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Optimal Stocking Density for Dual-Purpose Winter Wheat Production

Simeon Kaitibie, Francis M. Epplin, B. Wade Brorsen, Gerald W. Horn, Eugene G. Krenzer, Jr., and Steven I. Paisley

Dual-purpose winter wheat production is an important economic enterprise in the southern Great Plains of the United States. Because of the complex interactions involved in producing wheat grain and beef gain from a single crop, stocking density is an important decision. The objective of the research is to determine the stocking density that maximizes expected net returns from dual-purpose winter wheat production. Statistical tests rejected a conventional linear-response plateau function in favor of a linear-response stochastic plateau function. The optimal stocking density of 1.48 steers/ha (0.60 steers/acre) is 19% greater with a stochastic than with a nonstochastic plateau.

Key Words: dual purpose, response function, stochastic plateau, stocking density, wheat

JEL Classifications: R32, Q12, C29, D21

The use of winter wheat as a dual-purpose forage plus grain crop is important to the agricultural economies of southern Kansas, eastern New Mexico, Oklahoma, southeastern Colorado, and the Texas Panhandle (Redmon et al. 1995a). Pinchak et al. estimated that 30%–80% of wheat in the United States southern plains is grazed. True et al. reported that livestock grazed about 50% of Oklahoma wheat during the 1995–1996 growing season and that most wheat pasture is used for grazing young steers. The fall–winter wheat pasture produced by dual-purpose wheat is a valuable source of high-quality forage when perennial pastures are dormant.

One of the most economically important decisions for dual-purpose wheat pasture producers is the selection of the number of animals to stock on a given land area. Low stocking densities could lead to underutilization of forage, whereas high stocking densities could result in low gain per animal. The stocking density decision can be made on the basis of a measure of the quantity of forage available prior to stocking. Thus, initial standing crop of forage can be used as a decision criterion. The objective of this study is to determine the stocking density that would maximize expected net returns from dual-purpose winter wheat production on the basis of the quantity of forage available immediately prior to placing animals on the pasture in late October or early

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Simeon Kaitibie is former research assistant, Francis M. Epplin is professor, and B. Wade Brorsen is regents professor and Jean and Patsy Neustadt Chair, Department of Agricultural Economics, Oklahoma State University, Stillwater, OK. Gerald W. Horn is professor, Department of Animal Science, and Eugene G. Krenzer, Jr., is professor, Department of Plant and Soil Sciences, Oklahoma State University, Stillwater, OK. Steven I. Paisley is assistant professor, Department of Animal Science, University of Wyoming, Laramie, WY.

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November. The effect of stocking density on wheat grain yield and average daily gain of steers was determined by experimental data. The economically optimal stocking density was determined from the producer's expected net returns function.

The decision as to how many animals to purchase for placement on the wheat must be made before the season's weather is revealed. After the steers are placed on the wheat pasture, weather conditions might be more or less favorable than average. If weather conditions are better (worse) than average, then expected steer weight gains could be better (worse) than average. This difference in weight gain is due in part to differences across years in wheat forage growth after the steers have been placed on the wheat and in part to a direct weather influence on the steers.

In the model developed in this paper, production uncertainty is captured in a linear-response stochastic plateau model. For a given level of initial forage quantity, steer average daily gain is uncertain. This is a departure from standard deterministic response function analysis, under which, for a given level of forage, gain is assumed to be known with certainty.

Data

Data used to estimate steer average daily gain response to winter wheat forage were obtained from a stocking density experiment conducted for 7 years at the Oklahoma State University wheat pasture research unit in Logan County, Oklahoma. The Kirkland silt loam soil at the wheat pasture research unit is typical of much of the cropland in north central Oklahoma. The research unit included 16 pastures that ranged in size from 7.3 to 9.7 ha (18–24 acres). The research facility enabled close approximation to farm production practices.

The stocking density studies were conducted beginning with the 1992–1993 wheat pasture season and continued through the 1999–2000 wheat pasture season with the exception of the 1995–1996 season. After acquisition, the steers were transported to the research facility and placed in a receiving program. During the receiving program, the animals were vaccinated, treated for parasites, acclimated to the climate, and implanted with a combination estradiol-progesterone implant. Following the receiving program, the steers were weighed and placed on pastures. Stocking densities ranged from 0.82 to 2.87 steers/ha (0.33–1.16 steers/acre).

Table 1. Means of Average Daily Gain, Initial Forage Allowance, Grazing Pressure, and Stocking Density in the Stocking Density Experiments at the Wheat Pasture Research Unit, 1992–2000

Item	Unit of Measure	Mean
Average daily gain[a]	kg/steer-d	0.99
	lbs./steer-d	2.18
Initial forage allowance	Mg/steer-d	0.0086
	lbs./steer-d	18.96
Grazing pressure	steer-d/Mg	116.75
Stocking density[b]	steers/ha	1.60
	steers/acre	0.65

[a] This is the average daily gain of the steers that were stocked on wheat pasture for an average of 120 grazing days with an initial weight of 228 kg/steer (503 lbs./steer).

[b] Stocking density in the pastures ranged from 0.82 to 2.87 steers/ha (0.33–1.16 steers/acre).

Mean placement weight for the steers at the beginning of the grazing period was 228 kg (503 lbs.). During the pasture season, the steers were provided free-choice access to water and a high-calcium commercial mineral mixture but received no other supplemental feed except for limited amounts of alfalfa hay when snow covered the wheat fields. Steers were only removed from the pastures for weighing. More detailed information regarding activities at the wheat pasture research unit has been reported by Horn et al. (1995a, 1996, 1997, 1999); Kaitibie; Paisley; and Paisley et al.

Initial standing crop measurements were made prior to placement. This involved clipping a 0.5-m^2 area of forage to the soil surface from each of 10 quadrats randomly selected from each of the 16 pastures. The forage was dried to constant weight in a 55°C (131°F) oven, and yields were expressed as dry weight. Means of selected variables are provided in Table 1.

The available data enabled an analysis appropriate for producers who make the stocking density decision in the fall when the only available information is the current condition of the growing winter wheat (quantity of initial standing forage). The data enabled an analysis appropriate for producers who make a stocking density decision on the basis of initial standing forage and who maintain the stocking density throughout the grazing period. This situation describes that faced by many dual-purpose winter wheat producers in the region who do not have access to alternative forage during the winter.

Previous research has found that if winter wheat grazing is properly managed, stocking density will not adversely affect grain yield (Christiansen, Svejcar, and Phillips; Winter, Thompson, and Musick; Worrell, Undersander, and Khalilian). If livestock placement on the winter wheat is delayed until the plant roots are well anchored, if soil fertility is adequate, and if livestock are removed from the wheat prior to development of the first hollow stem stage, stocking density is not expected to influence grain yield. The field research was conducted consistent with these practices, so no effect on grain yield was expected.

Analytical Framework

Several studies have modeled animal response from grazing dual-purpose winter wheat (Horn et al. 1995b; Mader et al.; Pinchak et al.; Redmon et al. 1995b; Rodriguez et al.). However, these studies did not determine optimal stocking density on the basis of quantity of standing crop forage at placement time.

Hart et al. (1988b), studying rangeland stocking decisions, measured grazing intensity differences as either forage allowance (*FA*) or as grazing pressure (*GP*) (Hart et al. 1988b; Vallentine; Volesky et al.). Grazing pressure, *GP*, is the ratio of animal unit days to the weight of dry matter forage per unit area, whereas forage allowance, *FA,* is the available forage per animal unit or animal unit day. Therefore, when properly defined, *FA* is the inverse of *GP*. The definition of *GP* here is based on the definitions of Hart et al. (1988a) and Torell, Lyon, and Godfrey, so that

$$(1) \qquad GP = \frac{t \times SD}{F},$$

where *GP* is grazing pressure in steer-days per million grams of forage (steer-d/Mg, where Mg = metric ton = 1,000 kg), *t* is length of grazing period in days, *SD* is stocking density (steers/ha), and *F* is quantity of forage produced (Mg/ha). Since forage production (*F*) was determined immediately prior to placement, reference to *GP* and *FA* implies initial *GP* and initial *FA*.

The Response Function

Past research has estimated the effect of *GP* on average daily gain (Hart et al. 1988a,b; Torell, Lyon, and Godfrey; Volesky et al.) and the effect of *FA* on average daily gain (Pinchak et al.; Redmon et al. 1995b). These studies generally postulated a linear-response plateau function. The average daily gain (*ADG*) response declines to the right of the plateau for *GP,* whereas *ADG* increases to the left of the plateau for *FA,* as in the following univariate linear-response plateau functions.

$$(2) \qquad ADG = \begin{cases} \lambda_0 + \lambda_1 GP + \epsilon, & \text{if } GP > GP_{\text{critical}} \\ ADG_{\max} + \epsilon, & \text{otherwise} \end{cases}$$

and

$$(3) \qquad ADG = \begin{cases} \alpha_0 + \alpha_1 FA + \epsilon, & \text{if } FA < FA_{\text{critical}} \\ ADG_{\max} + \epsilon, & \text{otherwise.} \end{cases}$$

GP_{critical} is the critical initial grazing pressure, FA_{critical} is the critical initial forage allowance, and $ADG_{\max}$ is the maximum average daily gain represented by the plateau. The linear-response plateau function is assumed to be continuous such that $ADG_{\max} = \lambda_0 + \lambda_1 GP_{\text{critical}}$ (or $ADG_{\max} = \alpha_0 + \alpha_1 FA_{\text{critical}}$ in the case of initial forage allowance) represents the spline point.

The true form of the response function is not known. Intuitively, the choice of functional form for the response function is less important than the location of the plateau (FA_{critical}

and $ADG_{\max}$). However, given that weather and other uncontrollable factors that influence livestock weight gain vary from year to year, Berck and Helfand and Tembo, Brorsen, and Epplin raise the possibility of a response function with a stochastic plateau. Accordingly, the model error, ϵ, is linearly decomposed into a pure random error, ϵ^*, with mean 0 and variance $\sigma^2_{\epsilon^*}$ and year random effects, u, with mean 0 and variance σ^2_u. A third random error term v allows FA_{critical} to change by year. This third error has mean 0 and variance σ^2_v. The three error terms are assumed to be independent. This specification allows for the random effects to be estimated in a nonlinear mixed model.

Average daily gain was estimated as a function of initial FA rather than GP. The nonlinear mixed procedure in SAS was used to estimate a linear-response stochastic plateau function and a conventional linear-response (nonstochastic) plateau function. The conventional linear-response plateau function is nested in the linear-response stochastic plateau function. The likelihood ratio test, which is invariant to nonlinear transformations, was used to discriminate between the two models.

Profit-Maximizing Stocking Density

In dual-purpose winter wheat production, revenue is derived from both wheat grain and beef gain. To formulate the producer's profit function, the effect of stocking density on wheat grain yield was needed. Data from the grazed pastures were used to test wheat grain yield response to stocking density and wheat grain yield response to initial FA. Consistent with other studies (Christiansen, Svejcar, and Phillips; Kaitibie; Redmon et al. 1996; Winter, Thompson, and Musick; Worrell, Undersander, and Khalilian), it was determined that stocking density had no effect on grain yield. Therefore, only expected net returns from beef gain was considered to determine the optimal stocking density.

The formulated expected net returns function derives revenue from expected total gain. Total gain expresses steer gain per hectare for the length of the grazing season. It is obtained by multiplying ADG by GP (steer-d/ha). When the response function has a stochastic plateau, variability in total gain increases as GP increases. On the basis of the linear-response plateau function in Equation (3), total gain is expressed as[1]

$$(4)\qquad TG = ADG \times GP = \begin{cases} \alpha_0 GP + \alpha_1 + \epsilon GP, \\ \quad \text{if } FA < FA_{\text{critical}} \\ ADG_{\max} GP + \epsilon GP, \\ \quad \text{otherwise,} \end{cases}$$

where $FA_{\text{critical}} \sim N[(ADG_m - \alpha_0)/\alpha_1, \sigma^2_v/\alpha^2_1]$, and ADG_m is the mean average daily gain. $TG_{\max} = ADG_{\max} \times GP$ is the maximum total gain, and $FA^{-1} = GP$ when the determinants are expressed in identical units. Based on Equation (4), the total gain function can be rewritten using an indicator function, such that

$$\begin{aligned}(5)\qquad TG = \{&(\alpha_0 GP + \alpha_1)[1 - I_{-\infty,GP^{-1}}(FA_{\text{critical}})] \\ &+ ADG_{\max} GP(I_{-\infty,GP^{-1}})(FA_{\text{critical}})\} \\ &+ \epsilon GP,\end{aligned}$$

where the indicator function is defined as

$$(6)\qquad I_{-\infty,GP^{-1}}(FA_{\text{critical}}) = \begin{cases} 1, & \text{if } FA_{\text{critical}} \le GP^{-1} \\ 0, & \text{otherwise.} \end{cases}$$

On the basis of the assumption that the expected value of the error term is zero, expectations of the total gain function in Equation (5) can be taken to obtain the following

$$\begin{aligned}(7)\qquad &E(TG\,|\,GP) \\ &= (\alpha_0 GP + \alpha_1)E[1 - I_{-\infty,GP^{-1}}(FA_{\text{critical}})] \\ &\quad + E[ADG_{\max} GP(I_{-\infty,GP^{-1}})(FA_{\text{critical}})],\end{aligned}$$

where the expected value of the indicator function is defined as

$$\begin{aligned}(8)\qquad E[I_{-\infty,GP^{-1}}(FA_{\text{critical}})] &= \text{prob}(FA_{\text{critical}} \le GP^{-1}) \\ &= F(GP^{-1}).\end{aligned}$$

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[1] Other variables such as $ADG_{\max}$, $GP_{\text{critical}} = FA^{-1}_{\text{critical}}$, or $TG_{\max}$ can be used as spline criterion, rather than FA_{critical}. However, because FA_{critical} is normally distributed, it is more convenient.

$F(\cdot)$ is the cumulative density function of FA_{critical} evaluated at GP^{-1}. Because of the nonlinearity of the linear-response stochastic plateau function, the expectations must be maintained throughout the derivation. On the basis of the distributional assumption of FA_{critical} in Equation (4), the normal density function of FA_{critical} is expressed as

$$(9)\quad f(FA_{\text{critical}}) = \frac{1}{(2\pi\sigma_v^2/\alpha_1^2)^{1/2}}\exp\left[-\frac{(FA_{\text{critical}} - \mu_{FA})^2}{2\sigma_v^2/\alpha_1^2}\right],$$

where the parameter μ_{FA} is the mean critical initial FA (Mg forage/steer-d) associated with the plateau level ADG. Executing the expectations in Equation (7) gives

$$(10)\quad E(TG\,|\,GP) = (\alpha_0 GP + \alpha_1)[1 - F(GP^{-1})] + \int_{-\infty}^{GP^{-1}} (\alpha_0 + \alpha_1 FA_{\text{critical}}) \times GP f(FA_{\text{critical}})\, dFA_{\text{critical}},$$

where $F(GP^{-1})$ is the cumulative density function, defined as $\int_{-\infty}^{GP-1} f(FA_{\text{critical}})$. For the normal probability density function, $F(\cdot)$ does not have a closed-form solution. When the response function is a linear-response stochastic plateau, the profit-maximizing decision-maker's objective is equivalent to

$$(11)\quad \max E(\pi\,|\,GP) = p\{(\alpha_0 GP + \alpha_1)[1 - F(GP^{-1})]\} + p\left\{\int_{-\infty}^{GP^{-1}} (\alpha_0 + \alpha_1 FA_{\text{critical}}) \times GP f(FA_{\text{critical}})\, dFA_{\text{critical}}\right\} - rGP,$$

where π is net returns (\$/ha), p is the value of steer gain (\$/kg), and r is the marginal cost of the steer-grazing enterprise (\$/steer-d). To obtain the profit-maximizing level of GP, the first-order condition can be obtained by differentiating Equation (11) with respect to GP, so that

$$(12)\quad \frac{\partial E(\pi\,|\,GP)}{\partial GP} = p\left\{\frac{\partial(\alpha_0 GP + \alpha_1)[1 - F(GP^{-1})]}{\partial GP}\right\} + p\left\{\frac{\partial}{\partial GP}\int_{-\infty}^{GP^{-1}} (\alpha_0 + \alpha_1 FA_{\text{critical}}) \times GP f(FA_{\text{critical}})\, dFA_{\text{critical}}\right\} - r = 0.$$

We use the chain rule to evaluate the first term and the Liebnitz integral rule (Khuri; Tembo, Brorsen, and Epplin) to evaluate the second, so that

$$(13)\quad \frac{\partial E(\pi\,|\,GP)}{\partial GP} = p\left\{\alpha_0[1 - F(GP^{-1})] + \int_{-\infty}^{GP^{-1}} (\alpha_0 + \alpha_1 FA_{\text{critical}}) \times f(FA_{\text{critical}})\, dFA_{\text{critical}}\right\} - r = 0.$$

Because the cumulative density function does not have a closed-form solution, Equation (13) cannot be solved analytically. A grid search procedure was used to obtain the GP that maximizes expected net returns.

Results

Table 2 includes estimates of parameters and variance components for both response functions. The response functions showed similar expected maximum gains per steer-day. The spline point occurs (FA_{critical}) at 0.0116 Mg/steer-d (26 lbs./steer-d or 86 steer-d/Mg) and 0.0105 Mg/steer-d (23 lbs./steer-d or 95 steer-d/Mg) for the conventional linear-response plateau function and the linear-response stochastic plateau function, respectively. Their respective expected maximum average daily

Table 2. Average Daily Gain Response to Initial Forage Allowance for Different Functional Forms at the Wheat Pasture Research Unit, Marshall, OK, 1992–2000

Regressor/Error Component	Symbol	Linear-Response Plateau	Linear-Response Stochastic Plateau
Intercept	α_0	0.6002 (0.1019)	0.4812 (0.1038)
Initial forage allowance (Mg/steer-d)	α_1	49.32 (9.68)	66.47 (9.59)
Expected maximum gain (kg/steer-d)	$ADG_{\max}$	1.1740 (0.0734)	1.1798 (0.0997)
Initial forage allowance at maximum gain (Mg/steer-d)	FA_{critical}	0.0116 (0.0010)	0.0105 (0.0011)
Variance of year random effects	σ_u^2	0.0321 (0.0181)	0.0384 (0.0214)
Variance of error term	$\sigma_{\epsilon^*}^2$	0.0160 (0.0026)	0.0123 (0.0021)
Variance of plateau level gain	σ_v^2		0.0022 (0.0163)
−2 log likelihood		−83.9	−92.3

Notes: The dependent variable is average daily gain (kg) of steers with an initial weight of 228 kg (503 lbs.); standard errors are in parentheses.

gains were 1.17 and 1.18 kg/steer-d (2.58 and 2.60 lbs./steer-d).

The likelihood ratio test ($\chi^2 = 8.40$) showed that the conventional linear-response plateau function can be rejected at the 5% probability level ($\chi^2 = 3.84$). The economically optimal stocking density was based on the linear-response stochastic plateau and compared to that derived from the conventional linear-response plateau function.

The parameter values for α_0 and α_1 in the linear-response stochastic plateau are 0.4812 and 66.47, respectively. The value of α_1 is further adjusted to transform initial forage allowance (Mg/steer-d) into hectares per steer-day (ha/steer-d). This new value of α_1 is obtained when 66.47 is multiplied by the average initial standing crop of 1,732 kg/ha and divided by 1,000 kg/Mg, giving a value of 115.13 for α_1.

The steer sale price and steer carrying costs were estimated for the 1999–2000 wheat-growing season on the basis of data obtained from records of the experiment and the USDA. The average steer sale price at the wheat pasture research unit trials was $75/45.5 kg (100 lbs.), whereas the purchase price was $86/45.5 kg. For the initial steer weight of 228 kg (503 lbs.), an average *ADG* of 0.99 kg/steer-d (2.18 lbs./steer-d) and a grazing period of 120 days, the value of gain was estimated as $1.20/kg ($0.54/lb.) with the Equation (14)

$$\text{(14)}\quad p = \{[\text{sale price} \times (\text{initial wt} + \text{grazing period} \times ADG)] - (\text{initial wt} \times \text{pur. price})\} \div (\text{grazing period} \times ADG).$$

Steer production costs were determined from cost data obtained from the wheat pasture research unit. The steer carrying costs include order buyer fees ($4.97/steer), shipping to pasture ($9.95/steer), receiving program ($9.53/steer), hay during inclement weather ($1.44/steer), high-calcium mineral mixture ($0.76/steer), and veterinary care and medicine ($9.00/steer). It also covers shipping to market and sales commission ($14.90/steer), machinery costs ($10.00/steer), and labor ($7.50/steer). Interest on operating capital was

Table 3. Optimal Grazing Pressure and Stocking Density by Type of Response Function

Item	Unit	Linear-Response Plateau	Linear-Response Stochastic Plateau		
		$r = 0\text{–}1.40$[a]	$r = 0.67$	$r = 1.01$	$r = 1.40$
Grazing pressure	Steer-d/ha	149	178	162	144
	Steer-d/acre	60	72	66	58
Stocking density[b]	Steers/ha	1.24	1.48	1.35	1.20
	Steers/acre	0.50	0.60	0.55	0.49

Note: The value of gain is assumed to be $1.19/kg ($0.54/lb.) and the initial standing forage is assumed to be 1,732 kg/ha (1,547 lbs./acre).

[a] The letter r represents marginal steer carrying costs ($/steer-d). For the conventional linear-response plateau function, the optimal stocking density is either at the spline point or at zero. If the value of gain per steer per day ($p \times$ average daily gain) is greater than the marginal steer carrying cost, r, then the optimal point occurs at the spline point. Under the assumptions of a value of $1.19/kg ($0.54/lb.) gain and 1,732 kg/ha (1,547 lbs./acre) of initial standing forage, the optimal stocking density for the conventional linear-response plateau function is 1.24 steers/ha (0.50 steers/acre) when the marginal steer carrying costs are less than $1.41/d.

[b] Stocking density is based on a 120-day grazing period.

based on a 9.5% interest rate, resulting in $13.37/steer for a 228-kg (503 lbs.) steer purchased at $428/steer, and grazed for approximately 120 days. The steer carrying costs sum up to approximately $81.42/steer. Dividing $81.42 by 120 days yields a marginal steer carrying cost, r, of $0.67/steer-d.

Substituting for p, α_0, α_1, and r in Equation (13) and using a grid search procedure in MAPLE 7 (Wright) yields an economically optimal grazing pressure of 178 steer-d/ha (72 steer-d/acre). On the basis of a 120-day grazing period, this *GP* translates into a stocking density of 1.48 steers/ha (0.60 steers/acre).

For comparison purposes, the optimal *GP* was also derived from the estimated conventional linear-response plateau function. For the conventional linear-response plateau function, the optimal stocking density is either at the spline point or at zero. If the value of gain per steer per day ($p \times$ average daily gain) is greater than the marginal steer carrying cost per steer-day, r, then the optimal stocking density occurs at the spline point. Under the assumptions of p = $1.20/kg ($0.54/lb.) and 1,732 kg/ha (1,547 lbs./acre) of initial standing forage, the optimal stocking density for the conventional linear-response plateau function is 1.24 steers/ha (0.50 steers/acre), as long as the marginal steer carrying costs, r, are less than $1.41/d. If $r >$ $1.41, then the optimal stocking density is zero. Table 3 shows optimal grazing pressures and stocking densities by type of response function.

Additional analyses were carried out to determine how changes in the cost-price structure affect optimal stocking density for the linear-response stochastic plateau function. The marginal steer carrying costs were arbitrarily increased to $1.01 and then to $1.40. At r = $1.01, the optimal *GP* declined to 162 steer-d/ha (66 steer-d/acre). When r was further increased to $1.40, the optimal *GP* declined to 144 steer-d/ha (58 steer-d/acre). The results suggest that with the expected levels of p and r, the stochastic plateau specification leads to an optimal *GP* that is greater than that indicated by a nonstochastic plateau, but this depends on the ratio of the marginal steer carrying costs to the value of steer gain. For example, as shown in Table 3, if r is increased from its expected level of $0.67/steer-d to $1.40/steer-d, with a constant expected value of gain of $1.20/kg ($0.54/lb.), the optimal stocking density declines from 1.48 steers/ha (0.60 steers/acre) to 1.20 steers/ha (0.49 steers/acre). For these price levels, the estimated optimal stocking density is greater (1.24 steers/ha [0.50 steers/acre]) for the conventional linear-response plateau function.

Table 4 includes a summary of optimal stocking densities for selected levels of initial standing forage and value of gain given the expected marginal steer carrying cost of

Table 4. Effects of Changes in Initial Standing Forage (kg/ha) and Value of Gain on Optimal Stocking Density (steers/ha) for the Linear-Response Stochastic Plateau Function

Value of Gain ($/kg)	Initial Standing Forage (kg/ha) 1,200	1,732	2,000
	Stocking Density (steers/ha)		
0.99	0.99	1.43	1.65
1.19	1.04	1.48	1.73
1.34	1.11	1.58	1.83

Notes: Optimal stocking density is based on a 120-day grazing period. The marginal steer carrying cost is assumed constant at $0.67/steer-d.

[a] The mean initial standing forage quantity across pastures across years was 1,732 kg/ha (1,547 lb./acre).

Table 5. Expected Cost of Nonoptimal Stocking Densities, Given Expected Prices, the Mean Level of Initial Standing Forage, and a 120-day Grazing Period

Stocking Density (steers/ha)	Expected Net Returns[a] ($/ha)	Expected Cost of Nonoptimal Stocking Density ($/ha)
1.98	116.14	4.79
1.73	118.63	2.30
1.60	120.19	0.74
1.48[b]	120.93	—
1.36	118.50	2.92
1.23[c]	110.24	10.69
0.99	88.33	32.60

Note: The optimal stocking density with an estimated value of gain, *p,* of $1.19/kg ($0.54/lb.); a 120-day grazing period; a marginal steer carrying cost, r, constant at $0.67/steer-d); and an initial standing forage of 1,732 kg/ha (1,547 lbs./acre) is 1.48 steers/ha (0.60 steers/acre) at 228 kg steer (503 lb. steer).

[a] These are expected net returns to the grazing component of the dual-purpose winter wheat production enterprise and do not include returns from wheat grain.

[b] The optimal stocking density derived with the linear-response stochastic plateau model is 1.48 steers/ha (0.60 steers/acre).

[c] The optimal stocking density derived with the conventional linear-response plateau model is 1.23 steers/ha (0.50 steers/acre). The difference in expected returns at the budgeted prices is $10.69/ha ($4.33/acre).

$0.67/steer-d. As expected, optimal stocking density increases with an increase in initial standing forage. Optimal stocking density also increases with an increase in the expected value of gain.

Table 5 includes the expected net returns from the grazing component of the dual-purpose wheat production enterprise for the optimal stocking density on the basis of the linear-response stochastic plateau model and six nonoptimal stocking densities. The expected net returns to the grazing component for the optimal stocking density of 1.48 steers/ha (0.60 steers/acre) is $120.93/ha ($48.96/acre). This finding is based on an estimated value of gain, *p,* of $1.19/kg ($0.54/lb.); a 120-day grazing period; a marginal steer carrying cost, *r,* constant at $0.67/steer-d; and an initial standing forage of 1,732 kg/ha (1,547 lbs./acre). The expected net returns from a stocking density of only 1.24 steers/ha (0.50 steers/acre; as suggested by the conventional linear-response plateau function) is $110/ha ($44.63/acre), or $10.70 ($4.33) less than optimal.

The model suggests that the cost of understocking is relatively more expensive than overstocking. For example, overstocking by 0.5 (range 1.48–1.98) steers/ha (0.2 steers/acre) costs $4.79/ha ($1.94/acre). However, understocking by 0.5 steers/ha (range 1.48–0.8) costs $33.00/ha ($13.35/acre). Unlike perennial pastures, overstocking of dual-purpose winter wheat is not expected to have negative consequences in subsequent periods. Hence, in general, over the range of stocking densities considered, having too few cattle and permitting forage to go unused is relatively more costly than having too many cattle. The model suggests that producers should ensure that there are sufficient cattle to eat all of the available forage.

The analysis has several shortcomings. First, only production risk is considered. If expected utility maximization were considered rather than expected net returns maximization, other sources of variability, such as steer purchase price and steer sale price, would become important. Second, in the model it is assumed that the cost to determine the initial quantity of standing forage is zero. This is clearly not the case. Prior to adoption of the model as a management decision aid, research would be required to develop a reliable and inexpensive means to measure the initial quantity of standing winter wheat forage in the fall of the year after the wheat plants have become anchored in the soil. Third, methods for appropriately incorporating this material into extension education programs remain to be developed.

Summary and Conclusions

Producers in the southern Great Plains cultivate much of their wheat crop for dual-purpose production. This study found the stocking density that maximizes expected net returns on the basis of quantity of standing winter wheat forage at placement time and a priori knowledge of the length of the grazing period. The response of average daily gain to the standardized grazing input, initial forage allowance, was evaluated with conventional linear-response plateau and linear-response stochastic plateau functions. Statistical tests rejected the conventional linear-response plateau function in favor of the linear-response stochastic plateau function.

Under management conditions used at the wheat pasture research unit, when grazing is delayed until plants are anchored, fertilization is adequate; when grazing is terminated prior to development of first hollow stem, it was determined that, over the range of stocking densities used in the study, grain yield is independent of stocking density. Therefore, the rational producer's stocking decision is to select the stocking density that maximizes expected net returns from the steer production enterprise while ensuring that wheat grazing begins after proper root formation and ceases prior to the development of the first hollow stem.

On the basis of a linear-response stochastic plateau function, the economically optimal grazing pressure was estimated at 178 steer-d/ha, yielding a stocking density of 1.48 steers/ha (0.60 steers/acre) on the basis of a 120-day grazing period. This grazing pressure was higher than that indicated by a conventional linear-response plateau function. Uncertainty leads to higher stocking densities, depending on the cost-price structure of the steer-grazing enterprise. The higher stocking density in the stochastic plateau is essentially a result of the producer making sure that there are enough cattle to eat all of the forage available.

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