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**The Shape of the Optimal Hedge Ratio:
Modeling Joint Spot-Futures Prices
using an Empirical Copula-GARCH Model**

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The Shape of the Optimal Hedge Ratio: Modeling Joint Spot-Futures Prices using an Empirical Copula-GARCH Model

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The Shape of the Optimal Hedge Ratio: Modeling Joint Spot-Futures Prices using an Empirical Copula-GARCH Model

Abstract

Commodity cash and futures prices have been rising steadily since 2006. As evidenced by the April 2008 Commodity Futures Trading Commission Agricultural Forum, there is much concern among traditional futures and options market participants that the usefulness of commodity derivatives has been compromised. When basis risk is particularly high, dynamic hedging methods may be helpful despite their complexity and higher transaction costs. To assess the potential benefits of dynamic hedging in volatile times, this paper proposes a novel, empirical copula-based method to estimate GARCH models and to compute time-varying hedge ratios. This approach allows a nonlinear, asymmetric dependence structure between cash and futures prices. The paper addresses four principal questions: (1) Does the empirical copula-GARCH method overcome traditional limitations of dynamic hedging methods? (2) How does the empirical copula-GARCH hedging approach perform, for storable agricultural commodities, compared with traditional GARCH and Minimum Variance (static) hedging methods? (3) Is dynamic hedging more or less effective in the post-2006 biofuels expansion time period? (4) How sensitive is the ranking of methods to the hedging effectiveness criterion used? Preliminary findings suggest that the empirical copula-GARCH approach leads to superior hedging effectiveness based on some, but not all, risk criteria.

INTRODUCTION

Agricultural commodity prices have risen sharply since late 2006 (Figures 1 and 2), partly due to the Federal mandate for biofuels (mainly ethanol) and the resulting demand-side pressure on corn and supply-side pressure on soybeans, wheat and cotton. As an unintended effect, rising commodity prices attracted nontraditional investors such as mutual and pension funds. As a consequence, as of mid-2008, the relationship between spot and futures prices seems to have undergone substantial changes. In particular, the cash-futures price basis for certain commodities and locations has been observed to be much weaker than the historical norm. From a practical standpoint, this makes the effectiveness of futures hedging a timely problem.

This paper concerns the estimation of optimal dynamic hedge ratios for price risk management, e.g. by a grain elevator purchasing corn, wheat and soybeans. The conventional approach to the problem is to use a multivariate GARCH (MGARCH) model to estimate conditional (co)variances. However, an important limitation of MGARCH models is the typical assumption of joint multivariate normality, despite the empirical evidence against elliptical distributions in price returns. This paper adopts a different approach by combining univariate GARCH models of spot and futures prices

with empirical (nonparametric) copulas to characterize the higher order moment dependence between the two.

This paper is interested in the *shape of the optimal hedge ratio* in the sense that the hedge ratio depends not only on cash and futures price (co)variances but also on the level of prices, the sign of the basis (asymmetry), and other characteristics that may be best captured by describing the full joint dependence structure of cash and futures prices.

Empirical applications are presented for hedging corn and soybean meal from the perspective of a Texas feedlot operator with attention paid to the problem of increasing dimensionality (e.g. several simultaneous long or short hedges). As a secondary contribution, the paper shows how empirical copulas can be used to improve the results in the case of a small data sample by providing an arbitrarily large number of draws from the underlying distribution.

The paper also discusses the importance of using an appropriate hedging effectiveness criterion, which a few recent papers have noted, and suggests adopting “coherent measures” to obtain appropriate benchmarks when comparing different hedging approaches.

DYNAMIC HEDGING: CONCEPTS AND ISSUES

Futures and futures options are commonly used in agribusiness to hedge commodity price risk. Cash and futures prices for a given commodity move closely together over time but the difference between the two (i.e. basis) is itself time-varying. Due to such basis risk, a naïve hedge position (equal and opposite) is unlikely to be successful. Moreover, it has been well understood at least since Cecchetti, Cumby and Figlewski (1988) and Baillie and Myers (1991) that a constant (OLS) hedge ratio may be inappropriate when prices are possibly nonstationary, and that conditional rather than unconditional (co)variances should be used. Myers and Thompson (1989) proposed a general framework to estimate dynamic hedging.

The Engle (1982) and Bollerslev (1986) GARCH framework allows for the estimation of the conditional variance in a univariate case. Since the dynamic hedge ratio under min-variance criterion is the ratio of the conditional cash/futures covariance over the conditional futures variance at time t , a natural approach would be to estimate a bivariate GARCH model of cash and futures prices. Consequently, a large number of papers have applied this framework to estimate time-varying hedge ratios (e.g. Baillie and Myers, 1991; Bera, Garcia and Roh, 1997; Garcia, Roh and Leuthold, 1995; Moschini and Myers, 2002). In particular, Moschini and Myers (2002) devised a test enabling them to reject the null hypothesis of a time-constant hedge ratio for corn cash and futures prices, in favor of a time-varying hedge ratio.

Most of the empirical results, however, provide only weak evidence of any significant improvements in hedging effectiveness (Collins, 1997; Lence, 1995; Lien, 2005). To explain this apparent failure of dynamic hedging models, research on hedging

has examined cointegration (e.g., Haigh and Holt, 2000, 2002), parameter and model uncertainty (Dorfman and Sanders, 2005; Lence and Hayes, 1994a,b; Manfredo and Sanders, 2004) as well as business risk (Turvey and Baker 1989, 1990; Brorsen 1995).

Moreover, an issue that has been well noted in the multivariate GARCH literature and which extends beyond the problem of hedging is that the number of parameters increases rapidly with the dimensionality of multivariate GARCH models. For example, in the case of the widely used full MGARCH-BEKK (Engle and Kroner, 1995), the two-variable (one commodity) problem involves only eleven parameters, but a three-commodity problem implies 42 parameters and a seven-commodity problem implies 497 parameters. This is an important concern for a number of agribusiness risk management problems including multiple input/output price risk, currency risk or shipping cost risk.

A second principal issue raised in the dynamic hedging literature is the appropriate measure of hedging effectiveness. Early papers concluded that GARCH dynamic hedges were inferior to minimum variance hedges, but this comparison was likely misleading because the standard hedging effectiveness criterion (minimum variance) is designed for unconditional (co)variances and is therefore ill-suited to evaluate the usefulness of GARCH dynamic hedges (Lien, 2005). Moreover, several authors have noted that only downside risk should be minimized (e.g. Lien and Tse, 1998). More generally, Cotter and Hanly (2006) show that the ranking of hedging models is highly sensitive to the criterion used, to the point of this paper that better measures of hedging effectiveness ought to be considered such as recently-developed so-called “coherent measures” of risk (e.g. Acerbi, 2008).

The present paper proposes an empirical copula-GARCH model to better describe the joint comovement of variables in a portfolio of several cash and futures prices. This allows us to determine, for example, whether the failure of GARCH models to provide useful dynamic hedging is due to the possibly unrealistic assumption of joint multivariate normality. The latter is generally necessary to maintain model tractability but may be unduly restrictive. Indeed, recent papers (Bertram, Taylor and Wang, 2007; Jondeau and Rockinger, 2006; Lee and Long, 2007; Hsu, Tseng and Wang, 2008; Fernandez, 2008) have explored copula-GARCH approaches and have concluded that improving modeling of the joint distribution (i.e. through a copula) provides greater overall hedging efficiency gains than does improved modeling of the price dynamics (i.e. through GARCH). However, all of these papers take a parametric copula approach, and often use the Gaussian copula, implicitly assuming an elliptical dependence structure that is a function of only one parameter, and which further ignores higher order moments. In contrast our contribution is to propose a nonparametric, data-driven framework. The principal weakness is that the empirical copula is not itself time-varying, thus it is implicitly assumed that all dynamics are accurately captured in the marginals (i.e. univariate GARCH models).

The GARCH Estimation Framework

This section presents a review of essential concepts in the GARCH estimation framework. Consider a time series variable P_t such as a commodity cash price sampled

at a weekly frequency. A convenient measure of variation is the continuously compounded log-return, defined as a log-change in the case of commodity prices. The log-change is $r_t = \ln(P_t) - \ln(P_{t-1})$. Let the unconditional mean and variance be defined by μ and σ^2 and let the *conditional* mean and variance be:

$$m_t = \mathbf{E}[r_t | \mathcal{F}_{t-1}]$$

$$h_t = \mathbf{E}[(r_t - m_t)^2 | \mathcal{F}_{t-1}]$$

where \mathcal{F}_{t-1} is the information set (filtration) at time (t-1). Then we may write:

$$r_t = m_t + \sqrt{h_t} \varepsilon_t$$

where ε_t is the standardized innovation (error) at time t , which has conditional mean zero (0) and unit conditional variance. The basic GARCH model assumes the distribution of ε_t to be Gaussian Normal, but likelihood functions are available for a large number of other distributions, notably including Student-t, GED and Skewed Student-t. In this paper the Student-t distribution is assumed for the innovations, based on the results of Likelihood Ratio tests that reject the Normal distribution and Kolmogorov-Smirnov tests that find no support for the GED distribution.

The simplest GARCH model specification allows one ARCH parameter α and one GARCH parameter β . Although any combination GARCH(p,q) is admissible, previous research has found that the (1,1) specification performs very well as long as the most appropriate distribution is specified for the innovations (see above). Therefore, the model of conditional variance is:

$$h_t = \omega + \beta h_{t-1} + \alpha (r_{t-1} - \mu)^2$$

with $(\alpha + \beta) < 1$ to ensure stationarity. Note that the persistence of volatility shocks can be estimated based on the values taken by α and β . The appropriate likelihood is maximized using a nonlinear solver. Robust standard errors are computed following Bollerslev and Wooldridge's (1992) method.

The dynamic hedge ratio at each date in time can be computed as the ratio of the conditional cash-futures covariance over the conditional futures variance. Although a simple univariate GARCH model provides the latter, a method to obtain the conditional covariance is needed. The main challenge with multivariate GARCH models is how to specify a stable parametric structure that is also computationally solvable in a reasonable amount of time. A large number of methods have been suggested, but the BEKK specification of Engle and Kroner (1995) in particular has become widely used. It has the advantage of imposing positive semidefiniteness of the conditional covariance matrix, which is helpful to avoid a numerical failure to converge. Unfortunately, it requires that a relatively large number of parameters be estimated, indeed far more do than the individual univariate GARCH models.

An alternative approach used in this paper is to nonparametrically estimate the joint density, recover the empirical copula dependence structure and compute numerically the conditional covariance as follows:

$$h_{SF,t} = h_{S,t} h_{F,t} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \varepsilon_{S,t} \varepsilon_{F,t} f(\varepsilon_{S,t}, \varepsilon_{F,t} | \mathfrak{S}_{t-1}) dx dy$$

where $h_{S,t}$ is the conditional cash price variance, $h_{F,t}$ is the conditional futures price variance, and f is the joint density of cash and futures price innovations (errors).

As an empirical application of this novel approach, we consider the dynamic hedging problem of a Texas feedlot operator who must purchase corn and soybean meal for livestock feed. An advantage of this approach is that, unlike many parametric multivariate GARCH methods, it extends well to higher-dimensionality problems.

COPULA PROCEDURES

Copulas provide an alternative way to model joint distributions of random variables with greater flexibility both in terms of marginal distributions and the dependence structure. Copulas have been used in financial literature for quite sometime (see, for example, Embrechts et al. 2002; Cherubini, Luciano and Vecchiato, 2004; Chen and Huang, 2007; Fernandez, 2008), but have not made their way yet to the agricultural economics literature. What is more, theory was until recently inadequate to support the application of copulas to stochastic processes (i.e. time series), as argued by Mikosch (2006). Recent advances have focused on extending the copula concept to the stochastic process (time series) setting (Chen and Fan, 2006; Ibragimov, 2007; Patton, 2006). These theoretical results support empirical applications of copula theory to the case of time series assuming certain conditions are satisfied, and this includes in particular stationary Markov processes, of which martingales (which describe a number of financial asset price return series) are a special case. The Markov assumption is appropriate if we model the copula on the dependence structure *after* having estimated the GARCH model, in what is therefore a two-step solution method.

This paper's choice of empirical copula (analogous to nonparametric kernel density estimation) rather than parametric (e.g. Gaussian or Student) copula is motivated by the paucity of theoretical economic justifications for a specific copula form (see e.g. deVries and Zhou, 2006).

Overview of Copulas¹

The connection between copulas and joint distributions is established by the Sklar's Theorem (Nelsen, 2006, p. 15), which states that any distribution function $H(x, y)$ with margins $F(x)$ and $G(y)$ can be represented as

$$H(x, y) = C(F(x), G(y)), \quad (1)$$

where $C(\cdot, \cdot)$ is a uniquely determined copula function. The theorem also states that any two distribution functions $F(x)$ and $G(y)$ combined with an arbitrary copula C according to (1) result in a joint distribution function $H_c(x, y)$ with the margins F and G .

If the distribution functions and the copula in (1) are continuous, Sklar's theorem can be restated in terms of the probability densities as

$$h(x, y) = c(F(x), G(y)) \cdot f(x) \cdot g(y), \quad (1')$$

where $h(x, y) = \frac{\partial^2 H(x, y)}{\partial x \partial y}$, $f(x) = F'(x)$, $g(y) = G'(y)$, and $c(u, v) = \frac{\partial^2 C(u, v)}{\partial u \partial v}$ is the copula density. Eq. (1') often referred to as the canonical representation essentially decomposes the joint distribution of two variables into a product of marginal densities and the dependence structure captured by the copula density (Cherubini, Luciano and Vecchiato, 2004).

From the practical standpoint, (1') allows both derivation of copulas from a known distribution and construction of a joint distribution given marginal distributions and the copula. For example, if h is a bivariate standard normal distribution with the correlation ρ and the standard normal margins, then (1') implies the Gaussian copula density

$$c(u, v) = \frac{1}{\sqrt{1 - \rho^2}} \exp\left(\frac{(\Phi^{-1}(u))^2 + (\Phi^{-1}(v))^2}{2} + \frac{2\rho\Phi^{-1}(u)\Phi^{-1}(v) - (\Phi^{-1}(u))^2 - (\Phi^{-1}(v))^2}{2(1 - \rho^2)} \right), \quad (2)$$

where $\Phi(\cdot)$ is the cumulative density function of the standard normal distribution. The Gaussian copula is parameterized by a single parameter, which can be estimated from the historical data in a straightforward fashion.

The real advantage of copulas, however, comes from the fact that once the copula is derived or estimated, it can be applied to *any* pair of marginal distributions, not necessarily those implied by the original joint distribution. For instance, the Gaussian density (2) can be combined in (1') with a beta distribution f and a Student distribution g to result in a joint density function h , which is neither bivariate normal, nor beta, nor Student.

¹ The following is a brief summary of theory behind the copulas limited to two-dimensional copulas for brevity sake. A more formal and thorough presentation on the topic can be found in Nelsen, 2006.

For all its flexibility, the copula approach has one serious shortcoming. Generally speaking, there are an infinite number of copulas that can be used to generate joint distributions in (1'). Several parametric copulas have been frequently used in financial literature including the Gaussian copula (2) (Cherubini, Luciano and Vecchiato, 2004). Relative performance of different copulas can be measured against each other, but there is no constructive way to determine the "optimal" copula function (Kole, Koedijk and Verbeek, 2007).

Kernel Copula

An alternative to parametric copulas is a nonparametric kernel copula, which can be constructed from (1') by setting h equal to the kernel density estimate of the joint distribution, and f and g to the kernel density estimates of the corresponding marginals. A general form kernel density estimator of a univariate probability density function f can be written as

$$\hat{f}(x, \tau) = \frac{1}{n\tau} \sum_{i=1}^n K\left(\frac{x - X_i}{\tau}\right), \quad (3)$$

where $\{X_i\}_{i=1}^n$ are observations (i.i.d. draws from the distribution being estimated), $K(\cdot)$ is a kernel function, and τ is a smoothing parameter called bandwidth.²

A bivariate analog of (3) can be written as

$$\hat{h}(x, y, \tau_1, \tau_2) = \frac{1}{n\tau_1\tau_2} \sum_{i=1}^n K\left(\frac{x - X_i}{\tau_1}, \frac{y - Y_i}{\tau_2}\right), \quad (4)$$

where all the notation corresponds to (3), except that the kernel function now has two arguments and different smoothing parameters can be used along each dimension. There are several options for choosing the bivariate kernel function, but the most straightforward way is to use the product of two univariate (although not necessarily the same) kernels (Wand and Jones, 1995).

Based on (1'), (3), and (4), the overall procedure for estimating the kernel copula from a series of historical data $\{X_i, Y_i\}_{i=1}^n$ can be outlined as follows.³

Step 1. Construct the kernel density estimates of marginal distributions f and g according to (3) using appropriate kernels K_j and bandwidths τ_j .

Step 2. Calculate the cumulative density functions corresponding to f and g (e.g. by numerical integration)

$$\hat{F}(x) = \frac{1}{n\tau_1} \int \sum_{i=1}^n K_1\left(\frac{\xi - X_i}{\tau_1}\right) d\xi \quad \text{and} \quad \hat{G}(y) = \frac{1}{n\tau_2} \int \sum_{i=1}^n K_2\left(\frac{\eta - Y_i}{\tau_2}\right) d\eta \quad (5)$$

Step 3. Construct kernel density estimate of the joint density h according to (4) using the product kernel and the same bandwidths as in Step 1.

² The theory behind the kernel density estimator and the choice of the kernel function and bandwidth is beyond the scope of the present paper. A more detailed exposition can be found in (Wand and Jones, 1995).

³ Note that in the rest of the section the dependence of the estimated kernel density functions on the bandwidth is suppressed for brevity sake.

Step 4. Estimate the copula density at any given point (u,v) based on (1'), namely

$$\hat{c}(u,v) = \frac{\hat{h}(\hat{F}^{-1}(u), \hat{G}^{-1}(v))}{\hat{f}(\hat{F}^{-1}(u)) \cdot \hat{g}(\hat{G}^{-1}(v))}, \quad (6)$$

where $\hat{F}^{-1}(u)$ and $\hat{G}^{-1}(v)$ are inverse functions to the cumulative densities estimated in (5), which can be obtained by solving numerically the root-finding problems $\hat{F}(x)=u$ and $\hat{G}(y)=v$ for given u and v , respectively.

Once estimated, the kernel copula can be combined with any estimates of the marginal distributions of f and g , either parametric or nonparametric.

APPLICATION: HEDGING CATTLE FEED PRICES

An important agribusiness problem in the Western U.S. states is how to manage feed price risk in livestock operations. A number of papers have examined the problem of jointly hedging feed price risk and selling price risk (Shafer, Griffin and Johnston, 1978; Garcia, Leuthold and Sarhan, 1984; Leuthold and Peterson, 1987). Consider a typical feedlot pen with 100 heads of cattle, a starting weight of 800 lbs/head and a finished weight of about 1200 lbs/head at the end of a 17-week feeding period. Each head of cattle consumes 50 bushels of corn over this period (for a total of 5000 bu.) and about 120 lbs of a source of protein for which soybean meal is a reasonable proxy. This is a baseline problem that omits a number of relevant issues including hedging cattle prices, (feeder and fed/live) as well as credit liquidity risk, financial leverage and taxes. The impact of these concerns will be addressed in a future draft of this work

For the purposes of the paper, we use Texas triangle area corn cash prices, Decatur, Ill. soybean meal cash prices, and Chicago Board of Trade (CME Group) corn and soybean meal futures prices. All prices are sampled weekly on Thursdays. The observations range from 1/6/2000 to 1/17/2008 for a total of $T = 420$.

To compare hedging effectiveness before and after the large structural change in agricultural commodity markets due to the biofuels boom, we determine that a structural break in grain and oilseed prices is located in October 2006 and separate the data into two samples: pre- and post-October 2006. The analysis is completed separately in each sample and the results are then compared.

Transaction costs

Dynamic hedging involves changing one's futures position (ratio of bushels equivalent of futures contracts to bushels in cash position) based on how the GARCH hedge (cash/futures conditional covariance over futures conditional variance) varies. Since very few hedgers can be assumed to be members of the Chicago Board of Trade (for grains and oilseeds) or Chicago Mercantile Exchange (for livestock), a measure of broker fee or transaction cost should be included to reflect the price paid to change the futures position frequently. As a simple measure of transaction costs, it is assumed that there is a

proportional and constant fee of \$0.01 per bushel each time the futures position is adjusted. Although this proxy is a simplification, it appears to be a reasonable value based on communications with practitioners.

EMPIRICAL RESULTS AND HEDGING EFFECTIVENESS

First we discuss the GARCH model estimates for both the kernel copula approach and the benchmark MGARCH-BEKK model. We also include minimum variance static hedge ratio results. Second we present evidence of hedging effectiveness based on portfolio risk criteria as well as portfolio returns for hypothetical scenarios.

Results for Dynamic Hedge Ratios

The estimated dynamic hedge ratios are presented, together with the minimum-variance (static) hedge ratio, in Figure 3 for soybean meal and in Figure 4 for corn. First, for both corn and soybean meal the static hedge ratio increases in the post-October 2006 time period reflecting relatively higher volatility of cash prices. For corn, the GARCH-BEKK approach produces a dynamic hedge ratio that is higher, over most of the time period, than the static hedge ratio. In contrast, the kernel copula GARCH produces a hedge ratio that is generally smaller than the After October 2006, both the BEKK and kernel copula GARCH hedge ratios become closer to the static hedge ratio. For soybean meal, the two dynamic hedge ratios are relatively close to the static hedge ratio, but over most of the entire time period the BEKK GARCH hedge ratio is greater than the kernel copula GARCH hedge ratio.

Hedge Effectiveness and Portfolio Returns

As a baseline, we begin with a simple portfolio mean-variance analysis. Figures 5 and 6 present the frequency with which each of the three hedging strategies (static, GARCH-BEKK, kernel copula-GARCH) led to portfolio returns in various intervals. Note that interestingly the kernel copula GARCH approach leads to the overall best *returns*, but understandably that is not the main objective of hedging. Regarding the cost of broker fees associated with weekly updating of futures positions, the kernel copula GARCH dynamic hedge ratio ends up being less costly than is hedging with the BEKK GARCH approach.

Figures 7 and 8 present similar results using the measure of portfolio standard deviation, a simple measure of risk. Here, the kernel copula GARCH approach generally does well in reducing “tail risk” (i.e. probability of very large gains or losses), which is consistent with Fernandez’s (2008) finding that a copula approach to hedging is generally optimal according to the Value-at-Risk criterion. However, the kernel copula approach does not perform as well in reducing variance, which is also consistent with the original objective of using copulas to address higher-order moment risk.

CONCLUSIONS

The present paper proposes the use of empirical (kernel) copulas as a means of estimating multivariate GARCH models without imposing the restrictive assumption of joint normality. The joint model is estimated as a combination of univariate (marginal) GARCH models and a kernel copula to characterize the joint dependence structure and in particular capture non-elliptical higher order moment relationships.

In addition to being a methodological contribution to improve the estimation of MGARCH models in a tractable framework, this paper also presents a simple empirical application that may be generalized to a number of agribusiness hedging and risk management problems. We find that the kernel copula GARCH approach is a promising method but that the current, preliminary results are mixed. In the empirical application, this new approach provides lower broker fees, superior hedge returns and lower tail risk, but only mediocre variance reduction.

There are several directions in which this research may be extended to obtain better results. We note two areas of concern. First, the assumption of a stationary copula may be overly restrictive. While time-varying parametric copulas have been developed (e.g. Patton, 2006), it remains a computationally difficult problem to extend this to the nonparametric case. Second, we follow Cotter and Hanly (2006) and emphasize the importance of using appropriate risk criteria, as the interpretation of the hedging effectiveness results is highly sensitive to the criterion used. One increasingly used set of criteria are the Lower Partial Moments (e.g. Mattos, Garcia and Nelson, 2008; Turvey and Nayak, 2003). Lastly, a more general framework in which to nest the different criteria and their comparisons may be the recently defined “coherent measures of risk” (e.g. Acerbi, 2008).

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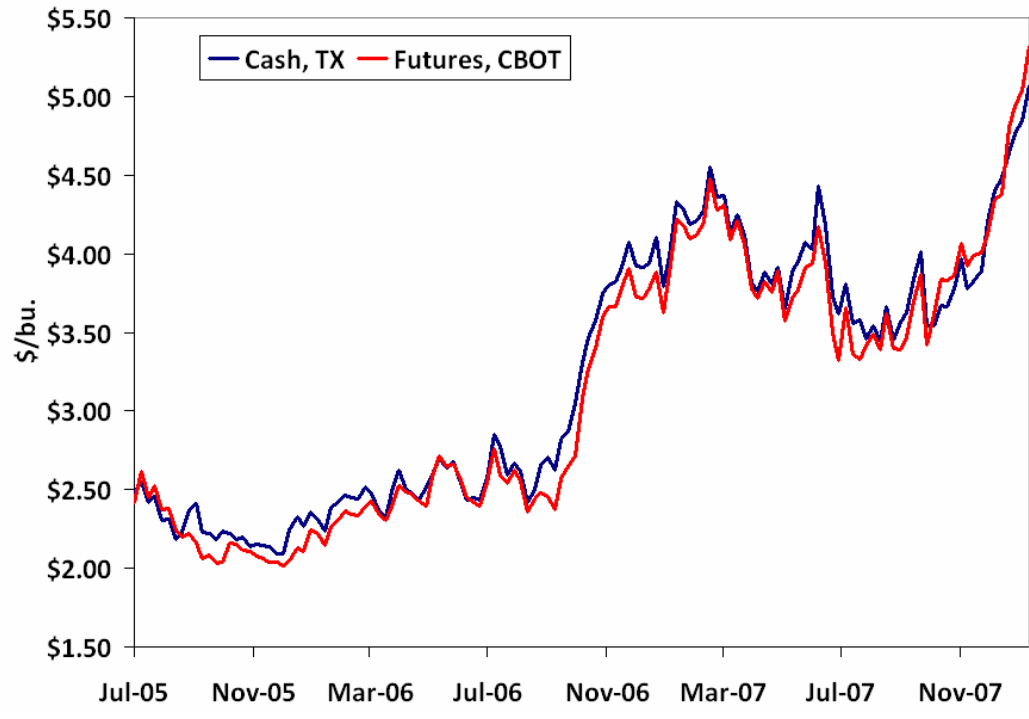


Figure 1: Corn futures and cash prices (Texas basis), 7/2005-12/2007

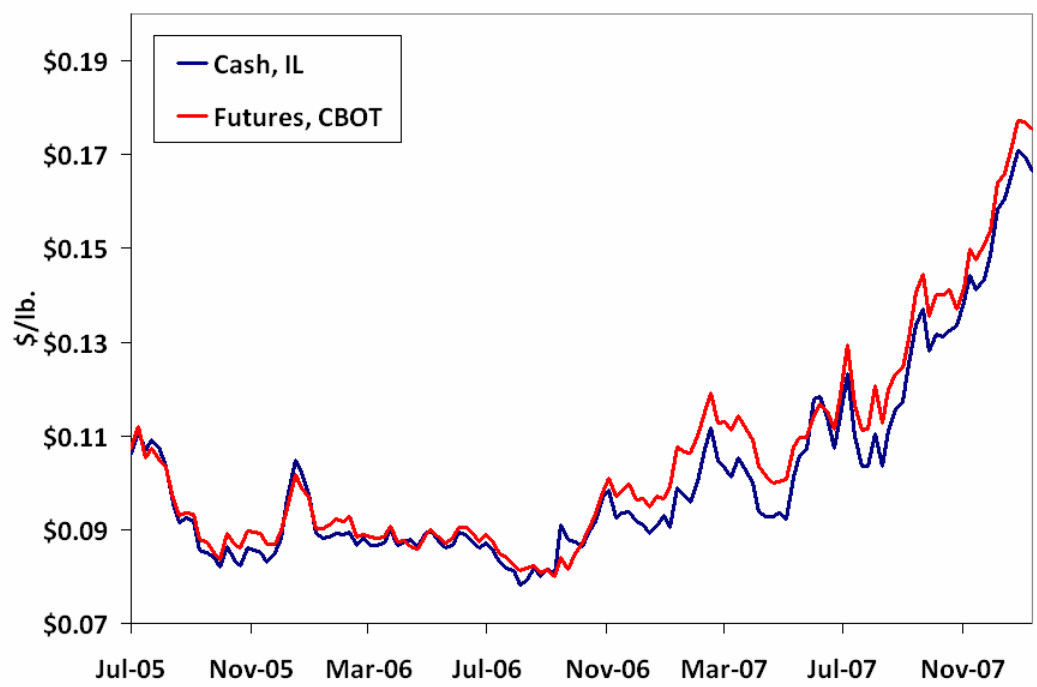


Figure 2: Soybean meal futures and cash prices (Illinois basis), 7/2005-12/2007

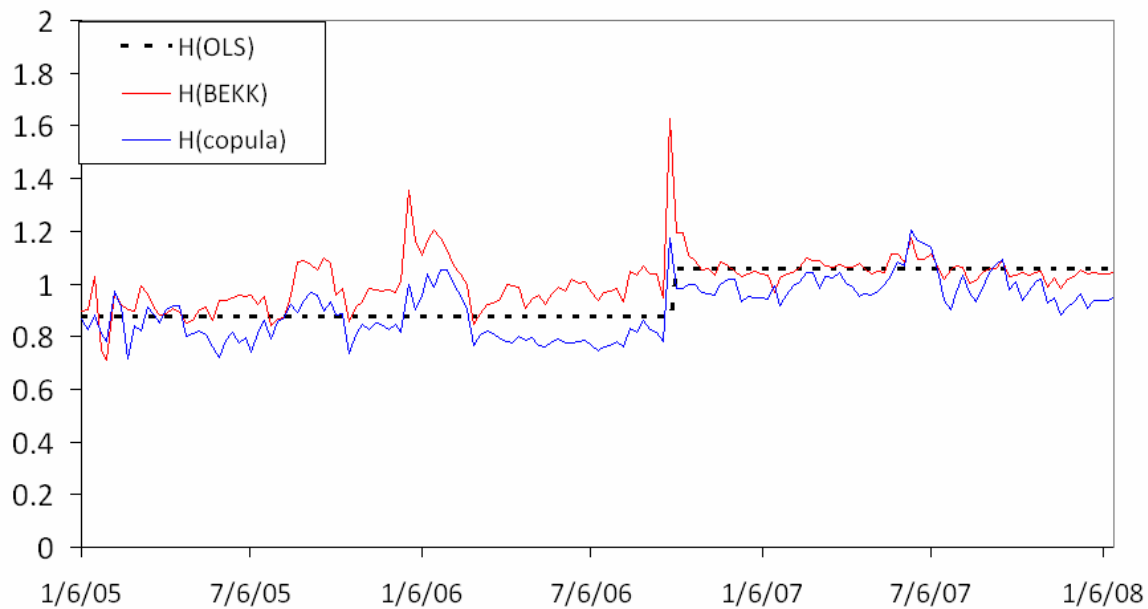


Figure 3: Static and dynamic hedge ratios, soybean meal, GARCH-BEKK, GARCH-kernel copula

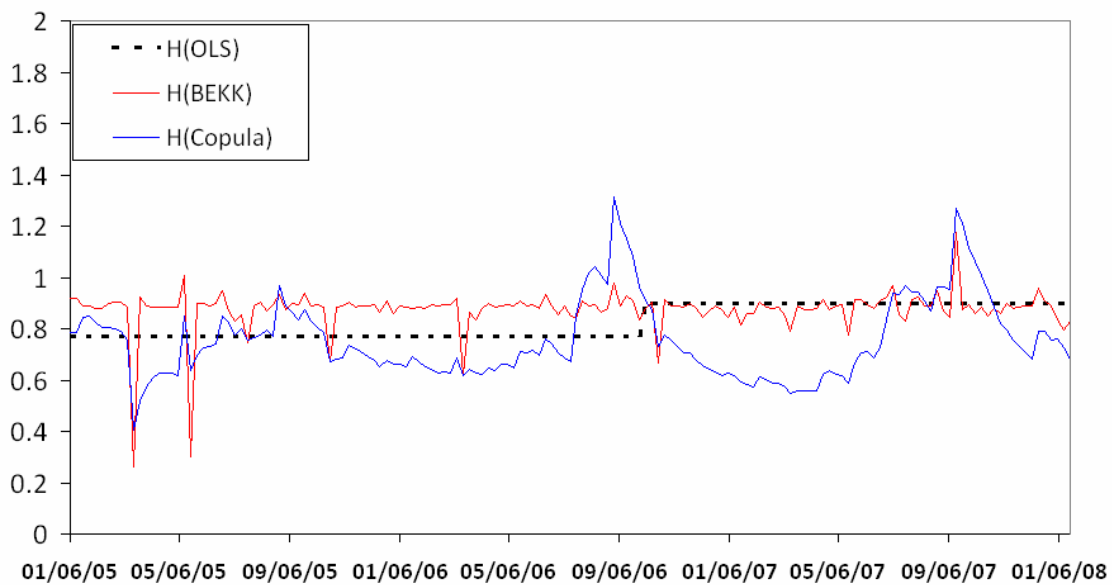


Figure 4: Static and dynamic hedge ratios, corn, GARCH-BEKK, GARCH-kernel copula

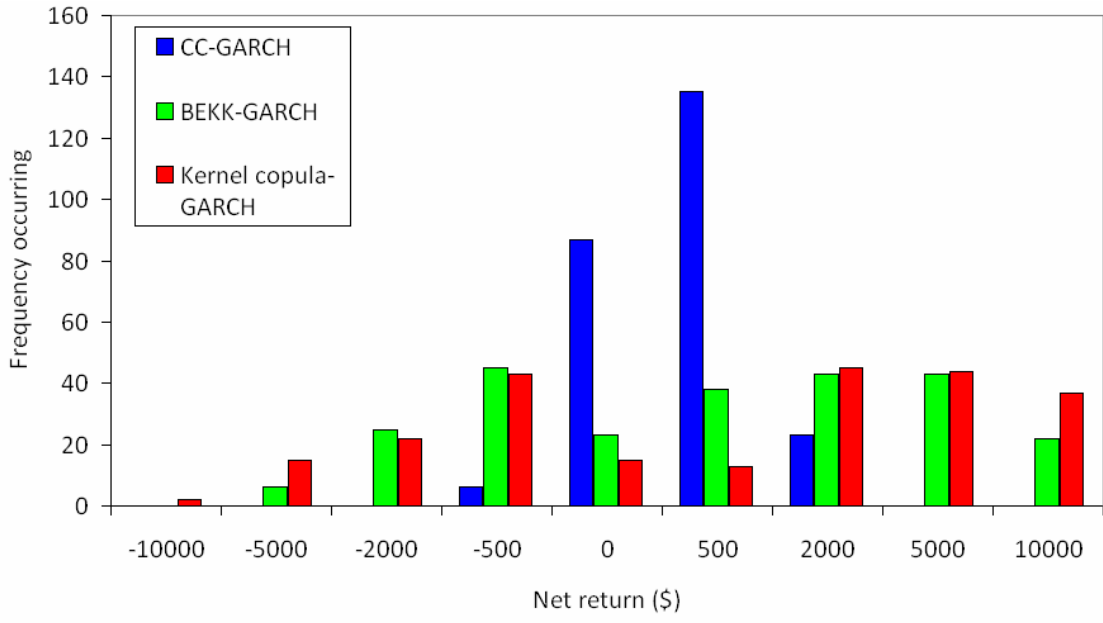


Figure 5: Feed Storage Hedge *Net Return*, Number of Hypothetical Cases Occurring in Each Bracket, over 1/2000-9/2006

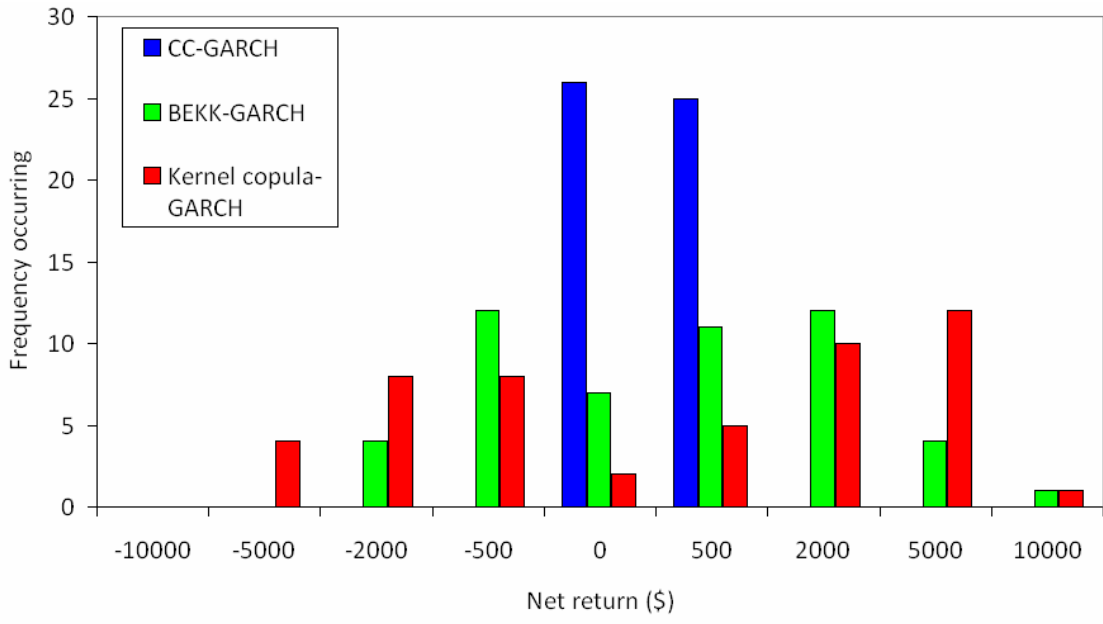


Figure 6: Feed Storage Hedge *Net Return*, Number of Hypothetical Cases Occurring in Each Bracket, over 10/2006-1/2008

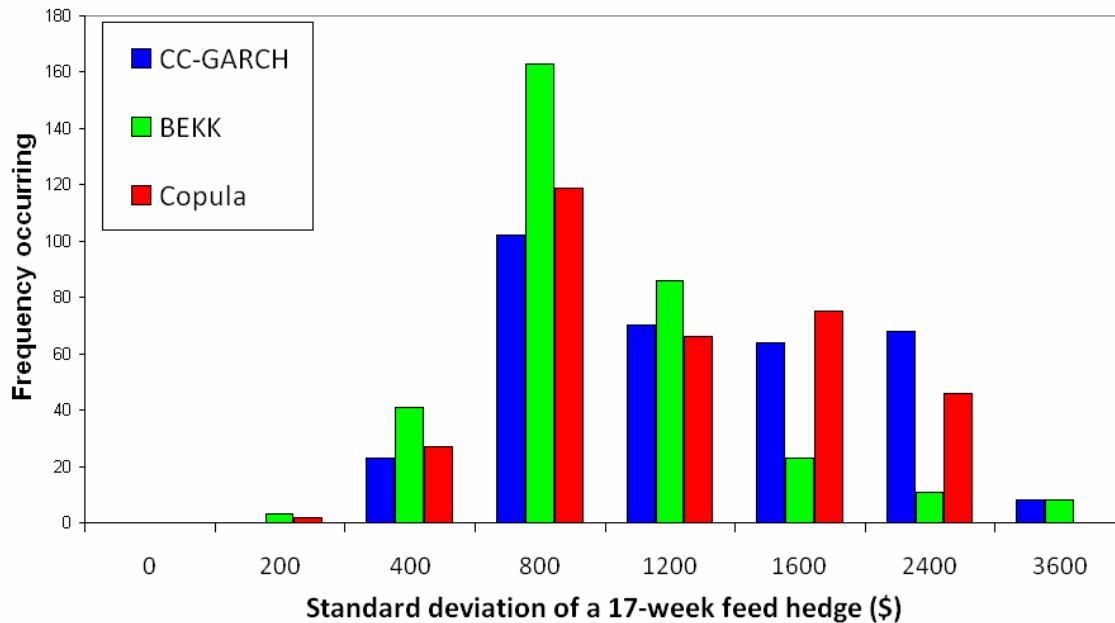


Figure 7: Feed Storage Hedge *Portfolio Standard Deviation*, Number of Hypothetical Cases Occurring in Each Bracket, over 1/2000-9/2006

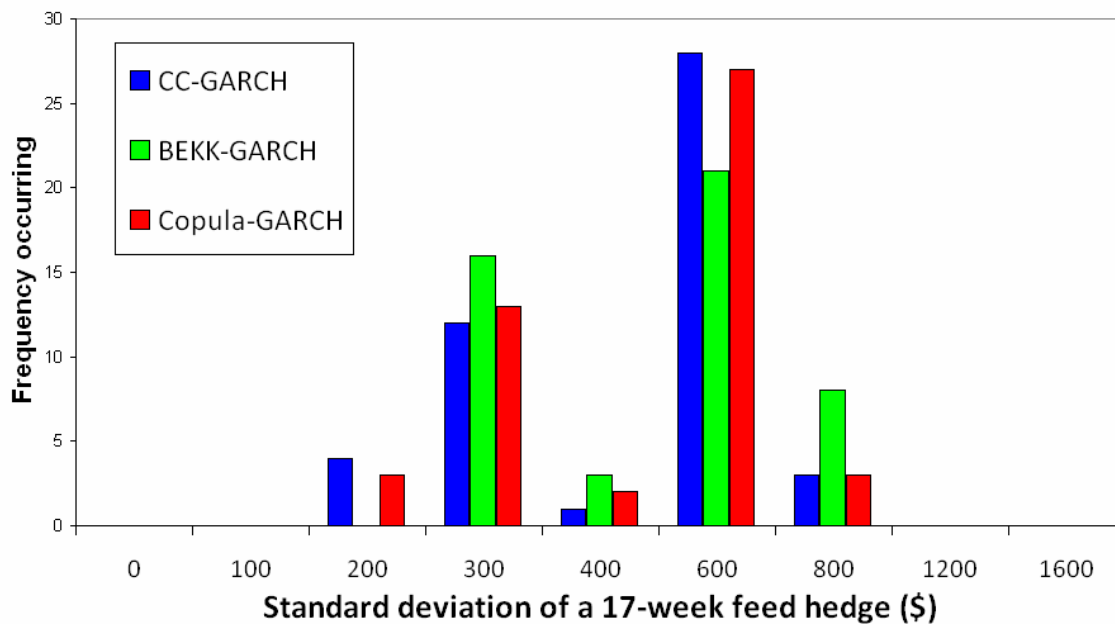


Figure 8: Feed Storage Hedge *Portfolio Standard Deviation*, Number of Hypothetical Cases Occurring in Each Bracket, over 10/2006-1/2008