Sorting and blending under quality uncertainty:
Application to wheat protein

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Abstract: Quality uncertainty is of considerable interest to the grain industry. In this paper, uncertainty is incorporated in a model of grain blending decisions. A mixed-integer, nonlinear optimization problem is developed. Simulations illustrate the effect of crop quality on blending decisions, and the effect of blending activities on the distribution of protein within a wheat marketing channel.

Key words: Wheat, grain quality, uncertainty, blending, elevators, protein premiums.

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I. Introduction

An extensive literature has developed on the economics of grain quality. Several studies have applied mathematical programming models to grain quality issues, often from the perspective of elevators or processors who face blending decisions.[1] Other studies have addressed quality premiums and discounts and the implications of different pricing schedules for blending incentives.[2] Common to most of these studies is an assumption that qualities to be blended (or sorted) are known with certainty.[3]

Quality uncertainty has emerged as an important issue for the U.S. wheat industry.[4] Given diverse growing conditions and the number of wheat varieties with different attributes that enter the grain handling system, some amount of quality variability is inescapable. However, there has been little analysis of the effects of routine handling practices—specifically, sorting (segregation) and blending activities—on quality variability within a marketing channel. That is the focus of this paper.

The analysis proceeds from two main insights. First, opportunities for segregation limit the ability of an elevator to control variation in a quality parameter, such as protein. As the number of segregations increase (e.g., from 2 to 3, 4, 5 or more), quality variation within individual grain bins is reduced. Grain loaded into an elevator is 'stacked' in order of arrival. If protein is normally distributed within individual truckloads (with different mean values, and possibly different standard deviations), the protein within the elevator's bin (containing multiple truckloads) will be a mixture of normal distributions.

Second, blending allows elevators to control not only the mean value of quality attributes but also their dispersion. This has implications for the sampling risk associated with standard

[1] Early examples include Schruben (1968) and Ladd and Martin (1976). Others include Johnson and Wilson (1993), and Yoon, Brorsen, and Lyford (2002).
[2] Hennessy (1996) and Hennessy and Wahl (1997) relate the curvature of premium/discount schedules to blending/segregation decisions. Sivaraman, Lyford and Brorsen (2002) develop a general framework for grain blending and segregation.
[3] An exception is a paper by Wilson and Preszler (1992). They used chance-constrained programming to analyze the effects of quality uncertainty on wheat procurement decisions by a flour miller.
[4] Wilson and Dahl (1999) cite "variability in quality due to sampling and grade errors [as] a major concern for exports, domestic shipments, and farmer deliveries" (p. 210). They also point to earlier USDA-ERS and OTA reports, which suggested that inconsistent quality might place U.S. wheat exports at a competitive disadvantage in world markets.

tests for grain quality. When grain from two sources is blended together in known proportions, the resulting variance is lower than a weighted average of input variances. (This is analogous to variance reduction in stock portfolios.)

The objectives of this paper are: 1) to assess how sorting and blending activities affect the variability of grain quality attributes; and 2) to develop a firm-level optimization model that incorporates quality uncertainty, and analyze the effects of this uncertainty on blending decisions. The model is framed from the perspective of an elevator that seeks to maximize the value of its wheat sales, given blending opportunities and market premiums for protein. Special features cause this to be specified as a mixed-integer, nonlinear programming problem. A simulation analysis, based on crop-quality data from the Northern Plains region, provides insight into the interplay between price schedules, blending, and quality variability.

The paper is organized as follows. The next section provides background relevant to the analysis—on protein premiums, quality uncertainty, and the economics of blending. The mathematical programming problem is specified in the third section. The fourth section presents simulation results, which illustrate blending margins and the effects of blending on protein distributions within a marketing channel. The paper concludes with some discussion and ideas for further research.

II. Background and Concepts

Premium and discount schedules

In cash grain markets, prices for wheat protein are usually specified through step schedules, with premiums and discounts at specified break points. Figure 1 shows two variants of a protein schedule for hard red spring (HRS) wheat. The reference grade is 14% protein (grade number 1, milling quality). Both schedules provide a 30 c/bu premium for protein in excess of 15% and a 20 c/bu discount for protein less than 14%; however, one schedule uses percentage-point steps while the other uses smaller increments. Increments of a half, fifth, or tenth percentage point are used at different times by industry participants (i.e., millers or grain merchandisers), depending on procurement needs and market conditions.

Step functions pose some difficulties for economic analysis. Because they are discontinuous and not differentiable at breakpoints, they do not lend themselves to the type of comparative statics used by Hennessy (1996) in his analysis of blending incentives. A study by Hennessy and Wahl (1997) showed that piecewise linear functions could capture the incentives provided by step functions (as applied to dockage discounts and wheat cleaning decisions).[5] Other authors have also used linear relationships to approximate the value of wheat protein. [6] Figure 2 provides an illustration of piecewise linear (spline) schedules. Both schedules shown imply a 30-cent premium for 15% protein, and a 20-cent discount for 13% protein (as in Figure 1). However, one schedule is capped at these values, while the other extends in both directions.

Although linear approximations can be made to pass through given breakpoints, they do not imply the same expected values (premium or discount) as step functions when qualities are random. Consider an example where protein is normally distributed, with mean 14.5% and standard deviation 0.5. Under the whole-percentage schedule shown in Figure 1, the expected monetary value of protein would be

$$\begin{aligned}\text{Expected value (c/bu)} &= (-20)\ \text{prob}(\ x < 14) + (30)\ \text{prob}(\ x \geq 15) \\ &= (-20)\ \Phi(14;\ 14.5,\ 0.5) + (30)[1 - \Phi(15;\ 14.5,\ 0.5)] \\ &= 1.6\end{aligned}$$

where $\Phi(x\ ;\ \mu\ ,\ \sigma)$ is the cumulative distribution function for a normal variable. Similar expressions, with more numerous terms, apply to step functions with smaller protein intervals (half, fifth, or tenth percent). Contrast this expression to those for the spline functions shown in Figure 2. When premiums and discounts are unbounded, we have

$$\begin{aligned}\text{Expected value (c/bu)} &= (-20)\ [14 - E(\ x \mid x \leq 14)] + (30)[E(\ x \mid x \geq 14) - 14] \\ &= (-20)\ [\ 14 - 13.737] + (30)\ [14.644 - 14] \\ &= 15.4\end{aligned}$$

where $E(x \mid \cdot\ )$ is a conditional expectation and 14 is the joint of the spline function.[7] When premiums and discounts are bounded as shown, the value is given by

$$\begin{aligned}\text{Expected value (c/bu)} &= (-20)\ [14 - E(\ x \mid x \leq 14)] + (30)[E(\ x \mid x \geq 14) - 14] \\ &\quad - (-20)\ [13 - E(\ x \mid x \leq 13)]\Phi(\ 13\ ;\ 14.5,\ 0.5) - (30)[E(\ x \mid x \geq 15) - 15]\Phi(\ 15\ ;\ 14.5,\ 0.5) \\ &= 14.2.\end{aligned}$$

[5] As noted by Sivaraman et al. (2002, p. 158), the rationale for a piecewise linear approximation developed by Hennessy and Wahl does not apply to attributes like wheat protein. Physically, dockage can be removed from wheat and then recombined in arbitrary proportions. Mixing of protein (two or more lots) is not reversible.
[6] An example is the study by Baker, et al. (2003), which analyzes trade-offs between wheat yield and protein values.
[7] Greene (pp. 898 -899) provides formulas for moments of truncated normal distribution.

Figure 3 compares the expected value under different versions (step-wise and linear) of the premium/discount schedule, and with different assumptions about the standard deviation of protein. Step functions will be used in the remainder of the paper to characterize protein premiums and discounts.

Distribution of protein: blends vs. mixtures

When two lots of grain are blended together, the mean value of protein (and most other quality attributes) is a weighted average. If the two lots are each normally distributed, the blend—as a linear combination of normal variables—will also be normal. Formulas for the mean and variance of a blend are shown in Table 1.

Also shown in Table 1 are formulas for grain that is mixed but not blended.[8] Unblended mixtures occur when different lots of grain are loaded into the same storage container or vessel in sequence, rather than simultaneously. Within an individual elevator bin, for example, there may be multiple truckloads of wheat with different levels of protein—their physical positions reflecting order of arrival, with most recent grain at the top of the bin. In this situation, the distribution of protein can be described as a mixture of (normal) distributions.[9]

The distinction between blends and mixtures is illustrated in Figure 4. Two lots of wheat are represented. Protein in the first lot is distributed $Z_1 \sim N(14.5, 0.3)$, and protein in the second lot is distributed $Z_2 \sim N(15.5, 0.3)$. With (arbitrary) blending proportions of 20/80, the blend has mean protein of $0.2\,(14.5) + 0.8\,(15.5) = 15.3$, and standard deviation[10] of

$$\sigma_B = [(0.2)^2\,(0.3)^2 + (0.8)^2\,(0.3)^2]^{1/2} = 0.247.$$

In this example, the blend has lower variance than the individual lots. The mixed distribution, using the same proportions, has a much larger standard deviation, $\sigma_C = 0.5$, although the mean is identical to that of the blend. As shown in Table 1, the variance of protein in the mixture is calculated as the mean of conditional variances plus the variance of conditional means.[11]

Effects of segregation

[8] Mixed grain might also be called 'commingled'.
[9] McLachlan and Peel (2000) provide a comprehensive introduction to statistical applications of mixture models.
[10] By assumption, errors (in measurement or sampling of protein) for the two lots are uncorrelated.
[11] See Lindgren, p. 130.

Grain elevators routinely segregate spring wheat based on protein content. This creates opportunities for blending to capture protein premiums or meet contract specifications. Elevators with more numerous bins have greater latitude in their blending and marketing decisions. The number of storage bins can also affect the variability of protein in an elevator's grain shipments.

To illustrate this point, a simulation was performed in which 120 lots of grain (truckloads) were allocated to bins and then blended together. Protein levels differed across lots, but the 120 lots were drawn from a distribution representative of actual protein levels in 2004.[12] Lots were allocated to bins in two different ways—randomly (without regard to protein), and based on protein content. By assumption, wheat from all bins is blended together prior to shipment.[13] Interest centers on the standard deviation of protein in outbound shipments, and the effect of different numbers of storage bins.

Results are shown in Figure 5. As the number of bins increases, the standard deviation of protein in outbound shipments declines, even when individual lots are assigned to bins randomly. Assigning lots based on protein content leads to a (relatively) small additional reduction in standard deviation. The implication is that blending, rather than segregation based on protein levels, accounts for most of the observed reduction in variance.

A simple blending problem

Suppose there are two lots of wheat with protein distributed normally:

$$Z_1 \sim N(14.5, 0.3)\ ;\ \ Z_2 \sim N(15.5, 0.3)$$

A 30 c/bu premium applies for protein at or above 15%, and a 20 c/bu discount applies for protein below 14%. If protein were known with certainty, it would be optimal to blend the two lots in equal proportions, as shown in Figure 6. Based on a 50/50 blend, protein would be exactly 15% and the higher price applies. But given uncertainty about protein, the objective is to maximize the *expected* value of sale premiums. Assume both lots consist of one bushel, and blending is costless. The problem is:

[12] The analysis used crop quality data for Northeast North Dakota (CRD 3) in 2004, as analyzed by the Cereal Science Department of North Dakota State University. A mixture of two normal distributions was used to approximate the cdf implied by actual grain samples. For individual lots, a standard deviation of 0.5 was assumed.
[13] Note that blending does not follow any economic criteria in this illustration.

$$\max_{x_1,x_2,y_1,y_2} \quad B[prob(z_B \le 14)(-20) + prob(z_B \ge 15)(30)]$$
$$+ C[prob(z_C \le 14)(-20) + prob(z_C \ge 15)(30)]$$

subject to

$B = x_1 + x_2$	(total blend quantity)
$C = y_1 + y_2$	(total unblended quantity)
$x_1 + y_1 = 1$	(quantity available, lot 1)
$x_2 + y_2 = 1$	(quantity available, lot 2)
$x_1, x_2, y_1, y_2 \ge 0$	(non-negativity constraints)

where x_i is the quantity allocated to the blend from lot i (i=1,2); y_i is the residual quantity from lot i (not blended); z_B is the protein % in the blend, and z_C is protein in grain not blended prior to sale. Formulas for probabilities are provided in Table 1. The solution values are: x_1=0.469, x_2 = 1, y_1 = 0.531, y_2 = 0. Mean protein in the blend is $\bar{z}_B = 15.18$. Note that this is higher than in the certainty case—even though the decision-maker is assumed to be risk-neutral.

Figure 7 provides some insight into the relative value of blends versus unblended grain. Using optimal blending proportions from the same problem, the figure shows cumulative distribution functions for the blend and an unblended mixture. For comparison, cdfs for the two original lots are also shown. Vertical lines are placed at breakpoints in the protein price schedule. Note that the cdf for the blend lies below that for the mixture at 15% protein, indicating that the blend has a greater probability of earning the 30 c/bu price premium. Conversely, the mixture cdf lies above the blend cdf at 14% protein, indicating a higher probability of triggering the 20 c/bu discount.

The two vertical lines divide Figure 7 into three sections. These have significance for the relative returns of blends and (unblended) mixtures. If the cross-over point (where cumulative probabilities are equalized) for the blend and mixture occurs in the first (left-most) section, then the mixture offers a higher return. If the cross-over point occurs in the third (right-most) section, as shown, the blend is superior. If the cross-over point occurs in the middle section, then relative returns depend on the relative magnitudes of the premium and discount.[14]

A limitation of Figure 7 is that it assumes fixed proportions, both for the blend and the mixture. Figures 8-11 provide a different perspective of the blending problem, one showing the overall return from different blending alternatives. These figures show a sequence of blending problems, each with a different assumption about protein in the first lot. Thus in Figure 8, z_1 is distributed as N(12, 0.3); other parameters are identical to those shown earlier. Higher levels of mean protein are assumed in Figures 9, 10, and 11. Each figure is a contour graph showing expected sales value, including values of both blended and unblended grain. Coordinates in the graph are the amounts blended. Thus, (0,0) corresponds to a zero blend; (0.5, 1.0) corresponds to a blend of one-half bushel from lot 1 with 1 bushel from lot 2; and the upper right corner, (1,1) represents complete blending of available grain.

Optimal solutions are marked with an asterisk. In Figure 8, three corner solutions are equivalent; they include zero values for one or both lots, implying no blending. In Figure 9, with 13% expected protein in lot 1, blending is approximately optimal[15] in proportions (0.1 , 1.0). A small amount of lower-protein wheat is blended with all of the high protein wheat. Note the non-convexities in the payoff surface; as discussed below, non-convexities are a feature of blending under uncertainty that adds to the computational difficulty of optimization models. In Figures 10 and 11, expected protein in lot 1 is increased in further 1% increments. The optimal solution includes higher proportions of lower-protein wheat until finally (Figure 11) all grain is blended. This last solution is driven by the variance reduction in blended wheat. Given the assumed price schedule with both lots having at least 15% mean protein, there would have been no gains from blending in a certainty model.

To summarize, blending incentives depend on quality distributions and on market premiums or discounts. Blending proportions affect not only the mean, but also the standard deviation of quality in blended grain.

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[14] Expected returns are given by $\pi_i = prob(z_i \le 14)D + prob(z_i \ge 15)P \quad (i = B, C)$, where P is the premium and D is the discount (D<0). Expected returns for the blend and mixture are equalized if

$$\frac{prob\ (z_C \le 14) - prob\ (z_B \le 14)}{prob\ (z_B \ge 15) - prob\ (z_C \ge 15)} = \frac{P}{D}\ .$$

III. A Mathematical Programming Model of Blending

The optimization model presented in this section adds complexity to the blending problem. The model adopts the perspective of a country elevator with wheat in two storage bins. Each bin is assumed to contain several lots (truckloads) with different mean protein levels. Lots are placed in bins in a specific order corresponding to farmer deliveries, and must be removed (either for blending or direct loading into railcars) in the same order; this constrains the elevator's blending opportunities. The elevator manager seeks to maximize the expected value of wheat sales given a known price schedule for protein.

Formally, the function to be maximized is the expected value of sales revenue minus costs associated with blending:

$$R = \sum_k \left(\begin{array}{c} p + PROB(Z_k \le b_1)\, prem_1 \\ + \sum_{m=2}^{M-1} \left[PROB(Z_k \ge b_m) - PROB(Z_k \ge b_{m+1}) \right] prem_m \\ + PROB(Z_k \ge b_M)\, prem_M \end{array} \right) XK_k \quad - \quad \sum_i \sum_j \sum_k c \cdot Y_{ijk} \qquad (1)$$

Endogenous variables are indicated by capital letters and fixed parameters by small letters; all are defined in Table 2. The expression in large parentheses is the probability-weighted price (\$/bu), and the last term applies a fixed cost to each blend. The maximization (1) is subject to the following constraints:

$$\sum_k X_{ijk} \le q_{ij} \qquad \forall\, i, j \qquad (2)$$

$$XK_k = \sum_i \sum_i X_{ijk} \qquad \forall k \qquad (3)$$

$$PL_k = \sum_i \sum_j X_{ijk} \cdot pro_{ij} \,/\, XK_k \qquad \forall k \qquad (4)$$

$$PS_k^2 = \sum_i \sum_j X_{ijk}^2 \cdot sdp_{ij}^2 \,/\, XK_k^2 \qquad \forall k \qquad (5)$$

$$ALPHA_{km} = (b_m - PL_k) / PS_k \qquad \forall\, k, m \qquad (6)$$

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[15] Figures 8-11 were developed by evaluating the expected sales value over a predetermined grid. Hence the

$$PROB(Z_k \le b_m) = \Phi(ALPHA_{k\,m}) \qquad \forall k, m \tag{7}$$

$$PROB(Z_k \ge b_m) = 1 - \Phi(ALPHA_{k\,m}) \qquad \forall k, m \tag{8}$$

$$Y_{i\,j\,k} \cdot q_{i\,j} \ge X_{i\,j\,k} \qquad \forall i, j, k \tag{9}$$

$$\sum_j Y_{i\,j\,k} \le 1 \qquad \forall i, k \tag{10}$$

$$\sum_{k'<k} X_{i\,j'k'} \ge Y_{i\,j\,k} \cdot q_{i\,j'} \qquad \forall i, j, k\ ; \forall j' < j \tag{11}$$

$$X_{i\,j\,k} \ge Y_{i\,j\,k} \qquad \forall i, j, k \tag{12}$$

$$X_{i\,j\,k} \ge 0\ ;\ Y_{i\,j\,k}\ binary. \tag{13}$$

Constraints (2) and (3) ensure material balance. Constraint (4) defines the mean protein level in blend k, and constraint (5) defines the standard deviation of protein. Constraints (6) through (8) define probabilities contained in the objective function; Φ signifies the cumulative normal distribution function. Constraints (9) through (12) involve a binary variable, $Y_{i\,j\,k}$, indicating whether grain has been allocated to a given blend. Constraint (9) ensures that if $X_{i\,j\,k} > 0$, then $Y_{i\,j\,k} = 1$. Constraint (10) ensures that blends contain grain from no more than one lot per bin. The next constraint (11) makes sure that lots within bins are used up in proper sequence. Finally, (12) imposes a minimum (1 bushel) quantity when grain allocations are non-zero.[16]

The model divides the elevator's grain sales into a series of blends. These include degenerate blends—those consisting of grain from only one lot. By construction, all blends are distributed normally. With the introduction of binary variables (for the sequencing constraint), the model becomes a mixed-integer, nonlinear programming problem (MINLP).

This is a challenging class of problems from a computational point of view. Experience with the blending model confirms that selection of feasible starting values can be essential to the success of optimization routines. Even then, because of non-convexities in the constraints, there is no assurance that the optimum reported by a solver is actually global. This suggests a strategy of experimentation with different, feasible starting values, and comparing results to identify the starting values most likely to yield a global optimum.[17]

"optimal" solutions are only approximate.

[16] Ruling out extremely small allocations appears to improve model performance.

[17] Placing reasonable bounds on variables, and scaling of variables and equations, are also important. To prevent 'division by zero' errors, equations (4), (5), and (6) can be rearranged, with variables in denominators moved to the left hand side.

An alternative would be to ignore the differences in protein levels within individual grain bins, i.e., assume that grain within each bin is drawn from a single, normal distribution. In that case, the optimization model would be similar to that shown earlier—a nonlinear problem, but with only continuous variables. Simulations reported in the next section will illustrate the difference between these approaches.

IV. Simulation Analysis

To make the analysis concrete, the problem is set up for an elevator with two bins (I=2), each containing three lots of wheat (J=3). The price schedule for protein is shown in Figure 12; these show values relative to the nearby Minneapolis futures price quoted on September 30, 2004 by Market FAX, a news service of *Milling and Baking News*. Protein was in relatively short supply; this contributed to relatively high protein premiums for the 2004 spring wheat crop. Rather than using a single set of quality parameters, the contents of the bins are determined through a simulation exercise. The simulation involves drawing six lots of grain, each with different mean protein, from a distribution that is representative of crop quality in a given region.

Two regions were chosen for illustrative purposes: NASS Crop Reporting District 2 in Montana, and CRD 3 in North Dakota. For each region, empirical cdfs of protein were developed from sample data collected as part of the annual spring wheat quality assessment.[18] The protein distribution was then approximated by a mixture of two normal distributions; this provided a reasonably good fit to the empirical data. Figure 13 shows the empirical distribution and the mixed-normal approximation for CRD 2 in Montana.

In each round of the simulation, six protein levels (lots of grain) are drawn from the mixed-normal distribution for a region. These can be interpreted as expected protein levels (pro_{ij}) for individual lots, based on results of protein tests conducted on grain delivered to the elevator. Protein levels in each lot are assumed to have the same standard deviation, and errors (sampling and measurement) are uncorrelated across lots. Lots are divided into two groups; the highest three protein levels are assigned to bin 1, and the lowest to bin 2. No sorting occurs within the two groups so that there is no necessary order of protein levels (lots) within individual bins. The model is solved for each drawing of six protein levels, and this is repeated 100 times.

[18] Data are from the Department of Cereal Science, North Dakota State University, Fargo.

This provides a way to estimate the *expected* returns to blending, given a price schedule and distribution of protein.

In addition to the MINLP model, a nonlinear programming (NLP) model is also solved. This omits the detailed information about individual lots, treating grain within each bin as though it were drawn from a single normal distribution. For comparability, the NLP model uses average protein levels and standard deviations (for 2 bins) that are consistent with information contained in the MINLP problem.[19]

Perceived opportunities for blending are different in the two problems, leading to different solutions. The NLP problem uses less information and involves less exacting quality control. It is introduced to help quantify the value of more refined information about grain quality. This involves three steps:

- The NLP problem is solved, yielding quantities blended (or sold without blending) from each storage bin.
- Blend proportions and quantities from the NLP problem are imposed as constraints in the MINLP problem. This allows correct measurement of expected sales revenue, given the actual distribution of protein within individual lots and the associated blends.
- The MINLP problem is solved without these constraints.

The difference in expected sales revenue between steps (2) and (3) can be attributed to better information and quality management, including an ability to adapt blending proportions in response to changing qualities within bins.

Results from simulations vary across regions. Using the protein distribution for CRD 2 in Montana, results suggest that with superior quality management, elevator margins could be increased by about 2.2 cents per bushel, averaged over all grain handled. Results for CRD 3 in North Dakota provide a sharp contrast. Using that protein distribution, the change in elevator margins is only 0.2 cents per bushel. The difference is explained by low levels of protein in North Dakota's CRD 3, which reduced opportunities for profitable blending. Blending was much more frequent in the Montana simulation.

The distribution of protein within a marketing channel can change as a result of blending incentives. This point is illustrated in Figure 14. The line marked 'unblended' is the cdf for protein in Montana's CRD 2, based on a mixed-normal approximation. The line marked ‘blended’ is the result of the simulation exercise, reflecting the cumulative effects of blending operations in response to market price signals. Blending has shifted the distribution of protein—upwards at high protein levels (above 15%), and downwards at lower levels. In effect, blending operations have reduced the amount of high-protein wheat sold directly to buyers in terminal markets.

V. Discussion and Ideas for Future Research

Protein premiums display great variability over time due to changing supply conditions. The 2004 Hard Red Spring (HRS) crop was large, thanks to record yields, but was characterized by low average protein content. In combination with lower supplies of Hard Red Winter wheat (a competing class), the shortage of high-protein spring wheat in 2004 led to rapid escalation of premiums in the post-harvest period—reaching $1.70/bu for 15% protein in early November (delivered Minneapolis, basis nearby futures). Price spreads of this magnitude would seem to offer strong incentives for blending. Yet as the simulations reported here show, much depends on the distribution of grain quality. Elevators in Montana CRD 2 were better positioned to take advantage of blending opportunities than those in North Dakota CRD 3 due to differences in local quality conditions. This leads to two observations. First, elevators (and spring wheat producers) may choose to hold stocks of high-protein wheat for one or more years. Such a strategy pays off handsomely in a year like 2004. Second, there is an important spatial dimension to grain quality. Presumably, handlers with facilities in multiple locations are able to capture returns to blending that are not available to smaller, local grain elevators.

The mathematical programming model omits much of the detail and complexity of actual grain handling and blending operations. Elevators typically sort and blend wheat according to multiple quality criteria. In addition to protein, there are both grade (e.g., test weight, damage, defects) and non-grade factors (falling number) that determine whether wheat is suitable for milling. The model included only one price schedule. A more realistic setting would include different marketing alternatives—price schedules, destinations, and (potentially) shipping modes.

The model could be extended into a principal-agent framework. The principal (a milling company) is interested in obtaining grain with particular characteristics—mean protein, say, with specified (maximum) risk of protein falling below some level. The agent (elevator) assembles grain for shipment, blending in response to price signals. The agent has marketing alternatives.

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[19] A formula for variance of protein in commingled (not blended) grain is shown in Table 1.

Hence, the principal's problem is to choose a price schedule that will induce participation by the agent and reward quality management (blending or not blending) at minimum cost To extend the model in this direction, a second price schedule could be added. Parameters of the second schedule (that of the principal) could be varied, and the agent's revenue-maximization problem solved, in an iterative way until optimality (from the principal's perspective) is established. Essentially, this turns price schedules into an endogenous variable. It provides a way to analyze price schedules, and their effects on quality, from both buyer and seller perspectives.

Much of this paper has been devoted to the economics of blending. Segregation is another aspect of grain handling that poses difficult analytical problems when uncertainty is introduced. When grain is delivered to an elevator it must be allocated to one of several storage bins. Suppose deliveries are heterogeneous in quality terms; then in allocating a lot (truckload) to a particular bin, the elevator manager changes the distribution of quality factors within that bin. Allocations to bins change the elevator's blending opportunities. Framing this as an optimization problem, while taking into account the likely pattern of future deliveries and sale opportunities, would provide useful insights into segregation strategies.

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Table 1. Distribution of protein in blends and unblended mixtures.

Notation

$z_i \sim N(\bar{z}_i\ ,\sigma_i)$ $(i=1,2)$	Protein level in lot i, normally distributed. Assume that measurement and sampling errors are uncorrelated across lots.
x_i	Quantity blended.
$s_i = \dfrac{x_i}{x_1 + x_2}$	Share in blend.
y_i	Quantity not blended.
$w_i = \dfrac{y_i}{y_1 + y_2}$	Share in grain not blended.
$Q_i = x_i + y_i$	Quantity available in lot i.
$\Phi(k\ ;\mu\ ,\sigma)$	Normal cumulative distribution function: integral from -∞ to k, given mean μ and standard deviation σ.

Formulas

	Quantity (bu)	Mean protein (%)	Variance of protein	Prob ($z \le k$)
blend	$B = x_1 + x_2$	$\bar{z}_B = s_1\,\bar{z}_1 + s_2\,\bar{z}_2$	$\sigma_B^2 = s_1^2\sigma_1^2 + s_2^2\sigma_2^2$	$\Phi(k\ ;\ \bar{z}_B\ ,\sigma_B)$
commingled, not blended	$C = y_1 + y_2$	$\bar{z}_C = w_1\,\bar{z}_1 + w_2\,\bar{z}_2$	$\sigma_C^2 = [w_1\sigma_1^2 + w_2\sigma_2^2] + [w_1(\bar{z}_1 - \bar{z}_C)^2 + w_2(\bar{z}_2 - \bar{z}_C)^2]$	$w_1\Phi(k\ ;\bar{z}_1\ ,\sigma_1) + w_2\Phi(k\ ;\bar{z}_2\ ,\sigma_2)$
total grain	$T = B + C$	$\bar{z}_T = \left(\dfrac{B}{T}\right)\bar{z}_B + \left(\dfrac{C}{T}\right)\bar{z}_C$	$\sigma_T^2 = \left[\left(\dfrac{B}{T}\right)\sigma_B^2 + \left(\dfrac{C}{T}\right)\sigma_C^2\right] + \left[\left(\dfrac{B}{T}\right)(\bar{z}_B - \bar{z}_T)^2 + \left(\dfrac{C}{T}\right)(\bar{z}_C - \bar{z}_T)^2\right]$	$\left(\dfrac{y_1}{T}\right)\Phi(k\ ;\bar{z}_1\ ,\sigma_1) + \left(\dfrac{y_2}{T}\right)\Phi(k\ ;\bar{z}_2\ ,\sigma_2) + \left(\dfrac{B}{T}\right)\Phi(k\ ;\bar{z}_B\ ,\sigma_B)$

Table 2. Notation in mathematical programming model.

Sets	
$i = 1, \ldots, I$	Storage bins
$j = 1, \ldots, J$	Lots
$k = 1, \ldots, K$	Blends
$m = 1, \ldots, M$	Steps in protein schedule
Endogenous variables	
X_{ijk}	Grain (bu) from bin i, lot j, allocated to blend k
XK_k	Total grain (bu) allocated to blend k
PL_k	Mean protein level (%) in blend k
PS_k	Protein standard deviation (%) in blend k
$ALPHA_{km}$	Intermediate variable used in calculation of probabilities
$PROB(\ )_{km}$	Probability of specified protein level
Y_{ijk}	Binary variable indicating inclusion of X_{ijk} in blend k
Fixed parameters	
q_{ij}	Quantity available (bu) in bin i, lot j
p	Base price ($/bu)
$prem_m$	Premium ($/bu) associated with step m in protein schedule
b_m	Breakpoint (%) in protein price schedule
c	Fixed cost ($) applied to each blend
pro_{ij}	Protein level (%) in bin i, lot j
sdp_{ij}	Standard deviation of protein (%) in bin i, lot j

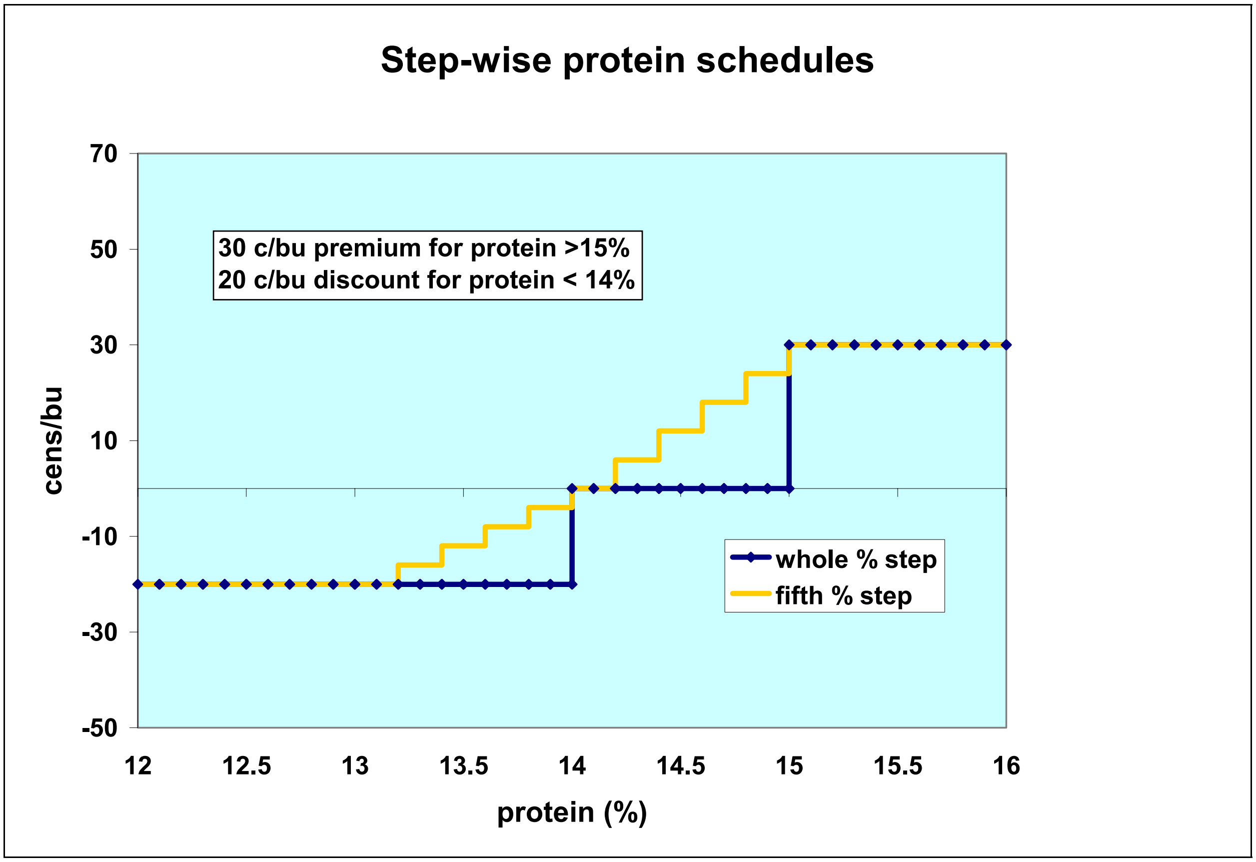

Figure 1. Step-wise protein schedules.

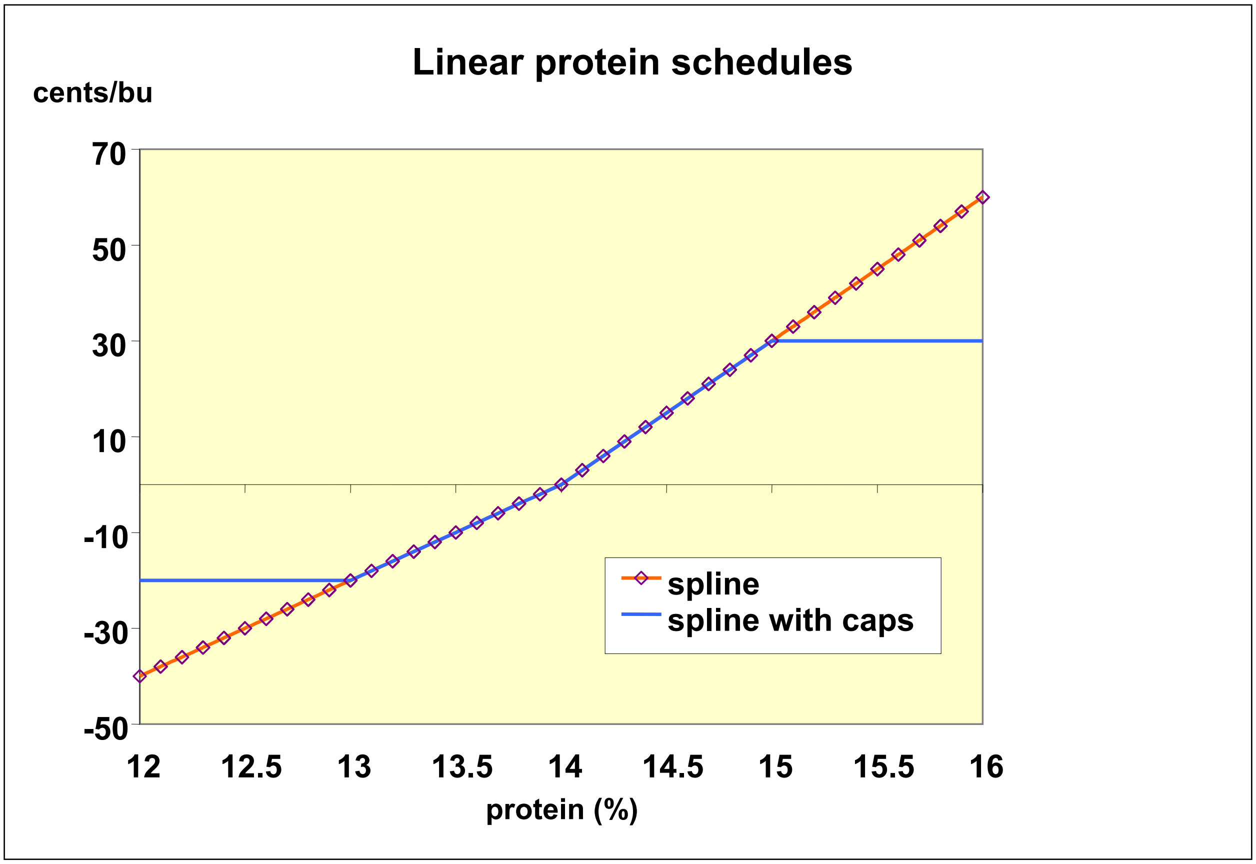

Figure 2. Linear approximations to protein schedules.

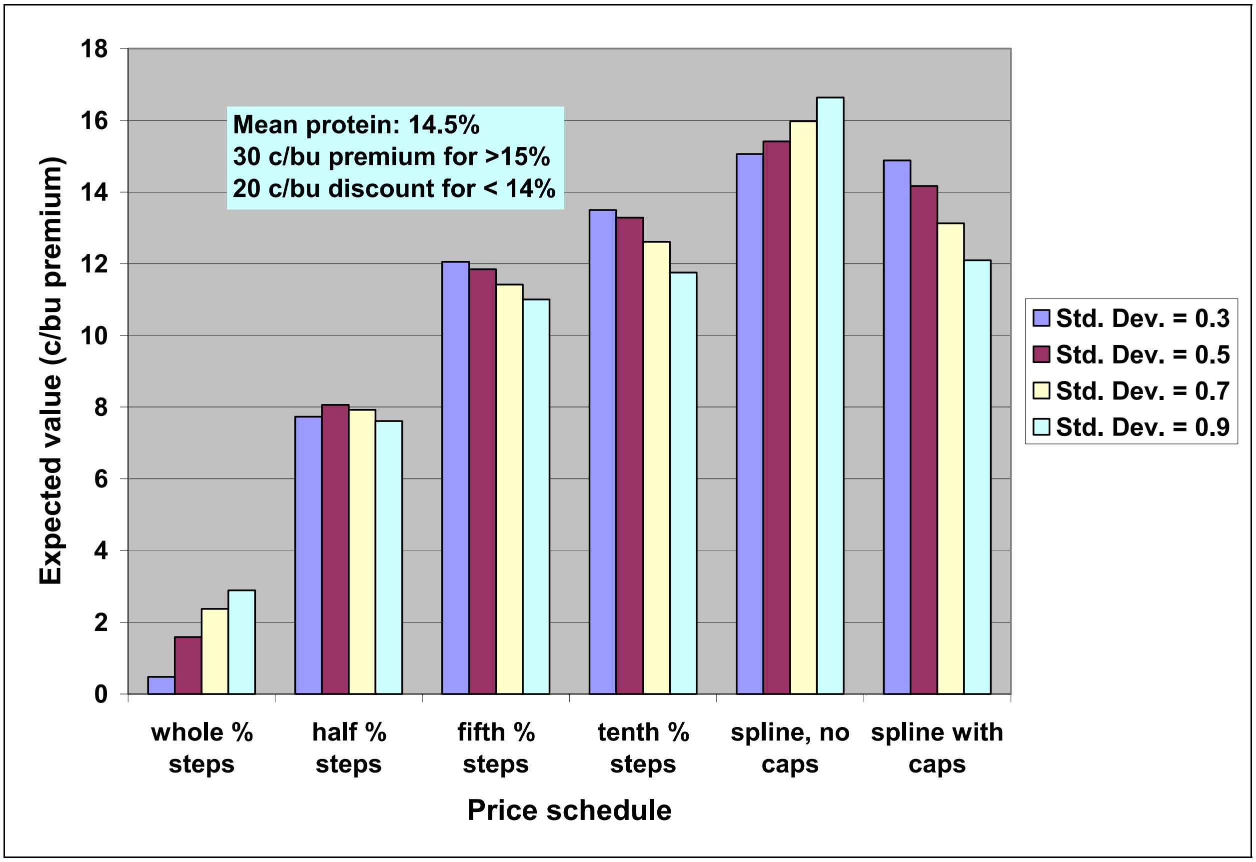

Figure 3. Expected monetary value of protein under different price schedules.

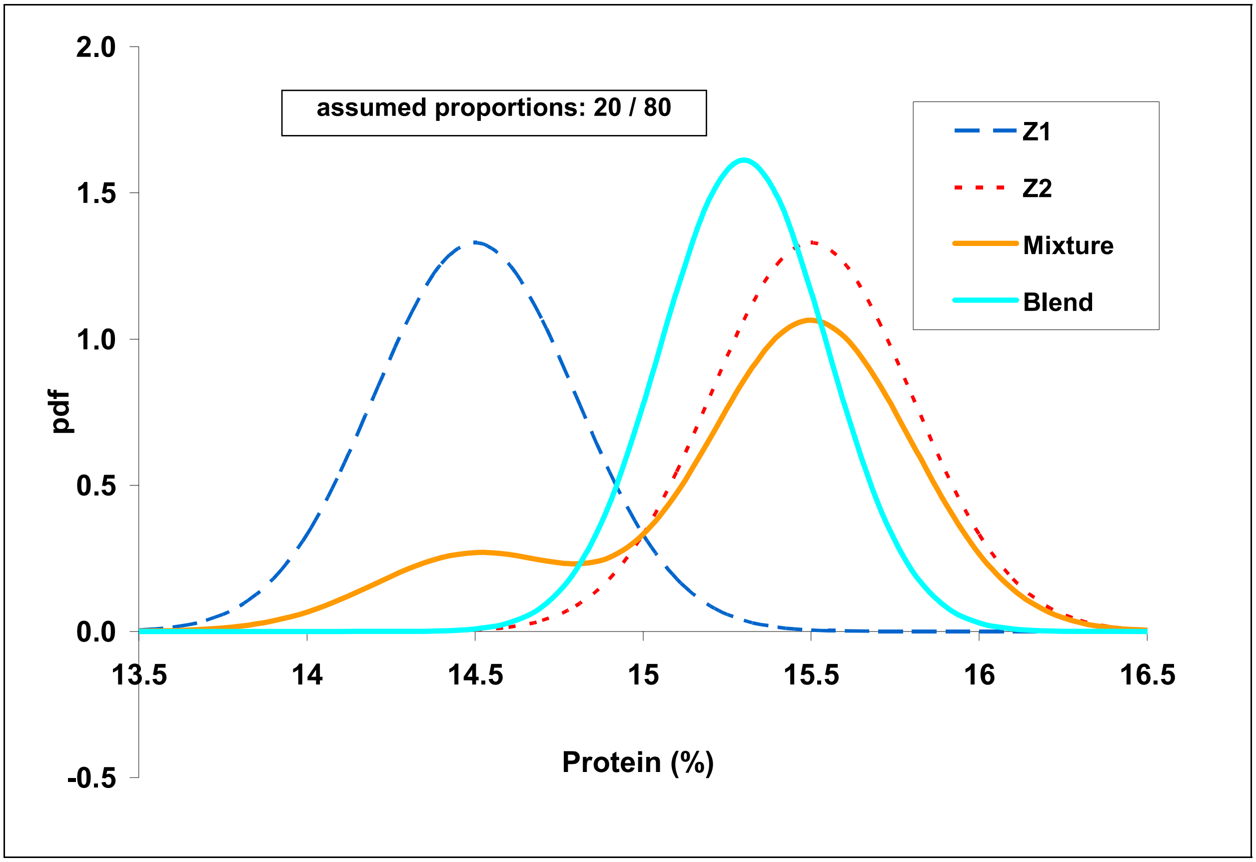

Figure 4. Distributions of blend and unblended mixture.

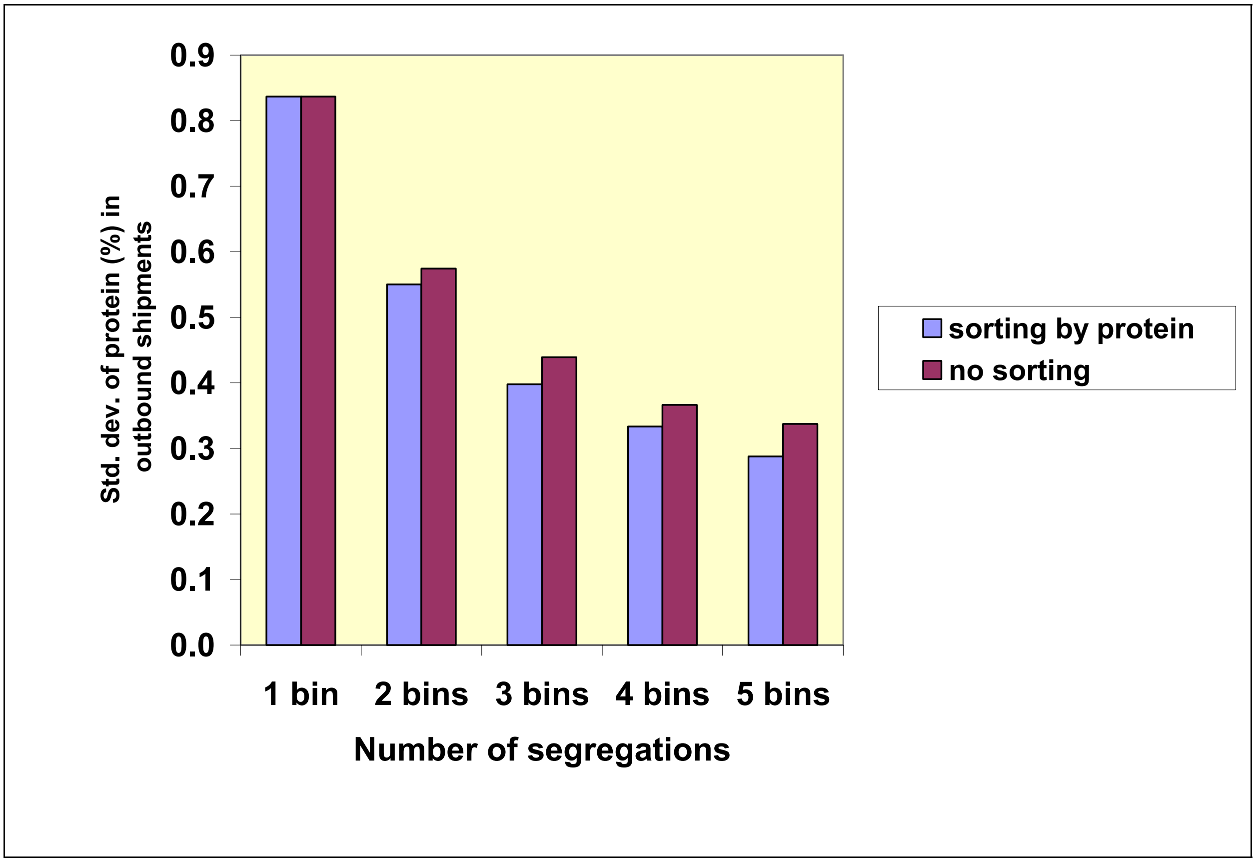

Figure 5. Impact of segregation on variability of protein

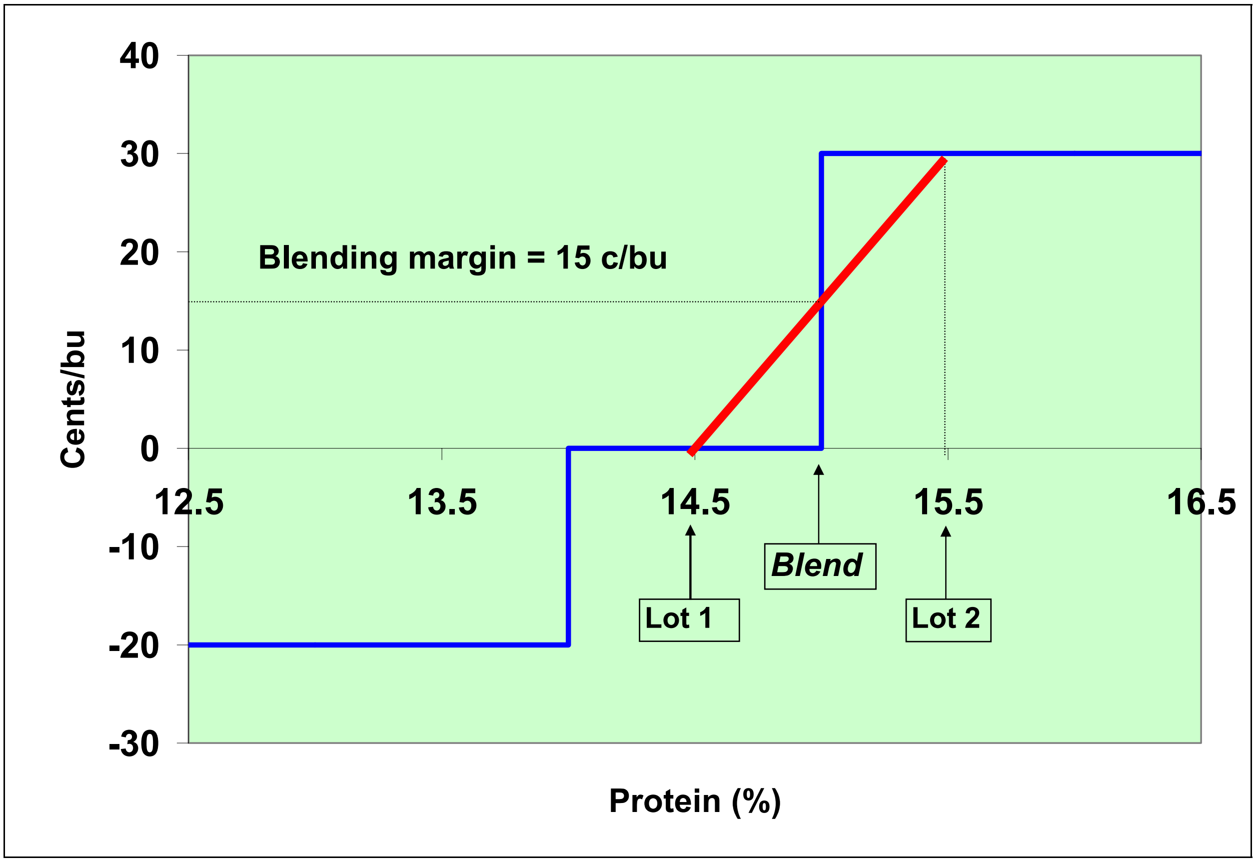

Figure 6. Returns to blending under certainty.

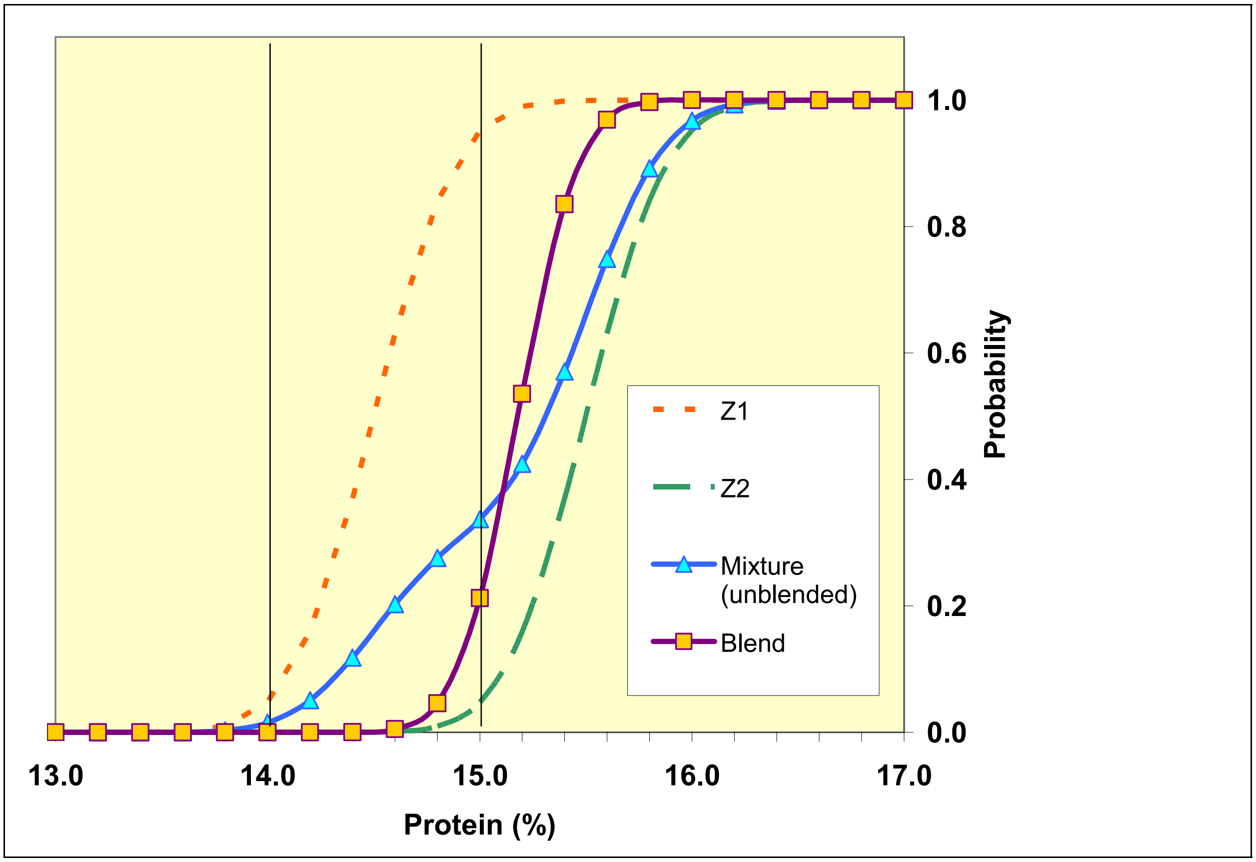

Figure 7. CDFs of blended and unblended grain.

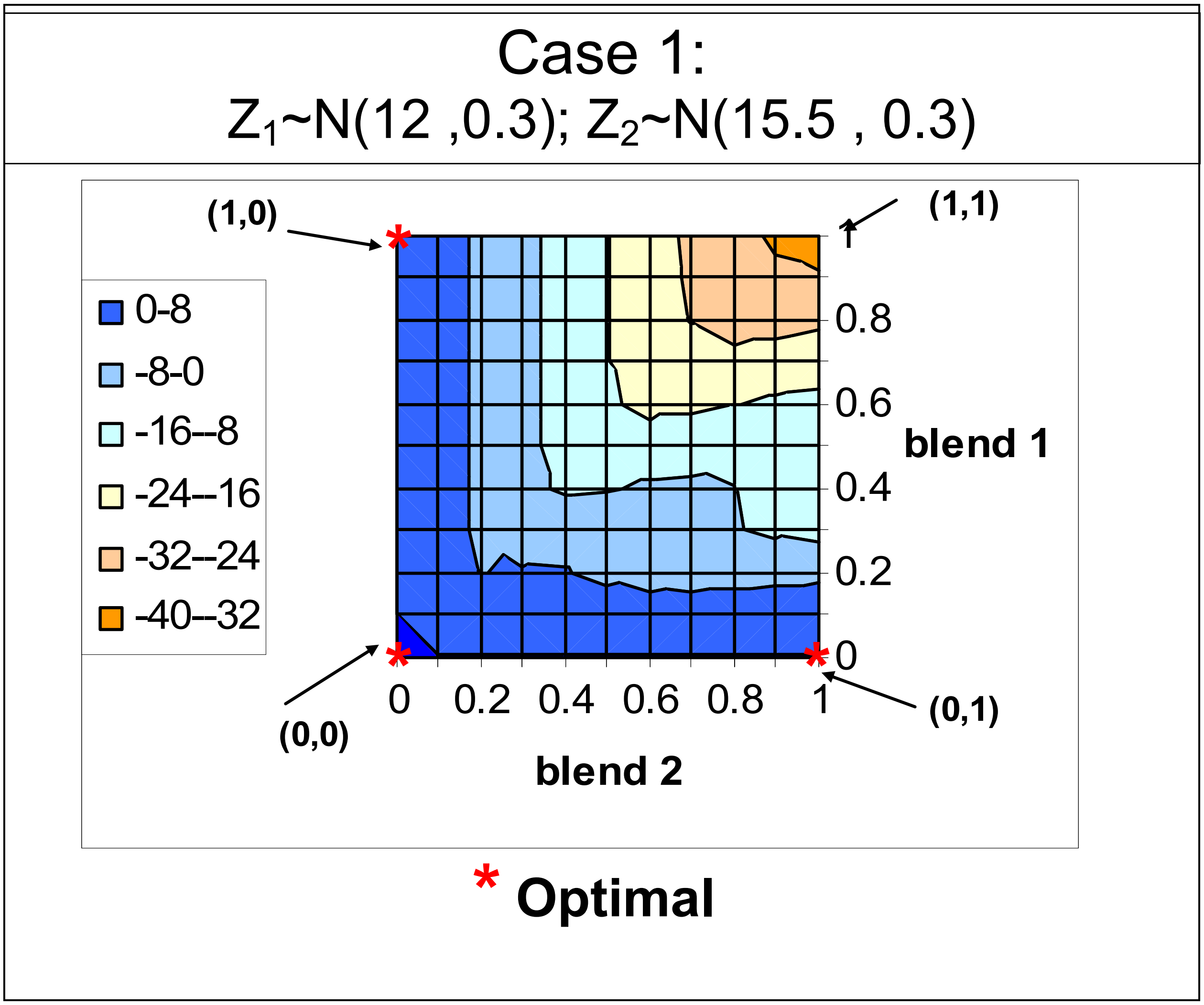

Figure 8. Contour graph of expected sales revenue, case 1.

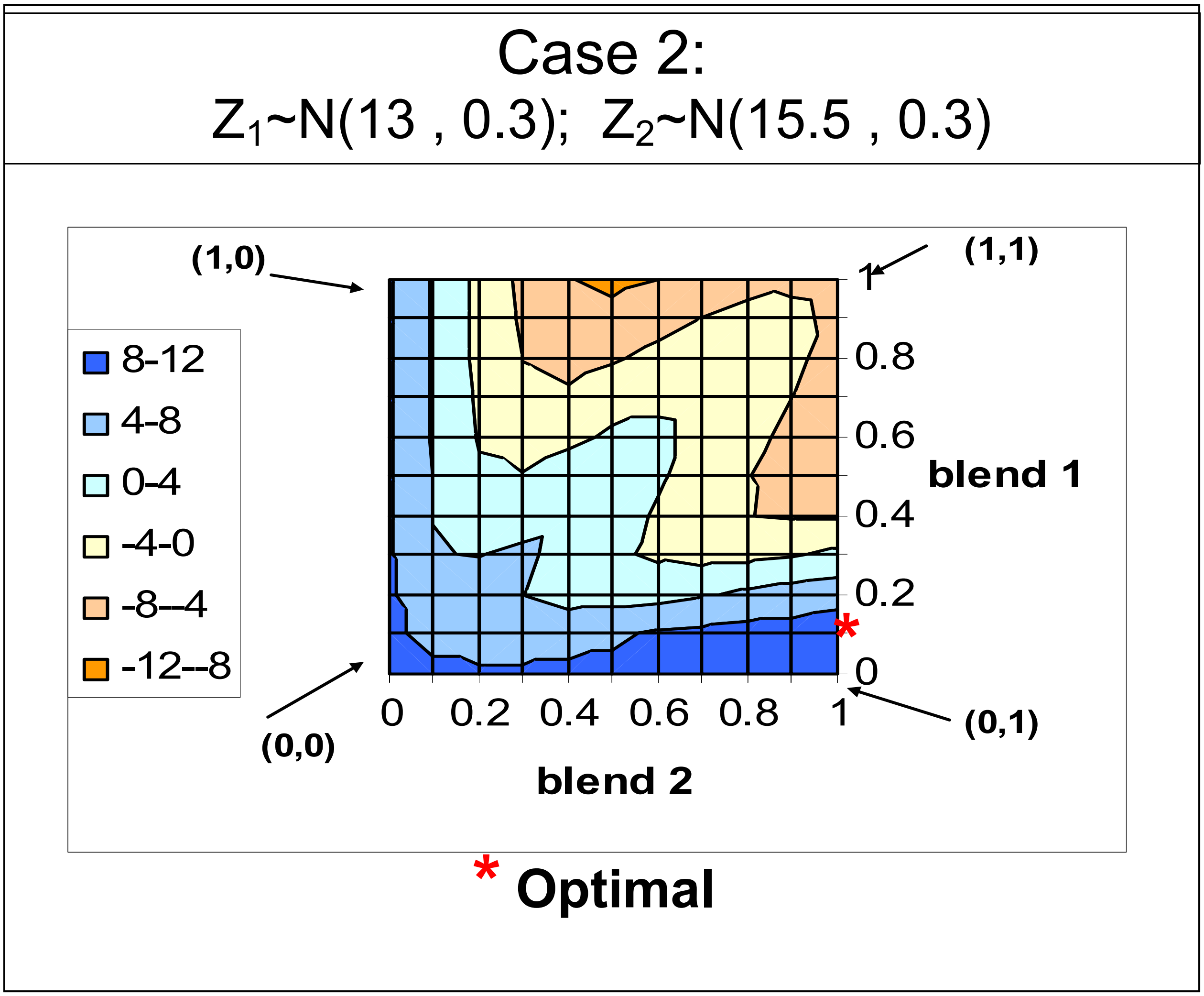

Figure 9. Contour graph of expected sales revenue, case 2.

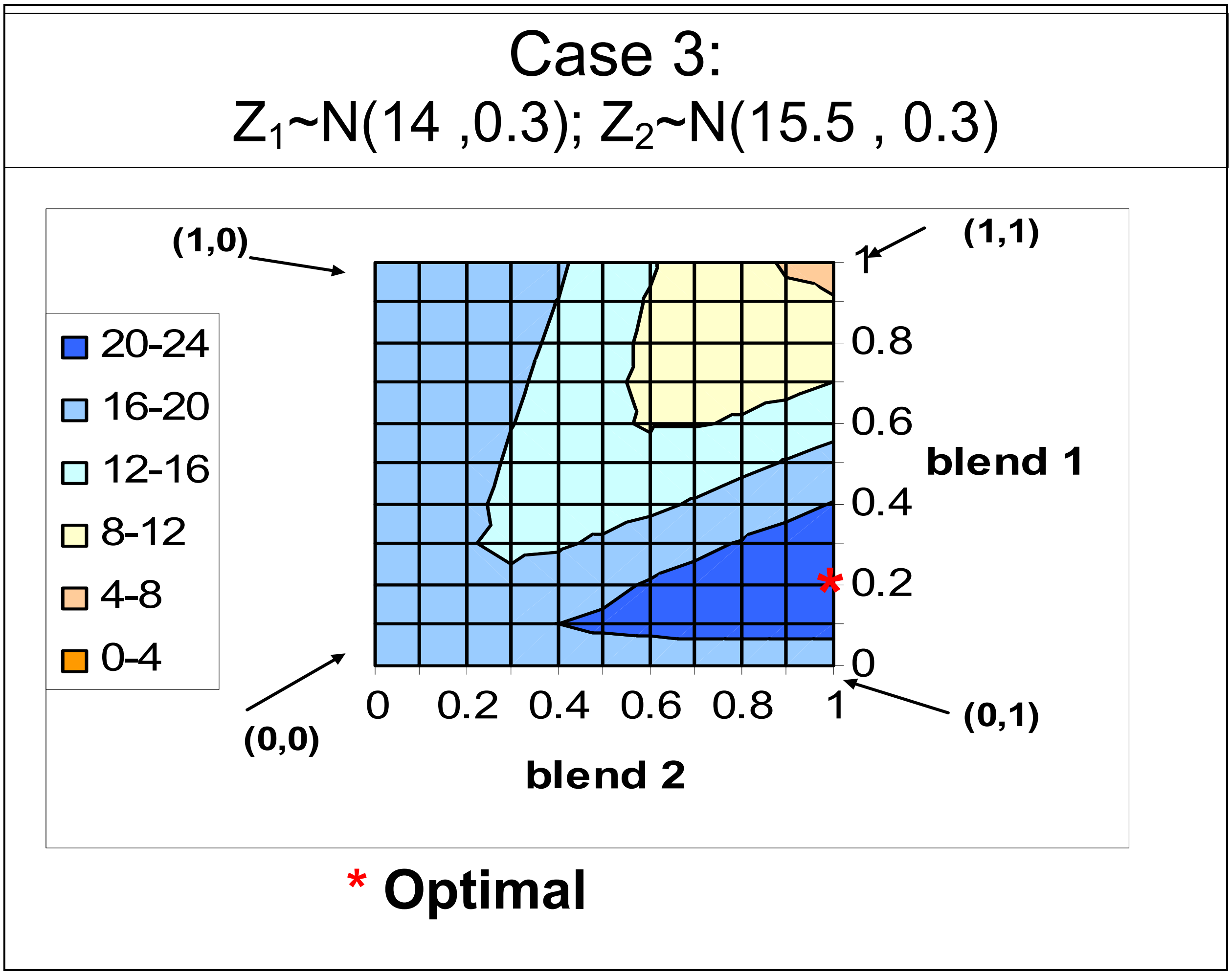

Figure 10. Contour graph of expected sales revenue, case 3.

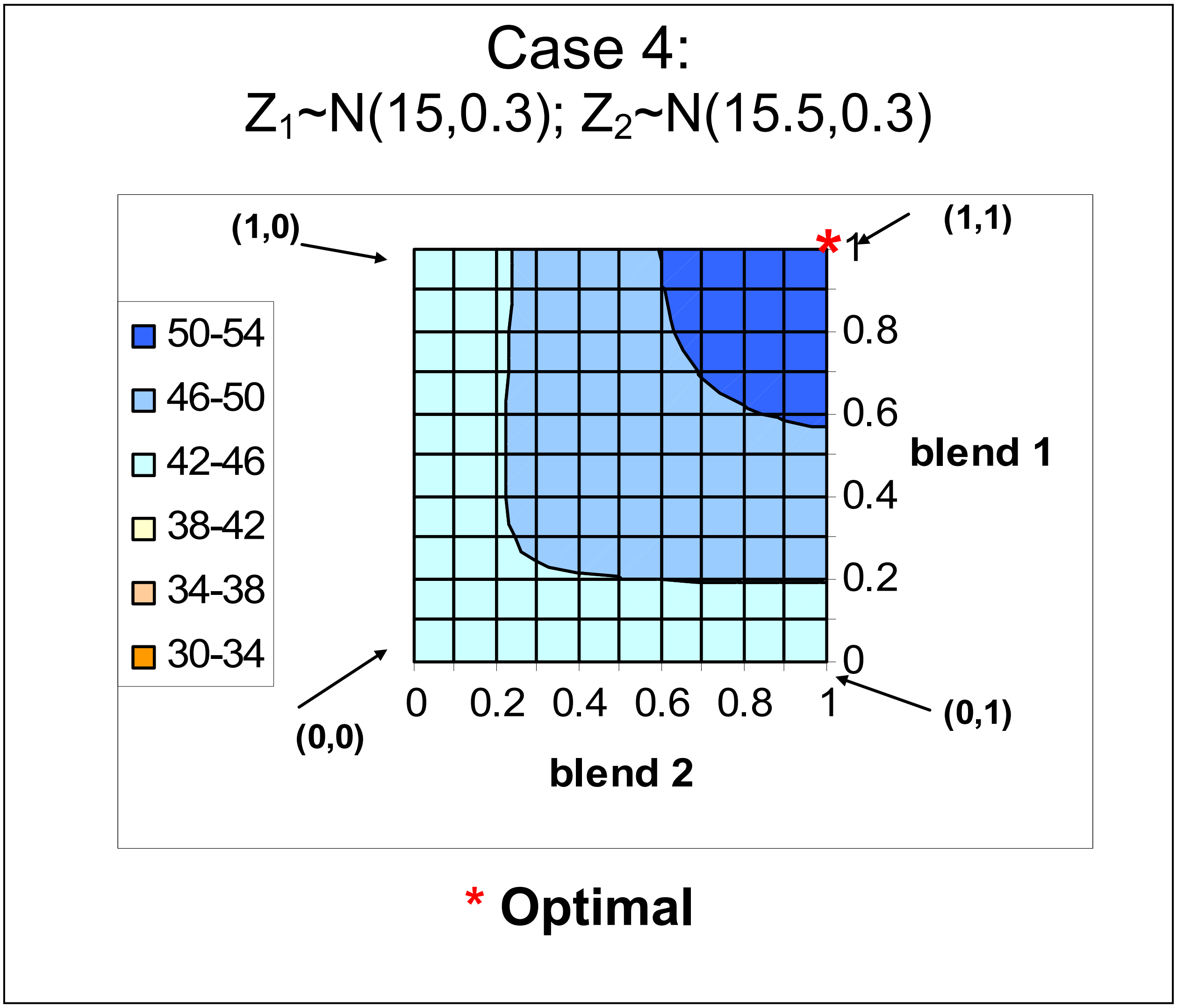

Figure 11. Contour graph of expected sales revenue, case 4.

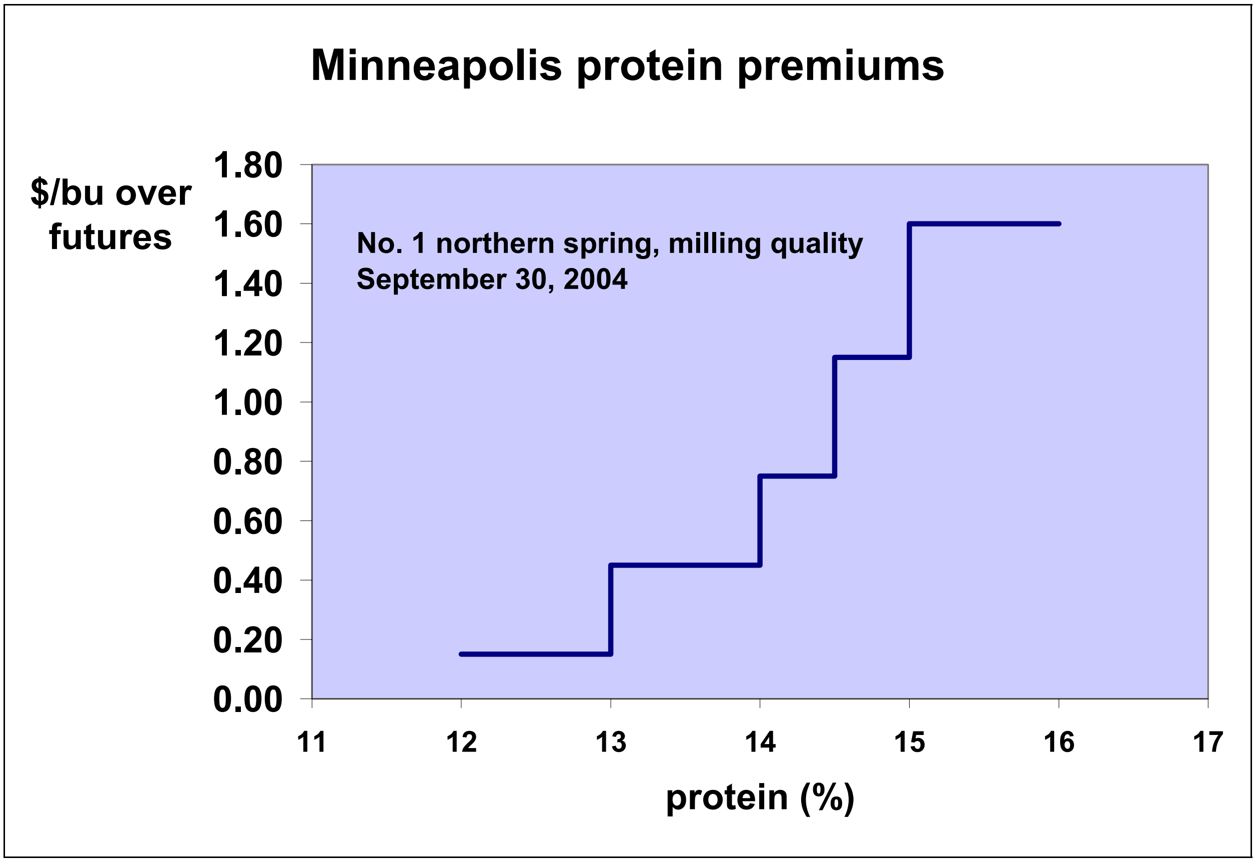

Figure 12. Protein premiums for 2004 spring wheat crop, quoted end September.

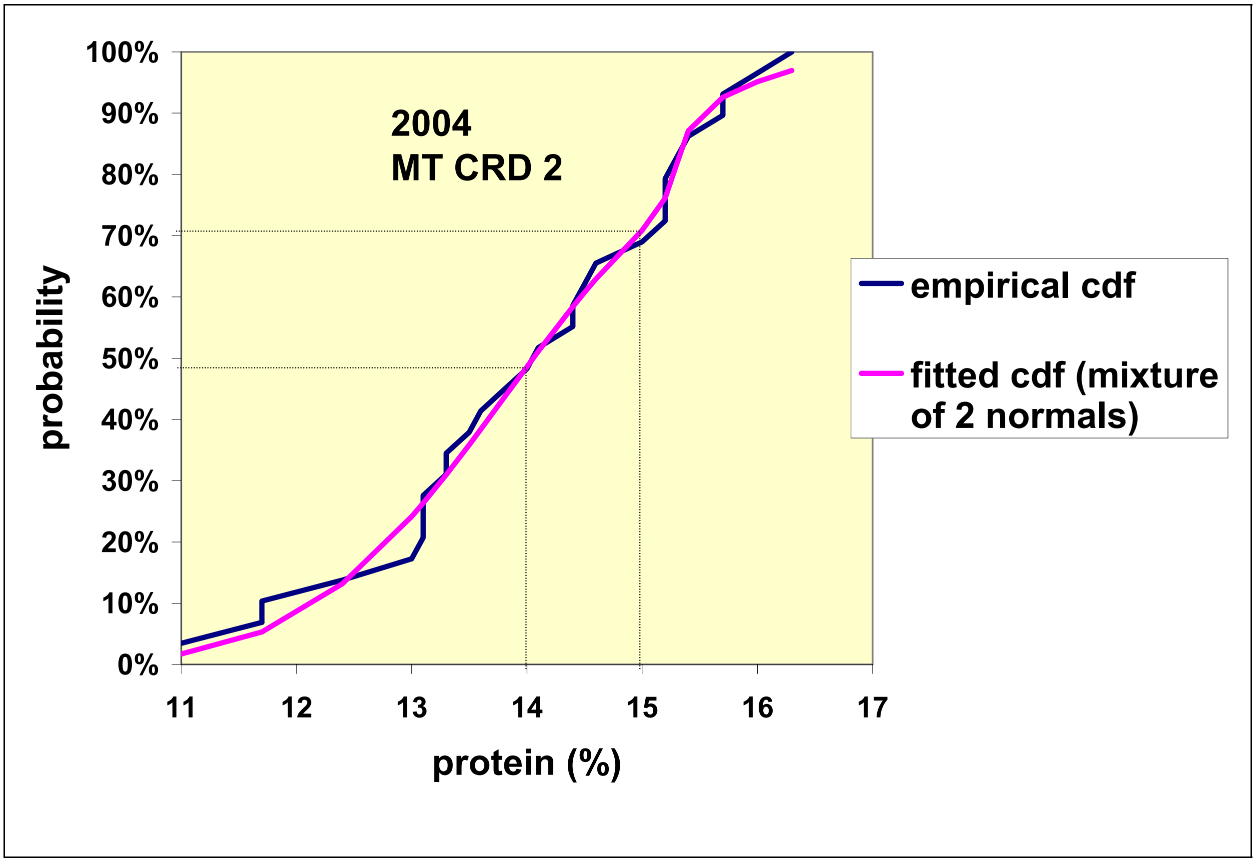

Figure 13. Empirical and fitted cdfs for protein, CRD 2 in Montana.

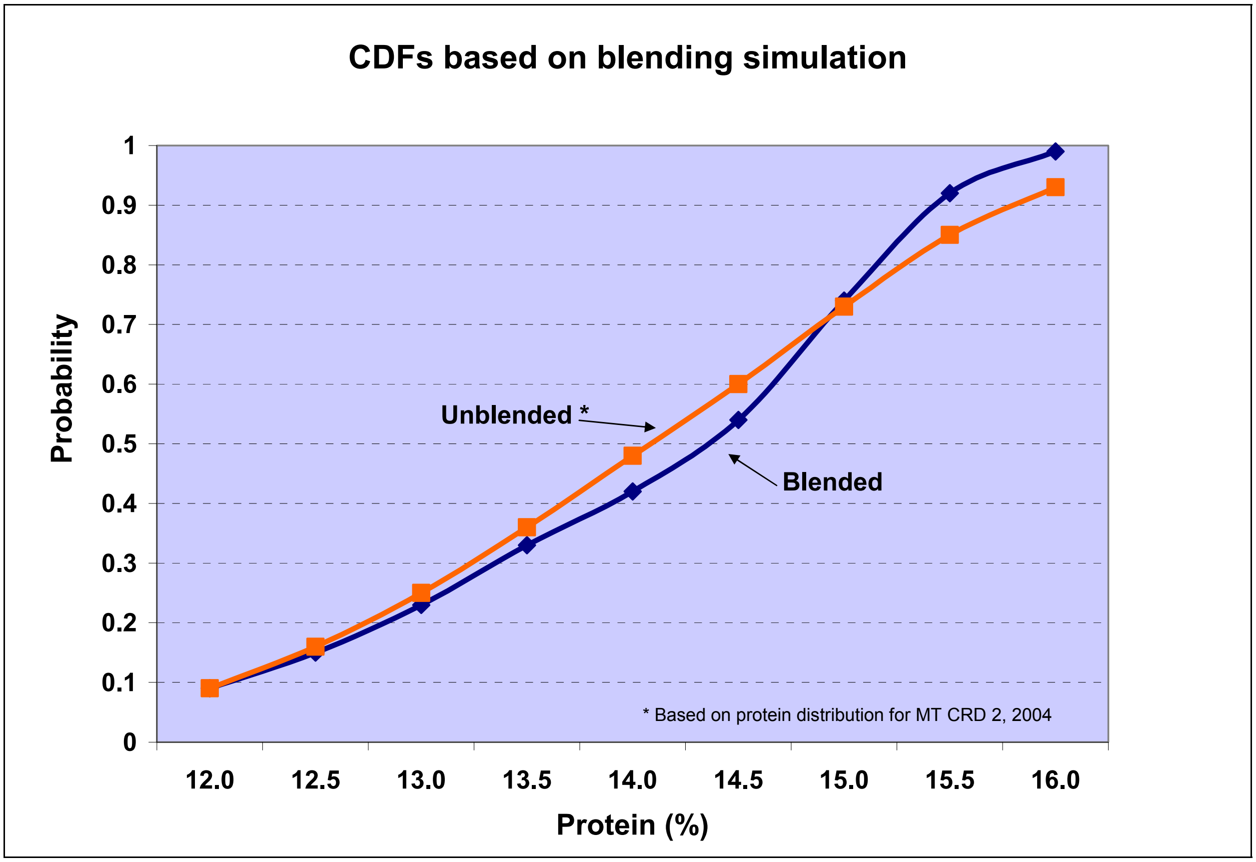

Figure 14. Impact of blending on distribution of protein.