Target MOTAD for Risk Lovers: An Alternative Version

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Abstract

An earlier paper presented a direct expected utility maximization version of Target MOTAD for risk lovers. This paper presents an indirect expected utility maximization version which is more like Tauer's Target MOTAD model. This alternative version is illustrated by applying it to the same problem which Tauer considered.

Target MOTAD for Risk Lovers: An Alternative Version

In an earlier paper, we presented a Target MOTAD model which may be appropriate for decision makers who prefer risk (McCamley and Rudel). That model differs from Tauer's (1983) Target MOTAD model in several ways. One difference is that our model is a direct expected utility maximization formulation while Tauer's model involves indirect expected utility maximization. We resorted to direct expected utility maximization partly because we didn't fully understand an important implication of preference for risk.

We think that, at least for risk lovers, direct expected utility maximization formulations tend to be better than indirect expected utility maximization. However, for the sake of completeness (and symmetry) this paper presents an indirect expected utility maximizing version of Target MOTAD for risk lovers. It is more similar to Tauer's model than our original Target MOTAD model for risk lovers.

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Motivation

Agricultural economists use various techniques to evaluate risky alternatives. When the decision variables are continuous, risk neutrality or risk aversion is almost always assumed and mathematical programming is used to find optimal solutions. By contrast, when the number of choices is small, the assumptions about risk preferences are sometimes less restrictive and stochastic dominance criteria are often applied. It is fairly common to identify efficient sets for decision makers who prefer risk as well as for decision makers who are risk averse and/or approximately risk neutral. The study by Larson and Mapp is one of several for which this was done.

Friedman and others have suggested that risk seeking behavior should be rare (or limited) in financial or production decision making because it is possible to buy risk at very low prices through gambling. This view is partly supported by empirical studies of risk attitudes. These studies usually find that more decision makers are risk averse than risk loving. Nonetheless, many of them also find that some persons are risk seekers (Tauer (1986); Love and Robison; King and Oamek). Robison and Lev (1986) argue that even risk averse persons may sometimes act as if they prefer risk. Particularly, when things like bankruptcy laws and ad hoc disaster payments which can affect the relevant probability distributions are ignored and/or incompletely considered.

It is appropriate to give more attention to risk aversion than to risk seeking. However, it does not seem reasonable to consider both risk aversion and preference for risk when a few discrete alternatives are feasible but to almost completely ignore preference for risk when the decision variables are continuous.

We suspect that consideration of preference for risk may be more acceptable if the criteria and models used are analogous to those used when risk aversion is assumed. Target MOTAD is somewhat popular with agricultural economists.[1] One of the desirable features of Target MOTAD for risk averse persons is that it is an easily solved linear programming model. Unfortunately, this feature is not shared by risk lovers' versions of

Target MOTAD. However, the optimal solutions for risk averse and risk lovers' versions do share some characteristics.

Initial Model

One objective of our Target MOTAD models for risk lovers is to have models which are similar to Tauer's version of Target MOTAD. We also want models which are at least as consistent with expected utility maximization as Tauer's model is.

Tauer's version of Target MOTAD is consistent with a family of piecewise linear utility functions. Each member of the family either has, or is a (positive) linear transformation of a function having, the following form:

(1) $U(z) = z + \alpha[\min(0, z - T)]$

where z is return, α is a parameter having a value no smaller than 0 and T is a target level.

It seems reasonable to use a similar family of utility functions as a basis for Target MOTAD for risk lovers. For risk lovers, α must be larger than -1 to ensure positive marginal utility and no larger than 0 to be consistent with preference for risk (and the limiting case of risk neutrality). Note that for both utility function families, the relevant deviations are below target level deviations. For risk lovers, considering deviations above a target or threshold level is an attractive alternative. We consider below target deviations largely to be consistent with Tauer's model.

The (DEUM) model presented in our earlier paper was essentially

(2) Maximize $\Sigma p_i\{y_i + \alpha[\min(0, y_i - T)]\}$

subject to

(3) $Cx - y = 0$,

(4) $Ax \leq b$, and

(5) $x \geq 0$.

In (2), summation (in (2)) is over i, p is an s-element column vector of probabilities associated with the various states of nature, y is an s-element column vector of net returns associated with the states of nature, α is a risk aversion parameter which is larger than -1 and no larger than 0, min is the minimum function and T is a target return level. C is an s by n matrix of returns associated with the enterprises for the various states of nature, x is an n-element vector of enterprise levels, A is an m by n matrix of resource or technical requirements, b is an m-element column vector of resource or technical levels, n is the number of enterprises, m is the number of resource or technical constraints and s is the number of states of nature. We assume that the set of feasible y vectors is bounded from above.

Our initial alternative (INITIAL) model is

(6) Maximize $p'y^k$

subject to

(7) $\Sigma p_i\{abs[min(0, y_i^k - T)]\} \geq \gamma^k$,

(8) $Cx^k - y^k = 0$,

(9) $Ax^k \leq b$, and

(10) $x^k \geq 0$.

In (7), the summation index is i, abs is the absolute value function, γ^k is the k^{th} lower limit on the absolute value of expected deviations and k is an index (not an exponent). A role for k will, we hope, become clearer later. (7) is analogous to the upper limit on expected deviations in Tauer's model. Unlike the expected deviations constraint in Tauer's model, (7) cannot be stated as a set of linear inequalities. This is related to the fact that (7) usually defines a nonconvex set. An undesirable, but apparently unavoidable, aspect of Target MOTAD for risk lovers is that it is a nonlinear rather than a linear programming model.

The Problem

We consider a problem based on data from Anderson, Dillon and Hardaker (ADH). Tauer used the same problem to illustrate his model. Just as he did, we assume that each state of nature is equally likely to occur.

In the balance of this paper, the feasible set defined by (9) and (10) is called the resource feasible set. We use this name to differentiate this set from the (generally) smaller completely feasible set defined by (7) through (10). The most relevant features of the resource feasible efficient set are four corners, A through D, and three edges; AB, AC and AD.

One Solution Strategy

We begin by applying a solution strategy which shows some of the difficulties of solving the model, reveals some characteristics of solutions and is reasonably consistent with a better solution strategy. Consider the problem(s) of finding optimal solutions for three different combinations of T and γ^k; (900, 20), (1000, 55) and (1140, 130).

The best solution for (900, 20) is an "edge" solution on edge AC. The best solution for (1000, 55) is also on edge AC but it is not consistent with expected utility maximization because it violates the requirement that margin utility must be positive. The best solution for (1140, 130) appears to be on edge AB. However, it is inferior to what can be regarded as a mixed strategy for which (a "pure" strategy) corner A is selected with a probability of (approximately) .448 and (another "pure" strategy) corner B is selected with a probability of (approximately) .552.

An Extended Model

The INITIAL model is extended to allow mixed strategies by replacing (6) with

(6') Maximize $p'\Sigma\pi^k y^k$,

restating (7) in equivalent form

(7') $\Sigma p_i\{abs[\min(0, y_i^k - T)]\} - \gamma^k \geq 0$,

and adding constraints

(11) $\Sigma\pi^k\gamma^k \geq \Gamma$,

(12) $\Sigma\pi^k = 1$ and

(13) $\pi^k \geq 0$ for all k.

The index for the summations in (6'), (11) and (12) is k; in (7'), it is i. Each π^k is the probability associated with the pure(r) strategy, k. Note that the γ^k's have changed roles.

They are now variables rather than user selected parameters. Their role as parameters has been assumed by Γ.

The positive marginal utility condition can be stated as

(14) $E^{*} + \Gamma > E' + \Gamma'$

for all

(15) $\Gamma' \leq \Gamma$

where E^{*} is the expected return which solves the extended model ((6'), (7') and (8) through (13)), E' is the optimal expected return associated with the same model but when Γ is replaced by Γ'.

Although our statements of the extended model and the positive marginal utility condition make them seem formidable, they are usually relatively easy to solve and verify (or refute), respectively.

A Better Solution Strategy?

Corners of the resource feasible set are important for risk lovers. If the relevant corners are known, it is relatively easy to solve the revised model and to determine whether the positive marginal utility condition is satisfied.

A Solution Procedure

The following procedure is suggested.

1. Identify the corners of the resource feasible set.[2] Or perhaps just the vector efficient corners.

2. If desired, screen these corners using any criterion(ia) for which the associated set of utility functions includes the family of utility functions which we associate with Target MOTAD for risk lovers.

3. For each (surviving) corner, compute the expected deviations and expected return. Let $\Sigma p'y^k$ ($= E^k$) and γ^k equal the expected return and expected deviations, respectively, for the k^{th} corner.

4. Solve a model consisting of (6') and (11) through (13). It should be obvious that (7'), (8), (9) and (10) will be satisfied. It is true, but possibly less obvious, that enough of the feasible solutions to (7'), (8), (9) and (10) are included.

5. Determine whether the positive marginal utility condition is satisfied by determining whether (14) and (15) are satisfied. In practice, it should neither be necessary to do this for every feasible (for (15)) value of Γ' nor to explicitly solve the extended model for every (or any) feasible value of Γ'. It should be sufficient to consider those corners of the resource feasible set which have expected deviations no larger than Γ and expected returns greater than E^*.

Steps 4 and 5 can be done simultaneously. A potential difficulty with doing that is interpreting any infeasible solutions. Is there no feasible solution because the extended model is infeasible for the selected combination of T and Γ? Or does the optimal solution of the extended model fail to satisfy the positive marginal utility condition?

Steps 1 through 5 find the optimal mixed strategy(ies) if it (they) exists. The next step is optional.

6. Determine whether there is (are) any deterministic (non-random) optimal solution(s). For a deterministic solution to be optimal, its expected deviations must be greater than or equal to Γ and its expected return must equal E^*.[3] In practice, this can be verified (or refuted) by computing a deterministic enterprise activity level vector, x^d, as

(16) $x^d = \Sigma\pi^k x^k$ and

computing the expected deviations associated with x^d. If the expected deviations are greater than or equal to Γ, then x^d is an optimal deterministic (edge or surface) solution.

Relation to Analysis of Simple Example

The resource feasible set has eight corners. Three of them, A, B and C, survived screening by convex set stochastic dominance implementation of the second stochastic inverse dominance criterion (SSID).[4] This validates our previous consideration of only corners A, B, C and D. We have already effectively applied the balance of the suggested solution strategy for three T, Γ combinations. We merely summarize the results now. The best solutions for the first combination are an edge solution on AC and a mixed strategy based on corners A and C. They have the same expected return. There is no solution for the second combination. For the third combination, the best solution is a mixed strategy based on corners A and B.

Comparisons

Comparison with Our First Target MOTAD Model for Risk Lovers

Our (two) versions of Target MOTAD for risk lovers are, in a sense, equivalent and have the same solutions. The current version gives more prominence to mixed strategy solutions. The fact that we are not very comfortable with mixed strategy solutions is one reason why we prefer our first version of Target MOTAD for risk lovers. Another reason is that direct expected utility maximization versions of Target MOTAD for risk lovers and risk averters provide better understanding of the behavior of nearly risk neutral persons.

Comparison with Tauer's Model

Although Tauer's model and the model which we present here are based on different assumptions about risk preferences, they and their solutions have some common characteristics.

Their optimal solutions tend to have the same expected utility as other feasible solutions. For our current model, optimal mixed strategies and optimal edge/surface solutions have, in order to be optimal, the same expected utility as the corners associated with the optimal solutions.

The optimal solutions for risk lovers can be very similar to, or mostly different from, the optimal solutions for risk averters. They are rather similar for the ADH problem. If we compared the optimal solutions for all feasible target levels and limits on expected deviations for risk lovers and risk averters we would find that the set for risk averters is a subset of the set of solutions for risk lovers. For each feasible solution on AB and AC, there is a T, Γ combination for which it is optimal for risk lovers. No optimal solutions for Tauer's model are on the upper part of AB. We illustrated our first version of Target MOTAD for risk lovers by applying it to a problem based on Hazell's data. For that example, only two optimal solutions were shared by Tauer's model and ours. Thus, the degree of similarity of solutions for risk lovers and risk averters depends on the specific problem being considered.

The solutions to Tauer's model and ours also tend to differ in some ways. For our model and other models for risk lovers, an edge or surface solution can never be better than the best mixed strategy solution; the former can at best be just as good. For Tauer's and other risk (averse) programming models, a mixed strategy solution of the type which we describe in this paper can never be better than the best edge, surface or interior solution; the former can at best be just as good.

Concluding Remarks

We have chosen to develop models for risk lovers partly because they represent a neglected area and partly because of our interests. We present them more to provide balance than to encourage others to use them. In fact, we would encourage anyone interested in applying a model for risk lovers to note that Target MOTAD for risk lovers is not much easier to apply than other models for risk lovers.

As we noted earlier in the paper, it would be possible to formulate our Target

MOTAD for risk lovers models on above target rather than below target deviations. Doing that would have made it unnecessary to have a separate positive marginal utility condition but would have slightly complicated the solution maps.

Footnotes

1. An EconLit search identified 16 publications which apply and/or discuss Target MOTAD; an Agricola search identified 29. All of them seem to describe agricultural applications and/or be authored by agricultural economists.

2. It is sometimes more efficient to consider corners in y space because there are usually fewer of them. A corner in y space must correspond to a corner in x space. The converse is not always true. It is true for the ADH problem.

3. The expected return cannot be larger than E^*.

4. Zaraś' (1987, 1989) SSID criterion is (for risk lovers) analogous to second degree stochastic dominance (for risk averse persons).

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