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Fuzzy Logic and Preference Uncertainty in Non-market Valuation

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FUZZY LOGIC AND PREFERENCE UNCERTAINTY IN NON-MARKET VALUATION

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Abstract

In seeking to value environmental amenities and public goods, individuals often have trouble trading off the (vague) amenity or good against a monetary measure. Valuation in these circumstances can best be described as fuzzy in terms of the amenity valued, perceptions of property rights, and the numbers chosen to reflect values. In this paper, we apply fuzzy logic to contingent valuation, employing a fuzzy clustering approach for incorporating preference uncertainty obtained from a follow-up certainty confidence question. We develop a Fuzzy Random Utility Maximization (FRUM) framework where the perceived utility of each individual is fuzzy in the sense that an individual's utility belongs to each cluster to some degree. The model is then applied to a Swedish survey that elicited residents' willingness to pay for enhanced forest conservation. The results from fuzzy models are generally 'better' than those obtained using the traditional random utility framework.

JEL Classification Numbers: Q51, C35

Keywords: random utility maximization and fuzzy logic; contingent valuation and preference uncertainty; c-means clustering; forest conservation

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1. INTRODUCTION

The impact of uncertainty on contingent valuation estimates is both a theoretical and empirical concern. McFadden (1973) first incorporated uncertainty about individuals' preferences using a random utility maximization (RUM) framework. The RUM model postulates that, from the point of view of the analyst, an individual's utility consists of a deterministic component plus an unobservable random error term. Hanemann (1984) subsequently applied this idea to the valuation of non-market amenities using a contingent valuation device where a respondent is faced with a choice to accept or reject an offered payment ('bid') for an improvement in the level of an environmental amenity or public good. This approach addresses uncertainty on the part of the investigator, not preference uncertainty on the part of the respondent.

Preference or respondent uncertainty arises in many different ways. Uncertainty might originate with the non-market commodity or contingency that is to be valued; respondents may be uncertain about what it is that they are valuing, having no experience with it and perhaps never having 'seen' it. The value an individual assigns to the specified non-market amenity is influenced by prices of both substitutes and complements, if they even exist, and markets for these goods may behave in ways that cannot be predicted by the individual (Wang 1997). Uncertainty can also originate with the questionnaire used to elicit information, although this problem can be overcome to some extent by improved survey design. Nonetheless, it is generally accepted that the contingent valuation method (CVM) contributes to potential measurement error, because it relies on hypothetical scenarios (Loomis and Ekstrand 1998). Over and above the hypothetical nature of CVM, individuals may simply be unable to make a tradeoff between the amenity in question and monetary value. They may not understand the proposed contingency and the way it is to be achieved, perhaps even unsure about the success that the public program (e.g., setting aside more habitat for a species) or government policy (e.g., tax, subsidy) will have in bringing about the change. Further, they may not understand or may even object to the proposed payment mechanism.

While some preference uncertainty can be resolved by better informing respondents, or working with them one-on-one, some uncertainty can never be resolved. This is why some prefer situations where a facilitator helps stakeholders identify their preferences and/or enables disparate groups of stakeholders to make a decision concerning environmental amenities (Gregory, Lichtenstein and Slovic 1993).

A number of methods have developed for incorporating preference uncertainty in empirical applications while maintaining the RUM framework. The first to do so were Li and Mattsson (1995) who used a follow-up question to ask respondents how certain or confident they were of the 'yes'/'no' answer they provided to the preceding valuation question. The same 'follow-up' strategy for addressing preference uncertainty was employed by a number of other researchers (e.g., Champ et al. 1997; Blumenschein et al. 1998; Johannesson, Liljas and Johansson 1998; Loomis and Ekstrand 1998; Ekstrand and Loomis 1998; Ready, Navrud and Dubourg 2001), but the seemingly ad hoc methods used for converting the follow-up responses for inclusion in the RUM econometric framework varied considerably.[1] Another approach imbedded information about

[1] Note that the follow-up questions used in this literature are not designed to increase the confidence of the estimated welfare measure, as with the double-bounded approach (Kanninen 1993). They are meant specifically to address respondent uncertainty.

preference uncertainty directly in the response options to the valuation question, thereby jettisoning the straightforward 'yes'/'no' choice (Ready, Whitehead and Blomquist 1995; Wang 1997; Welsh and Poe 1998; Alberini, Boyle and Welsh 2003). This enabled the researchers to employ an ordered probability distribution function, such as ordered probit or logit, instead of the standard binary one.[2] Despite the somewhat makeshift manner in which responses are often treated, what these lines of inquiry did recognize is the need to address respondent uncertainty.

Our view is that the apparent precision of standard WTP or WTA estimates may mask the underlying vagueness of preferences and lead to biased outcomes. Valuation can best be described as fuzzy in terms of perceptions about the property rights to the good, the amenity being valued (vagueness about what it is), and the actual tradeoffs between the amenity and the money metric. Although widely applied in engineering, computer science and bioinformatics, fuzzy logic has been largely ignored in economics, particularly in the area of non-market valuation where its use might be considered most appropriate. Paliwal et al. (1999) and van Kooten, Krcmar and Bulte (2001) may have been the first to apply fuzzy logic in this context. The former proposed a fuzzy hedonic method to value land degradation as explanatory factors – suitability, compatibility and operability – were assessed by experts using linguistic terms that were represented by fuzzy numbers. The researchers found that fuzzy as opposed to conventional regression significantly improved the mean squared error. Van Kooten et al. applied fuzzy logic in the context of the contingent valuation method. Using the same data as Li and Mattsson (1995, hereafter L&M), they specified the fuzzy sets willingness to pay ($W\widetilde{T}P$) and

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[2] Ready, Whitehead and Blomquist (1995) were an exception as they converted their responses back to a binary-type framework.

willingness not to pay ($W\widetilde{N}TP$), and then found an aggregated measure of the change in welfare. Their estimates of the value of forest preservation in Sweden were about half those of L&M's original measures. Differences in the nature of the preference uncertainty assumptions and measures of welfare were the main reasons for the different estimated values in these two studies, although, in this paper, we show that the differences are in fact much smaller than indicated.

In the current paper, instead of fuzzifying WTP and WNTP on the basis of responses to a dichotomous choice with follow-up certainty confidence procedure, we fuzzify respondent utility functions from the beginning. We employ a fuzzy clustering approach for incorporating preference uncertainty based on follow-up certainty confidence information and develop a Fuzzy Random Utility Maximization (FRUM) framework where the perceived utility of each individual is fuzzy in the sense that an individual's utility belongs to each cluster to some degree.

Cluster analysis is commonly used for pattern recognition (Bezdek 1982), soft learning (Karayiannis 2000), information control (Ruspini 1969), signal analysis (Leski 2005) and other engineering applications. In economics, it is mainly used to segment markets by incorporating heterogeneous preferences. Thus, in modeling choice of shopping trips, Salomon and Ben-Akiva (1983) classified people into different lifestyle clusters based on social, economic and demographic information, while Swait (1994) segmented individuals choosing beauty aids according to latent socio-demographic and psycho-graphic variables. In both cases and more generally, results indicate that the explanatory power of the latent segmentation model is greater than that of traditional approaches. In the context of non-market valuation, Boxall and Adamowicz (2002) applied latent segmentation to the choice of wilderness recreation sites, identifying latent classes by incorporating motivation, perceptions and individual characteristics. They found significant differences in welfare measures with the segment model. Fuzzy clustering analysis provides an alternative to the latent segmentation model that addresses non-linearity in a flexible way and avoids identification problems.

In this study, we use a fuzzy clustering approach that incorporates certainty confidence information to construct a fuzzy random utility maximization model. The model is then applied to L&M's survey of Swedish residents' willingness to pay for enhanced forest conservation. To demonstrate the feasibility, effectiveness and advantages of the proposed FRUM approach, the fuzzy results are compared with those obtained from a traditional RUM model, as well as L&M's model. Results indicate that the FRUM 'performs' as well or better than traditional methods of non-market valuation.

The paper is organized as follows. In the next section, we present a brief background to fuzzy logic and apply it to individuals' preferences. Our empirical model is described in section 3, where we introduce fuzzy c-means clustering and Takagi-Sugeno fuzzy inference. These concepts are applied in the context of a fuzzy random utility maximization model, which is also developed in section 3. The empirical results are provided in section 4, followed by some conclusions and further discussion.

2. FUZZY SET THEORY AND FUZZY PREFERENCES

Multivalued or fuzzy logic was first introduced in the 1920s and 1930s to address indeterminacy in quantum theory. The Polish mathematician Jan Lukasiewiccz introduced three-valued logic and then extended the range of truth values from {0, ½, 1} to all rational numbers in [0,1], and finally to all numbers in [0,1]. In the late 1930s, quantum philosopher Max Black used the term 'vagueness' to refer to Lukasiewicz' uncertainty and introduced the concept of a membership function (Kosko 1992, pp.5-6). Subsequently, in 1965, Lofti Zadeh introduced the term 'fuzzy set' and the fuzzy logic it supports. The theory was refined and further developed by Kaufman (1975), Kandel and Lee (1979), Dubois and Prade (1980), and many others.

Fuzzy Set Theory

Fuzzy logic starts with the concept of a fuzzy set. A fuzzy set has no crisp or clearly defined boundary as it can contain elements that have only partial membership in the set. Consider the set of "tall" people as an example. Most would agree that someone taller than two meters is an element of the set "tall". What about someone who is only 1.8 meters tall? To a basketball player, this person is not tall, although someone who is 1.5 m would consider them to be "tall". The point is that a person who is 1.8 m is not a member of the set "tall" to the same extent as someone who is more than 2 m tall (the former is a partial member of the set "tall"), while a person who is 1.5m is simply not a member of the set "tall" (or a partial member with very low degree of membership). Fuzzy logic is valuable because it permits the truth of any statement to be a matter of degree.

Consider the idea of fuzzy set and partial membership more formally. An element x of the universal set X is assigned to a fuzzy set $\widetilde{A}$ via the membership function $\mu_{\widetilde{A}}$, such that $\mu_{\widetilde{A}}(x) \in [0,1]$.[3] Thus, the closer the value of $\mu_{\widetilde{A}}(x)$ is to unity, the higher the grade of membership of x in $\widetilde{A}$. When A is an ordinary set, its membership function can

[3] We use a ~ to denote a fuzzy set; thus, A denotes an 'ordinary' set, while $\widetilde{A}$ denotes a fuzzy set.

take on only two values, 0 and 1, with $\mu_A(x) = 1$ or 0 according as the element does (full membership) or does not (no membership) belong to *A*.

The intersection and union of two fuzzy sets $\tilde{A}$ and $\tilde{B}$ are defined by Zadeh (1965) as:

(1) Intersection: $\mu_{\tilde{A}\cap\tilde{B}}(x) = \min\{\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)\} \forall x \in X$,

(2) Union: $\mu_{\tilde{A}\cup\tilde{B}}(x) = \max\{\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)\} \forall x \in X$,

The intersection $\tilde{A} \cap \tilde{B}$ is the largest fuzzy set that is contained in both $\tilde{A}$ and $\tilde{B}$, and union $\tilde{A} \cup \tilde{B}$ is the smallest fuzzy set containing both $\tilde{A}$ and $\tilde{B}$. Both union and intersection of fuzzy sets are commutative, associate and distributive as is the case for ordinary or crisp sets. Further, the complement $\tilde{A}^c$ of fuzzy set $\tilde{A}$ is defined as:

(3) $\mu_{\tilde{A}^C}(x) = 1 - \mu_{\tilde{A}}(x)$.

Fuzzy logic deviates from crisp or bivalent logic because, if we do not know $\tilde{A}$ with certainty, its complement $\tilde{A}^c$ is also not known with certainty. Thus, $\tilde{A}^C \cap \tilde{A} \neq \phi$ (ϕ is the null set) unlike crisp sets where $A^C \cap A = \phi$, so fuzzy logic violates the 'law of non-contradiction'. It also violates the 'law of the excluded middle' because the union of a fuzzy set and its complement does not equal the universe of discourse – the universal set. $\tilde{A}$ is properly fuzzy if and only if $\tilde{A}^C \cap \tilde{A} \neq \phi$ and $\tilde{A}^C \cup \tilde{A} \neq X$, where X is the universal set (Kosko 1992, pp.269-72).

A fuzzy number $\tilde{F}$ is defined on the real line, and has a membership function $\mu_{\tilde{F}}(x) \in [0,1]$, while a fuzzy variable has fuzzy numbers as its values. It is in this form that fuzzy set theory is used to define fuzzy utility, which is modeled as a fuzzy number with a certain membership function.

Fuzzy Preferences

Consumers often reveal their preferences using verbal statements such as: "I prefer the car with dark blue color." "I like that restaurant very much." "I would prefer to see more protection of forestland." Everyday statements about preferences are expressed in a fuzzy manner, as 'fuzziness' is inherent in human thinking, especially where people are asked to state a preference for one item over another (where one of them is a money metric), as opposed to making the actual choice itself. Stated preferences are different than revealed preferences, and it is the former that contingent valuation surveys address. Over and above the hypothetical nature of CVM, individuals may simply be unable to make a tradeoff between the amenity in question and a monetary value. Further, they may not understand the environmental quality change in question and the manner in which the questionnaire proposes that it would be achieved or paid for. Valuation in these circumstances can best be described as fuzzy.

Let X be a finite collection of alternatives and let $x,y \in X$. Traditionally, we define the preference relation as x weakly dominates y if $x \succeq y$, and x strongly dominates y if $x \succ y$. On the same set of alternatives X, the fuzzy preference relation $\tilde{R}(x,y)$ is defined as a fuzzy set, with membership function $\mu_{\tilde{R}}(x,y)$ representing the degree to which x is at least as good as y. It is clear that the crisp preference relation is the limit of the fuzzy preference relation where membership $\mu_{\tilde{R}}(x,y)$ can only take on values 0 (y strongly preferred to x) or 1 (x strongly preferred to y). A fuzzy preference relation that satisfies the following properties is called a fuzzy preference ordering:

- Reflexivity: $\forall\, x \in X,\ \mu_{\tilde{R}}(x,x) = 1$.
- Connectedness (completeness): $\forall\, x,y \in X,\ \mu_{\tilde{R}}(x,y) + \mu_{\tilde{R}}(y,x) \geq 1$.
- Max-min transitivity: $\forall\, x,y,z \in X,\ \mu_{\tilde{R}}(x,z) \geq \min[\mu_{\tilde{R}}(x,y), \mu_{\tilde{R}}(y,z)]$.

Accordingly, the individual's utility function, indifference curve and compensating/ equivalent surplus are fuzzy as well.

A graphical illustration of a fuzzy indifference curve is provided in Figure 1. The figure is also used to illustrate fuzzy compensating surplus. Income and the amount of the environmental amenity are assumed to be well defined or crisp. Representative fuzzy indifference curves are provided in the figure for two individuals (*A* and *B*) faced with the opportunity of paying an amount *W* to increase the availability of an environmental amenity from E_0 to E_1, or remaining at the status quo level of the amenity (E_0) at point *K*. Combinations of income and the environmental amenity located on the dark lines have memberships equal to 1.0 in the fuzzy indifference sets, $\tilde{I}(A)$ and $\tilde{I}(B)$. Points located off the dark lines but in the respective bounded areas have a degree of membership in the fuzzy indifference level that is less than 1.0 but greater than 0. For the respondent with fuzzy indifference curve $\tilde{I}(B)$, the new consumption set represented by β has a membership in $\tilde{I}(B)$ of 1.0, while $\mu_{\tilde{I}(B)}(\gamma) = 0.70$, say, and $\mu_{\tilde{I}(B)}(\alpha) = 0$. For the individual with fuzzy indifference curve $\tilde{I}(A)$, $\mu_{\tilde{I}(A)}(\alpha) = 0.30$, say.

<Insert Figure 1 about here>

When a respondent's indifference curve is crisp (i.e., described only by the dark line), then *W* will be accepted ('yes' answer) when the indifference curve at E_1 is below the line *m–W*. This is the case for respondent *B*, who would be expected to answer 'yes' because her compensating surplus (=θβ) exceeds *W*, but not for respondent *A*, whose crisp compensating surplus is less than *W*. Figure 1 illustrates the potential problems in answering a dichotomous-choice question regarding the bid amount *W* when a respondent's indifference curve and hence compensating surplus *S* is also fuzzy. Respondent *A* will always reject the opportunity to pay *W* for more of the environmental amenity. For the environmental amenity level E_1, respondent *B*'s fuzzy indifference curve intersects the interval that contains the *m–W* value. Consequently, some points of the intersecting interval are below and others above the line *m–W*; thus, $0 < \mu_{\tilde{S}(W)}(\gamma) < 1$, where $\tilde{S}(W)$ is the fuzzy set "compensating surplus equals *W*". *B*'s response to a dichotomous-choice question is therefore subject to the individual's interpretation of the verbal description of the contingency, the vagueness of the tradeoff, and so on. These factors dictate her 'yes'/'no' response, with either answer consistent with her preferences. The RUM model based on crisp preferences (utility) may be misleading in these circumstances.

3. EMPIRICAL MODEL

Fuzzy c-Means Clustering and Takagi-Sugeno Fuzzy Inference

The fuzzy c-means clustering (FCMC) algorithm was proposed by Bezdek (1973) as an improvement over an earlier 'hard' c-means algorithm to classify inputs into *c* categories. In contrast to the crisp classifications of 'hard' c-means clustering, fuzzy c-means clustering allows each data point to belong to a cluster to a degree specified by a grade of membership and allows a single data point to be a member of more than one cluster.

The objective of the FCMC algorithm is to partition a collection of n data points x_k, k=1, …, n, into c fuzzy sets or clusters ($\widetilde{A}_1$, …, $\widetilde{A}_c$) in a way that best fits the structure of the data. Let $\mu_{\widetilde{A}_i}(x_k)$ be the degree of membership of data point x_k in cluster $\widetilde{A}_i$, where the sum of degrees of belonging for a data point always equal unity by imposing the following normalization:

(4) $$\sum_{i=1}^{c} \mu_{\widetilde{A}_i}(x_k) = 1, \forall k = 1,\ldots,n.$$

The objective function is then to minimize the criterion function:

(5) $$J_m(U,V;X) = \sum_{i=1}^{c}\sum_{k=1}^{n}\left(\mu_{\widetilde{A}_i}(x_k)\right)^m \left\|v_i - x_k\right\|^2,$$

where $0 \leq \mu_{\widetilde{A}_i}(x_k) \leq 1$ and U is the matrix of possible memberships; $v_i \in V$ is the cluster center of the fuzzy set i with V the vector of all cluster centers; $\left\|v_i - x_k\right\|$ is the Euclidean distance between the i^{th} cluster center and k^{th} data point; and $m \in [1,\infty)$ is a weighting exponent. There is no prescribed value for m, but it is common to choose m=2 (Giles and Draeseke 2003). In the case of crisp sets m=1.

Minimization of (5) subject to condition (4) yields two necessary first-order conditions that can be solved to give:

(6) $$\mu_{\tilde{A}_i}(x_k) = \frac{1}{\sum_{j=1}^{c}\left(\frac{\|v_i - x_k\|}{\|v_j - x_k\|}\right)^{2/(m-1)}}.$$

(7) $$v_i = \frac{\sum_{k=1}^{n}\left(\mu_{\tilde{A}_i}(x_k)\right)^m x_k}{\sum_{k=1}^{n}\left(\mu_{\tilde{A}_i}(x_k)\right)^m}.$$

The FCMC algorithm consists of iterations alternating between (6) and (7) that converges either to a local minimum or saddle point of J_m (Bezdek 1973). It involves the following steps (Giles and Draeseke 2003; Jang 1997):

1. Fix the number of clusters c, $2 \le c \le n$, and the threshold level ξ.
2. Initialize the cluster centers v_i.
3. Compute the membership matrix according to (6).
4. Update the cluster centers by calculating $\bar{v}_i$ according to equation (7).
5. Calculate the defect measure: $D = |\bar{v}_i - v_i|$.
6. Stop if D< ξ; otherwise, go to step 4.
7. Defuzzify the results by assigning every observation to that cluster for which it has maximum membership value – the 'home' cluster.

Next consider Takagi-Sugeno fuzzy inference. Suppose that we can classify inputs x into c fuzzy sets, $\tilde{A}_1$, …, $\tilde{A}_c$, with associated membership functions $\mu_{\tilde{A}_1(x)}$, …, $\mu_{\tilde{A}_c(x)}$. Suppose further that we can assign crisp functions to each of the clusters such that, if $x \in \tilde{A}_i$, then $y = f_i(x)$. Then, according to Takagi-Sugeno fuzzy inference, the combined effect is represented by (Takagi and Sugeno 1985):

(8) $$y = \frac{\sum_{i=1}^{c} \mu_{\widetilde{A}_i}(x) f_i(x)}{\sum_{i=1}^{c} \mu_{\widetilde{A}_i}(x)}.$$

The conjunction of the FCMC method with Takagi-Sugeno fuzzy inference enables construction of models in a flexible way. Giles and Draeseke (2003) employed this method to model econometric relationships. In their research, the sample observations for *x* were clustered into fuzzy sets using the FCMC algorithm such that the similarity within a set is larger than that among sets. Correspondingly it also defines an implicit partition of the data for output *y*. The relationship of interest is estimated over each set using the data for the set separately, and then with Takagi-Sugeno inference each sub-model is combined into a single overall model. We employ a similar approach in the empirical analysis to derive the Fuzzy Random Utility Maximization model.

Fuzzy Random Utility Maximization (FRUM) model

We proportion the sample observations into clusters based on information from the follow-up certainty confidence question using the fuzzy c-mean clustering method. That is, individuals with similar certainty confidence are grouped into one cluster, the 'home' cluster. These clusters have fuzzy boundaries because each observation can, at the same time, belong to other clusters to some degree smaller than their membership in the 'home' cluster.

The fuzzy random utility maximization model is based on Figure 1 in much the same way as the standard RUM model (Hanemann 1984). Individual *k*'s fuzzy utility function $\widetilde{u}_k$ can be specified as a function of a fuzzy deterministic component $\widetilde{w}_k$ and a crisp additive stochastic component ε_k :

(9) $\tilde{u}_k(z,m;s) = \tilde{w}_k(z,m;s) + \varepsilon_{z,k}$,

where $z \in \{0, 1\}$ is an indicator variable that takes on the value 1 if the individual accepts the proposed change in the amenity and 0 otherwise, m is income, s is a vector of the respondent attributes, and ε is the stochastic disturbance arising from uncertainty on the part of the observer.[4] Each individual's utility function is fuzzy in the sense that it belongs to every cluster to some degree. The probability of saying 'yes' for each observation is then:

$$
(10) \quad \begin{aligned} \Pr_k(yes) &= \Pr\{\tilde{w}_k(1,m;s) + \varepsilon_{1k} > \tilde{w}_k(0,m;s) + \varepsilon_{0k}\} \\ &= \Pr\{(\varepsilon_{1k} - \varepsilon_{0k}) > -[\tilde{w}_k(1,m;s) - \tilde{w}_k(0,m;s)]\} \end{aligned}
$$

Replacing $[\tilde{w}_k(1,m;s) - \tilde{w}_k(0,m;s)]/\sigma$ with $\Delta\tilde{w}_k$ and $(\varepsilon_{1k} - \varepsilon_{0k})/\sigma$ with ε_k, where $\varepsilon_k \sim$ N(0,1) is i.i.d. because ε_{1k} and ε_{0k} are i.i.d., yields the fuzzy probit model:

(11) $\Pr_k(yes) = \Pr(\varepsilon_k > -\Delta\tilde{w}_k) = F_\varepsilon(\Delta\tilde{w}_k)$

Assuming a linear utility function, the change in the 'deterministic' part of the utility function between the two states is then given as

(12) $\Delta\tilde{w}_k = \tilde{\alpha}_k + \tilde{\beta}_k M_k + \tilde{\gamma}_k^{\prime} s_k$,

which is estimated based on the information from each cluster. Once the sample observations are proportioned into c fuzzy clusters, we can use the data for each fuzzy cluster separately and specify each individual's utility at the 'home' cluster as:

[4] Notice that the error term ε addresses uncertainty on the part of the observer, while the fuzzy component (referred to as the deterministic component in standard RUM) deals with respondent or preference uncertainty.

(13) $u_{ij} = w_{ij} + \varepsilon_{ij}$; $j = 1, \ldots, n_i$; $i = 1, \ldots, c$.

Note that an individual's utility is fuzzy since it is estimated from the coefficient estimates for each cluster, as in equation (15) below, but the utility is assumed to be crisp within each cluster so that it is possible to employ a standard probability framework within each cluster. A linear specification of the indirect utility function can be assumed (as in RUM) and the change in the deterministic parts of the utility functions between the two states is then given as:

$$(14)\quad w_{ij} = \alpha_i + \beta_i M_{ij} + \gamma_i^{/} s_{ij} + \varepsilon_{ij}\,,$$

where M_{ij} is the bid, s_{ij} is a vector of observable attributes, ε_{ij} is a random component, and α, β and vector γ constitute parameters to be estimated. A standard probit (or logit) model can be estimated within each cluster. Using Takagi-Sugeno inference (8), the fuzzy indirect utility is then:

$$(15)\quad \Delta\widetilde{w}_k = \widetilde{\alpha}_k + \widetilde{\beta}_k M_k + \widetilde{\gamma}_k^{/} s_k = \frac{\sum_{i=1}^{c} \alpha_i \mu_{\widetilde{A}_i}(x_k)}{\sum_{i=1}^{c} \mu_{\widetilde{A}_i}(x_k)} + \frac{\sum_{i=1}^{c} \beta_i \mu_{\widetilde{A}_i}(x_k)}{\sum_{i=1}^{c} \mu_{\widetilde{A}_i}(x_k)} M_k + \frac{\sum_{i=1}^{c} \gamma_i^{/} \mu_{\widetilde{A}_i}(x_k)}{\sum_{i=1}^{c} \mu_{\widetilde{A}_i}(x_k)} s_k\,,$$

where $k = 1, \ldots, n$. And probability of saying 'yes' for each observation can be rewritten as:

$$(16)\quad \Pr_k(yes) = F_\varepsilon(\widetilde{\alpha}_k + \widetilde{\beta}_k M_k + \widetilde{\gamma}_k^{/} s_k) = F_\varepsilon\left[\frac{\sum_{i=1}^{c} (\alpha_i + \beta_i M_k + \gamma_i^{/} s_k)\mu_{\widetilde{A}_i}(x_k)}{\sum_{i=1}^{c} \mu_{\widetilde{A}_i}(x_k)}\right],$$

where $k = 1, \ldots, n$ and $F(.)$ is the cumulative distribution function of the stochastic term.

The median WTP of each individual based on FRUM is then given as:

$$(17)\quad WTP_k = \frac{(-\widetilde{\alpha}_i - \widetilde{\gamma}_i^{/} s_k)}{\widetilde{\beta}_i} = \frac{-\sum_{i=1}^{c} \alpha_i \mu_{\widetilde{A}_i}(x_k) - \sum_{i=1}^{c} \gamma_i^{/} \mu_{\widetilde{A}_i}(x_k) s_k}{\sum_{i=1}^{c} \beta_i \mu_{\widetilde{A}_i}(x_k)},\ k = 1, \ldots, n.$$

That is, based on the FRUM model, the predicted probability or median WTP is a certain form of weighted average information for the fuzzy clusters, with the weights varying continuously throughout the sample. This is different from the traditional RUM model, where the predicted probability or median WTP is derived from a homogeneous model with an underlying assumption that utility is crisp.

4. EMPIRICAL RESULTS

We apply the FRUM model to a survey of Swedish residents that asked respondents whether they would be willing to pay a stated amount to continue to visit, use and experience the forest environment found in the northern part of the country (Li and Mattsson 1995). Bid amounts took one of the following values: 50, 100, 200, 700, 1000, 2000, 4000, 8000 and 16,000 SEK. A follow-up question asked how certain the respondent was about her ‘yes’/‘no’ answer on a percent scale with 5% intervals. Some 14% of the ‘yes’ respondents and 11% of the ‘no’ respondents reported confidence levels below 50%. Only about 35% of the ‘yes’ and 16% of the ‘no’ respondents had complete confidence in their response to the valuation question. The survey also collected data on respondents’ age, gender, number of forest visits, education, and household income. The sample is identical to that of L&M and consists of 344 observations.

The results of assuming two to five fuzzy clusters are summarized in Tables 1 through 4, respectively. As in L&M, the regressors include respondents' average annual forest visits, education, household income and the interaction of income and education. From the tables, we see that the sub-model estimates based on the separate fuzzy clusters can differ fundamentally. In Table 5, we compare the four fuzzy models with a traditional RUM model that assumes respondent certainty (i.e., ignores the follow-up question) and with L&M's approach for incorporating the follow-up uncertainty responses. Approaches that include information about respondent uncertainty perform better than the RUM model that ignores such uncertainty. The fuzzy models with three and five clusters outperform the other models based on the percentage of correct predictions (76.4% and 77.0%, respectively), while the fuzzy model with five clusters also has the lowest root mean square error. L&M's model is the 'winner' when the comparison is based on lowest mean absolute error. The fuzzy model fits the data better than the traditional RUM model and is competitive with and may even be preferred to the approach of L&M. The membership functions for fuzzy regressions with various clusters are plotted in Figures 2 through 5.

<Insert Tables 1-5 and Figures 2-5 about here>

The derived sample means of the median WTPs from each model are also provided in Table 5. The sample mean of median WTPs using the L&M approach is SEK 3394, which differs significantly from L&M's original estimate of SEK 12,817 (based on overall mean) or SEK 8578 (using truncated mean). There are several reasons for this difference, but the most important is that L&M used mean WTP as a measure of welfare instead of median WTP. Further, they assumed a log-linear valuation function, while we use a linear specification of the indirect utility function. We employ the 'corrected' L&M measures for comparison purposes, rather than the original L&M estimates.

From Table 5, the sample mean of median WTPs for the fuzzy models range from SEK 1537 to SEK 3899, which is similar to the estimates provided by van Kooten, Krcmar and Bulte (2001). Of the fuzzy models, the one with five clusters performed 'best', and it provides an estimate of WTP of SEK 3176, which is lower than the estimate of SEK 3394 derived using L&M's method. It is also substantially lower than the estimate of SEK 3899 that one obtains from the certainty model. These results indicate that WTP estimates are lower if preference uncertainty is taken into account and, further, that the method used to take into account preference uncertainty matters.

5. DISCUSSION

Welfare measures based on revealed preferences entail little in the way of a methodological problem for economists (even though their estimation may be difficult), but analysts measuring the welfare of public goods on the basis of stated preferences are likely to encounter preference uncertainty. In the literature, such uncertainty was recognized in the framework of random utility maximization – the use of a dichotomous-choice rather than an open-ended format for the valuation question (Hanemann and Kriström 1995). However, the RUM model considers only uncertainty on the part of the observer, not the respondent. There have been attempts to incorporate respondent uncertainty into the RUM framework, but these have, for the most part, been ad hoc (e.g., Alberini, Boyle and Welsh 2003; Ready, Navrud and Dubourg 2001). In this study, an alternative approach was brought to bear on the issue, namely, one rooted in fuzzy logic that interprets uncertainty in contingent valuation in a fundamentally different way than the standard framework. By assuming that a respondent's utility is vague and can be represented by a fuzzy number in utility space, the fuzzy random utility maximization method addresses both imprecision about what is to be valued and uncertainty about values that are actually measured.

While this paper represents one of the earliest efforts to apply fuzzy set theory to nom-market valuation, it is clear that much research remains to be done. For example, it is necessary to examine whether a fuzzy interpretation of utility can shed light on the persistent differences between WTP and WTA that are observed in experimental markets and contingent valuation surveys (Horowtiz and McConnell 2002). Further, the application of fuzzy set theory to non-market valuation would seem especially appropriate given that the valuation of environmental amenities and public goods is likely best done using verbal language, as noted by Evans, Flores and Boyle (2003), and fuzzy set theory is best suited to quantitative analysis of language. Yet, no contingent valuation studies have attempted to employ only language in the assessment of the tradeoff between the environment and a money metric.

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Table 1: Fuzzy Regression Results (c=2; m=2)[a]

Fuzzy cluster	Obs (# yes)	Cluster center	β_0	β_1	β_2	β_3	β_4	β_5	# of correct predictions (%)
1	90	41.321	-1.637	-0.000	-0.000	0.227	0.006	-0.001	55
	(48)		(-1.24)	(-0.66)	(-0.86)	(1.91)	(0.76)	(-1.24)	(61.1%)
2	254	91.714	-2.672	-0.000	0.003	0.239	0.015	-0.001	202
	(132)		(-3.28)	(-6.89)	(2.68)	(0.07)	(3.18)	(-3.04)	(79.5%)

[a] The t-statistics associated with the estimated β s are provided in parentheses.

Table 2: Fuzzy Regression Results (c=3; m=2)[a]

Fuzzy cluster	Obs (# yes)	Cluster center	β_0	β_1	β_2	β_3	β_4	β_5	# of correct predictions (%)
1	36	20.655	-0.788	0.000	-0.002	0.113	0.006	-0.001	24
	(21)		(-0.31)	(1.22)	(-0.92)	(0.56)	(0.37)	(-0.52)	(66.7%)
2	81	60.628	-1.178	-0.000	0.001	0.155	0.004	-0.001	50
	(38)		(-0.87)	(-3.14)	(0.53)	(1.25)	(0.60)	(-0.78)	(64.2%)
3	227	94.189	-2.674	-0.000	0.003	0.249	0.015	-0.001	185
	(121)		(-3.15)	(-6.67)	(2.67)	(3.42)	(2.96)	(-2.93)	(81.5%)

[a] The t-statistics associated with the estimated β s are provided in parentheses.

Table 3: Fuzzy Regression Results (c=4; m=2)[a]

Fuzzy cluster	Obs (# yes)	Cluster center	β_0	β_1	β_2	β_3	β_4	β_5	# of correct predictions (%)
1	26	16.328	0.450	0.000	-0.003	-0.010	-0.004	-0.000	17
	(15)		(0.16)	(0.97)	(-1.15)	(-0.04)	(-0.20)	(0.23)	(65.39%)
2	55	50.702	-2.361	-0.000	-0.001	0.352	0.014	-0.002	38
	(30)		(-1.25)	(-1.92)	(-0.54)	(1.83)	(1.33)	(-1.67)	(69.09%)
3	83	77.851	-2.702	-0.000	0.001	0.251	0.010	-0.001	58
	(38)		(-1.50)	(-3.34)	(0.39)	(1.78)	(1.07)	(-1.20)	(69.88%)
4	180	96.406	-3.009	-0.000	0.005	0.246	0.017	-0.001	149
	(97)		(-3.19)	(-6.37)	(3.472)	(3.014)	(2.937)	(-2.67)	(82.78%)

[a] The t-statistics associated with the estimated β s are provided in parentheses.

Table 4: Fuzzy Regression Results (c=5; m=2)[a]

Fuzzy cluster	Obs (# yes)	Cluster center	β_0	β_1	β_2	β_3	β_4	β_5	# of correct predictions (%)
1	26	14.960	0.450	0.000	-0.003	-0.010	-0.004	0.000	17
	(15)		(0.16)	(0.97)	(-1.15)	(-0.04)	(-0.20)	(0.23)	(65.39%)
2	43	46.791	-3.102	-0.000	-0.003	0.406	0.018	-0.002	28
	(24)		(-1.48)	(-1.00)	(-0.85)	(1.94)	(1.51)	(-1.74)	(65.12%)
3	35	65.470	-1.675	-0.000	0.005	0.168	0.005	-0.001	24
	(15)		(-0.57)	(-1.43)	(1.62)	(0.62)	(0.30)	(-0.52)	(68.57%)
4	60	82.290	-2.102	-0.000	-0.002	0.238	0.011	-0.001	44
	(29)		(-0.98)	(-2.77)	(-0.78)	(1.45)	(0.94)	(-1.06)	(73.33%)
5	180	97.023	-3.009	-0.000	0.005	0.246	0.017	-0.001	149
	(97)		(-3.19)	(-6.37)	(3.47)	(3.01)	(2.98)	(-2.67)	(82.78%)

[a] The t-statistics associated with the estimated β s are provided in parentheses.

Table 5: Comparing Model Performance for Swedish Forest Protection Survey

Method of comparison	Standard RUM[a]	Fuzzy (c=2)	Fuzzy (c=3)	Fuzzy (c=4)	Fuzzy (c=5)	L&M
%RMSE	0.430	0.409	0.403	0.397	0.393	0.438
%MAE	0.373	0.340	0.336	0.325	0.321	0.311
# of correct predictions	253	254	263	260	265	254
(% correct)	(73.6%)	(73.8%)	(76.4%)	(75.6%)	(77.0%)	(73.8 %)
Mean WTP (SEK)	3899.01	3674.8	1536.6	3837.8	3176.4	3394.15

[a] Assuming crisp utility functions or certainty on the part of respondents, and estimated as a probit model.

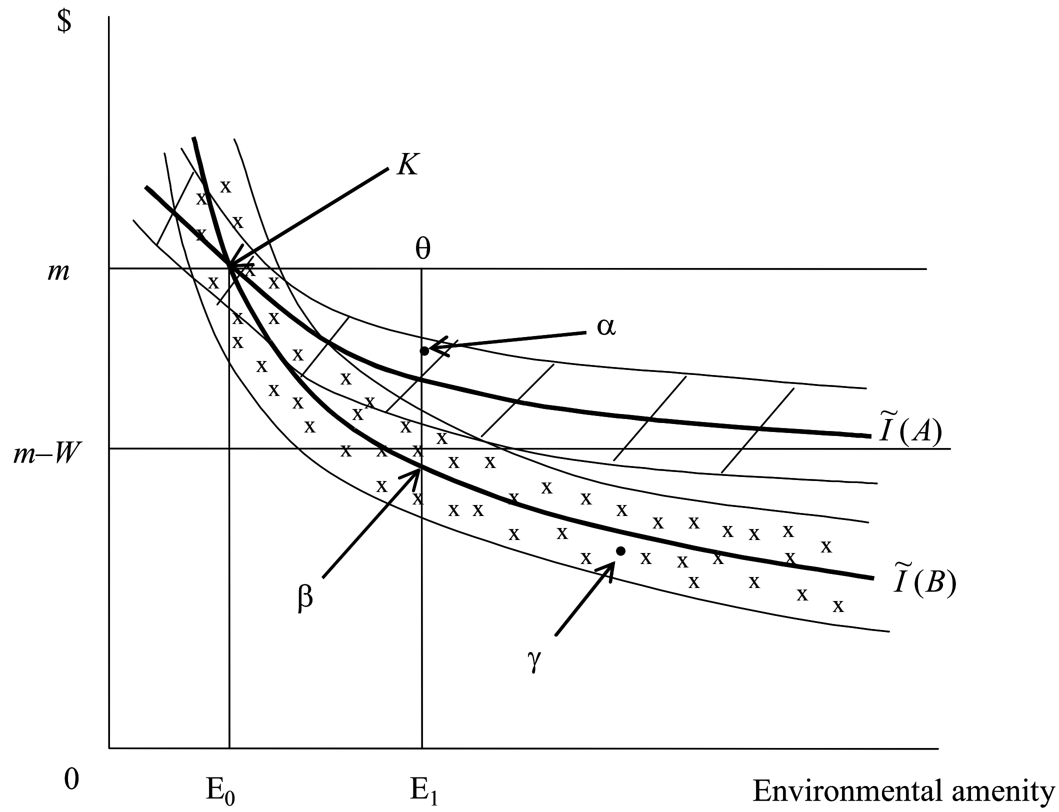

Figure 1: Interpretation of dichotomous-choice answers with fuzzy utility

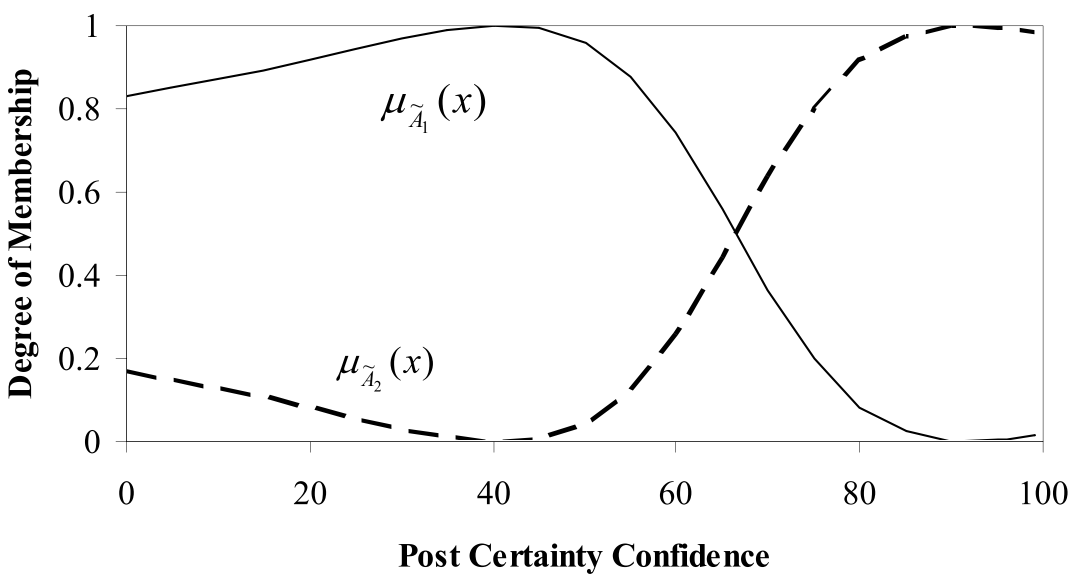

Figure 2: Membership Functions for Fuzzy Regression (c=2; m=2)

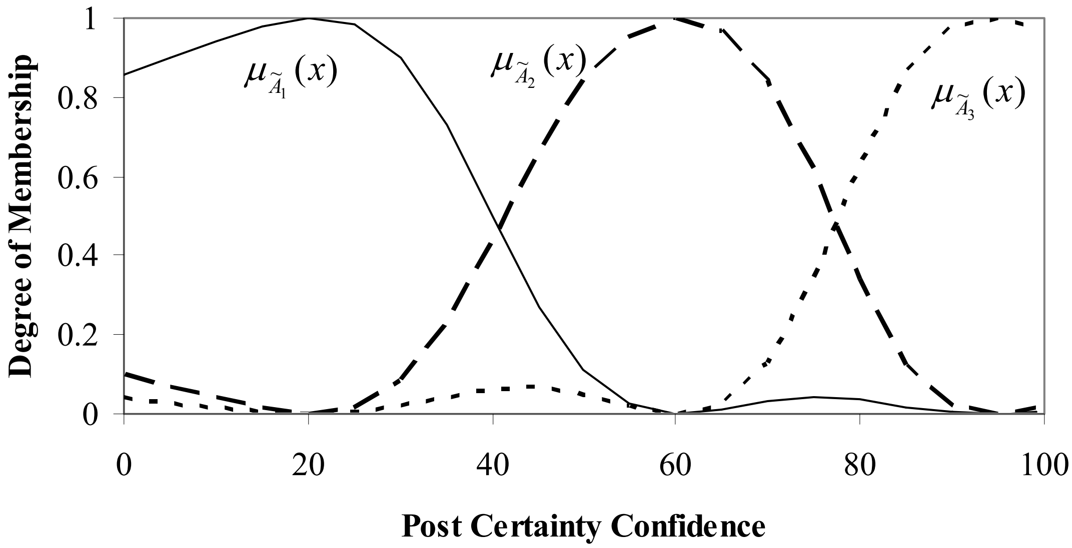

Figure 3: Membership Functions for Fuzzy Regression (c=3; m=2)

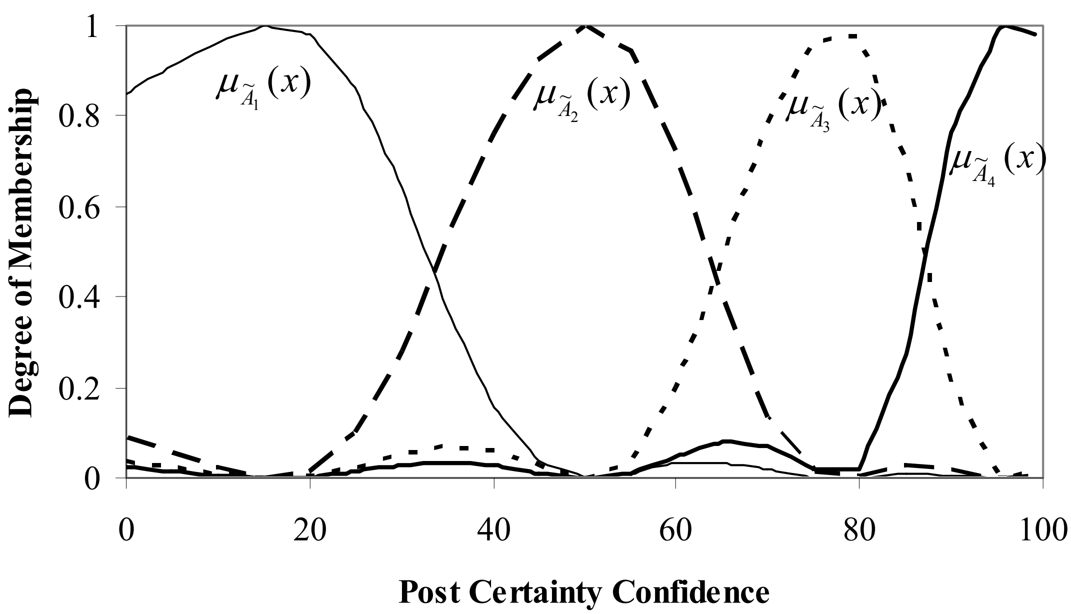

Figure 4: Membership Functions for Fuzzy Regression (c=4; m=2)

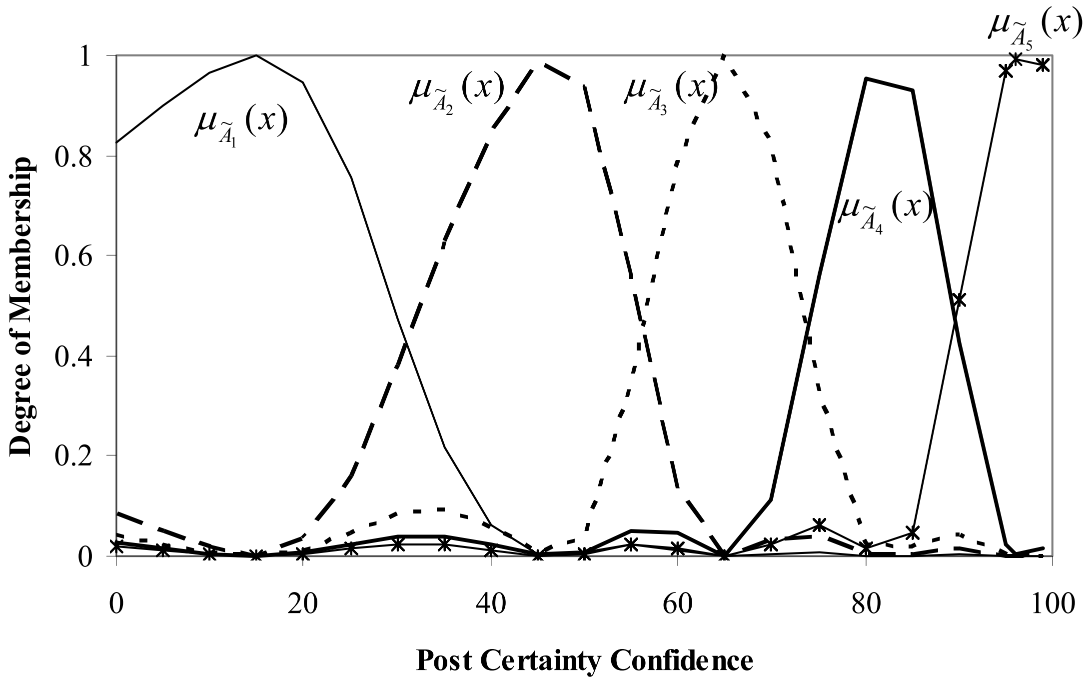

Figure 5: Membership Functions for Fuzzy Regression (c=5; m=2)