exhaustible resource (stock y, flow k). Social welfare--or the utility of a representative agent--is given by a function $u = g^{\gamma}$. This small economy can be viewed either as a competitive equilibrium problem, a planning problem, or a market equilibrium problem in which agents act to maximize present value of profits or utility subject to a present value budget constraint. Since there is only one consumer and no externalities, the solutions will be identical.

To find the market equilibrium, start with the consumer's problem. The consumer's problem in such a world would be to

$$\max \int_0^{\infty} U(g)\; e^{-rt}\, dt$$

subject to a wealth constraint,

$$W(t) = W(0) - \int_0^{t} zge^{-rt}\, dt \text{ and } W \geq 0.$$

As is pointed out by Shell and Stiglitz [15], this is not the same as maximizing utility subject to the investment possibilities available in the resource firms. For simplicity, the rate of time preference of the consumer is assumed to equal the economywide discount rate for goods. The Hamiltonian for the consumer's problem is

$$H = U(g)\; e^{-rt} - \lambda ge^{-rt}\; z \tag{22}$$

with necessary conditions $U' = \lambda z$, $\dot{\lambda} = 0$, and $\dot{W} = -zge^{-rt}$. The transversality condition is

$$\lim_{t\to\infty} W\lambda = 0.$$

For the chosen utility function, the solution--up to the choice of λ which depends on wealth through the transversality condition--is

$$z = \left(\frac{\gamma g}{\lambda}\right)^{(\gamma-1)} \text{ and } \lambda = \lambda(W). \tag{23}$$

The function g (z, W) is the demand for goods. The consumer's demand for goods gives rise to the firm's demand for its factors of production, which are the resources.

The g-producing firm's profit-maximizing problem is

$$\max_{h,k} zh^{\alpha} k^{\beta} - hp - ks, \tag{24}$$

and the first-order conditions are

$$\bar{\alpha}zh^{\bar{\alpha}-1} k^{\bar{\beta}} - p = 0 \tag{25}$$

$$\bar{\beta}zh^{\bar{\alpha}} k^{\bar{\beta}-1} - s = 0. \tag{26}$$

These conditions can be explicitly solved for the factor demands and the g-supply equations, and then the g-demand [Eq. (23)] and g-supply equation can be used to eliminate z. The result of that calculation is the derived demand for factors which are expressed using the convenient notation

$$\alpha = \bar{\alpha}\gamma \qquad \beta = \bar{\beta}\gamma; \tag{27}$$

and, in the Cobb-Douglas case, these are

$$h^D (p, s) = \left(\frac{\gamma}{\lambda}\right)^{\gamma d} \alpha^{(1-\beta)d} \beta^{\beta d} p^{(\beta-1)d} s^{-\beta d} \tag{28}$$

$$k^D (p, s) = \left(\frac{\gamma}{\lambda}\right)^{\gamma d} \beta^{(1-\alpha)d} \alpha^{\alpha d} s^{(\alpha-1)d} p^{-\alpha d}. \tag{29}$$

where $d = 1/(1 - \alpha - \beta)$. The exhaustible resource-owning firms are of the sort described by Hotelling [7]; they maximize present value,

$$\pi^x = \int_0^\infty e^{-rt} sk \, dt,$$

subject to $\dot{y} = -k$, $y \geq 0$, with the result that $\dot{s}/s = r$ or k takes a corner value. The initial value of s, s(0), is determined from

$$\int_0^\infty k^D \, dt = y(0),$$

a determination that depends on p and W.

The situation of the renewable resource-owning firms is exactly that of an industry faced by growing demand:

$$\max \int_0^\infty e^{-rt} ph \, dt$$

subject to $\dot{x} = f(x) - h$ where p is determined from the firm's first-order conditions and the demand equation h^D [p, s(0) e^{rt}, W]. Inspection of the form of h^D [Eq. (28)] shows that it is indeed of the form Q (pe^{-mt}) when $s = s_0 e^{rt}$ is substituted into the equation. Thus, the world with only a renewable and nonrenewable resource can be modeled in an explicit demand and supply framework, and that framework leads to a renewable resource industry facing a nonautonomous demand. Because the computation of consumer wealth (rents in the resource-holding industries plus the profits of the consumer good-producing firms) is difficult, it is easier to find the solution to this economy in terms of the equivalent planning problem.

Under these conditions, the planning problem is

$$\max \int_0^\infty u\ (h^{\bar{\alpha}} \cdot k^{\bar{\beta}})\ e^{-rt}\, dt \tag{30}$$

subject to

$$\dot{x} = f(x) - h$$

$$\dot{y} = -k$$

$$x,\ y \geq 0.$$

The Hamiltonian is

$$H\ (h,\ k,\ x,\ y,\ \lambda_1,\ \lambda_2) = h^{\alpha}\ k^{\beta}\ e^{-rt} + \lambda_1\ (f - h) + \lambda_2(-k). \tag{31}$$

Making the change to current time-price variables $p = e^{rt}\ \lambda_1$, $s = e^{rt}\ \lambda_2$, the necessary conditions are

$$\frac{\dot{p}}{p} = r - f' \qquad p \geq 0 \tag{32}$$

$$\frac{\dot{s}}{s} = r \qquad s \geq 0 \tag{33}$$

$$\dot{x} = f(x) - h \qquad x \geq 0 \tag{34}$$

$$\dot{y} = -k \qquad y \geq 0. \tag{35}$$

The maximum principle yields

$$\frac{\alpha h^{\alpha}\ k^{\beta}}{h} = p \qquad \frac{\beta h^{\alpha}\ k^{\beta}}{k} = s \tag{36}$$

and the transversality conditions are

$$\lim_{t\to\infty} x\, e^{-rt}\, p = 0 \tag{37}$$

$$\lim_{t\to\infty} y\, e^{-rt}\, s = 0. \tag{38}$$

Solving the two marginal utility conditions and substituting $s_0\, e^{rt}$ for $s(t)$ yields

$$Q = h^D = \alpha^{(\beta-1)d}\, \beta^{\beta d}\, s_0^{-\beta d} \left[pe^{-r\beta t/(\beta-1)}\right]^{(\beta-1)d} \tag{39}$$

which is an exponentially decreasing, nonautonomous demand curve of exactly the form discussed in the previous section. In this example, that section's parameter m is $r\beta/(\beta - 1)$ and, since it is negative, $r > m$ whatever the (permissible) values of α and β. Using the results of the section on nonautonomous demand curves, the renewable resource sector acts just as if its time preference rate were $r - m$, except that renewable resource prices constantly fall at rate m instead of being stationary. The exhaustible resource is not depleted in finite time. Asymptotically, its rate of extraction is given by

$$k(t) = h^{*\alpha/(\beta-1)}\, s_0^{1/(\beta-1)}\, e^{rt/(\beta-1)} \tag{40}$$

where h^* is the steady state renewable resource flow and s_0 is determined from the initial conditions and transversality. Clearly, k decreases at an exponential rate, as does u (h, k), revealing the increasing degree of misery promised in this model.

3. EXTERNALITIES

3.1. *Introduction*

Externalities are the more usual reasons cited for aiming for a steady state resource stock greater than that given by $f'(x^*) = r$. Vousden [18] and Lusky [10] have made analogous arguments for the pure theory of exhaustible resources, and Calish, Fight, and Teeguarden [1] use estimates of stock-provided externalities to determine the optimal stock in a Faustman-type forestry problem. The essence of the problem is that the stock of a resource provides benefits $a(x)$ to society but that these benefits are not captured by the resource owner. Examples in forestry include water, hiking, wildlife, and pretty views. To account for these externalities, a regulatory agency would try to induce the resource owner to hold a higher stock than he would under competition. The usual sorts of policy instruments are available. Standards are frequently enforced for reforestation but, except on public lands, are not used to determine the resource stock itself. Taxes are not currently employed to encourage an increase in forest stocks; to the contrary, the current taxes discourage the holding of even as much as the free market competitive resource stock [11].

Assumption 7--Planner's Objective. The planning agencies' objective is

$$\max \int_0^\infty [U(h) - C(x)h + a(x)]\, e^{-rt}\, dt. \tag{41}$$

The plan chosen by the agency is characterized in Theorem 3.

Theorem 3. *If Assumptions 1, 5, and 6 hold and the planner's object is Assumption 7, then*

a. *There is a steady state given by* $\dot{x} = \dot{p} = 0$ *and an optimal path* $\hat{h}$, $\hat{p}$, $\hat{x}$.

b. *The optimal path slopes downward in* $p - x$ *space--if* $x > x^*$, *then* $p < p^*$ *along the optimal path.*

c. *As* a' *increase, so does* x^*.

d. *As* a' *increases,* p^* *increases if* $x^* \geq x_{max}$ *and decreases otherwise.*

e. *Since harvest is a function of price* $h^D(p)$, h^* *decreases with* a' *if* $x^* \geq x_{max}$ *and increases otherwise.*

To show Theorem 3, form the Hamiltonian for this problem:

$$H = [U(h) + a(x) - C(x)h]\, e^{-rt} + \lambda\, (f - h). \tag{42}$$

The necessary conditions in terms of present price $[(p - c) \equiv \lambda e^{rt}]$ are

$$\dot{p} = (r - f')\ (p - c) + c'f - a' \tag{43}$$

and

$$\dot{x} = f(x) - h. \tag{44}$$

The maximum principle on the assumption $\lim_{h \to 0} U(h) = \infty$ yields

$$U'(h) = p. \tag{45}$$

The usual manipulation of the utility function gives the demand curve $h^D(p) = U'^{-1}(p)$ which is then used to reduce the equations to a two-equation system:

$$\dot{x} = f(x) - h^D(p), \tag{46}$$

$$\dot{p} = (r - f)\ (p - c) - c'f + a', \tag{47}$$

and the transversality condition

$$\lim_{t\to\infty} xpe^{-rt} = 0. \tag{48}$$

The $\dot{x} = 0$ locus is identical to that of Eq. (7) while the $\dot{p} = 0$ locus is simply displaced upwards from that of Eq. (6) by the inclusion of a'. The phase space is given in Figure 3. As drawn, there are three $\dot{p} = 0$ curves corresponding to increasing values of a' $[a_1(x) < a_2(x)\ \forall x]$. Taking $a_1(x) = 0$, examination of the phase diagram makes Theorem 3 plain.

Theorem 3 gives the answer to the planner's problem and shows that the optimum in the sense of utility less costs plus external benefits can be achieved by a command economy. The same results can easily be achieved by decentralization.

Theorem 4. *Under the assumptions of Theorem 3 and Assumptions 2, 3, and 4, a competitive economy with consumers maximizing*

$$\int U(h)\ e^{-rt}$$

subject to $W = pe^{-rt}\ h$ *and producers maximizing*

$$\int_0^\infty [(p - c)\ h + a(x)]\ e^{-rt}\ dt$$

subject to $\dot{x} = f(x) - h$ *will have the same time path of resource use as the planning problem of Theorem 3.*

The consumer's problem yields $U'(h) = \delta p$ where δ is a constant depending continuously on wealth. Since infinite wealth leads to $\delta \to 0$ and zero wealth

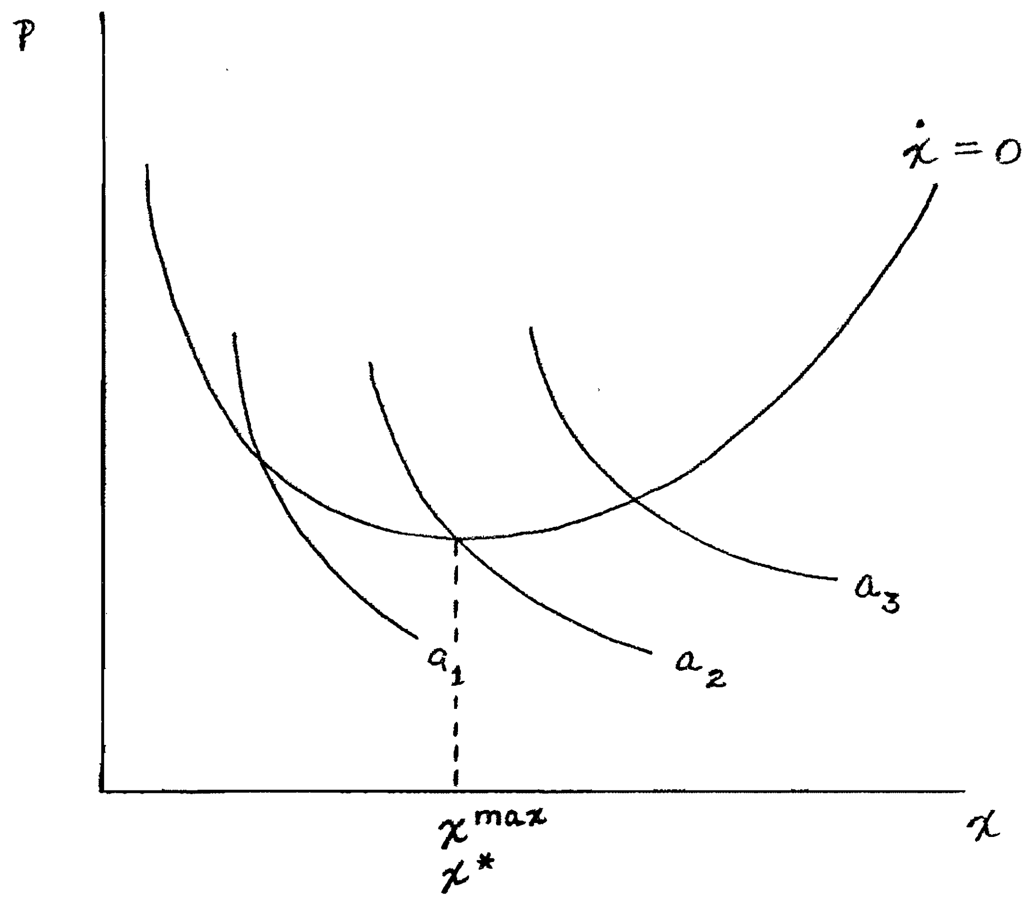


Figure 3.

leads to $\delta \to \infty$, there is some wealth W* such that $\delta = 1$. The first-order conditions for the producer are just Eqs. (43) and (44) so the competitive system with initial wealth W* leads to a differential equation system identical to that of the command system.

Theorem 4 shows that the economy can be decentralized by providing the resource owners with a subsidy of a'(x). Since subsidies are often politically unpalatable, two tools that are easier to use are discussed next. The next two subsections provide the details of policies of taxing resource flow and "irrational conservation" and discuss their efficacy in solving the valued-stock problem.

3.2. *Product Taxes*

Discouraging use of a commodity by taxing it is the economist's "Pigouvian" prescription. A tax on the flow of the resource would be relatively easy to collect; but, as Plott [12] recognized, taxing a commodity to influence the consumption of its factors of production may be less than successful. In the case of a product-taxes effect on the stock of a natural resource, the situation is not as bad as that of an ordinary factor market; at worst, the tax has no effect on the resource stock.

Theorem 5. *Under Assumptions 1 through 6 and a tax τ on the resource flow,*

a. A tax on product increases steady state stock when unit cost varies ($c' \neq 0$) and leaves it unchanged otherwise.

b. The trace of the path leading to the taxed equilibrium lies below that leading to the untaxed equilibrium.

c. A small tax on product when marginal cost varies increases product flow when $x^* < x_{max}$ and decreases it otherwise. When unit cost is constant, the tax has no effect on product flow.

The usual manipulation of the Hamiltonians yields:

$$\dot{p} = (r - f')\ (p - c) + c'f \tag{49}$$

$$\dot{x} = f(x) - Q\ (p + \tau). \tag{50}$$

The phase diagram, Figure 4, shows the downward shift of the $\dot{x} = 0$ curve and makes parts (a) and (c) of the theorem plain. To show part (b): at any point, $\partial\dot{p}/\partial\tau = 0$ while $\partial\dot{x}/\partial\tau > 0$; thus, the $(\dot{p}, \dot{x})$ vector field is warped upward--that is, trajectories point higher through given points in quadrant IV.

3.3. *Irrational Conservation*

Irrational conservation refers to the practice of increasing extraction costs to reduce the flow of a resource product. Crutchfield and Pontecorvo [4] examine salmon, an open-access resource, and conclude that restrictions of the type of gear cause large losses over a system of regulating catch directly. Indeed, there is no question that legislated inefficiency will not be an optimum policy for a solely owned resource; but since it is easy to legislate standards, it is still of interest to see what could be accomplished with such a policy. Let Ac(x) be nonzero unit costs where A is a technical change parameter. A policy of irrational conservation is one that increases A.

Theorem 6. *Under Assumptions 1 through 6 and irrational conservation,*

a. A cost increase, when unit cost varies (resp. is constant), increases (resp. leaves unchanged) the steady state stock.

b. When unit cost varies, a cost increase increases product flow when $x^* < x_{max}$ *and decreases it otherwise.*

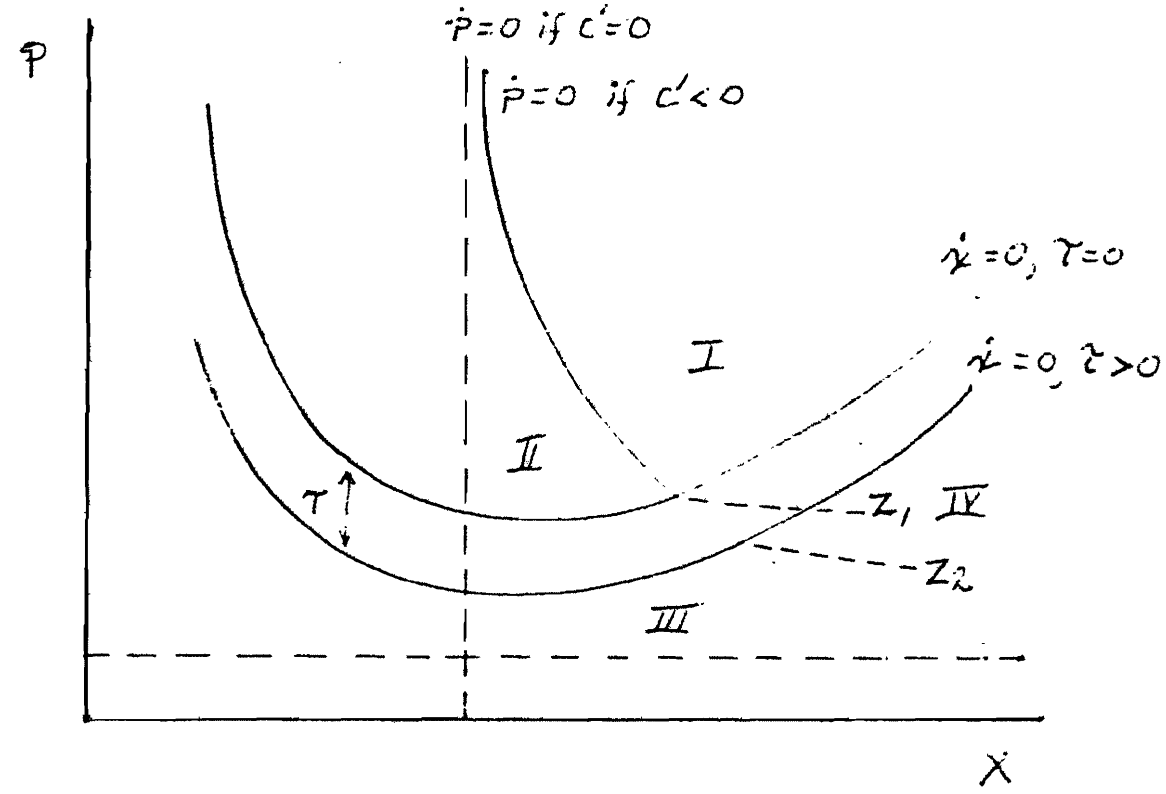

P
$\dot{p}=0$ if $c'=0$
$\dot{p}=0$ if $c'<0$
$\dot{x}=0, \tau=0$
$\dot{x}=0, \tau>0$
I
II
III
IV
τ
Z_1
Z_2
X

Figure 4.

Like Theorems 4 and 5, the proof is by reference by the appropriate phase diagram. The equations of motion are:

$$\dot{p} = (r - f')\ (p - Ac) + Ac'f \qquad (51)$$

$$\dot{x} = f(x) - Q(p). \qquad (52)$$

Since $\partial p/\partial A < 0$, the $\dot{p} = 0$ curve will be shifted upward wherever it slopes down.

When unit cost is constant, the cost increase has no effect on stock and product flow, or it results in a shutdown of the industry. Figure 5 gives the phase diagrams in this case. By an argument like that for the case of taxes, it can be shown that the new convergent arm lies everywhere above the old one.

4. CONCLUSION

Neither choosing the optimal long-run renewable resource stock nor attaining the planned goal is easy. Simply equating a rate of growth to a rate of time preference--as if determining the rate of time preference were simple--will not work if there is anticipated growth in demand or a significant externality attached to the resource stock. Failure to account for demand growth or an externality both result in too low a plan for the resource stock. In the case of an externality, a regulatory agency might find the traditional tax tool for correction of an externality to be very difficult to handle. Taxing the product--the easiest tax to levy--has an effect on stock that is critically mediated by the slope of the unit cost curve. If the curve is flat, the tax has little effect on ultimate stock. Regulation of production techniques will be inefficient, while stock subsidies are hard to administer because of the difficulty in measuring stock and the requirement for a dispersion of public monies. In short, the optimal regulation of or management of renewable resources cannot be meaningfully reduced to an $r = f'(x)$ or other such simple rule.

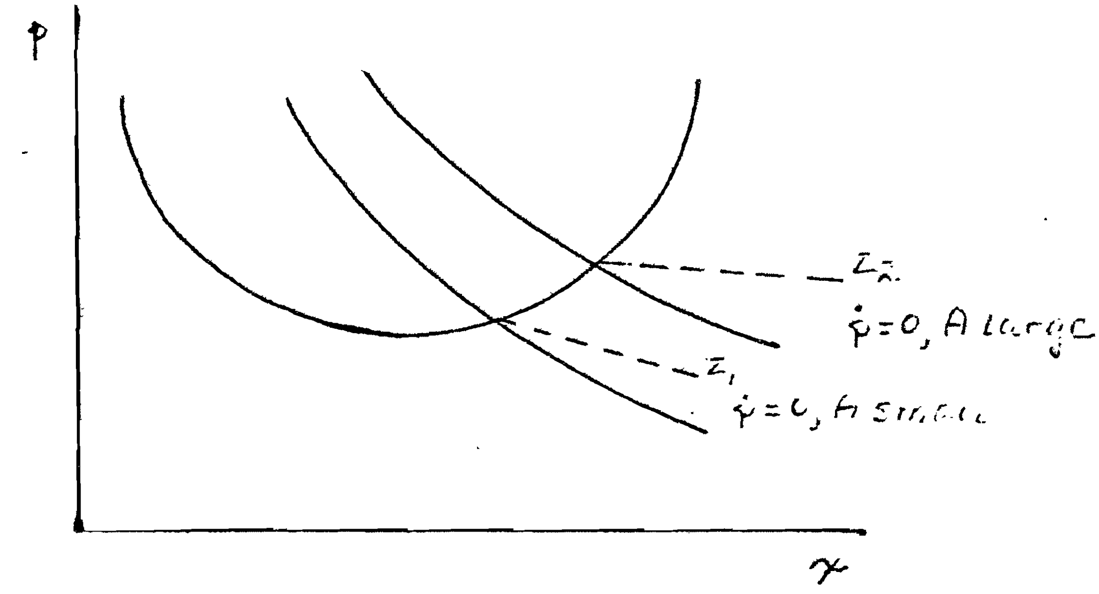


Figure 5.

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LIST OF SYMBOLS

α	alpha
β	beta
δ	lower case delta
λ	lambda
γ	gamma
τ	tau
∂	partial differential
∞	infinity
Δ	upper case delta
$\in$	element of
$\equiv$	identical with
$\rightarrow$	give, pass over to, or lead to
$\hat{}$	circumflex
$\sqrt{}$	square root
$>$	greater than
$<$	less than
$\geq$	greater than or equal to
$\leq$	less than or equal to
$\neq$	unequal to
$\forall$	for all