IS INVERSE DEMAND PERVERSE?
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Abstract. Our non-representative sample of 245 undergraduates had significantly lower scores on questions presented in the standard heterogeneous form (i.e., Direct Demand equation and Inverse Demand graph) than on questions presented in non-standard homogenous forms. This result, which holds for advanced students, highlights one reason why 95 percent of students in economics principles classes do not enter the major—economics can be gratuitously mathematical. We argue that the Inverse Demand standard hurts rather than helps economics when it is used in early courses, but that professors have no incentive to change their methods. We recommend that early classes use either no graphs or a homogenous combination of graph and equation. The “standard” should be introduced later, when benefits outweigh costs.

Most instructors teach the principles of economics (e.g., Economics 1) to undergraduates with a combination of intuition, algebra and graphs. Although the weight given to each component varies by instructor, some elements are universally accepted as “the standard.” Inverse Demand, the inverted graphical presentation of a Direct Demand function, with the independent variable, price, on the vertical axis (an inversion of mathematical convention) is one of them.\footnote{We say a question-answer pair are homogenous if the question and answer have the same form, i.e., $P(Q)$ or $Q(P)$. Thus, a Direct (Inverse) graph mapped to/from a Direct (Inverse) equation is homogenous. (We also consider equation-equation pairs.) If the question is heterogeneous, the graph maps to/from an equation in another form, i.e. Direct (Inverse) graph to Inverse (Direct) equation.} See Figure 1.

Key words and phrases. Economics teaching and education. JEL: A22, B1. 2,400 words. PhD Students, Department of Agricultural & Resource Economics, University of California, Davis. One Shields Avenue, Davis, CA 95616-8512. Carlo Russo is also affiliated with the Università degli Studi di Cassino, Dipartimento Economia e Territorio. Corresponding Author: David Zetland (david@primal.ucdavis.edu).

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Copyright 2005 by Russo, Yavapolkul and Zetland. All rights reserved. Readers may make verbatim copies of this document for non-commercial purposes by any means, provided that this copyright notice appears on all such copies. This paper is based on an earlier working paper (Russo et al., 2004) that is both thorough and long-winded. Readers interested in the details of surveys, history of Inverse Demand, or the formal model of switching standards can download that paper from http://www.agecon.ucdavis.edu/uploads/grad_papers/inversedemand.pdf.
Inverse Demand may be one reason why 95 percent of the students who take economics principles graduate in another major (Hansen et al., 2001). We claim that Inverse Demand is the result of historical inertia (not of theoretical necessity or pedagogical efficiency) and question the effectiveness of this standard. In this paper, we evaluate if Inverse Demand makes it more difficult for students to understand and apply demand theory. We find that it does and suggest ways to keep Inverse Demand while improving economic teaching by putting Inverse Demand in context.

1. The Backstory on Marshall and Walras

When Marshall wrote *Principles of Economics*, he used the contemporary analytical presentation of $P(Q)$ and graphically represented the market with price on the vertical axis.\(^2\) He kept to this notation, even in situations he knew to be

\(^2\) Marshall (1920, p 90): “There may be even more violent changes than this in the price of thing which is not necessary, if it is perishable and the demand for it is inelastic: thus fish may be very dear one day, and sold for manure two or three days later.”
Q(P), trading logic for consistency, while solidifying the standard. Léon Walras developed an alternative standard, following on Cournot, of Q(P). These “standards” reflected a deeper theoretical division. “The basic difference between Walras and Marshall, with regard to the market adjustment mechanism, is that Walras regarded price as the adjusting variable when markets are in disequilibrium whereas Marshall focused on quantity as the adjusting variable in the same circumstances” (Ekelund Jr. and Hebert, 1975, p 314). Mainstream economics intertwines both traditions, allowing for both P(Q) and Q(P). Graphically, however, economists have settled on the Marshallian presentation.

2. Empirics—Do Students Understand Inverse Demand?

Economists use Inverse Demand often and are comfortable with its idiosyncrasies. We suspected that Inverse Demand was not so easy for beginners, in the same way that idiomatic English can be confusing to non-native speakers. We set out to see how professors and students perceive and use Inverse Demand.

2.1. Survey of Professors. Most of the 13 professors surveyed in a non-representative sample said that Inverse Demand is an unusual standard, that students (“especially the smart ones”) have no problems with it, and that they (almost) always use the Inverse form for graphing—for the “usual network externality.”

2.2. Survey of Students. We surveyed 283 undergraduate students taking economics classes at UC Davis. We collected descriptive data on the students and asked eight multiple choice questions in three parts. (Appendix A is the survey.)

Part I asked them to look at a demand graph and choose the demand equation which matched it (graph → equation). Part II asked them to add two individual demand equations and choose the equation for aggregate demand (equation → equation). Part III asked them to add two individual demand equations and then choose the aggregate demand graph which matched the sum (equation → graph).

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3We had 245 observations after eliminating surveys with unanswered test questions. Our data are not representative; we prefer to think of our sample as a case study to provoke further thought.
4The answers are 1A, 2C, 3B, 4B, 5B, 6C, 7A, 8B.
Table 1. Survey Results. Sub-samples in three middle columns are cohorts who have taken the same number of economics classes.

Each part had questions and answers in Direct and/or Inverse form. If the forms were the same (e.g., Direct (Inverse) graph to Direct (Inverse) equation), we considered the question to be “homogenous” in form. If the form was Inverse graph and Direct equation, we considered the question to be in “heterogeneous” form. We hypothesized that students would perform worse with the heterogenous form.

2.2.1. Survey Results. We found two major results in the data:

1. Our sample students do significantly worse when answering a question with graph and equation in heterogenous form compared to a question in homogenous form. Thus, the status quo presentation of Inverse Demand may be sub-optimal for students. See Table 2.
Heterogeneous vs. Homogeneous (Direct)

<table>
<thead>
<tr>
<th>Part</th>
<th>gph → eqn</th>
<th>% Inverse → Direct</th>
<th>% Direct → Direct</th>
<th>Test Stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part I</td>
<td></td>
<td>62</td>
<td>81</td>
<td>4.82***</td>
</tr>
<tr>
<td>Part II</td>
<td></td>
<td>94</td>
<td>96</td>
<td>0.81</td>
</tr>
<tr>
<td>Part III</td>
<td></td>
<td>55</td>
<td>66</td>
<td>2.51**</td>
</tr>
</tbody>
</table>

Heterogeneous vs. Homogeneous (Inverse)

<table>
<thead>
<tr>
<th>Part I</th>
<th>gph → eqn</th>
<th>% Inverse → Direct</th>
<th>% Inverse → Inverse</th>
<th>Test Stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part I</td>
<td></td>
<td>62</td>
<td>82</td>
<td>4.94***</td>
</tr>
<tr>
<td>Part III</td>
<td></td>
<td>55</td>
<td>59</td>
<td>0.91</td>
</tr>
</tbody>
</table>

Homogeneous (Direct) vs. Homogeneous (Inverse)

<table>
<thead>
<tr>
<th>Part I</th>
<th>gph → eqn</th>
<th>% Direct → Direct</th>
<th>% Inverse → Inverse</th>
<th>Test Stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part I</td>
<td></td>
<td>81</td>
<td>82</td>
<td>0.12</td>
</tr>
<tr>
<td>Part III</td>
<td></td>
<td>66</td>
<td>59</td>
<td>1.59</td>
</tr>
</tbody>
</table>

* p < 0.1, ** p < 0.05, *** p < 0.01

Table 2. Total sample \((n = 245)\). Students have consistently and significantly higher success rates answering graph-equation questions in homogenous forms (either Direct-Direct or Inverse-Inverse) compared to heterogenous forms. There is no difference between the homogenous forms across the whole sample.

(2) There is a Learning Curve, but not the type we want. Students who have taken more economic classes generally perform better then those who have taken fewer classes, but Advanced students still perform significantly worse on questions in heterogenous form. See Figure 2, Table 3 and Table 4.

2.3. Interpretation.

- Students do not have difficulty with inverse form per se but the heterogeneous mix of graph and equation.
- A homogenous presentation of the material increases student success. The type of homogeneity does not usually matter.
- There is no inherent “math bias” of Direct-Direct over Inverse-Inverse for students used to seeing independent variables on the vertical axis.
- Advanced students work better with Inverse-Inverse than Direct-Direct, the likely result of learning that presentation form.
Table 3. **Learning Curve:** Advanced students performed significantly better than beginning students in graph → equation and equation → graph questions but not in the equation → equation questions. They still do poorly on heterogeneous forms. (See Table 4.)

<table>
<thead>
<tr>
<th>Part I (graph → equation)</th>
<th>Beginners</th>
<th>Advanced</th>
<th>Test Stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inverse → Direct</td>
<td>50</td>
<td>73</td>
<td>3.01***</td>
</tr>
<tr>
<td>Direct → Direct</td>
<td>64</td>
<td>94</td>
<td>4.66***</td>
</tr>
<tr>
<td>Inverse → Inverse</td>
<td>64</td>
<td>84</td>
<td>2.85***</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Part II (equation → equation)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Direct → Direct</td>
</tr>
<tr>
<td>Inverse → Inverse</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Part III (equation → graph)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inverse → Direct</td>
</tr>
<tr>
<td>Direct → Direct</td>
</tr>
<tr>
<td>Inverse → Inverse</td>
</tr>
</tbody>
</table>

* p < 0.1, ** p < 0.05, *** p < 0.01

Table 4. **Advanced Students Still Don’t Get It:** Students with more than four economics classes (n = 102) are still challenged by heterogeneous forms. They do better with the Inverse-Inverse over the Direct-Direct homogenous form, probably because of familiarity.

<table>
<thead>
<tr>
<th>Part I</th>
<th>% Inverse → Direct</th>
<th>% Direct → Direct</th>
<th>Test Stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>gph → eqn</td>
<td>73</td>
<td>84</td>
<td>2.06**</td>
</tr>
<tr>
<td>eqn → eqn</td>
<td>95</td>
<td>99</td>
<td>1.67*</td>
</tr>
<tr>
<td>eqn → gph</td>
<td>61</td>
<td>74</td>
<td>1.96***</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Part III</th>
<th>% Inverse → Direct</th>
<th>% Inverse → Inverse</th>
<th>Test Stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>eqn → gph</td>
<td>73</td>
<td>94</td>
<td>4.32***</td>
</tr>
<tr>
<td>gph → eqn</td>
<td>61</td>
<td>69</td>
<td>1.18</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Homogeneous (Direct) vs. Homogeneous (Inverse)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part I</td>
</tr>
<tr>
<td>gph → eqn</td>
</tr>
<tr>
<td>eqn → gph</td>
</tr>
</tbody>
</table>

* p < 0.1, ** p < 0.05, *** p < 0.01
3. EXPLAINING THE STATUS QUO

So—if our results are not specific to the data and indicate a larger pattern in economics instruction, why has this sub-optimal situation persisted? This was the exact question of our *Journal of Economic Education* reviewer:10

If there was such an interest in change because of some significant pedagogical problem then some enterprising textbook author would have exploited the problem and developed a book using direct demand graphs. That event has not happened.

Unfortunately, this critique misses our point—that students are the ones who lose from a confusing standard. Since professors use Inverse Demand often, when they (naturally) use it for teaching, they may not see that students have a harder time with Inverse Demand. Professors who do not compare performance under homogeneous vs. heterogeneous presentations (as we did) may only see that student performance with Inverse Demand improves over time (which we also found), not that it is always worse than the alternative.

10The paper was rejected.
Students are helpless. They know no alternative to Inverse Demand—only that it is the standard upon which their grade depends.\textsuperscript{11} Many quit economics when they fail to see the “intuition.”

3.1. \textbf{A Model, in Brief.} In our earlier paper (Russo et al., 2004), we created an elaborate model to show why a professor has no incentive to switch to Direct Demand from Inverse Demand. A Professor who switches receives students who care more about grades than economics.\textsuperscript{12} The quality of students falls, the number of students rises and the workload increases—three bads. Since the system does not award popularity (for good reason), the Professor gets little compensation for these bads. Thus, there is no incentive to switch to Direct Demand. We can summarize this result as:

<table>
<thead>
<tr>
<th>Benefit</th>
<th>Professor</th>
<th>Student</th>
</tr>
</thead>
<tbody>
<tr>
<td>More students (?)</td>
<td>Better understanding (or grades)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Cost</th>
<th>Effort to form new curriculum.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Cognative dissonance in departing from professional norms.</td>
</tr>
<tr>
<td></td>
<td>Knowledge incompatible with standards for further progress.</td>
</tr>
</tbody>
</table>

On further thought, we discovered that a smaller adjustment can help both students and professors, i.e., putting Inverse Demand in context.

4. \textbf{Suggestions for Change}

To work with, not against, Inverse Demand, we suggest that professors:

(1) Start without graphs so that $Q(P)$ algebra is clear.

(2) Use Direct Demand graphs to show how supply and demand equate. The gains from trade are now measured from the horizontal axis but easy to understand.

(3) Now introduce cost and utility functions that are optimized to give marginal cost (supply) and marginal benefit (demand) functions and curves that have quantity as independent variables. Map them to the vertical axis.

\textsuperscript{11}Our anecdotal (i.e., without statistical significance) impression from our research on this topic is that a number of students only memorize the Inverse Demand pattern; they do not learn the mathematical or economic principles or retain the concept beyond their intermediate microeconomics class.

\textsuperscript{12}We assume that students who care about economics stay with the standard because their future depends on it. Grade-seekers don’t care since they are leaving anyway.
(4) Use both algebraic forms (Direct Demand and Inverse Demand) as appropriate. This can help students understand the pros and cons of the inconsistency.

When they have mastered the basics of market equilibrium and are ready to derive profit- and utility-maximizing supply and demand curves from utility and cost functions, the place of the Inverse Demand graph within the whole will make sense.

5. Conclusion

Inverse Demand is a sub-optimal standard for students if test performance is correlated with student comprehension. This does not seem to be the case with Direct Demand. The problem arises when the graphical form is inconsistent with the equation form (heterogeneous). Rather than throw out the baby with the bathwater—Inverse Demand is very useful in other contexts—we suggest that Inverse Demand be introduced later in the student’s instruction and that either no graphs or homogenous graphs be used for early instruction—especially in principles classes.

We hope that our results provoke thought and discussion among economics teachers. If notation hinders students from understanding principles, economics loses.

References


APPENDIX A: ECONOMIC TEACHING METHODS EVALUATION

INSTRUCTIONS

This evaluation is voluntary and confidential. Do NOT write any identifying information on this page! Please try to answer all questions in the time allowed.

ABOUT YOU

Major: Are you now declared (or intending to declare) economics or managerial economics as your major? Please circle one . . . Yes No

Classes: Please circle all classes that you have taken or are currently taking:

ARE100A ARE100B ARE155 ARE156 ECN 1A ECN 1B
ECN 100 ECN 101 Math16A Math16B Math21A Math21B

GPA: What’s your cumulative GPA? _______

QUESTIONS ON DEMAND

Part I: Choose the multiple choice answer corresponding to the given graph (for the given axis combination)

(1) Choose the answer which matches the graph to the right ($Q_d$ on x-axis).
   (a) $Q_d = 10 - 2p$
   (b) $Q_d = 5 - \frac{1}{2}p$
   (c) $Q_d = 5 - 2p$

(2) Choose the answer which matches the graph to the right ($Q_d$ on x-axis).
   (a) $p = 7 - \frac{7}{2}Q_d$
   (b) $p = 7 - \frac{7}{2}Q_d$
   (c) $p = 2 - \frac{7}{2}Q_d$

(3) Choose the answer which matches the graph to the right ($P$ on x-axis).
   (a) $Q_d = 3 - \frac{1}{2}p$
   (b) $Q_d = 6 - 2p$
   (c) $Q_d = 6 - \frac{1}{2}p$
Part II Given the following individual demand equations \((q_1, q_2)\) and market price \((p)\), circle the correct value of aggregate demand, \(Q_d\) (where \(Q_d = q_1 + q_2\)).

(4) If \(q_1 = 3 - 2p\), \(q_2 = 3 - 2p\) and \(p = 1\), then:
(a) \(Q_d = 4\)
(b) \(Q_d = 2\)
(c) \(Q_d = 3\)

(5) If \(p = 4 - 3q_1\), \(p = 4 - 3q_2\) and \(p = 1\), then:
(a) \(Q_d = 4\frac{1}{2}\)
(b) \(Q_d = 2\)
(c) \(Q_d = 7\frac{1}{3}\)

Part III Circle the graph representing the aggregate demand function (for the given axis combination)

(6) If \(q_1 = 4 - p\) and \(q_2 = 4 - p\), then aggregate demand looks like (\(Q_d\) on x-axis):

(7) If \(q_1 = 3 - p\) and \(q_2 = 3 - p\), then aggregate demand looks like (\(P\) on x-axis):

(8) If \(p = 3 - 5q_1\) and \(p = 3 - 5q_2\), then aggregate demand looks like (\(Q_d\) on x-axis):
Appendix B: Empirics

**Methodology.** We assume that the answer for each question is distributed as multinomial with three possible outcomes per trial (student). \((y_i, p_i)\) describes the outcomes and probabilities for \(i = 1, 2, 3\), where 1 is the correct answer. A maximum likelihood estimator for \(p_k\), given observed \((y_1, y_2, y_3)\) is simply the number of students who answer the \(k^{th}\) choice divided by the total number of students. Thus, we are implicitly assuming the same \textit{a priori} \(p_i\)'s across all students.

We constructed an asymptotic z-statistic to test the difference in the rate of success between pairs of questions within parts for significance. We found the z-statistic with

\[
Z_z = \frac{\hat{p}_1^s - \hat{p}_1^t}{\sqrt{\left(\frac{\hat{p}_1^s(1-\hat{p}_1^s)}{n_s}\right) + \left(\frac{\hat{p}_1^t(1-\hat{p}_1^t)}{n_t}\right)}}
\]

which is \(\sim N(0, 1)\)

and where \(\hat{p}_1^s = \frac{y_1^s}{n_s}\) and \(\hat{p}_1^t = \frac{y_1^t}{n_t}\), for \(s \neq t\).

Note that \(\hat{p}_1^s\) and \(\hat{p}_1^t\) indicate an estimated parameter for the probability of choosing the right answer (choice 1) in two different questions (e.g. Direct-Direct and Direct-Inverse). This allows us to test hypotheses on student performance on various types of questions.