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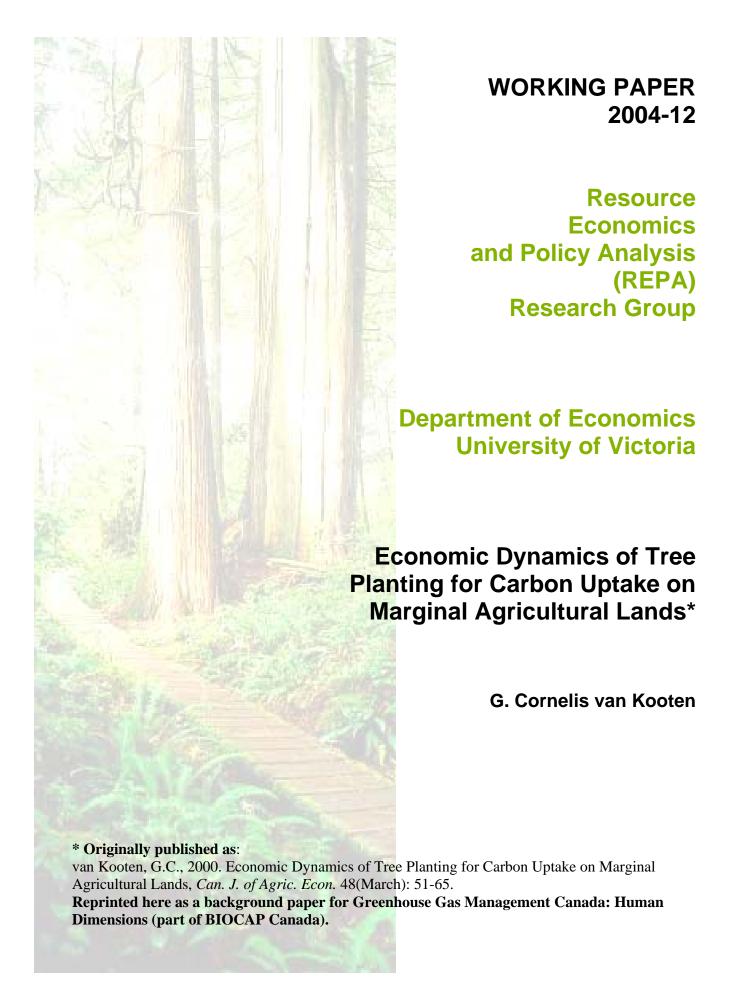
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# Economic Dynamics of Tree Planting for Carbon Uptake on Marginal Agricultural Lands

### **ABSTRACT**

As a result of the 1997 Kyoto Protocol, afforestation of agricultural lands can be expected to take on an important role in the CO<sub>2</sub>-emissions reduction policy arsenal of some countries. To date, identification of suitable (marginal) agricultural lands has been left mainly to foresters, but their criteria fail to take into account economic nuances. In this study, an optimal control model is used to determine the optimal level of afforestation in the western Canada. The results indicate that, while planting fast–growing trees for carbon uptake on marginal agricultural land may be important, the path dynamics matter in determining whether Canada can rely on afforestation to meet its obligations under Kyoto.

**Key words**: Afforestation and climate change; optimal control model of land use; economics of carbon sequestration

# Background

Concern about global climate change led to the United Nations' Framework Convention on Climate Change (FCCC) signed in Rio de Janeiro in June 1992. The Convention sought to stabilise atmospheric concentrations of carbon dioxide (CO<sub>2</sub>) by having developed countries reduce their CO<sub>2</sub> emissions to the 1990 level by 2000 (article 4). The Intergovernmental Panel on Climate Change (Houghton et al. 1996) created a sense of urgency that nations were not taking climate change seriously, especially since few nations would meet the Rio target. Thus, at a Conference of the Parties to the FCCC in December 1997 at Kyoto, Japan, developed countries further agreed to curtail their CO<sub>2</sub> emissions relative to the 1990 level. The EU committed to reduce emissions by 8%, the US by 7%, and Canada and Japan by 6%. Other developed countries agreed to other levels of CO<sub>2</sub>-emission controls. To date, the Protocol has not been ratified by many countries, most importantly the United States.

Forest policy is expected to play an important role in helping some countries meet their emission targets. Already in 1989, the Noordwijk Declaration proposed increasing global forest cover as a means of slowing climate change. The Kyoto Protocol allows countries to claim as a credit any carbon (C) sequestered as a result of afforestation (planting trees on agricultural land) and reforestation (planting trees on denuded forestland) since 1990, while C lost as a result of deforestation is a debit (article 3.3). The forest component of the Protocol has several interesting aspects, although each of these is under review as countries seek clarification on the Protocol's interpretation of terrestrial C sinks, especially forest sinks.

Deforestation is defined as a change in land use. When a site is harvested and subsequently regenerated, there is, according to some interpretations, no change in land use, so only the C uptake associated with reforestation is counted as a credit and not the debit associated with C release. For example, if a mature forest stand is harvested sometime after 1990 and subsequently replanted, only growth of the newly established stand is counted as a credit; the debit from harvest is not counted. Given that the actual commitment period is 2008–2012, only verifiable growth during this period (on stands planted since 1990) is counted as a credit, while only deforestation during 2008–2012 is counted as a debit. Finally, only the commercial (and measurable) component of the tree is counted, so changes in soil carbon, for example, might be ignored, although the Protocol leaves open the opportunity to include additional activities (article 3.4).

Forests store carbon by photosynthesis. For every tonne (t) of carbon sequestered in forest biomass, 3.667 t of CO<sub>2</sub> is removed from the atmosphere. In general, plantation forests are a cost–effective means of sequestering C (Sedjo et al. 1995; Adams et al.

1999). Hence, countries that have a large forest sector are interested in C credits related to reforestation, and those with large tracts of agricultural land are interested in afforestation as a means for achieving some of their agreed upon CO<sub>2</sub>–emissions reduction.

Countries are now unable to adopt large–scale afforestation programs before the millennium, and even reforestation of sites harvested since 1990 is unlikely to make much of an impact during the commitment period. For forests in Scandinavia, Russia, Canada and the US, the major producing countries, the increase in biomass over the first two decades after planting of indigenous commercial species is generally insignificant (Figure 1). In many instances, growth tables do not even begin until the third or fourth decade (see Thompson et al. 1992). Thus, any measure of C uptake by forests taken in the Protocol's accounting period 2008–2012 will be small, or biased upwards if mean annual increment (MAI) over the entire rotation is used as a proxy for actual growth. It would appear, therefore, that forest policies are important in the intermediate term, and not the short term of the Kyoto Protocol. High-yielding hybrid varieties might be an exception (Figure 1), but planting such species on a large scale could result in adverse environmental consequences associated with mono-cultures and may still not be in time to make much difference for the Protocol. In this study, the planting of high-yielding species is investigated further, but environmental externalities associated with such plantings are left to further research.

The purpose of this study is to examine the potential for planting hybrid poplar on marginal agricultural land in Canada as one method for achieving CO<sub>2</sub>–emissions reduction. In 1990, Canadian emissions of CO<sub>2</sub> amounted to 596 million tonnes (Mt) of

CO<sub>2</sub>—equivalent greenhouse gas emissions, or 162.5 Mt of C; in 1996 (the latest year for which data are available), emissions amounted to 669 Mt of CO<sub>2</sub>, or 182.4 Mt of C (Jacques 1998). Business as usual scenarios project annual emissions to remain stable to 2000, and then rise to 203.2 Mt C in 2010 and 225–230 Mt C in 2020 (see McIlveen 1998). To meet the Kyoto target, Canadian emissions must be 152.7 Mt C (560 Mt CO<sub>2</sub>), some 25% (or 50.5 Mt C) below the level projected for the commitment period. Canada expects a large part of its international commitment to reduce CO<sub>2</sub> emissions to come from forestry, with perhaps 25–40 percent of its Kyoto commitment coming via tree planting (Canadian Forest Service 1998; Nagle 1990).

The specific purpose here is to investigate the claims of foresters that afforestation of marginal lands in (mainly western) Canada can make a significant contribution to Canada's international commitments (e.g., Nagle 1990; Guy and Benowicz 1998). To do so, we employ a dynamic optimisation model that determines optimal levels of land conversion and, thus, the potential contribution that afforestation of marginal agricultural land can make to Canada's Kyoto commitment (assuming Canada takes the commitment seriously). The study area encompasses the Peace River region of British Columbia (BC) and all of Alberta.

# **Agricultural Values and Tree Planting Costs**

We investigate the potential of afforestation in Northeast BC and Alberta as a means for removing CO<sub>2</sub> from the atmosphere. Current agricultural land uses in the BC Peace River region and the eight Agricultural Reporting Areas (ARA) in Alberta are provided in Table 1. Improved land includes non-forage crops, forage crops, fallow, pasture and

other land, while unimproved land contains mainly pasture.

The agricultural land types considered suitable for afforestation are primarily those associated with forage production and pasture, although suitability depends on the value of lands in their current agricultural activity. Current use is assumed to be the best use and to continue indefinitely.<sup>3</sup> The land considered suitable for afforestation consists of forage (hay and alfalfa) and pasture (both improved and unimproved). We ignore lands in non-forage crops and fallow because of their high returns in agriculture (see We also ignore land in the "other" categories as there is insufficient information to enable a decision about their potential in tree plantations. In the northern areas of the study region (BC Peace region and Alberta ARAs 6 & 7), some unimproved pasture already has some tree cover. Without further information and to account for this, we eliminate from consideration for afforestation unimproved pasture in the BC Peace, and assume lower growth rates for trees on unimproved pasture in ARAs 6 & 7 (although tree-planting costs are not adjusted). Finally, ARAs 1 & 2 are characterised by irrigated forage production and considered too dry for planting trees. Therefore, they are also excluded from further analysis, although it may turn out that growing trees using irrigation may be an economically viable C uptake option. In total, it is estimated that some 7.033 million ha of agricultural land in the study region could be planted to hybrid poplar for the purpose of sequestering carbon.

For each agricultural activity and region it is necessary to have data on the net returns associated with the current agricultural activity (the opportunity cost of afforestation), the direct costs of afforestation, and the net change in C fluxes and sinks associated with the change from agriculture to forestry. Estimated net annual returns to

various agricultural activities and regions in Alberta and Northeast BC are provided in Table 2. For all regions, fast growing hybrid poplar is the only species considered for C sequestration purposes. Direct tree–planting costs for hybrid poplar are estimated to range from \$1270 ha<sup>-1</sup> to \$4000 ha<sup>-1</sup>.<sup>4</sup> Changes in carbon fluxes and sinks due to afforestation are considered in the next section.

# **Afforestation and Carbon Uptake**

Carbon is stored in trees (stem, branches, leaves and root), understory, forest litter and forest soils. Anticipating changes in the Kyoto provisions, we calculate storage of C in total tree biomass (bole, branches, leaves and roots) plus litter and soil. Calculation of the stream of C uptake over a specified time horizon requires estimates of tree growth (see Nagle 1990). For this, we employ the Chapman–Richards function:

(1) 
$$v(t) = \gamma (1 - e^{-kt})^m$$
,

where  $\gamma$  is maximum stem wood volume and k and m are parameters (Guy and Benowicz 1998). Volume is measured in cubic metres (m<sup>3</sup>).

Because of its rapid growth rates, hybrid poplar is the only viable choice for afforestation projects whose primary purpose is to sequester carbon. This is clear from Figure 1 where 40–year average growth for indigenous softwood and hardwood boreal species and hybrid poplar are provided. Growth is given by equation (1) using parameter values from Guy and Benowicz (1998). A problem with hybrid poplar is that many clones exist and "... quoted growth rates of hybrid poplar vary tremendously across

Canada and the northern USA making it difficult to estimate average values for each region" (Guy and Benowicz 1998, p.8). Available data on growth rates have been obtained under various management regimes, including fertilisation and irrigation. For varieties recommended for planting in western Canada, approximate values of the parameters in equation (1) are as follows: for the boreal region,  $\gamma$ =329 and k=0.156; for the prairie region,  $\gamma$ =270 and k=0.143; and m=3.0 for both zones (see Guy and Benowicz 1998). In the analysis below, we employ a finer parameter range to account for land quality and locational differences.

Let r be the social discount rate. For parameter values m=3.0,  $\gamma \in [270, 330]$ ,  $k \in [0.140, 0.160]$  and  $r \in [0.02, 0.08]$ , the financial rotation age for hybrid poplar is between 9 and 12 years.<sup>5</sup> If r is 2% or 4%, the rotation age is 11 years for parameters that yield the fastest rates of growth and 12 years for parameter values that yield somewhat lower rates of growth. Given that C uptake values will increase rotation ages slightly (see van Kooten, Binkley and Delcourt 1997; van Kooten, Thompson and Vertinsky 1993), a rotation age of 12 years is assumed for hybrid poplar.

Carbon flux needs to be calculated for six different accounts (see AACM International Pty Limited 1998). The most important account is likely the bole or merchantable component of the tree. Equation (1) provides the growth of volume for this component, which is translated into C by multiplying by 0.187 t C m<sup>-3</sup> (van Kooten, Thompson and Vertinsky 1993, pp.244–45). Carbon builds up in the bole until harvest time (12 years), when it is assumed to enter another account (e.g., wood products) or the atmosphere (by burning). A new stand of trees replaces the old, with the process assumed to continue indefinitely.

Next is above–ground biomass other than the bole; this consists mainly of branches and leaves. It is usually determined as a proportion of merchantable volume, with Guy and Benowicz (1998) employing a factor of 0.57. When trees are cut, all of the unused biomass is left on the site as slash. At that time, it enters the litter account (treated below). When a new stand of trees is planted, there is re–growth of the non–bole biomass. In this sense, the unused biomass is treated much like the bole.

Let  $\eta$  (=1.57) be an expansion factor that translates bole biomass into total aboveground biomass and  $\phi$  (=0.187) a factor that converts growth into carbon. The total discounted carbon per ha for the merchantable (M) plus related above-ground biomass (B) account can then be derived much like any other financial formula as:

(2) 
$$C_{M\&B} = \frac{\eta \phi \left( \int_{0}^{t} \dot{v}(s)e^{-rs}ds - v(t)e^{-rt} \right)}{1 - e^{-rt}},$$

where t=12 is the rotation age and  $\dot{v} = dv/ds$ . The first term in parentheses counts the (discounted) carbon that accumulates during the growing stage, while the second term measures the C released to another account at harvest time. Upon dividing by  $1-e^{-rt}$ , we obtain the sum of the infinite series of "returns" that accrue every t years, beginning after t years, or the end of the first rotation (see van Kooten, Binkley and Delcourt 1997).

Third, carbon in the root pool is calculated from the following relationship between root biomass (U) and above–ground biomass (G=M+B):

(3) 
$$U(G) = 1.4319 G^{0.639}$$
,

where U and G are both measured in  $m^3$  per ha (see Guy and Benowicz 1998). A one–time growth in roots is assumed, after which decay causes C to enter the soil pool at a rate exactly offset by the rate at which new growth adds to the root pool. Total discounted C per ha for the root account is given as:

(4) 
$$C_{R} = \phi \int_{0}^{t} \dot{U}(G) e^{-rs} ds.$$

Fourth, there is a change in soil C when agricultural land is converted to plantation forests. Data on soil C are difficult to obtain. Field trials in the northern Great Plains of the US indicate that sites with hybrid poplar have an average of 191 tonnes of C per ha in the top 1 metre of soil, row crops an average of 179 t of soil C, and grass that is regularly cut 157 t per ha (Hansen 1993, p.435). Guy and Benowicz (1998) note that forest soils in the study region store some 108 tonnes of C per ha compared to cropland that stores some 60 t. Soil C rebuilds only slowly when cultivation stops. Using Guy and Benowicz's data and assuming that 2% of the difference is sequestered each year when land is converted from agriculture to forestry, 48 t ha<sup>-1</sup> is added to soil over a 50-year period. It is assumed that annual build-up of soil C is constant and equal to 0.96 t ha<sup>-1</sup> for a period of 50 years, after which the soil is assumed to be in equilibrium (additions to soil C from roots and litter decay equals release to the atmosphere). It is difficult to determine soil C for different agricultural activities, and Hansen (1993) even finds row crops store more C than grassland that is regularly cut. However, marginal land eligible for tree planting is used only for grazing or growing forage crops. Therefore, it is

assumed that there is no difference in the C sink potential of different agricultural lands and agricultural activities. Total discounted C per ha in the soil (S) account is thus calculated as:

(5) 
$$C_{S} = c_{s} \left( \frac{1 - e^{-50r}}{r} \right),$$

where  $c_s$  (=0.96 t) is annual addition of C to the soil sink and the term in parentheses discounts an annual flow for a 50-year period to the present.

Fifth, the litter pool consists of dead or dying biomass on the forest floor that releases C to the atmosphere through fire and decay and to the soil pool. It is a relatively small pool of C that changes rapidly (AACM International Pty Limited 1998). It is assumed that the litter account grows by a constant amount each year for 50 years, after which it is in equilibrium. At that point it is assumed that the litter pool is one—half the non—bole biomass. In addition, there is a spike in the pool's biomass at harvest time. It is assumed that the slash component of the litter releases a constant amount of C into the atmosphere over the next 12 years (linear decay) so that it is depleted by the time of next harvest. This carbon spike and subsequent decay is important because physical C is discounted—it matters when C is removed from the atmosphere. Using normal financial formulae, the total discounted carbon per ha accruing to the litter account (C<sub>L</sub>) is calculated as:

(6) 
$$C_{L} = (\eta - 1)\phi \left[ c_{l} \left( \frac{1 - e^{-50r}}{r} \right) + \frac{\int_{0}^{t} \dot{v}(s)e^{-rs} ds - v(t)e^{-rt}}{1 - e^{-rt}} \right],$$

where  $c_1 = \frac{v(t)}{2t}$  is the constant annual addition to the litter pool. The first term constitutes the current "value" of the 50-year litter pool, while the second term is the discounted sum of the infinite deposit and subsequent decay of litter beginning with the current period and continuing every t (=12) years.

Finally, it is important to consider what happens to the bole (or commercial component of the tree). Two alternatives are considered for harvested timber: burning wood in place of an energy–equivalent amount of coal (thus saving CO<sub>2</sub> emissions from coal) or storing C in wood products. The latter alternative delivers the most C "removal" per dollar of costs, and is used here. It is assumed, however, that 20% of the bole is waste and burned, with 3.78 m³ wood substituting for 1 tonne coal, saving 0.707 t of C emissions and returning \$7.50 per m³ in revenue (van Kooten et al. 1999). The remaining 80% of the bole goes into paper products (3/4) and wood products (1/4), such as furniture, lumber, posts and OSB (Winjum, Brown and Schlamadinger 1998).

To obtain carbon fluxes for wood products, assume that proportion  $\rho$  ( $0 \le \rho \le 1$ ) of the C gets stored in products that decay (release C) at a rate  $\delta$  ( $0 \le \delta \le 1$ ) per year. Then, the total discounted C per ha stored in wood products at time of harvest plus the discounted emission savings resulting from the substitution of wood for coal in energy production at time of harvest can be calculated as:

(7) 
$$C_{W} = \phi v(t) \left[ \rho \left( \frac{r}{r+\delta} \right) + (1-\rho) \right] \left( \frac{e^{-rt}}{1-e^{-rt}} \right),$$

where C<sub>W</sub> refers to the discounted C uptake resulting from use of commercial timber. Each time wood is harvested, a proportion  $\rho$  of the C in the bole is stored immediately in wood products, but every year thereafter a proportion  $\delta$  is released. The first term in the square brackets in (7) gives the infinite sum of the total discounted C stored in wood products at each harvest (recall that harvests begin only at the end of the first rotation); the second term in brackets represents C saved by burning wood in place of coal (van Kooten et al. 1999). The final term in (7) is a factor that sums the "values" that accrue every t years over the infinite time horizon (see van Kooten, Binkley and Delcourt 1995). Skog and Nicholson (1998) argue that paper products have a half-life of one to six years, while lumber in housing has a half-life of 80 to 100 years. Winjum, Brown and Schlamadinger (1998), on the other hand, point out that oxidation rates are 0.02 per year for industrial roundwood products and 0.005 for paper products that end up in landfills. We assume that two-thirds of the paper products end up in landfills, releasing C at a very low rate, while the remainder releases C at a rate of 0.5; for other wood products, we assume a rate of decay of 0.02. The blended rate of decay, with 75% of wood going to paper and 25% to lumber and other building products, is 0.131. Thus,  $\rho$ =0.8 (since 20%) is waste) and  $\delta$ =0.131.

Discounted carbon uptake for selected values of the growth parameters are provided in Table 3 for the various accounts and a discount rate of 4%. Total discounted C uptake varies from 85.4 t per ha to 111.4 t per ha, while annualised C uptake varies from 3.4 to

4.5 t per ha (Table 3). Annualised values are provided so that we can compare C fluxes in some accounts that continue every year into the future with those that attain equilibrium at some future date. Annual values are used to construct the functions needed to determine the optimal level of afforestation. These values will vary by region and land quality, as determined by current land use (see Table 4), which causes average C fluxes to differ.

# **Economically Optimal Level of Afforestation**

In this section, we employ a dynamic optimisation model to provide an indication of the optimal amount of agricultural land to plant to trees for the purpose of removing carbon from the atmosphere. The model is not as detailed as the Forest and Agricultural Sector Optimisation Model (FASOM) employed by Adams et al. (1999) to determine the minimum costs of meeting various carbon uptake targets through afforestation in the US. The purpose of the optimal control model used here is to determine the optimal amount of marginal agricultural land in a particular region of Canada to afforest, and to examine the path dynamics, which will affect Canada's ability to rely on the tree planting option in contributing to the Kyoto targets (or any future targets Canada might agree to in the future).

## **Dynamic Optimisation Model**

The objective is to maximise the discounted flow of present and all future net benefits, including benefits of carbon uptake. The objective function can be written as follows:

(8) 
$$\max W = \int_{0}^{\infty} \pi(t) e^{-rt} dt,$$

where

(9) 
$$\pi = \int_{0}^{A(t)} B'(s) ds + \int_{0}^{A_0 - A(t)} [P F(z) + p_c C(z)] dz - \tau(R) R(t).$$

Here  $\pi(t)$  is economic benefits;  $A_0$  represents the initial stock of (marginal) agricultural land available for afforestation (7.03 million ha for the study area) and A(t) the land in agriculture at any time, so that  $A_0$ –A is land converted from agriculture to plantation forest for the purpose of sequestering C; R(t) is the agricultural area afforested at time t; B'(A) are the marginal benefits of agricultural production, which decline as more of the available agricultural land is retained in agriculture rather than converted to forest, B''(A) < 0, indicating that the poorest agricultural land is afforested first; P is the stumpage value of timber;  $p_c$  is the shadow price of carbon;  $sF(z) + p_cC(z)$  are the marginal benefits of afforestation; and r is the social rate of discount. The term  $\int_{0}^{A_0-A} [PF(z) + p_cC(z)]dz$ describes the total benefits for the  $A_0$ -A hectares of farmland that is afforested. Marginal benefits of tree planting equal the sum of the marginal commercial timber benefits, PF(z), and the shadow value of the marginal C uptake benefits,  $p_cC(z)$ . Recognising that  $z=A_0-$ A, F'(z) < 0 and C'(z) < 0. The function  $\tau(R)$  represents the cost of planting a hectare of farmland to trees, which increases as one attempts to plant more area in a given year.

The required functions are discussed further below.

The dynamic (subject to) constraint is

$$(10) \qquad \dot{A}(t) = -R(t),$$

where the dot over a variable indicates a time derivative. The focus is on conversion of agricultural land into plantation forest, because cost of converting land from forest to agriculture is ignored (see van Kooten and Folmer 1997).

Maximisation takes place subject to the equation of motion (10). The current value Hamiltonian (suppressing time notation) is defined as:  $H = \pi - \lambda R$ , where  $\lambda$  is the costate variable. Assuming an interior solution, the necessary conditions for an optimum solution are:

(11) 
$$\frac{\partial H}{\partial R} = 0 \Rightarrow \lambda = -\tau'(R) R - \tau(R)$$

(12) 
$$\dot{\lambda} = r\lambda - \frac{\partial H}{\partial A} \Rightarrow \dot{\lambda} = r\lambda - [B'(A) - P F(A_0 - A) - p_c C(A_0 - A)].$$

The interpretation of (11) is that the rate of conversion of agricultural land to forest should be chosen so that the discounted marginal net benefit from current conversion,  $\lambda$ , equals the marginal benefit (marginal costs avoided) of delaying conversion. The discounted marginal benefits of current conversion take into account the opportunity cost of lost agricultural production, while  $\tau$  could be constant. Equation (12) provides a

standard intertemporal arbitrage condition (see Clark 1990).

The steady state occurs when the co–state multiplier and the area retained in agricultural production are constant ( $\dot{\lambda} = \dot{A} = 0$ ) so no further afforestation takes place (R=0). The equation that describes the optimal amount of land to keep in agriculture in the steady state is:

(13) 
$$\frac{PF(A_0 - A^*) + p_c C(A_0 - A^*)}{r} - \tau(0) = \frac{B'(A^*)}{r}.$$

Equation (13) says that, in equilibrium, the present value of the benefits of afforestation minus planting costs must equal the discounted stream of benefits of keeping land in agricultural production at the margin. Included in benefits are the shadow costs and benefits of C uptake and release. The difficulty in solving (13) lies with determining the four functions  $F(A_0-A)$ ,  $C(A_0-A)$ ,  $\tau(R)$ , and B'(A).

#### **Parameter Values for the Model**

An exponential functional form is assumed for  $F(A_0-A)$ ,  $C(A_0-A)$  and B'(A), namely,

(14) 
$$f(x) = \alpha_i e^{\beta_i x}, (x = A_0 - A, A; i = F, C, B),$$

while a linear functional form is employed for the marginal planting cost function,

(15) 
$$\tau(R) = \alpha_{\tau} + \beta_{\tau} R.$$

For function (14), parameter values can be determined by calculation if f(0) and one other point on the function are known; for  $\tau(R)$ , parameter values can be calculated if any two points on the function are known. As already noted, estimates of planting costs for hybrid poplar vary from \$1270 to \$4000 per ha. It is assumed that  $\alpha_r = \tau(0) = 1200$  and that costs rise at a rate of \$0.005 per ha so that the 360,000<sup>th</sup> ha planted in a given year costs \$4000 to plant. Since R is measured in millions, however,  $\beta_r = 5000$ .

Land with the lowest agricultural value is planted to trees first, followed by increasingly valuable land. The parameters for B'(A) are found by assuming that marginal land in agriculture has an annual net return of \$10 per ha when all land is in agriculture, and \$350 per ha when it has all been afforested. These approximate the high and low values in Table 2.

When calculating  $F(A_0-A)$  and  $C(A_0-A)$ , it is also necessary to assume that agricultural land with the lowest values is afforested first. Timber growth varies by region and this is reflected in the parameter values of equation (1). For unimproved pasture in the BC Peace region, no growth is assumed possible, while low values of the growth parameters (k=0.140,  $\gamma$ =270) are used for unimproved pasture in northern Alberta (ARAs 6 & 7) to account for extant tree growth. These agricultural areas also correspond to the (nearly) lowest returns to agricultural activities (Table 2). For other regions, parameter values are chosen according to land quality, as measured by agricultural returns (Table 2), and location (boreal or prairie zone). Values for the growth parameters for the regions in our study area are provided in Table 4. From these, it is possible to calculate timber growth and associated (annualised) C uptake (using the relations in the previous section, but then

in annual or discrete terms). For the C uptake function, separate calculations are required for different assumptions about the rate used to discount physical carbon.

Parameter values for each of the functions in equations (14) and (15) are found in Table 5. Sensitivity analysis is used to determine the impacts of the various parameter values on optimal levels of afforestation. Areas A and  $A_0$  are measured in millions of hectares. Finally, it is necessary to multiply commercial timber by the annualised stumpage value (P), which depends on the discount rate. Van Kooten et al. (1999) employ harvest plus hauling costs that average \$22.50 per m³. Revenues amount to \$7.50 per m³ for waste wood that is burned and \$30.00 per m³ for timber used in wood products. Waste wood is burned despite costs exceeding revenues to enhance C uptake. Given that burning accounts for 20% of timber and wood products for 80%, the average stumpage value is \$3 per m³. Earnings are realised every t years, so annualised returns are \$ $\frac{3re^{-rt}}{1-e^{-rt}}$  per m³. The values of P are also given in Table 5.

## **Empirical Results**

The optimal steady–steady solution is found by solving (13). The results are provided in Table 6. These indicate that, for a shadow price of C not exceeding \$20 per tonne (a reasonable assumption), no more than about 50% of available marginal agricultural land should be planted to trees to meet Canada's Kyoto target. At shadow prices for C of \$50 per tonne or more, about three–quarters of marginal agricultural land can be afforested. To determine the sensitivity of the results to the assumptions, the marginal benefits of land in agricultural activities were increased (both the slope and intercept terms), and the returns to forestry were reduced (from an annualised \$0.1948 per m³ to \$0.15 per m³).

When the marginal benefit function for land in agricultural activities has a lower slope or larger intercept (so land in agriculture is slightly more valuable at the margin), the optimal amount of agricultural land to convert to forests declines by 0.5-1.5 million ha (for lower shadow prices of C). At a shadow price of C of \$20 per tonne, a decline in timber revenue of 1% results in a 0.27% decline in the optimal area to be afforested. Not surprisingly, the results are most sensitive to the value of  $\tau(0)$ , the marginal value of tree planting costs when R=0. If costs of planting hybrid poplar are significantly higher than assumed here (in Table 6 they are increased from \$1,200 ha<sup>-1</sup> to \$2,000 ha<sup>-1</sup>), it is possible that no more than one–quarter of available marginal agricultural land should be planted to trees for C uptake purposes. Indeed, if planting costs are \$2,950 per ha or more in the model, regardless of the type of agricultural land, no agricultural areas should be afforested.

We investigate the role of planting costs in greater detail by examining the dynamic approach path. Taking the time derivative of (11) gives:

(16) 
$$\dot{\lambda} = -\dot{R} \left[ \tau''(R) R + 2 \tau'(R) \right].$$

Substituting (11) and (16) into (12) and solving for  $\dot{R}$  gives:

(17) 
$$\dot{R} = \frac{r[\tau'(R)R + \tau(R)] + B'(A) - sF(A_0 - A) - p_c C(A_0 - A)}{\tau''(R)R + 2\tau'(R)}.$$

Assuming a 4% discount rate for both monetary values and physical carbon, and with  $p_c$ =\$20 per tonne, equations (10) and (17) can be used to construct the phase plane

diagram shown in Figure 2.

The optimal approach path could not be determined numerically for this autonomous, infinite horizon problem (see Conrad and Clark 1987). Nonetheless, we are able to shed some light on the problem using the phase–plane diagram. The optimal solution is necessarily a saddle point equilibrium (Leonard and Van Long 1992, pp.289–99). The  $\dot{R}=0$  isocline intersects the vertical line  $A_0$  at 350,000 ha, which corresponds to the maximum area that can be planted in one year without social benefits becoming negative. Along the optimal approach path, shown by the dotted line (separatrix), annual plantings cannot exceed some 200,000 ha. Even if 200,000 ha are planted annually, it will take some 18 years to achieve the optimal level of afforestation (3.5 million ha). However, plantings along the optimal path decline each year, so it is more likely an average of less than 100,000 ha per year would be planted along the optimal path, in which case it could take more than 35 years to achieve the optimal level of afforestation. Further, any other approach path will result in higher, probably unacceptable, carbon uptake costs.

### **Discussion**

Foresters are generally optimistic about Canada's ability to meet a significant proportion of its carbon uptake commitments by planting hybrid poplar on marginal agricultural land. This is partially confirmed by the results of this study, which show that, for a shadow price of C of \$20 per tonne, it may be optimal to afforest as much as 50% of identifiable marginal agricultural land. In that case, some 12.3 Mt of carbon will be sequestered per year in the study area, or nearly one–quarter of Canada's Kyoto commitment. If this result can be extended to marginal agricultural land in the rest of

Canada, then some 50–60% of Canada's Kyoto commitment could be attained through forestry policies. Of course, this is a most optimistic scenario. Under different assumptions, the optimal steady state level of afforestation would be lower. Even if it were half as much, afforestation remains an important, if not the most important, policy instrument available to Canada.

A different picture emerges if the path dynamics leading to the steady state are taken into account. In order to keep costs of C uptake at a reasonable (acceptable) level, one cannot afforest large areas of agricultural land all at once. Indeed, based only on rising planting costs, the optimal rate at which marginal agricultural land should be afforested is rather low—some 200,000 ha or less per year early on, but declining over time. If a planting program were implemented in 2000, then it is likely than not much more than one million ha of marginal agricultural land will be afforested by Kyoto's commitment period, if the optimal dynamic path is followed. In that case, afforestation in the study region would contribute only some 7–8% of the needed emissions reduction. Even so, if applicable to the rest of Canada, afforestation could account for slightly more than 15% of Canada's international commitments.

Several factors have been ignored in this study. First, there may be environmental costs to planting hybrid poplar on a large scale. These might include a reduction of wildlife habitat, particularly on non-cultivated agricultural lands, and loss of scenic amenities. These costs are not taken into account, but would increase the costs of carbon uptake.

Second is the problem of establishing proper incentives for landowners to grow hybrid poplar. Outright purchase of agricultural land will be infeasible because of budget limitations, while financial incentives (planting plus annual subsidies) may be difficult to implement as this will require drawing up contracts between landowners and the government agency responsible for the program. Contracting is not costless, and strategic behaviour by landowners could result in much higher costs than anticipated, as well as delays. However, the problem of contracting in such cases is rarely discussed and much less investigated.

Third, costs of monitoring growth and C uptake will be costly, and there do not now exist institutions in BC and Alberta (where public ownership of forestland exceeds 90%) that monitor growth and yield.

Finally, there is a great deal of uncertainty associated with planting of hybrid poplar on a large scale because this has not been done previously. In addition, there is uncertainty about the (current and future) prices of timber products (including what wood fetches as fuel) and agricultural output. One aspect is the problem of leakages (Sohngen and Sedjo 1999). Large–scale afforestation programs are bound to lower wood fibre prices, with current woodlot owners reducing their forest holdings (converting land back to agriculture) in anticipation.

Resolving each of these issues constitutes several research tasks.

#### **Notes**

- 1. For indigenous species, an exception may be the US South.
- 2. Other researchers have addressed the afforestation issue using a variety of modelling techniques. Adams et al. (1999) use a large, multi-period nonlinear program to investigate C uptake costs through afforestation in the US. Given that Canada is the largest wood products exporter in the world, their model would have been more realistic had it included Canada (or its regions). Plantinga, Mauldin and Miller (1999), on the other hand, employ an econometric approach to derive cost estimates for C uptake from afforestation programs for Maine, S. Carolina and Wisconsin.
- 3. Climate change may affect agricultural returns (viz., CO<sub>2</sub>-fertilisation affects), but it

- would also impact the ability to grow trees in these areas. Since this is a mitigation study, these aspects are not considered.
- 4. The BC Ministry of Agriculture, Fisheries and Food (1996) reports an establishment cost of \$1,270 per ha. Later studies place establishment costs of conventional species in BC at \$1,500 ha<sup>-1</sup> and hybrid poplar at \$4,000 ha<sup>-1</sup> (for a 12–year rotation), while US estimates for planting hybrid poplar are C\$1,050–1,250 ha<sup>-1</sup> (van Kooten et al. 1999). We choose a value that begins at \$1,200 ha<sup>-1</sup> and rises by amount planted (see below).
- 5. The rotation age (t) is found by solving:  $\frac{v'(t)}{v(t)} = \frac{mke^{-kt}}{1 e^{-kt}} = \frac{r}{1 e^{-rt}}.$

### References

- AACM International Pty Limited, 1998. Greenhouse Challenge Sinks Workbook. Quantifying the Greenhouse Benefits of Vegetation Management. Adelaide, Australia. Mimeograph. 58pp. Plus Appendices.
- Adams, D.M. R.J. Alig, B.A. McCarl, J.M. Callaway and S.M. Winnett, 1999. Minimum Cost Strategies for Sequestering Carbon in Forests, *Land Economics* 75(August): 360–74.
- British Columbia Ministry of Agriculture, Fisheries and Food, 1996. Hybrid Poplar Fraser Valley, Planning for Profit Enterprise Budget. Agdex #382–810. Abbotsford: Government of BC.
- Canadian Forest Service, 1998. Forest Sector Table Foundation Paper. Ottawa: National Climate Change Process. Mimeograph, September 28. 68pp.
- Clark, C.W., 1990. *Mathematical Bioeconomics*. 2<sup>nd</sup> editon. New York: John Wiley & Sons.
- Conrad, J.M. and C.W. Clark, 1987. *Natural Resource Economics. Notes and Problems*. Cambridge: Cambridge University Press.
- Guy, R.D. and A. Benowicz, 1998. Can Afforestation Contribute to a Reduction in Canada's Net CO<sub>2</sub> Emissions? Report prepared for the CPPA. Department of Forest Sciences, UBC. Mimeograph. March 25. 21pp.
- Hansen, E.A., 1993. Soil Carbon Sequestration beneath Hybrid Poplar Plantation in the North Central United States, *Boimass and Bioenergy* 5(6): 431–36.
- Houghton, J.T., Meira Filho, L.G., Callander, B.A., Harris, N., Kattenberg, A. and Maskell, K. (editors), 1996. *Climate Change 1995. The Science of Climate Change*. Cambridge, UK: Cambridge University Press.
- Jacques, A., 1998. Revised 1990 & 1996 Greenhouse Gas Emissions Estimates. Ottawa: Pollution Data Branch, Environment Canada. December 3.
- Leonard, D. and N. Van Long, 1992. *Optimal Control Theory and Static Optimization in Economics*. Cambridge, UK: Cambridge University Press.
- McIlveen, N., 1998. The Analysis and Modelling Group. The Emissions Outlook. Forest Sector Emissions Trend. Paper presented at the Forest Sector Table meeting, November 6, Montreal. Mimeograph.
- Nagle, G.S., 1990, Trees for Canada Program. Technical Background Paper, Prepared by Nawitka Renewable Resource Consultants Ltd., Victoria BC, for Forestry Canada.
- Plantinga, A.J., T. Mauldin and D.J. Miller, 1999. An Econometric Analysis of the Costs of Sequestering Carbon in Forests, *American Journal of Agricultural Economics* 81(November): 812–24.
- Sedjo, R.A., J. Wisniewski, A.V. Sample and J.D. Kinsman, 1995. The Economics of Managing Carbon via Forestry: Assessment of Existing Studies, *Environmental and Resource Economics* 6: 139–65.
- Skog, K.E. and G.H. Nicholson, 1998. Carbon Cycling through Wood Products: The Role of Wood and Paper Products in Carbon Sequestration, *Forest Products Journal* 48: 75–83.
- Sohngen, B. and R. Sedjo, 1999. Estimating Potential Leakage from Regional Forest Carbon Sequestration Programs. RFF Working Paper. Washington, DC. August. 26 pp.

- Statistics Canada, 1997a. *Agricultural Profile of Alberta*. Ottawa, ON: Agricultural Division, Government of Canada.
- Statistics Canada, 1997b. *Agricultural Profile of British Columbia*. Ottawa, ON: Agricultural Division, Government of Canada.
- Thompson, W.A., P.H. Pearse, G.C. van Kooten and I. Vertinsky, 1992. Rehabilitating the Backlog of Unstocked Forest Lands in British Columbia: A Preliminary Simulation Analysis of Alternative Strategies. Chapter 4 in *Emerging Issues in Forest Policy* (pp.99–130) edited by P.N. Nemetz. Vancouver: UBC Press.
- van Kooten, G.C. and H. Folmer, 1997. Economic Instruments and the Pasture–Crop–Forest Interface. Chapter 9 in *Climate–change Mitigation and European Land–use Policies* (pp.113–28) edited by W.N. Adger, D. Pettenella and M. Whitby. Wallingford, UK: CAB International.
- van Kooten, G.C., C.S. Binkley and G. Delcourt, 1995. Effect of Carbon Taxes and Subsidies on Optimal Forest Rotation Age and Supply of Carbon Services, *American Journal of Agricultural Economics* 77(May): 365–774.
- van Kooten, G.C., W.A. Thompson and I. Vertinsky, 1993. Economics of Reforestation in British Columbia When Benefits of CO<sub>2</sub> Reduction are Taken into Account. Chapter 12 in *Forestry and the Environment: Economic Perspectives* (pp.227–47) edited by W.L. Adamowicz, W. White and W.E. Phillips. Wallingford, UK: CAB International.
- van Kooten, G.C., E. Krcmar–Nozic, B. Stennes and R. van Gorkom, 1999. Economics of Fossil Fuel Substitution and Wood Product Sinks when Trees are Planted to Sequester Carbon on Agricultural Lands in Western Canada. *Canadian Journal of Forest Research* In press.
- Winjum, J.K., S. Brown and B. Schlamadinger, 1998. Forest Harvests and Wood Products: Sources and Sinks of Atmospheric Carbon Dioxide, *Forest Science* 44(2): 272-284.

Table 1: Farmland Area Classified by Land Use (ha)

	Improved land					Unimproved land			
Region	Non-	Forage	Fallow	Pasture	Other	Pasture	Other		
	forage								
	crops								
<b>BC Peace</b>	137,585	119,584	29,608	96,991	8,372	282,545	150,693		
Alberta Agricul	Alberta Agricultural Reporting								
Areas									
1(Southeast)	758,862	111,072	409,004	218,121	36,764	2,090,655	36,764		
2 (S central)	1,544,105	135,252	415,483	178,540	32,640	903,954	32,640		
3 (Southwest)	857,419	216,449	83,443	194,053	77,602	1,039,605	129,337		
4a (E central)	821,625	115,872	127,406	180,642	18,571	498,009	92,857		
4b (E central)	1,055,335	128,412	110,745	186,410	19,614	338,949	117,684		
5 (Central)	800,479	435,667	46,080	360,777	47,979	557,366	167,927		
6 (Northeast)	591,720	446,670	76,622	351,051	24,372	685,566	268,096		
7 (Northwest)	1,193,462	334,144	167,958	245,009	28,473	501,393	370,153		

Source: Statistics Canada (1997a, 1997b)

**Table 2: Net Annual Returns to Current Agricultural Activities (\$ per ha)** 

Region	Forage <sup>a</sup>	Improved Pasture	Unimproved	
			Pasture	
BC Peace	184.98	34.45	n.a.	
Alberta, ARA				
1(Southeast)	185.75 <sup>b</sup>	17.51	8.75	
2 (South central)	$304.04^{b}$	23.64	11.82	
3 (Southwest)	310.20	35.82	17.33	
4a (East Central)	101.47	24.84	12.42	
4b (East Central)	116.80	28.35	14.02	
5 (Mid Central)	260.56	46.93	20.26	
6 (Northeast)	168.63	58.01	21.04	
7 (Northwest)	178.75	34.45	15.15	

<sup>&</sup>lt;sup>a</sup> Forage is based on the net returns for hay and alfalfa, weighted by the production of each within the region.

Source: van Kooten et al. (1999)

<sup>&</sup>lt;sup>b</sup> ARAs 1 & 2 have irrigated forage production, are too dry for planting trees and are excluded from further analysis.

Table 3: Discounted Carbon per ha in Various Accounts, 4% Discount Rate<sup>a</sup>

_	Parameters for growth function, $m=3.0$					
Item	$\gamma=270$	$\gamma = 270$	$\gamma = 330$	γ=330		
	k=0.140	k=0.160	k=0.140	k=0.160		
Above ground biomass	13.32	16.37	16.28	20.01		
Roots	15.25	16.93	17.34	19.25		
Soil	20.62	20.62	20.62	20.62		
Litter	18.71	21.96	22.87	26.83		
Wood products and coal saving	17.52	20.21	21.41	24.70		
TOTAL	85.42	96.10	98.51	111.42		
Annualised Carbon (t C ha <sup>-1</sup> yr <sup>-1</sup> )	3.417	3.844	3.941	4.457		

<sup>&</sup>lt;sup>a</sup> Calculated from equations (1) through (7) for various growth parameters.

**Table 4: Parameter Values for Hybrid Poplar Growth Functions** 

	- J	1		
Region	egion Forage		Unimproved Pasture	
BC Peace	$\gamma$ =330, $k$ =0.16	$\gamma$ =330, $k$ =0.14	n.a.	
Alberta, ARA				
3 (Southwest)	$\gamma$ =300, $k$ =0.16	$\gamma$ =330, $k$ =0.14	$\gamma$ =270, $k$ =0.15	
4a (East Central)	$\gamma$ =300, $k$ =0.16	$\gamma$ =300, $k$ =0.14	$\gamma$ =270, $k$ =0.15	
4b (East Central)	$\gamma$ =300, $k$ =0.16	$\gamma$ =300, $k$ =0.14	$\gamma$ =270, $k$ =0.15	
5 (Mid Central)	$\gamma$ =300, $k$ =0.16	$\gamma$ =300, $k$ =0.16	$\gamma$ =270, $k$ =0.15	
6 (Northeast)	$\gamma$ =330, $k$ =0.15	$\gamma$ =300, $k$ =0.16	$\gamma$ =270, $k$ =0.14	
7 (Northwest)	$\gamma$ =330, $k$ =0.16	$\gamma$ =330, $k$ =0.14	$\gamma$ =270, $k$ =0.14	

**Table 5: Parameter Values for Functions (11.14) and (11.15)** 

Tuble of Tulumeter vu		unctions (11	11) una (11110)	
Function/Parameter	$\alpha_i^{\ b}$	$eta_i^{\ b}$	Values for calculating	
			parameters <sup>c</sup>	
B'(A)	350	-0.5055	(\$350 ha <sup>-1</sup> , \$10 ha <sup>-1</sup> )	
$\tau(R)$	1200	5000	see text	
$C(A_0 - A)$ at 2%	3.0	-0.0378	$(2.3 \text{ t ha}^{-1}, 3.0 \text{ t ha}^{-1})$	
$C(A_0 - A)$ at 4%	4.4	-0.0498	$(3.1 \text{ t ha}^{-1}, 4.4 \text{ t ha}^{-1})$	
$C(A_0-A)$ at 6%	5.3	-0.0400	$(4.0 \text{ t ha}^{-1}, 5.3 \text{ t ha}^{-1})$	
$F(A_0-A)$	205	-0.0492	$(205 \text{ m}^3, 145 \text{ m}^3)$	
Stumpage (\$ per m <sup>3</sup> ) <sup>a</sup>				
P at 2%	0.2	2212		
P at 4%	0.	1948		
P at 6%	0.	1707		

Table 6: Optimal Proportion of Total Available Marginal Agricultural Land in BC and Alberta to Plant to Trees for Carbon Uptake, Sensitivity Analysis

	Disco	ount Rate	(Base	_	-		Lower
Price of C	Para	meter Val	lues)	For $\tau(R)$	For B'(A)	For B'(A)	Stumpage
(\$ per t)	2%	4%	6%	$\alpha_{\tau}'=2000$	$\beta_{\tau}'=0.40$	$\alpha_B'=500$	P'=0.15
10	0.41	0.29	0.09	0	0.15	0.21	0.21
20	0.53	0.50	0.46	0.34	0.39	0.42	0.47
50	0.73	0.76	0.79	0.71	0.71	0.64	0.75
100	0.90	0.95	1.00	0.92	0.94	0.86	0.95

a Annualised values
b i=F, C, B,  $\tau$ c Calculated using data in Tables 2 and 4.

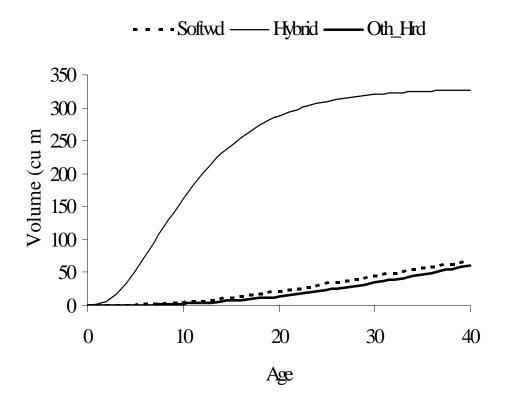


Figure 1: 40–Year Growth for Indigenous Softwoods and Hardwoods, and Hybrid Poplar, Boreal Region, Western Canada

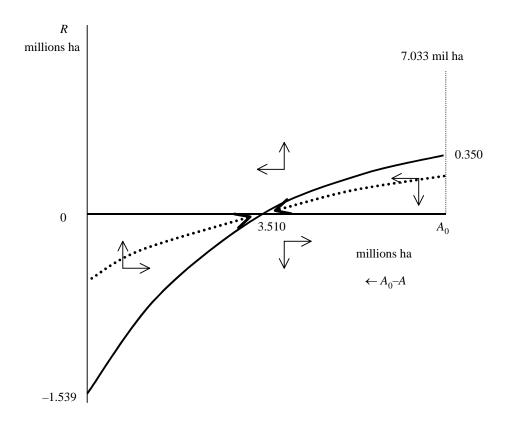


Figure 2: Phase–Plane Diagram and Optimal Approach Path