An Examination of Futures Price Determination though the Lens of Market Integration

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Abstract:

This paper develops a general model of cash and futures markets for a storable commodity. The cash market model is characterized by the incorporation of long hedging to establish contractual claims against existing inventories, which may be either short hedged or unhedged. The futures market model incorporates outright speculation as well as spread speculation. The paper then examines through mathematical analysis the characteristics these markets must possess if they are informationally efficient, if they are conformable for testing price discovery, and if they are integrated in the short or long run.

Thus far results indicate that informationally efficient futures markets are characterized by any one of five conditions: (1) perfectly inelastic utilization demand, (2) perfectly inelastic hedged inventory demand, (3) futures markets not used for hedging, (4) perfectly elastic speculation, or (5) infinitely elastic utilization demand and perfectly inelastic speculation. These conditions further imply that if futures markets are informationally efficient, their prices are not determined simultaneously with cash prices. The extreme assumptions associated with informational efficiency highlight the deficiency of the concept.

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Lists of the socially beneficial functions of futures markets typically include intertemporal resource allocation, risk transfer, and price discovery.[1] However, two seemingly unrelated observations create questions about the effectiveness of futures markets in filling these roles. First, in mid-August of 1999 the USDA reduced it forecast of the Florida orange harvest, causing the nearby September orange juice futures contract to increase by 17.2 cents per pound while the September 2000 contract increased by only five cents. Second, in response to a hypothetical reduction in the pending citrus harvest as part of a futures trading simulation in the author's course, one student related the advice of his father, a floor trader at one of the Chicago commodity exchanges. The father's advice was that when events cause changes in price expectations, the price-change for nearby futures contracts exceeds that of more distant maturities so the nearby contract should be traded. The inexplicable aspect of these two anecdotes is that in both cases anticipated harvests were reduced so that distant inventory expectations were more affected than current inventories. If one accepts the proposition that futures prices reflect expected futures spot prices, then one would expect under these scenarios that post-harvest futures prices would be more affected than nearby futures prices.

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1 Intertemporal resource allocation refers to the use of futures-spot price differences as signals on whether or not to store commodity. Risk transfer refers to the use of futures markets in hedging. Price discovery refers to the use of futures prices as market generated forecasts of future spot prices. With the passage of time, the time to contract maturity diminishes so these forecasts become more accurate. In this sense, the futures market has helped in the discovery of the spot price at contract maturity.

These two observations suggest several avenues for investigation. The possibilities include: (1) Do speculators more heavily trade nearby rather than distant maturity contracts when price expectations change? (2) Are nearby contract prices more heavily influenced by expectation-change than distant contract prices? (3) Is it logically consistent for them to be so? and (4) What are the implications of this phenomenon from the standpoint of market efficiency? This study seeks to bring together various definitions of market efficiency and to study the implications of efficiency so defined for futures market structure. The plan for this paper is as follows. First, efficiency concepts, including allocative efficiency, informational efficiency, price discovery, and market integration, will be reviewed. Then a mathematical model of spot and futures market price determination will be developed. In the section entitled implications, we will examine the conditions futures markets must satisfy in order to be found efficient and how the integration and efficiency concepts relate to each other. Finally, we will discuss the futures market characteristics that lead to the inexplicable conditions described above.

Because markets are defined with respect to time, product form, and space, market efficiency is also defined in these dimensions. Fackler and Goodwin point out that spatial market efficiency is used in the literature to encompass many distinct concepts. Intertemporal efficiency is likewise not clearly defined. Markets that present either spatial or intertemporal arbitrage opportunities are considered to be inefficient because reallocating resources away from low-priced locations to high-priced locations will increase social welfare. Thus, in the spatial context, allocative efficiency requires that price differences between two points on the site-price surface be less than or equal to arbitrage costs. Similarly, with delivery points arranged through time, allocative efficiency requires that price differences be less than or equal to intemporal arbitrage costs. Allocative efficiency in futures markets is defined as

$$F_{Tt} - P_t \leq C_{Tt} \text{ or } F_{Tt} \leq P_t + C_{Tt} \quad (1a)$$

where F_{Tt} is the currently prevailing futures price (at time t) for delivery at time T, P_t is the currently prevailing spot price, and C_{Tt} is the cost to arbitrage the spot market against the futures market.[2] If arbitrage occurs between two points in time or space, and if transactions are costless, then allocative efficiency requires equality in (1a). This equality maximizes the risk transfer capabilities of futures markets as futures and spot prices move in lockstep. When transactions costs (δ) are included, the allocative efficiency condition becomes

$$| F_{Tt} - P_t - C_{Tt} | \leq \delta. \quad (1b)$$

Fama's efficiency definition, which is widely applied in the finance literature, is "A market in which prices always 'fully reflect' available information is called 'efficient'." (p.383) Fama further defined available information so that a market is weak-form efficient if it fully reflects all information on past prices, volume and open interest, semi-strong-form efficient if it fully reflects all publicly available information, and strong-form efficient if it fully reflects all public and private information. As a practical matter, these definitions reflect the type of information sufficient to earn rents from speculative activity. Specifically, if markets are found to be inefficient in some sense, then speculators possessing the specified type of information can earn rents from speculative activity.

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2 One might be tempted to differentiate between spatial and intertemporal arbitrage based on the notion that spatial arbitrage is bi-directional, while storage only moves commodity forward through time. Such a distinction overlooks that a lack of storage incentives (i.e. $F_{Tt} - P_t < C_{Tt}$) is arbitraged by reallocating commodity away from storage to current consumption. This reallocation drives down the spot price and reduced inventories increase futures prices. These price movements are consistent with "reverse storage" arbitrage.

Bigman, Goldfarb and Schectman applied Fama's efficiency definition to futures markets with

$$E_t(P_T - F_{T,t} \mid \Omega_t) = 0 \tag{2a}$$

where Ω_t represents the information set at time t appropriate to the type of efficiency defined. Because $F_{T,t}$ is known at time t, efficiency requires

$$F_{T,t} = E_t(S_T \mid \Omega_t). \tag{2b}$$

The more general efficient market hypothesis provides a related efficiency condition

$$F_{T,t} = E_t(F_{T,t+1} \mid \Omega_t). \tag{2c}$$

(2b) and (2c) will both be referred to as informational efficiency conditions to distinguish them from allocative efficiency conditions. Informational efficiency is more general than allocative efficiency in that a futures market for a seasonally produced commodity may be allocatively efficient for a pre-harvest contract but not for a post-harvest contract and yet be informationally efficient in both.[3] We distinguish between (2b) and (2c) by referring to them as long-run and short-run informational efficiency, respectively, because (2b) involves current expectations about contract maturity while (2c) involves current expectations about the next market period. Convergence of futures and spot prices at contract maturity makes (2b) and (2c) equivalent.[4]

Many researchers have used Fama's definitions to investigate the efficiency of futures markets. Early studies (Bigman, Goldfarb and Schectman; MacDonnald and Hein) used regres-

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3 Ω_t can include S_t and C_{Tt} so that for a full carrying-cost market $E_t(S_T \mid \Omega_t) = S_t + C_{Tt}$.

4 If $F_{Tt} = E_t(F_{T,t+1} \mid \Omega_t)$, then $F_{T,t+1} = E_t(F_{T,t+2} \mid \Omega_t)$, $F_{T,t+2} = E_t(F_{T,t+3} \mid \Omega_t) \ldots$ and $F_{T,T-1} = E_t(F_{T,T} \mid \Omega_t)$. Continuous substitution gives $F_{Tt} = E_t(F_{T,T} \mid \Omega_t)$. To establish equivalence, we need either $F_{TT} = P_T$ or $E_t(F_{T,T} \mid \Omega_t) = E_t(P_T \mid \Omega_t)$.

sion analysis of the form $S_T = \alpha + \beta F_{Tt} + \varepsilon_t$ to test the null, efficient-market hypothesis that $\alpha = 0$ and $\beta = 1$. Variations on these analyses include the possibility of a risk premium which would occur if the hypothesis $\alpha = 0$ is rejected. Recent refinements account for developments in econometric techniques for analyzing integrated time series data[5] and the realization that both futures and spot price series are integrated (Bessler and Covey; Chowdhury; Lai and Lai). Current interpretations of the efficient-market hypothesis necessitate that a linear combination of the spot and futures prices is stationary (Enders, p. 357). Alternatively stated, "The concept of cointegration posits that two efficient markets for the same asset whose prices are each nonstationary by themselves should have an equilibrium relationship which is stationary." (Schroeder and Goodwin, p. 686)

A related family of studies investigates the price discovery function of futures markets (Garbade and Silber; Schroeder and Goodwin; Khoury and Yourougou; Witherspoon; and Ollermann, Brorsen and Farris). These studies use a model such as

$$\begin{bmatrix} P_t \\ F_t \end{bmatrix} = \begin{bmatrix} \alpha_p \\ \alpha_f \end{bmatrix} + \begin{bmatrix} 1-\beta_p & \beta_p \\ \beta_f & 1-\beta_f \end{bmatrix} \begin{bmatrix} P_\ell \\ F_\ell \end{bmatrix} + \begin{bmatrix} e_t \\ e_t \end{bmatrix} \qquad 0 \le \beta_f, \beta_p \le 1 \qquad (3)$$

to determine whether futures markets lead the spot market to equilibrium, or vice versa. These studies argue that if $\beta_p = \beta_f = 0$, then spot and futures prices follow separate random walks, but if $\beta_p \neq 0$, then previous period futures prices influence or "help discover" spot prices. One efficiency implication of this approach is that if $\beta_f > 0$, then previous period futures and spot prices help explain current period futures prices. Such a market is defined as weak-form inefficient.

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5 Basically time series data that follow a random walk.

Market integration has been used to study the connectedness of spatial markets and to determine their spatial extent (Delgado; Huff and Rust; Dublin; Monke and Petzel; Ravallion; Uri and Rifkin). Application to intertemporal markets is a natural extension of this technique because cash and futures market integration is required for futures markets to effectively transfer risk. The advantages of the integration approach are its intuitive nature, its derivation from structural models of the underlying markets, and its focus on market connectedness through arbitrage. The basic methodology (Ravallion, and Dahlgran and Blank) is to let the prices determined in N related markets be represented as

$$P_i = f_i(P^c_i, X_i) \qquad i=1, 2, \ldots N \qquad (3a)$$

where P_i is the commodity's price in market i, P^c_i is the complementary set of N-1 prices (one price for each of the other markets in the system), and X_i is the set of nonprice exogenous variables that influence the supply and demand for the commodity in market i. Because market integration focuses on how price changes in one market cause or are influenced by price changes in other markets, (3a) is rewritten as

$$\Delta P_{it} = \nabla f_i[\Delta P^c_{it}, \Delta X_{it}] \qquad i=1, 2, \ldots N \qquad (3b)$$

where Δ represents the difference operator and ∇f_i is a row vector of partial derivatives.[6]

The econometric specification of (3b) (Dahlgran and Blank)

$$\Delta P_{it} = \sum_{\tau=1}^{p_i} a_{i\tau}\Delta P_{i,t-\tau} + \sum_{i\neq j}^{N}\sum_{\tau=0}^{q_{ij}} b_{ij\tau}\Delta P_{j,t-\tau} + c_i \Delta X_{it}^{T} + \varepsilon_{it} \qquad i=1, 2, \ldots, N \qquad (4)$$

leads to standard market integration tests which are formulated to measure price reactions of different markets to events affecting both markets. These tests are represented as

H_1: Market i is segregated from other markets, i.e.,

$$b_{ij\tau} = 0 \text{ for } \tau = 0, 1, 2, \ldots, q_{ij}; j = 1, 2, \ldots i\text{-}1, i\text{+}1, \ldots, N \tag{4a}$$

H_2: Market i's and market j's prices are not simultaneously determined if

$$b_{ij0} = 0 \tag{4b}$$

H_3: Market j displays weak-form short-run integration with market i, i.e.,

$$b_{ij0} = 1 \quad \text{and} \quad \sum_{\tau=1}^{p_i} a_{i\tau} + \sum_{\tau=1}^{q_{ij}} b_{ij\tau} = 0 \tag{4c}$$

and/or strong-form short-run integration with market i, i.e.,

$$b_{ij0} = 1 \text{ and } a_{i\tau} = 0 \text{ for } \tau = 1, 2, \ldots, p_i, \text{ and } b_{ij\tau} = 0 \text{ for } \tau = 1, 2, , \ldots, q_{ij} \tag{4d}$$

H_4: Market j displays long-run integration with market i, i.e.,

$$\sum_{\tau=1}^{p_i} a_{i\tau} + \sum_{\tau=0}^{q_{ij}} b_{ij\tau} = 1 \tag{4e}$$

In the next section, a structural model of cash and futures markets will be developed. This model will include both spot (i.e. physical) and futures markets with multiple contract maturities. The model will provide a basis for examining integration and efficiency in cash and futures markets.

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6 Differencing the data is advantageous because it transforms first order integrated time series data into white noise.

Theoretical Model

The continuously required allocative choice in commodity markets is whether to currently utilize a commodity or hold it for later use. These choices require equilibrium between current demands and current supplies represented as[7]

$$I_t + D_t = I_{t-1} + Q_t \tag{5}$$

where I_t represents ending commodity inventories at time t, D_t represents commodity utilization, consumption or other removal from inventories in time t, and Q_t represents the new supplies of the commodity that become available at time t.

Commodity utilization is modeled with a demand function

$$D_t = D(z_t) \tag{6a}$$

where D_t represents utilization at time t, $z_t = \log(P_t / V_t)$, P_t represents the commodity's spot price at time t, and V_t represents the price of the product produced with the commodity. Differentiating (6a) gives

$$dD_t = D'(z_t)\, d\log z_t \tag{6b}$$

where $D'(z_t) \equiv \partial D(z_t)/\partial z_t \leq 0$.[8]

The inclusion of futures markets allows a refined conceptualization of inventories in (5). These inventories can be classified as either claimed or unclaimed where claims against existing inventories are established by long hedging (LH_t). Inventories not claimed by long hedging must

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7 The inclusion of inventories in this equilibrium condition is not part of the Garbade and Silber price discovery model nor models derived therefrom. This assumption imparts properties on our model that are not present in the Garbade and Silber "type" models.

8 Functional arguments are expressed in logarithmic form so differentials are F' dlog x. Alternatively, these arguments could be expressed as nonlogarithmic functions and the differential would be F' dx. Such a modification does not change our overall conclusions.

be allocated either to current utilization (D_t), to continued storage as unhedged inventories (U_t), or to continued storage as short-hedged inventories (SH_t). Thus, inventory-provided supplies for the current period are either unhedged or hedged, but these supplies must be reduced by amounts already claimed in long hedges,

$$I_{t-1} = U_{t-1} + SH_{t-1} - LH_{t-1}. \tag{7a}$$

Likewise, the demand for carryout inventory consists of demands for hedged and unhedged stocks, but these demands are reduced by demands already satisfied with inventory claimed in long hedges, so

$$I_t = U_t + SH_t - LH_t. \tag{7b}$$

The difference between short- and long-hedged inventories (SH_t-LH_t) in (7a) and (7b) is the physical-inventory representation of net hedging, defined by Telser as the difference between hedgers' short and long futures positions. As each hedge must be executed in a contract with a specific maturity, net hedging at time t in the futures contract that matures in period T is represented as

$$H_{Tt} \equiv (SH_{Tt} - LH_{Tt}) = H_T(h_{Tt}) \tag{7c}$$

where H_{Tt} represents the net of short- and long-hedged inventories at time t, hedging is for delivery at time T, T is a member of a set of possible contract maturities, $M = \{ T : T \in M \}$, and $h_{Tt} \equiv \log [F_{Tt}/P_t\ e^{(T-t)(r_{Tt}+\gamma)}]$. Long inventory positions (short hedges) are represented by positive quantities and short inventory positions (long hedges) are represented by negative quantities. If short hedging (long inventories) exceeds long hedging (short inventories), then $H_{Tt} > 0$ reflecting the net long inventory balances, while if long hedging exceeds short hedging, then $H_{Tt} < 0$ reflecting the short inventory balances.

The behavioral assumptions of (7c) are that the hedging decision involves comparing the currently prevailing futures contract price for maturity at time T with the current spot price plus the costs to store the commodity from now until contract maturity. The current spot price plus storage costs are computed in the denominator where the terms in the exponential function consist of the storage term (T-t) multiplied by the daily financing cost for the hedge term (r_{Tt}) plus the daily deterioration and insurance charge (γ). If h_{Tt} is positive, inventories will be held and covered by a short futures market position, whereas if h_{Tt} is negative, inventories will not be held in a short hedge but those with a short spot market position will long hedge anticipated commodity purchases in the futures market. Accordingly, $H'(h_{Tt}) \equiv \partial H_T(h_{Tt})/\partial h_{Tt} > 0$.

Unhedged inventories, held at time t for possible use at time T, are designated as U_{Tt} and represented by the behavioral relationship

$$U_{Tt} = U_T(u_{Tt}) \tag{8a}$$

where $u_{Tt} \equiv \log[\, P^*_{Tt}/\, P_t\, e^{(T\,-\,t)(r_{Tt}\,+\,\gamma)}]$. The numerator of the function's argument is the spot price (P) currently (t) expected (*) to prevail at the end of storage period (T). Like the hedging functions, the denominator contains the current spot price compounded to account for storage costs. If the spot price expected to prevail at the end of the storage term rises, more commodity will be stored, while if the current spot price or storage costs increase, less commodity will be stored. Hence, $U'(u_{Tt}) \equiv \partial U_T(u_{Tt})/\partial u_{Tt} > 0$.

Unlike hedged inventory holdings where the futures contract's maturity defines the time horizon for inventory holding, unhedged inventory holdings have ambiguous time horizons that could either be daily, or correspond to a futures contract's maturity, or correspond to a date between the maturity of two successive futures contracts. However, if unhedged-inventory holding is treated as day-to-day speculative activity with no predefined time horizon, then unhedged-inventory holding can be represented as

$$U_t = U[\log P^*_t / P_t\, e^{(r_{0t} + \gamma)}] = U[u_t] \tag{8b}$$

where $u_t \equiv \log[P^*_t / P_t\, e^{(r_{0t}+\gamma)}]$, P^*_t represents the one-day-ahead price expectation, r_{0t} represents the daily one-day interest rate, and $U' > 0$.

Total inventory holdings are now defined as

$$I_t \equiv U_t + \Sigma_{T \in M} H_{Tt}. \tag{9a}$$

This relationship permits substitution between hedged and unhedged inventories, and long-hedged future commodity acquisitions ($H_{Tt} < 0$) can be currently held by other agents as either unhedged or short-hedged inventory. Differentiating (9a) gives

$$dI_t = U' \, d\log u_t. + \Sigma_{T \in M} H'_T \, d\log h_{Tt}. \tag{9b}$$

Substituting (9b) and (6b) into the differential of (5), and incorporating the extended definitions for dlog u_t and dlog h_{Tt} gives

$$D'(p_t - v_t) + U' \, [p^*_t - p_t + (T\text{-}t)\, dr_{0t} - (r_{0t}+\gamma)\, dt\,] +$$

$$\Sigma_{T \in M} H'_T \, [f_{Tt} - p_t + (T\text{-}t)\, dr_{Tt} - (r_{Tt}+\gamma)\, dt] \tag{10}$$

$$= U' \, [p^*_{\prime} - p_{\prime} + (T\text{-}t_{\prime})\, dr_{0\prime} - (r_{0\prime}+\gamma)\, dt_{\prime}] + \Sigma_{T \in M} H'_T \, [f_{T\prime} - p_{T\prime} + (T\text{-}t)\, dr_{T\prime} - (r_{T\prime}+\gamma)]\, dt_{\prime} + dQ_t$$

where $f_{Tt} \equiv$ dlog F_{Tt}, $p_t \equiv$ dlog P_t, $v_t \equiv$ dlog V_t, $p^*_t \equiv$ dlog P^*_t, and ℓ indicates the previous observation.[9]

Futures market speculation includes both spreading and position trading. Spreading requires contracts with differing maturities (designated as T and T′) so we specify

$$S_{Tt} = S_T[\, \log F_{Tt}/F^*_{Tt}\, ,\ \log\, (F_{Tt}/F^*_{Tt})/(F_{T't}/F^*_{T't})\,] \tag{11}$$

where S_{Tt} represents speculators' net futures market positions at time t in the T-maturity contract. Speculators can be long ($S_{Tt} > 0$) or short ($S_{Tt} < 0$), but a *ceteris paribus* increase in a futures contract's price will generate more short (fewer long) positions, while a *ceteris paribus* increase in the current expected futures price will tend to generate more long (fewer short) positions. Thus, $S'_T < 0$ for either argument.

Equilibrium for the T-maturity futures contract requires

$$S_{Tt} = \rho_T\, H_{Tt} \tag{12a}$$

where ρ_T, the market-wide hedge ratio ($\rho_T > 0$), converts net-hedged inventory holdings into T-maturity futures contracts. If hedgers in the T-maturity contract are net short (i.e. on balance using short futures positions to hedge long inventory ($H_{Tt} > 0$) positions), then futures market equilibrium in the T-maturity contract requires speculators to be net long by $\rho_T\, H_{Tt}$ units. Substituting (11) and (7c) into (12a) and differentiating the result gives

$$S'_{T,1}\, [f_{Tt} - f^*_{Tt}\,] + S'_{T,2}\, [f_{Tt} - f^*_{Tt} - f_{T't} + f^*_{T't}] =$$

$$\rho_T\, H'_T\, [f_{Tt} - p_t - (T\text{-}t)dr_{Tt} - (r_{Tt} + \gamma)dt] \qquad T \in M. \tag{12b}$$

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[9] Lag ℓ is used instead of t-1 because weekends and holidays create unequally-spaced observations.

Equations (10) and (12b) constitute the core of the theoretical model for examining the implications of efficiency, price discovery, and/or market integration. For expository purposes, suppose the commodity has just two futures contract maturities, the nearby maturity designated as T=1 and a distant maturity designated as T'=2. The structural equations in (10) and (12b) are represented as

$$\begin{bmatrix} 1 & -\alpha_1 & -\alpha_2 \\ -\phi_1 & 1 & -\psi_1 \\ -\phi_2 & -\psi_2 & 1 \end{bmatrix} \begin{bmatrix} p_t \\ f_{1t} \\ f_{2t} \end{bmatrix} = \begin{bmatrix} (\alpha_0+\alpha_1+\alpha_2) & -\alpha_1 & -\alpha_2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} p_\ell \\ f_{1\ell} \\ f_{2\ell} \end{bmatrix} +$$

$$\begin{bmatrix} -\alpha_0 & -\alpha_1 & -\alpha_2 & \alpha_0 & \alpha_1 & \alpha_2 & -\eta & (1-\alpha_0-\alpha_1-\alpha_2) \\ 0 & \phi_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \phi_2 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} dr_{0t} \\ (T_1 - t)dr_{1t} - (r_{1t}+\gamma)dt \\ (T_2 - t)dr_{2t} - (r_{2t}+\gamma)dt \\ dr_{0\ell} \\ (T_1 - t_\ell)dr_{1\ell} - (r_{1\ell}+\gamma)dt_\ell \\ (T_2 - t_\ell)dr_{2\ell} - (r_{2\ell}+\gamma)dt_\ell \\ dQ_t \\ v_t \end{bmatrix}$$

$$+\begin{bmatrix} \alpha_0 & 0 & 0 \\ 0 & (1-\phi_1) & -\psi_1 \\ 0 & -\psi_2 & (1-\phi_2) \end{bmatrix} \begin{bmatrix} p_t^* - p_\ell^* \\ f_{1t}^* \\ f_{2t}^* \end{bmatrix} \tag{13}$$

where the definitions of the parameters α_T, η, ϕ_T, and ψ_T are summarized in table 1. By construction, the parameters are restricted to the space $\theta = \{ (\alpha_0, \alpha_1, \alpha_2, \eta, \phi_1, \phi_2, \psi_1, \psi_2) \mid \alpha_0, \alpha_1, \alpha_2, \eta, \phi_1, \phi_2, \psi_1, \psi_2 \text{ all} \geq 0 \text{ and } \alpha_0+\alpha_1+\alpha_2 \leq 1 \text{ and } \phi_1+\psi_1 \leq 1 \text{ and } \phi_2+\psi_2 \leq 1 \}$.

The stochastic shocks in this model are percentage changes in expected futures prices, f^*_{Tt} and the second-order percentage change in the expected spot price, $p^*_t - p^*_l$. This second order change represents the notion that futures price changes are due to *changes in expectations* of future spot prices. These terms have expected values of zero, unique variances and are generated by the random occurrence of price-influencing events and the random arrival of new information to the markets.

The structural model in (13) is represented as

$$G\,Y_t = C\,Y_L + B\,X_t + H\,z_t, \tag{14a}$$

so its reduced form is

$$Y_t = G^{-1}C\,Y_L + G^{-1}B\,X_t + G^{-1}H\,z_t.\text{[10]} \tag{14b}$$

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[10] Some useful intermediate mathematical results:

$$G^{-1} = \frac{1}{|G|}\begin{bmatrix} 1-\psi_1\psi_2 & \alpha_1+\alpha_2\psi_2 & \alpha_2+\alpha_1\psi_1 \\ \phi_1+\phi_2\psi_1 & 1-\alpha_2\phi_2 & \alpha_2\phi_1+\psi_1 \\ \phi_2+\phi_1\psi_2 & \alpha_1\phi_2+\psi_2 & 1-\alpha_1\phi_1 \end{bmatrix},$$

$|G| = 1-\psi_1\psi_2-\alpha_1(\phi_1+\phi_2\psi_1)-\alpha_2(\phi_2+\phi_1\psi_2)$,

$$G^{-1}H = \frac{1}{|G|}\begin{bmatrix} \alpha_0(1-\psi_1\psi_2) & \alpha_1(1-\psi_1\psi_2-\phi_1)-\alpha_2\phi_1\psi_2 & \alpha_2(1-\psi_1\psi_2-\phi_2)-\alpha_1\phi_2\psi_1 \\ \alpha_0(\phi_1+\phi_2\psi_1) & 1-\psi_1\psi_2-\phi_1-\alpha_2(\phi_2+\phi_1\psi_2-\phi_1\phi_2) & \alpha_2(\phi_1+\phi_2\psi_1-\phi_1\phi_2)-\phi_2\psi_1 \\ \alpha_0(\phi_2+\phi_1\psi_2) & \alpha_1(\phi_2+\phi_1\psi_2-\phi_1\phi_2)-\phi_1\psi_2 & 1-\psi_1\psi_2-\phi_2-\alpha_1(\phi_1+\phi_2\psi_1-\phi_1\phi_2) \end{bmatrix}$$

and $$G^{-1}C = \frac{1}{|G|}\begin{bmatrix} (\alpha_0+\alpha_1+\alpha_2)(1-\psi_1\psi_2) & -\alpha_1(1-\psi_1\psi_2) & -\alpha_2(1-\psi_1\psi_2) \\ (\alpha_0+\alpha_1+\alpha_2)(\phi_1+\phi_2\psi_1) & -\alpha_1(\phi_1+\phi_2\psi_1) & -\alpha_2(\phi_1+\phi_2\psi_1) \\ (\alpha_0+\alpha_1+\alpha_2)(\phi_2+\phi_1\psi_2) & -\alpha_1(\phi_2+\phi_1\psi_2) & -\alpha_2(\phi_2+\phi_1\psi_2) \end{bmatrix}$$

$$= \frac{1}{|G|}\begin{bmatrix} 1-\psi_1\psi_2 \\ \phi_1+\phi_2\psi_1 \\ \phi_2+\phi_1\psi_2 \end{bmatrix}\begin{bmatrix} (\alpha_0+\alpha_1+\alpha_2) & -\alpha_1 & -\alpha_2 \end{bmatrix}$$

In order for a solution to exist, $|G| \neq 0$ giving four parameter subsets excluded the solution set, $R_1 = \{ \theta \mid (\alpha_0, \alpha_1, \alpha_2, \phi_1, \psi_1) = (0, 1, 0, 1, 0) \}$, $R_2 = \{ \theta \mid (\alpha_0, \alpha_1, \alpha_2, \phi_2, \psi_2) = (0, 0, 1, 1, 0) \}$, $R_3 = \{ \theta \mid (\phi_1, \psi_1, \phi_2, \psi_2) = (0, 1, 0, 1) \}$ and $R_4 = \{ \theta \mid \alpha_1+\alpha_2=1 \text{ and } \phi_1+\psi_1=1 \text{ and } \phi_2+\psi_2=1 \}$.[11] The reduced form can also be expressed as

$$Y_t = \Pi_0 Y_L + \Pi_1 X_t + \Pi_2 z_t \tag{14c}$$

where the elements of Π_1 and Π_2 respectively indicate immediate-period price reactions to changes in the exogenous variables and price expectations. Successive back substitution for Y_L gives

$$Y_t = \Pi_0^{\infty} Y_{t-\infty} + \sum_{i=0}^{\infty} \Pi_0^i \Pi_1 X_{t-i} + \sum_{i=0}^{\infty} \Pi_0^i \Pi_2 z_{t-i} \tag{14d}$$

so that the respective cumulative effects of permanently changing the values of X and z at time t are $\sum_{i=0}^{\infty} \Pi_0^i \Pi_1 = (I - \Pi_0)^{-1}\Pi_1$ and $\sum_{i=0}^{\infty} \Pi_0^i \Pi_2 = (I - \Pi_0)^{-1}\Pi_2$. The structure of Π_0 simplifies the computation of the cumulative multipliers to $\Pi_1+(1-\lambda)^{-1}\Pi_0\Pi_1$ and $\Pi_2+(1-\lambda)^{-1}\Pi_0\Pi_2$ where $\lambda = 1-(1-\alpha_0-\alpha_1-\alpha_2)(1-\psi_1\psi_2)/|G|$.[12] Dynamic stability requires that $\lambda \neq 1$ or that $(1-\alpha_0-\alpha_1-\alpha_2)(1-\psi_1\psi_2) \neq$

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[11] For $| G | = 0$ there must exist a linear combination among the columns or rows of G and a solution to $| G | =0$ subject to the parametric restrictions $\{ 0 \leq \alpha_1, \alpha_2, \phi_1, \phi_2, \psi_1, \psi_2 \leq 1, \alpha_1+\alpha_2 \leq 1, \phi_1+\psi_1 \leq 1, \text{ and } \phi_2+\psi_2 \leq 1\}$. Linear combinations of columns are suggested by each of the rows of G. Letting $\alpha_1+\alpha_2 = 1$ and solving $| G | = 0$ subject to $\phi_1+\psi_1 \leq 1$ gives $(\alpha_1, \alpha_2, \phi_1, \psi_1) = (1, 0, 1, 0)$ with rank(G)=2. Letting $\phi_1+\psi_1 = 1$ and solving $| G | = 0$ subject to $\phi_2+\psi_2 \leq 1$ gives $(\phi_1, \psi_1, \phi_2, \psi_2) = (0, 1, 0, 1)$ with rank(G)=2. Letting $\phi_2+\psi_2 = 1$ and solving $| G | = 0$ subject to $\alpha_1+\alpha_2 \leq 1$ gives $(\alpha_1, \alpha_2, \phi_2, \psi_2) = (0, 1, 1, 0)$ with rank(G)=2. Finally, if linear combinations exist among the three columns of G, such that $\alpha_1+\alpha_2 = 1$, $\phi_1+\psi_1 = 1$, and $\phi_2+\psi_2 = 1$, then rank(G)=1 and $| G |$ is also zero.

[12] To avoid inverting $I - \Pi_0$, use $\Pi_0 = \frac{1}{|G|}\begin{bmatrix} 1-\psi_1\psi_2 \\ \phi_1+\phi_2\psi_1 \\ \phi_2+\phi_1\psi_2 \end{bmatrix}\left[(\alpha_0+\alpha_1+\alpha_2) \quad -\alpha_1 \quad -\alpha_2\right]$,

It can be shown that $\Pi_0^2 = [1-(1-\alpha_0-\alpha_1-\alpha_2)(1-\psi_1\psi_2)/|G|]\ \Pi_0 = \lambda\ \Pi_0$. Hence, $\sum_{i=0}^{\infty} \Pi_0^i \Pi_1 = (\Pi_0^0 + \Pi_0^1 + \Pi_0^2 + \Pi_0^3 + \ldots)\Pi_1 = [I + (1+\lambda+\lambda^2+\lambda^3+\ldots)\Pi_0]\Pi_1 = \Pi_1 + 1/(1-\lambda)\Pi_0\Pi_1$.

0. Thus, additional parameter subsets excluded from the solution set are $R_5 = \{ \theta \mid \alpha_0+\alpha_1+\alpha_2 = 1 \}$ and $R_6 = \{ \theta \mid \psi_1 \psi_2 = 1 \}$. R_6 is redundant because it is equivalent to the previously defined R_3. Therefore, stable solutions are $A = \{ \theta \mid \text{not } (R_1 \cup R_2 \cup R_3 \cup R_4 \cup R_5) \}$. Within this set, the impact multipliers, Π_1 and Π_2, and the lagged adjustments, $(1-\lambda)^{-1}\Pi_0\Pi_1$ and $(1-\lambda)^{-1}\Pi_0\Pi_2$, reflect the efficiency characteristics of each market, and the integration characteristics of market pairs (cash and futures, nearby and distant futures) with respect to cash market supply and demand shocks and with respect to changes in price expectations. These multipliers are summarized in table 2.

Implications

We can now use the model developed in the previous section to examine the implications of efficiency and market integration, if detected, for the structure of the underlying markets. We will find that efficiency and/or integration impose rather severe restrictions on the structure of the underlying markets. Our findings are summarized in table 3.

Implications for Informational Efficiency: (2a) and (2b) show two different statements of futures-market informational efficiency. The short-run condition is equivalent to $d \log F_{T,t} = d \log E_t(F_{T,t+1} \mid \Omega_t) \ \forall\ T, T \in M$. In the context of the multipliers in table 2 these conditions imply

$$[1-\psi_1\psi_2 -\alpha_2(\phi_2+\phi_1\psi_2) -\phi_1(1-\alpha_2\phi_2)] / \mid G \mid = 1, \text{ and} \qquad (15a)$$

$$[1-\psi_1\psi_2 -\alpha_1(\phi_1+\phi_2\psi_1) -\phi_2(1-\alpha_1\phi_1)] / \mid G \mid = 1. \qquad (15b)$$

These equations have solutions $\phi_1=\phi_2=0$ and $\phi_1= [(1-\alpha_1)(1-\alpha_2)-\alpha_1\alpha_2\psi_1\psi_2)] \;/\; [\alpha_1(1-\alpha_1)+\alpha_2^2\ \psi_2]$, $\phi_2 = [(1-\alpha_1)(1-\alpha_2)-\alpha_1\alpha_2\psi_1\psi_2)] \;/\; [\alpha_2(1-\alpha_2)+\alpha_1^2\ \psi_1]$.[13] For ϕ_1 and ϕ_2 to take non-zero solution values α_1 and α_2 must sum to 1.[14]

These conditions have differing implications for the underlying market structure. $\Sigma_{T\in M}\ \alpha_T$ =1 implies -D′+U′=0 or that both unhedged inventory balances and stock utilization are perfectly price inelastic. In this case, unhedged inventory holders do not respond to expected prices while hedged inventory holders do. Allowing unhedged inventory holders to display price responsiveness requires $\phi_T = 0\ \forall\ T,\ T\in M$. As $\phi_T \equiv \rho_T H'_T/(\rho_T H'_T - S'_{T,1} - S'_{T,2})$, $\phi_T=0$ implies that either $\rho_T=0$, or $H'_T=0$, or $(-S'_{T,1}-S'_{T,2}) \rightarrow \infty$. If $\rho_T=0$, then the T-maturity futures contract is not used for hedging, and the contract fills no risk transfer function. If $H'_T = 0$, then α_T is also zero, hedging in the T-maturity contract has no price responsiveness, and consequently the futures markets play no role in intertemporal resource allocation. If $-S'_{T,1}-S'_{T,2} \rightarrow \infty$, then all speculators in the T-maturity contract have the same price expectations. Furthermore, $\phi_T = 0\ \forall\ T,\ T\in M$ makes futures prices exogenous to the model in (13).

Futures market efficiency with respect to the expected spot price at contract maturity requires $d \log F_{T,t} = d \log E_t(\,P_T \mid \Omega_t\,) = 1$ which, according to the multipliers in table 2, requires

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13 Solving (15a) for ϕ_2 gives $\phi_2 = \phi_1\ [(1-\alpha_1) \;/\; (\alpha_2\phi_1+\alpha_1\psi_1)]$ which is substituted into (15b) to get $[\alpha_1(1-\alpha_1)+ \alpha_1^2\psi_2]\ \phi_1^2 - [1-\alpha_1-\alpha_2+\alpha_1\alpha_2(1-\psi_1\psi_2)]\ \phi_1 = 0$.

14 $\psi_1 \leq 1-\phi_1$ implies $\phi_1 \leq 1-\psi_1$. When this restriction is imposed on $\phi_1 = [(1-\alpha_1)(1-\alpha_2) - \alpha_1\alpha_2\psi_1\psi_2)] \;/\; [\alpha_1(1-\alpha_1)+\alpha_2^2\psi_2]$, the result is $-\alpha_1\psi_1+\alpha_2^2\psi_2 \;/\; (1-\alpha_1)+\alpha_2(\alpha_1-\alpha_2)\ \psi_1\psi_2 \;/\; (1-\alpha_1) \geq 1-\alpha_1-\alpha_2$. A similar restriction imposed on the solution for ϕ_2 gives $\alpha_1^2\psi_1 \;/\; (1-\alpha_2) + \alpha_2\psi_2+\alpha_1(\alpha_2-\alpha_1)\psi_1\psi_2 \;/\; (1-\alpha_2) \geq 1-\alpha_1-\alpha_2$. Letting $\alpha_1+\alpha_2+s_0=1$ with $s_0 \geq 0$, and solving each equation for ψ_1 gives
$\psi_1 \leq \{\ s_0 - [\alpha_2^2\ \psi_2 /(\alpha_2+s_0)]\ \} \;/\; \{\ -(1-\alpha_2-s_0) + [\alpha_2(1-2\alpha_2-s_0\,)\ \psi_2 \;/\; (\alpha_2+s_0\,)]\ \}$, and
$\psi_1 \geq \{-s_0-\alpha_2\psi_2\} \;/\; \{\ -[(1-\alpha_2-s_0)^2/(1-\alpha_2)] + [(1-\alpha_2-s_0\,)(1-2\alpha_2-s_0)\ \psi_2 \;/\; (1-\alpha_2)]\ \}$. Both inequalities cannot hold over the region $0 \leq \psi_2 \leq 1$ unless $s_0 = 0$. Hence, the solution requires $\alpha_1+\alpha_2=1$.

$\alpha_0 (\phi_1 + \phi_2 \psi_1) / |G| = 1$, and $\alpha_0 (\phi_2 + \phi_1 \psi_2) / |G| = 1$. (16a,b)

The behavioral conditions implied by these conditions are found by solving for ϕ_1 and ϕ_2 to get

$\phi_1 = (1 - \psi_1) / (\alpha_0 + \alpha_1 + \alpha_2)$, and $\phi_2 = (1 - \psi_2) / (\alpha_0 + \alpha_1 + \alpha_2)$.[15] (16c,d)

By definition $0 \leq \psi_T \leq 1-\phi_T$, which also implies $\phi_T \leq 1-\psi_T \ \forall \ T, T \in M$. This inequality is satisfied in (16c,d) only if $\alpha_0+\alpha_1+\alpha_2 \geq 1$. But by definition $\alpha_0+\alpha_1+\alpha_2 \leq 1$, so the only feasible solution is $\alpha_0+\alpha_1+\alpha_2 = 1$. Because $\alpha_0+\alpha_1+\alpha_2-D'\eta = 1$, this solution requires $-D'\eta = 0$. In short, the perfect translation of spot price expectation changes into futures price changes requires a perfectly inelastic commodity demand.

A final efficiency condition results from combining equivalent simultaneous changes in futures and spot price expectations as would occur if hedgers and speculators had identical conditional expectations. Specifically, suppose $d \log E_t(F_{1,t+1}|\Omega_t) = d \log E_t(F_{2,t+1}|\Omega_t) = d \log E_t(P_T|\Omega_t)$. Now the impact of an expectational change on an individual price is the sum of the three individual impacts. According to the multipliers in table 2, efficiency requires

$\lambda + D' \eta \, (k_1-k_0) / |G| = 1$, and $\lambda + D' \eta \, (k_2-k_0) / |G| = 1$. (17a,b)

The solutions to (17a) and (17b) are either $D'\eta = 0$ or $\phi_T=0 \ \forall \ T, T \in M$.[16] Hence, when hedgers and speculators have identical expectations and information sets, the requirements for informa-

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15 (16a) and (16b) can be arranged as $\begin{bmatrix} \alpha_0+\alpha_1 & \alpha_2 \\ \alpha_1 & \alpha_0+\alpha_2 \end{bmatrix} \begin{bmatrix} \phi_1+\phi_2\psi_1 \\ \phi_2+\phi_1\psi_2 \end{bmatrix} = \begin{bmatrix} 1-\psi_1\psi_2 \\ 1-\psi_1\psi_2 \end{bmatrix}$

which is solved as $\begin{bmatrix} \phi_1+\phi_2\psi_1 \\ \phi_2+\phi_1\psi_2 \end{bmatrix} = \dfrac{1}{\alpha_0+\alpha_1+\alpha_2} \begin{bmatrix} 1-\psi_1\psi_2 \\ 1-\psi_1\psi_2 \end{bmatrix}$.

Substituting $\phi_1= -\phi_2\psi_1+[(1-\psi_1\psi_2)/(\alpha_0+\alpha_1+\alpha_2)]$ into $\phi_2+\phi_1\psi_2=[(1-\psi_1\psi_2)/(\alpha_0+\alpha_1+\alpha_2)]$ gives (16c) and (16d).

16 Focusing on (17a), $\lambda + D'\eta \, (k_1-k_0) / |G| = 1$
$\Rightarrow \{1-(1-\alpha_0-\alpha_1-\alpha_2)(1-\psi_1\psi_2) / |G|\} + D'\eta \, (\phi_1+\phi_2\psi_1-1+\psi_1\psi_2) / |G| = 1$

tional efficiency are either inelastic utilization demand, inelastic hedging responses, or perfectly elastic speculation responses.

Implications for price discovery: The price discovery model in (3) is directly related to our reduced forms, (14b) and (14c), in that the price discovery coefficients correspond to Π_0. If the lagged price coefficients in the spot price equation sum to unity, then $\alpha_0(1-\psi_1\psi_2) / |G| = 1$, which requires $\alpha_0 = 1$.[17] Implicit in this condition is that $\alpha_1=\alpha_2= -D'\eta = 0$ which implies that all inventory adjustments are carried out by nonhedgers and that hedgers play no role in bringing equilibrium to the spot market. $\alpha_1=\alpha_2=0$ is consistent with the price discovery coefficients on futures prices in the cash price equation being zero.

The lagged price coefficients in the futures price equation in (3) also sum to unity. This condition is equivalent to $\alpha_0(\phi_1+\phi_2\psi_1) / |G| = 1$ and $\alpha_0(\phi_2+\phi_1\psi_2) / |G| = 1$ for contracts 1 and 2. These equations are solved as $\phi_1(1-\psi_2) = \phi_2(1-\psi_1)$ or upon further substitution as $\rho_1 H'_1/S'_{1,1} = \rho_2 H'_2/S'_{2,1}$. This condition insures that when futures prices change, the number of new contracts in short (long) positions in the nearby maturity are equal to the number of new contracts in long (short) positions in the distant maturity. Spread speculators, who take offsetting positions in the different maturities, require this relationship.

$\Rightarrow D'\eta\ (\phi_1+\phi_2\psi_1-1+\psi_1\psi_2) = (1-\alpha_0-\alpha_1-\alpha_2)(1-\psi_1\psi_2)$
$\Rightarrow -D'\eta\ (1-\psi_1\psi_2-\phi_1-\phi_2\psi_1) = -D'\eta\ (1-\psi_1\psi_2)$.
This equality holds over solution set A if $-D'\eta = 1-\alpha_0-\alpha_1-\alpha_2 = 0$. If $-D'\eta > 0$, $\phi_1 = 0$ and $\phi_2\psi_1=0$ is the solution but consideration of the other contract maturity requires $\phi_2 = 0$ and $\phi_1\psi_2=0$. Hence, the solution if $D'\eta \neq 0$ is $\phi_1=\phi_2=0$.

17 The lagged price coefficients in the spot price equation are the coefficients in the first row of $G^{-1}C$. These coefficients are easily seen to sum to $\alpha_0(1-\psi_1\psi_2) / |G|$.

Implications for Market Integration: Simultaneity between cash and futures prices is defined by the αs and the ϕs in the coefficient matrix G in (12). If either $\alpha_1=\alpha_2=0$ or $\phi_1=\phi_2=0$, then the system is block recursive with one market leading the other in unidirectional price discovery. Specifically, if $\alpha_1=\alpha_2=0$ but not $\phi_1=\phi_2=0$, then spot prices are instrumental in the discovery of futures prices. The corresponding mathematical conditions that either $-D' \rightarrow \infty$, or $U' \rightarrow \infty$, or both $-D'$ and $U' \rightarrow \infty$, imply that the futures market's role in intertemporal resource allocation is minor and most stockholding/utilization decisions are based on current spot prices and expectations about those prices. Alternatively, if $\phi_1=\phi_2=0$ but not $\alpha_1=\alpha_2=0$ then futures markets are the source of spot price discovery. This requires also concluding that either (a) the market-wide hedge ratio is zero and that the risk transfer function of the futures market for the commodity is unutilized, or (b) the net speculation function is perfectly elastic and futures prices are exogenously determined. If both $\alpha_1=\alpha_2=0$, and $\phi_1=\phi_2=0$ then the system is composed of two logarithmic random walks influenced by the other exogenous variables.

Market segregation is a stronger condition than nonsimultaneity in that it requires nonsimultaneity between each market in the pair and neither current nor lagged price changes from one market can affect the other. The structure of the matrix C indicates that futures prices are not affected by lagged spot or futures price changes. Thus, futures market nonsimultaneity with the cash market ($\phi_T=0$) is tantamount to futures market segregation. Cash market nonsimultaneity with the futures market ($\alpha_T=0$) is also equivalent to segregation as the coefficients in C are zero. The significance of this observation is that the adoption of the weaker nonsimultaneity condition is equivalent to adopting the much stronger and empirically dubious proposition that the cash and futures markets are segregated.

Analysis of the multipliers in table 2 can reveal integration relationships for three market pairs: (1) cash and nearby futures, (2) cash and distant futures, and (3) nearby and distant futures. In addition, the three types of integration (strong-form short-run, weak-form short-run, and long-run) can be examined. Recognizing symmetries in the multipliers for the two futures markets, we limit our attention to the integration characteristics of the cash and nearby futures markets and the nearby and distant futures markets knowing that the integration characteristics of the cash and distant futures markets parallel those of the cash and nearby futures markets. Furthermore, because the model is a first-order difference equation, lagged effects decay exponentially so that weak-form short-run integration is of limited concern. The solutions required for various integration results are summarized in table 3.

Strong-form, short-run, cash-nearby futures integration in response to supply shocks means that futures and spot prices move in lockstep and move immediately to a new equilibrium when supply shifts. The requirements for this condition are $\eta \neq 0$, $1-\psi_1\psi_2 = \phi_1+\phi_2\psi_1 \neq 0$, and $\lambda = 0$. $\eta \neq 0$ implies that none of the spot-market excess-supply components can have an infinitely elastic price response. $1-\psi_1\psi_2 = \phi_1+\phi_2\psi_1 \neq 0$ requires $\phi_1+\psi_1=1$ but excludes $(\psi_1, \psi_2) = (1, 1)$ so that all futures market equilibration must be due either to hedging or spreading, implicitly excluding outright speculation. $\lambda = 0$ requires one of five conditions: $A_1 = \{\theta \mid (\alpha_0, \alpha_2, \phi_1, \psi_1) = (0, 0, 1, 0)\}$, or $A_2 = \{\theta \mid (\alpha_0, \alpha_1, \phi_2, \psi_2) = (0, 0, 1, 0)\}$, or $A_3 = \{\theta \mid (\phi_1, \psi_1, \phi_2, \psi_2) = (0, 1, 0, 1)\}$, or $A_4 = \{\theta \mid \alpha_0=0, \alpha_1+\alpha_2<1, \phi_1+\psi_1=1, \phi_2+\psi_2=1\}$, or $A_5 = \{\theta \mid (\alpha_0, \alpha_1, \alpha_2) = (0, 0, 0)\}$.[18]

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18 $\lambda = 0 \Rightarrow |G| -(1-\alpha_0-\alpha_1-\alpha_2)(1-\psi_1\psi_2) = 0$
$\Rightarrow (\alpha_0+\alpha_1+\alpha_2)(1-\psi_1\psi_2)-\alpha_1(\phi_1+\phi_2\psi_1)-\alpha_2(\phi_2+\phi_1\psi_2) = 0$

A_3 is excluded because it is identical to R_3 which was shown earlier to give $|G| = 0$, and because it results in $k_0=0$. $\alpha_0=0$ is common to the four remaining conditions and is necessary (but not sufficient) for distinguishing strong-form, short-run from long-run cash-futures market integration in response to supply shocks. If unhedged inventory holders' respond to price changes following supply shocks, then immediate adjustment to new equilibrium prices is precluded. The more important these adjustments are relative to the other components of excess spot-market supply (larger values of α_0) the more gradual is the equilibration of the markets and in the limit when $\alpha_0=1$, (no demand or hedged inventory adjustments) supply shocks cause no price adjustments.

The requirements for distant-nearby futures market integration in response to supply shocks are similar to cash-futures market integration requirements except that $\phi_1+\phi_2\psi_1 = \phi_2+\phi_1\psi_2 \neq 0$ replaces $1-\psi_1\psi_2 = \phi_1+\phi_2\psi_1 \neq 0$. $\phi_1+\phi_2\psi_1 = \phi_2+\phi_1\psi_2 \neq 0$ requires that $\phi_1+\psi_1=1$ and $\phi_2+\psi_2=1$ and excludes $(\phi_1, \psi_1, \phi_2, \psi_2) = (0, 1, 0, 1)$. While these conditions preclude price-responsive outright speculation as a condition for nearby-distant futures integration they do not preclude price responsive outright speculation in the nearby and distant contracts by different agents when the responses are offsetting. α_0 plays the same critical role in determining λ and which distinguishes between long-run and short-run integration of the two futures markets.

$\Rightarrow |G^*| = 0$ where $G^* = \begin{bmatrix} (\alpha_0+\alpha_1+\alpha_2) & -\alpha_1 & -\alpha_2 \\ -\phi_1 & 1 & -\psi_1 \\ -\phi_2 & -\psi_2 & 1 \end{bmatrix}$. Divide the first row of G^* by $\alpha_0+\alpha_1+\alpha_2$ and let $\alpha^*_1 = \alpha_1/(\alpha_0+\alpha_1+\alpha_2)$ and $\alpha^*_2 = \alpha_2/(\alpha_0+\alpha_1+\alpha_2)$ so that $\left|G^*\right| = (\alpha_0+\alpha_1+\alpha_2)\begin{vmatrix} 1 & -\alpha_1^* & -\alpha_2^* \\ -\phi_1 & 1 & -\psi_1 \\ -\phi_2 & -\psi_2 & 1 \end{vmatrix}$. The problem of finding parametric values such that $|G^*| = 0$ is now identical to the earlier problem of finding $|G| = 0$ but with $(\alpha_0+\alpha_1+\alpha_2) = 0$ as an added solution. The solutions are therefore: $(\alpha^*_1, \alpha^*_2, \phi_1, \psi_1) = (1, 0, 1, 0)$, $(\alpha^*_1, \alpha^*_2, \phi_2, \psi_2) = (0, 1, 1, 0)$, $(\phi_1, \psi_1, \phi_2, \psi_2) = (0, 1, 0, 1)$, $(\alpha^*_1+\alpha^*_2, \phi_1+\psi_1, \phi_2+\psi_2) = (1, 1, 1)$, and $(\alpha_0, \alpha_1, \alpha_2) = (0, 0, 0)$.

Cash-futures market-integration requirements in response to demand shocks are the same as the corresponding integration requirements in response to supply shocks but with the additional requirement that $D' \neq 0$. Thus, if utilization demand is perfectly inelastic then the cash and futures markets cannot display any type of integration in response to a change in a demand-related price.

Market integration in response to a change in spot-price expectations with no corresponding change in futures-price expectations is determined similarly to the previous two cases except that $\alpha_0 \neq 0$ is required for a price response to exist. This condition is contrary to $\alpha_0 = 0$ which is required for the disappearance of lagged price effects. Hence, strong-form short run integration between price pairs in response to changes in futures spot price expectations is not possible.

Cash-futures integration in response to a change in futures-price expectations with no corresponding change in spot-price expectations is determined by $\alpha_1 k_0 - \phi_1(\alpha_1+\alpha_2\psi_2) = k_0 - \alpha_2 k_2 - \phi_1(1-\alpha_2\phi_2) \neq 0$. This relationship is satisfied by $A_6 = \{ \theta \mid (\alpha_0, \alpha_1, \alpha_2) = (0, 1, 0) \}$, $A_7 = \{ \theta \mid (\phi_1, \psi_1) = (1, 0) \}$, $A_8 = \{ \theta \mid (\alpha_0, \alpha_2, \phi_2, \psi_2) = (0, 1-\alpha_1, 1, 0) \}$ or $A_9 = \{ \theta \mid (\phi_1, \phi_2, \psi_2) = (1-\psi_1, 0, 1) \}$.[19] Because as $D'\eta \leq 0$, lagged adjustments offset the initial impact so that in the absence of other events, the effect of changes in expectations of futures prices tend to dissipate.

Short-run, strong form integration between nearby and distant futures contracts in response to changes in expectations of the nearby contract price requires $k_0 - \alpha_2 k_2 - \phi_1(1-\alpha_2\phi_2) = \alpha_1 k_2 - \phi_1(\alpha_1\phi_2+\psi_2) \neq 0$. This condition is solved by $A_{10} = \{ \theta \mid (\alpha_0, \alpha_2, \phi_1, \psi_1) = (0, 1-\alpha_1, 1, 0)$

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[19] $\alpha_1 k_0 - \phi_1(\alpha_1+\alpha_2\psi_2) = k_0 - \alpha_2 k_2 - \phi_1(1-\alpha_2\phi_2)$ can be expressed as $(1-\alpha_1)(1-\phi_1)[1-\alpha_2\phi_2/(1-\alpha_1) - \psi_1\psi_2/(1-\psi_1)] = 0$. The solutions follow from solving $1-\alpha_1 = 0$ or $1-\phi_1 = 0$ or $1-\alpha_2\phi_2/(1-\alpha_1) - \psi_1\psi_2/(1-\psi_1) = 0$ subject to the restrictions of the allowable parameter space.

and $\phi_2+\psi_2<1$ } or A_{11} = { $\theta \mid (\alpha_0, \alpha_2, \phi_2, \psi_2) = (0, 1-\alpha_1, 1, 0)$ and $\phi_1+\psi_1<1$ }.[20] These conditions also define strong-form, short-run integration because both solutions require $\alpha_0 = 0$ and $\alpha_2 = 1 - \alpha_1$ with the result that $D'\eta = 1 - \alpha_0 - \alpha_1 - \alpha_2 = 0$. On a more intuitive level, unhedged inventory holders do not adjust inventory levels ($\alpha_0 = 0$) so that immediate period price effects are the full and final effects. Similar to the case for cash-futures market integration, speculative bubbles are unsustainable as changes in the expected futures price tend to dissipate and the markets mode back toward their original equilibrium.

If futures prices reflect expected futures spot prices, then $d \log P^*_t = d \log F^*_{1t} = d \log F^*_{2t}$. Conditions for strong-form short-run integration between the cash and futures markets are that $D'\eta\ (k_1-k_0) = 0$ while $\lambda \neq 0$. These conditions require either perfectly inelastic utilization demand ($D' = 0$), or perfectly elastic cash market excess supply ($\eta = 0$), or spreading as the sole speculative reaction ($k_0 = k_1 \Rightarrow 1-\psi_1\psi_2 = \phi_1+\phi_2\psi_1 \Rightarrow \phi_1+\psi_1 = 1$ and $\phi_2+\psi_2=1$). The requirement of speculative spread reactions is actually a requirement of no speculative reactions because outright position speculation implies either $\phi_1+\psi_1 \leq 1$ or $\phi_2+\psi_2 \leq 1$ while the spread arguments of the net speculation function utilizes ratios of expected futures prices. As the changes in expected futures prices are the same, the ratio is unchanged. Hence, cash-futures market integration in the presence of equivalent expected price changes requires either perfectly inelastic utilization demand, or perfectly inelastic speculative responses.

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20 $k_0-\alpha_2k_2-\phi_1(1-\alpha_2\phi_2)=\alpha_1k_2-\phi_1(\alpha_1\phi_2+\psi_2)$ can be expressed as $(1-\phi_1)(1-\phi_2)-\psi_1\psi_2 = -(1-\alpha_1-\alpha_2)[\phi_1\psi_2+\phi_2(1-\phi_1)]$. As $(1-\phi_1)(1-\phi_2)-\psi_1\psi_2 > 0$ for $\phi_1+\psi_1 < 1$ and $\phi_2+\psi_2 < 1$ and $-(1-\alpha_1-\alpha_2)[\phi_1\psi_2+\phi_2(1-\phi_1)] \leq 0$, the only possibilities of equality require $-(1-\alpha_1-\alpha_2)[\phi_1\psi_2+\phi_2(1-\phi_1)] = 0$ and $(1-\phi_1)(1-\phi_2)-\psi_1\psi_2 = 0$ subject to the definitional restrictions on η and the requirements for solution existence and stability.

In order for cash and futures price to display short-run strong-form integration, lagged price adjustments must disappear so that $\alpha_0 + \lambda\, D'\eta = 0$. As $\alpha_0 = U'\eta$ this condition requires $U' = -\lambda D'$. From the standpoint of market structure, this requirement means that adjustments of unhedged inventory relative to consumption must be λ. Otherwise lagged effects will send the markets toward a new equilibrium.

Similar conditions describe integration between the nearby and distant futures contracts. Conditions for integration are $\lambda\, |G| + D'\eta\, (k_1\text{-}k_0) = \lambda\, |G| + D'\eta\, (k_2\text{-}k_0)$ which readily reduces to $D'\eta\, (k_1\text{-}k_2) = 0$. Thus, integration requires either $D'\eta = 0$ or $\phi_1+\phi_2\psi_1 = \phi_2+\phi_1\psi_2$. This equality is satisfied if and only if $\phi_1+\psi_1 = 1$ and $\phi_2+\psi_2 = 1$ but $(\phi_1, \psi_1, \phi_2, \psi_2) = (0, 1, 0, 1)$ is excluded. As was previously the case, position speculation as a price response is not allowable and spread speculation as a price response is not utilized because the ratio of price expectations is unchanged. The requirements for short-run strong-form integration as the same as in the previous case in that demand adjustments must offset unhedged inventory adjustments.

Conclusions

This paper has focused on pricing behavior of futures markets. In several instances we have seen that informational efficiency of futures markets is potentially inconsistent with the socially beneficial functions of futures markets, so that a conclusion of informational efficiency is tantamount to concluding that the markets do not deliver the other social benefits that are frequently ascribed to them. This paper has also identified a number of market price non-responses (unhedged inventory holding, commodity utilization, etc.) that are required for informational efficiency to prevail. The severity of the assumptions required for concluding that a market is informationally efficient does not lead to the conclusion that informational efficiency is a flawed concept. Instead, the methodology of testing for efficiency without a precise model of the market relationships that are being tested should be examined. Specifically, efficiency investigations that examine the cointegration of price data typically employ no specific structural model of arbitrage, so there is no notion of how the price series should relate to each other. Fundamental concepts of spot and futures price convergence, and expectation time targets are frequently ignored.

In closing, we return to the original anecdotes about the pricing behavior of orange juice futures that motivated this study. Closer inspection of market news during this time indicted that a "Trading Places" scenario was playing out. That is, government forecasts of the Florida orange crop were being revised downward and as a result frozen concentrated orange juice futures prices increased. However, the non-spot futures contracts were restricted by price limits while the spot futures contract was not. Hence, the spot futures contract displayed larger price increases than the distant contracts. Price limits, when binding, imply *de facto* that futures markets are not efficient. Furthermore, the extent to which spot futures maturities are not constrained by price limits while non-spot maturities are, gives floor traders incentive to trade nearby contracts even though theory suggests that a disruptive event will cause the greatest price impact on distant maturity futures contract prices.

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Table 1. Parameter definitions, interpretations, and implications at limits.

Parameter and definition	Range	Interpretation	Limits	Implication
Spot market				
$\eta \equiv 1/[-D'+U'+\Sigma_{T\in M} H'_T]$	$\eta \geq 0$	Inverse of spot-market excess supply response w.r.t. d log P_t | d log F_{Tt} =0.	$\eta = 0$	At least one the functions D, U, or H_T $T \in M$ is infinitely elastic.
$\alpha_0 \equiv U' \eta$	$0 \leq \alpha_0 \leq 1$	Nonhedgers' relative contribution to excess supply w.r.t. log P_t | F_{Tt}.		
$\alpha_T \equiv H'_T \eta$	$0 \leq \alpha_T \leq 1$	Hedgers' relative contribution to excess supply w.r.t. log P_t | F_{Tt}.	$\Sigma_{T\in M} \alpha_T = 1$	For dlog $P_t \neq 0$ | dlog $F_{Tt} = 0$, hedgers generate all (unhedged inv. holders and consumers generate no) excess supplies.
			$\Sigma_{T\in M} \alpha_T < 1$	For dlog $P_t \neq 0$ | dlog $F_{Tt} = 0$, unhedged inv. holders or consumers generate some excess supplies.
$\alpha_0+\Sigma_{T\in M} \alpha_T - D'\eta = 1$		Relative contributions sum to unity.		
Futures market				
$\phi_T \equiv \rho_T H'_T / (\rho_T H'_T - S'_{T,1} - S'_{T,2})$	$0 \leq \phi_T \leq 1$	Hedgers' relative contribution to excess supply of T-maturity contracts when dlog $F_{Tt} \neq 0$ | dlog $P_t = 0$.	$\phi_T = 1$ ($\phi_T = 0$)	For dlog $F_{Tt} \neq 0$ | dlog $P_t = 0$, T-maturity hedgers (speculators) generate all excess supply of T-maturity contracts. $\phi_T = 0$ due to either $\rho_T = 0$, or $H'_T = 0$, or $-S'_{T,1}-S'_{T,2} \rightarrow \infty$.
$\psi_T \equiv -S'_{T,2}/(\rho_T H'_T - S'_{T,1} - S'_{T,2})$	$0 \leq \psi_T \leq 1-\phi_T$	Spreaders' relative contribution to excess supply of T-maturity contracts when dlog $F_{Tt} \neq 0$ | dlog $P_t = 0$.	$\psi_T = 0$ ($\psi_T = 1$)	For dlog $F_{Tt} \neq 0$ | dlog $P_t = 0$, spreaders generate no (all) excess supply of T-maturity contracts.
$1 - \phi_T$		Spreaders' and outright speculators' relative contribution to excess supply of T-maturity contracts when dlog $F_{Tt} \neq 0$ | dlog $P_t = 0$.	$\phi_T + \psi_T = 1$	For dlog $F_{Tt} \neq 0$ | dlog $P_t = 0$, hedgers and spreaders generate all (outright speculators generate no) excess supply of T-maturity contracts.

Table 2. Summarization of impact multipliers and lagged adjustments.[a]

Endog. Var	Source of shock					
	dQ_t	$d \log V_t$	$d \log P^*_t$	$d \log F^*_{1t}$	$d \log F^*_{2t}$	$d \log P^*_t = d \log F^*_{1t} = d \log F^*_{2t}$
				Impact Multipliers ($\times \lvert G\rvert^{-1}$)		
$d \log P_t$	$-\eta\, k_0$	$-D'\,\eta\, k_0$	$\alpha_0\, k_0$	$\alpha_1 k_0 - \phi_1(\alpha_1+\alpha_2\psi_2)$	$\alpha_2 k_0 - \phi_2(\alpha_2+\alpha_1\psi_1)$	$\lambda \lvert G\rvert$
$d \log F_{1t}$	$-\eta\, k_1$	$-D'\,\eta\, k_1$	$\alpha_0\, k_1$	$k_0 - \alpha_2 k_2 - \phi_1(1-\alpha_2\phi_2)$	$\alpha_2 k_1 - \phi_2(\alpha_2\, \phi_1+\psi_1)$	$\lambda \lvert G\rvert + D'\,\eta\,(k_1-k_0)$
$d \log F_{2t}$	$-\eta\, k_2$	$-D'\,\eta\, k_2$	$\alpha_0\, k_2$	$\alpha_1 k_2 - \phi_1(\alpha_1\phi_2+\psi_2)$	$k_0 - \alpha_1 k_1 - \phi_2\,(1-\alpha_1\phi_1)$	$\lambda \lvert G\rvert + D'\,\eta\,(k_2-k_0)$
				Lagged Adjustments ($\times \lvert G\rvert^{-1}$)		
$d \log P_t$	$-\eta\, k_0\, \lambda\, /\, (1-\lambda)$	$-D'\,\eta\, k_0\, \lambda\, /\, (1-\lambda)$	$\alpha_0\, k_0\, \lambda\, /\, (1-\lambda)$	$D'\,\eta\, k_0\, [\alpha_1 k_0 - \phi_1(\alpha_1+\alpha_2\psi_2)]\, /\, (1-\lambda)$	$D'\,\eta\, k_0\, [\alpha_2 k_0 - \phi_2(\alpha_2+\alpha_1\psi_1)]\, /\, (1-\lambda)$	$(\alpha_0+\lambda\, D'\,\eta)\, k_0\, /\, (1-\lambda)$
$d \log F_{1t}$	$-\eta\, k_1\, \lambda\, /\, (1-\lambda)$	$-D'\,\eta\, k_1\, \lambda\, /\, (1-\lambda)$	$\alpha_0\, k_1\, \lambda\, /\, (1-\lambda)$	$D'\,\eta\, k_1\, [\alpha_1 k_0 - \phi_1(\alpha_1+\alpha_2\psi_2)]\, /\, (1-\lambda)$	$D'\,\eta\, k_1\, [\alpha_2 k_0 - \phi_2(\alpha_2+\alpha_1\psi_1)]\, /\, (1-\lambda)$	$(\alpha_0+\lambda\, D'\,\eta)\, k_1\, /\, (1-\lambda)$
$d \log F_{2t}$	$-\eta\, k_2\, \lambda\, /\, (1-\lambda)$	$-D'\,\eta\, k_2\, \lambda\, /\, (1-\lambda)$	$\alpha_0\, k_2\, \lambda\, /\, (1-\lambda)$	$D'\,\eta\, k_2\, [\alpha_1 k_0 - \phi_1(\alpha_1+\alpha_2\psi_2)]\, /\, (1-\lambda)$	$D'\,\eta\, k_2\, [\alpha_2 k_0 - \phi_2(\alpha_2+\alpha_1\psi_1)]\, /\, (1-\lambda)$	$(\alpha_0+\lambda\, D'\,\eta)\, k_2\, /\, (1-\lambda)$

a/ Let $k_0 = 1-\psi_1\psi_2$, $k_1 = \phi_1+\phi_2\psi_1$, $k_2 = \phi_2+\phi_1\psi_2$, $\lvert G\rvert = 1-\psi_1\psi_2 - \alpha_1(\phi_1+\phi_2\psi_1) - \alpha_2(\phi_2+\phi_1\psi_2)$. $\lambda \equiv 1 - (1-\alpha_0-\alpha_1-\alpha_2)(1-\psi_1\psi_2)/\lvert G\rvert = 1 + D'\eta(1-\psi_1\psi_2)/\lvert G\rvert$ as $1-\alpha_0-\alpha_1-\alpha_2 = -D'\,\eta$.

Table 3. Summarization of results.

Condition		Parametric Requirement	Solutions
Futures market informational efficiency with respect to			
Expected futures price $(d\log F_{Tt}/d\log E_t(F_{T,t+1}\mid\Omega_t)=1)$		$[k_0-\alpha_2k_2-\phi_1(1-\alpha_2\phi_2)]/\lvert G\rvert=[k_0-\alpha_1k_1-\phi_2(1-\alpha_1\phi_1)]/\lvert G\rvert=1$	$\phi_1=\phi_2=0$, or $\alpha_1+\alpha_2=1$
Expected future spot price $(d\log F_{Tt}/d\log E_t(P_T\mid\Omega_t)=1)$		$\alpha_0 k_1/\lvert G\rvert=\alpha_0 k_2/\lvert G\rvert=1$	$D'=0$
Expected identical futures and spot prices		$\lambda+D'\eta(k_1-k_0)/\lvert G\rvert=\lambda+D'\eta(k_2-k_0)/\lvert G\rvert=1$	$D'=0$, or $\eta=0$, or $(\phi_1,\phi_2)=(0,0)$
Price Discovery			
Spot market coefficients sum to 1		$\alpha_0(1-\psi_1\psi_2)/\lvert G\rvert=1$	$[\alpha_0,\alpha_1,\alpha_2]=[1,0,0]$
Futures market coefficients sum to 1		$\alpha_0(\phi_1+\phi_2\psi_1)/\lvert G\rvert=1$ and $\alpha_0(\phi_2+\phi_1\psi_2)/\lvert G\rvert=1$	$\phi_1(1-\psi_2)=\phi_2(1-\psi_1)$
Market Integration			
Simultaneity		G not block diagonal	$[\alpha_1,\alpha_2]\neq[0,0]$ and $[\phi_1,\phi_2]\neq[0,0]$
Segregation		G not block triangular	$[\alpha_1,\alpha_2]=[0,0]$ or $[\phi_1,\phi_2]=[0,0]$
w.r.t. dQ_t or $d\log V_t$			
Cash/nearby futures	- LR	$-D'\eta k_0=-D'\eta k_1\neq 0,\ \lambda\neq 0$	$\alpha_0+\alpha_1+\alpha_2<1,\ \phi_1+\psi_1=1$
	- SR Strong	$-D'\eta k_0=-D'\eta k_1\neq 0,\ \lambda=0$	$\alpha_0+\alpha_1+\alpha_2<1,\ \phi_1+\psi_1=1,$
Nearby/distant futures	- SR Strong	$-D'\eta k_1=-D'\eta k_2\neq 0,\ \lambda=0$	
	- LR	$-D'\eta k_1=-D'\eta k_2\neq 0,\ \lambda\neq 0$	
w.r.t $d\log P^*_t$			
Cash/nearby futures	- SR Strong	$\alpha_0k_0=\alpha_0k_1\neq 0,\ \lambda=0$	
	- LR	$\alpha_0k_0=\alpha_0k_1\neq 0,\ \lambda\neq 0$	
Nearby/distant futures	- SR Strong	$\alpha_0k_1=\alpha_0k_2\neq 0,\ \lambda=0$	
	- LR	$\alpha_0k_1=\alpha_0k_2\neq 0,\ \lambda\neq 0$	
w.r.t $d\log F^*_{1t}$			
Cash/nearby futures	- SR Strong	$\alpha_1k_0-\phi_1(\alpha_1+\alpha_2\psi_2)=k_0-\alpha_2k_2-\phi_1(1-\alpha_2\phi_2)\neq 0,\ D'\eta k_0=D'\eta k_1=0$	
	- LR	$\alpha_1k_0-\phi_1(\alpha_1+\alpha_2\psi_2)=k_0-\alpha_2k_2-\phi_1(1-\alpha_2\phi_2)\neq 0,\ D'\eta k_0\neq 0,\ D'\eta k_1\neq 0$	
Nearby/distant futures	- SR Strong	$k_0-\alpha_2k_2-\phi_1(1-\alpha_2\phi_2)=\alpha_1k_2-\phi_1(\alpha_1\phi_2+\psi_2)\neq 0,\ D'\eta k_1=D'\eta k_2=0$	
	- LR	$k_0-\alpha_2k_2-\phi_1(1-\alpha_2\phi_2)=\alpha_1k_2-\phi_1(\alpha_1\phi_2+\psi_2)\neq 0,\ D'\eta k_1\neq 0,\ D'\eta k_2\neq 0$	
w.r.t $d\log P^*_t=d\log F^*_{1t}=d\log F^*_{2t}$			
Cash/nearby futures	- SR Strong	$\lvert G\rvert+D'\eta k_0=\lvert G\rvert+D'\eta k_1\neq 0,\ (\alpha_0+\lambda D'\eta)k_0=(\alpha_0+\lambda D'\eta)k_1=0$	$\phi_1+\psi_1=1,\ \phi_2+\psi_2=1,\ U'=-\lambda D'$
	- LR	$\lvert G\rvert+D'\eta k_0=\lvert G\rvert+D'\eta k_1\neq 0,\ (\alpha_0+\lambda D'\eta)k_0\neq 0,\ (\alpha_0+\lambda D'\eta)k_1\neq 0$	$\phi_1+\psi_1=1,\ \phi_2+\psi_2=1$
Nearby/distant futures	- SR Strong	$\lvert G\rvert+D'\eta k_1=\lvert G\rvert+D'\eta k_2\neq 0,\ (\alpha_0+\lambda D'\eta)k_1=(\alpha_0+\lambda D'\eta)k_2=0$	$\phi_1+\psi_1=1,\ \phi_2+\psi_2=1,\ U'=-\lambda D'$
	- LR	$\lvert G\rvert+D'\eta k_1=\lvert G\rvert+D'\eta k_2\neq 0,\ (\alpha_0+\lambda D'\eta)k_1\neq 0,\ (\alpha_0+\lambda D'\eta)k_2\neq 0$	$\phi_1+\psi_1=1,\ \phi_2+\psi_2=1$