Valuing a Novel Biotechnology with Outcome Variability and Uncertainty:

The Case of Mycorrhizal Biofertilizers

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Valuing a Novel Biotechnology with Outcome Variability and Uncertainty: The Case of Mycorrhizal Biofertilizers

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Abstract

While phosphorus is an essential nutrient for plant growth, excess demand for phosphate fertilizers has contributed to extremely volatile fertilizer prices and caused billions of dollars in environmental damages. Over 80% of terrestrial plants have evolved to form symbiotic relationships with soil-dwelling fungi known as *mycorrhiza*, which help them to absorb phosphorus, and agricultural producers may be able to improve P-efficiency by encouraging this symbiosis. However, mycorrhiza cannot be directly observed and the magnitude of their effect on P absorption is context-dependent and subject to scientific debate. This paper estimates the value of the ecosystem services provided by mycorrhiza to agricultural operators through improved P-efficiency using a dynamic programming model with varying levels of state stochasticity and uncertainty. We compare several different management strategies under which an agricultural producer chooses to add fertilizer and/or a commercial mycorrhizal inoculum to maximize profit and yield. The first objective is to calibrate a deterministic dynamic programming model, wherein the biological and physical processes are well-understood. To this, stochasticity will added, including biological stochasticity for the biomass of mycorrhiza and the host plant. Lastly, distributions will be added to key parameters, representing uncertainty about the biomass of mycorrhiza, the stock of phosphorus, and the effects of symbiosis on plant biomass.

1 Introduction

Next to nitrogen, phosphorus is one of the most crucial nutrients required for plant growth (Prasad et al., 2017; Czarnecki et al., 2013). The introduction of mineral fertilizers have allowed unprecedented yields, helping to mitigate hunger in a world facing ever-growing populations (Pinstrup-Andersen and Hazell, 1985). However, the massive global demand for these fertilizers has strained known mineral reserves as well as aquatic ecosystems.

Some scientists estimate that current reserves will fall below current needs by 2040, while others project that known reserves will last 300-400 years (Nedelciu et al., 2020; Daneshgar et al., 2018; McDowell et al., 2023). Even if known reserves are not endangered, global events such as the Russia-Ukraine War and the 2025 trade war can cause extreme price spikes, endangering the food security of subsistence farmers in low-income countries and disrupting supply chains in countries like the United States (Bailey, 2020; White, 2025; Goodman and Times, 2023).

Moreover, fertilizer does not exhibit increasing returns to scale. Nutrients applied over a crop's needs are not absorbed by the plant and translated into larger yields; rather, this excess is susceptible to runoff (Janssen and Guiking, 1990; Tarkalson and Mikkelsen, 2004). Phosphorus runoff from agricultural sources has led to the eutrophication of waterways, which causes blooms of toxic algae that create dead zones in waterways or can end up in drinking water supplies (Khan and Mohammad, 2014; Cho, 2013). This process alone causes significant environmental damages: one estimate places the annual cost of phosphorus pollution in the United States as high as \$2.2 billion (Dodds et al., 2009).

Fortunately, most agricultural soils contain ample phosphorus, so-called legacy phosphorus (Pavinato et al., 2020). However, this phosphorus is frequently in forms that make it inaccessible to plants (Sharpley et al., 2013). As a result, risk-averse agricultural producers typically over-apply fertilizer to compensate for their uncertainty about soil P stocks as well as the rate at which this legacy P will be made available to the crop (McDowell and Haygarth, 2025). Reducing fertilizer overuse, then, requires improving the efficiency of P uptake, particularly through the conversion of so-called immobile P (inaccessible or unusable to the plant) to available P (Hyland et al., 2005; McDowell et al., 2024).

One potential solution already lies within the soil: mycorrhizal fungi. These obligate symbionts[1] are found all over the world, and over 80% of terrestrial plants associate with one or more types of mycorrhiza (Prasad et al., 2017). While indigenous mycorriza may provide benefits to host crops, there are also an increasing number of low-cost commercial inocula that farmers can use to increase the presence of mycorrhiza in their soils (Schroeder-Moreno, 2021).

Just like soil P, however, mycorrhiza are difficult to monitor, as current methods require several days, lab equipment, and subjective interpretation to measure mycorrhizal presence and diversity (Stoian et al., 2019). The exact magnitude and extent of the benefits that

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[1] An organism that depends on symbiosis with another species for survival (Relyea, 2021). In this case, mycorrhiza are unable to survive without their plant hosts.

mycorrhiza provide to their hosts are also subject to some scientific debate. Moreover, since mycorrhiza are biotic agents, their performance is subject to some level of stochasticity. Profit-maximizing producers therefore face tradeoffs between fertilizer price volatility and yield variability due to randomness in mycorrhizal performance. Further, it is difficult for producers to incorporate mycorrhiza and legacy P into their production decisions when both inputs are impossible to observe directly without specialized testing.

This paper estimates the value of the ecosystem services provided by mycorrhiza to agricultural operators through improved P-efficiency using a dynamic programming model with varying levels of state stochasticity and uncertainty. We compare several different management strategies under which an agricultural producer chooses to add fertilizer and/or a commercial mycorrhizal inoculum to maximize profit and yield. Under the first regime, the operator is unaware of the role that mycorrhiza play in plant nutrition. Under the second and third regimes, the operator knows about mycorrhizal symbioses but faces uncertainty about the presence of mycorrhiza in their fields (mycorrhizal biomass) and about the size of the effect of mycorrhiza on P uptake. The farmer, then, uses passive or active adaptive management strategies (PAM and AAM, respectively), which reveal information about the presence and role of mycorrhiza, and this information is used to update their beliefs before fertilizer and inoculum decisions are made next season.

Under PAM, managers acknowledge that there may be uncertainty about key biological parameters. These choices may reveal more information about the true underlying states, and managers may update their beliefs after they have implemented their optimal management strategies for the period. However, they choose their optimal strategies based on their current best estimates of these parameters alone. This differs from AAM strategies, wherein managers explicitly consider the fact that resolving their uncertainty about biological processes sooner may have economic value. They incorporate the so-called ”value of information” into their optimal strategies, thus making their management decisions based not only on their immediate goals such as profit maximization but also on the potential benefits of learning through experimentation (Williams, 2011; LaRiviere et al., 2018; Walters and Hilborn, 1976).

2 Background

Soil is one of humanity's most precious natural resources, though one that is easy to neglect as more of us become increasingly removed from agriculture. Since humans began farming nearly 12,000 years ago, we have been largely dependent on the soils that allow us to produce high-yield, calorically dense produce (Carey, 2023). But soil is not just dirt, comprised of pulverized sand or clay (Jensen and Anderson, 1995). Rather, the soil beneath our feet is teeming with life: plant roots, insects, bacteria, and fungi all compose the rich communities that sustain human life.

Today, scientists are revealing the incredible complexity of soil microbial life, one of the most remarkable of which is the symbiotic relationship between plants and so-called mycorrhiza. The name *mycorrhiza*, given by the German scientist Albert Frank in 1885, reveals its function: *fungus-root* (Frank, 2005; Sheldrake, 2020). Mycorrhiza are fungi which attach to plant roots and spread through the soil in web-like structures, essentially acting as extensions of the root system in exchange for the products of photosynthesis such as carbohydrates (Smith and Read, 2008; van der Heijden et al., 2015). Over 80% of plants form associations with mycorrhiza, and they are found in ecosystems all over the world, from deserts to the tundra (van der Heijden et al., 2015).

Of the four types of mycorrhiza that have so far been discovered, Arbuscular Mycorrhiza (AM) are the most common (Karimi-Jashni and Yazdanpanah, 2023). AM fungi attach by injecting *hyphae*, which are hairlike filaments, into the tip of a plant's root (Prasad et al., 2017) and webbing out into the surrounding soil (indeed, the word *hypha* is derived from the Latin for *web* (Oxford English Dictionary, 2024)). These fungal webs organize into so-called networks, which can stretch up to one hundred times further than the host plant's roots (Sheldrake, 2020). Around 300 species of AM fungi have been described by scientists, though it is estimated that as many as 1,600 or more species likely exist (Větrovský et al., 2023).

Due to their ubiquity and diversity, the effects of mycorrhiza on plants are typically dependent on the particulars of the species of plant and fungi involved and their exposure to biotic and abiotic stressors (Hoeksema et al., 2010). As a result, the scientific literature has produced a wide range of often-contradictory findings about the impact of mycorrhizal associations on biomass, yield, and stress tolerance. In general, however, it is agreed that the primary benefit of mycorrhiza to plants is nutrient transport, especially phosphorus (P) and nitrogen (N) and especially forms of these nutrients that plants are unable to reach or metabolize independently (Read and Perez-Moreno, 2003). By some estimates, plants can get up to 80% of their N and P from mycorrhiza (van der Heijden et al., 2015).

However, levels of available phosphorus that exceed a plant's needs often inhibit the growth of mycorrhizal networks (Grant et al., 2005). Overuse of fertilizers can inhibit association with mycorrhiza, in part because the plants do not send signals which attract the fungi (Grant et al., 2005; Czarnecki et al., 2013). Not only does this mean that there fertilizer may represent redundant costs to the producer as the plant substitutes the P absorbed via mycorrhiza for the P available from fertilizer, but the plant also may miss out on other benefits they receive from mycorrhizal associations. These additional benefits include resistance to diseases and pests, increased drought tolerance, and reduced absorption of heavy metals (Akinsanya et al., 2020; Benedetti et al., 2021; Spagnoletti et al., 2020; Augé, 2001; Bahadur et al., 2019; Zou et al., 2021; Yang et al., 2016; Karagiannidis and Hadjisavva-Zinoviadi, 1998; Rozpadek et al., 2014; Rask et al., 2019; Mitra et al., 2022).

3 Deterministic Model

The first objective is to calibrate a deterministic dynamic programming model, wherein the biological and physical processes are well-understood. To this, stochasticity will added, including biological stochasticity for the biomass of mycorrhiza and the host plant. Lastly, distributions will be added to key parameters, representing uncertainty about the biomass of mycorrhiza, the stock of phosphorus, and the effects of symbiosis on plant biomass. The conceptual model of the full process, including the sources of parameter uncertainty, are outlined in Figure 1.

In this model, a profit-maximizing, risk-neutral agricultural producer decides the level

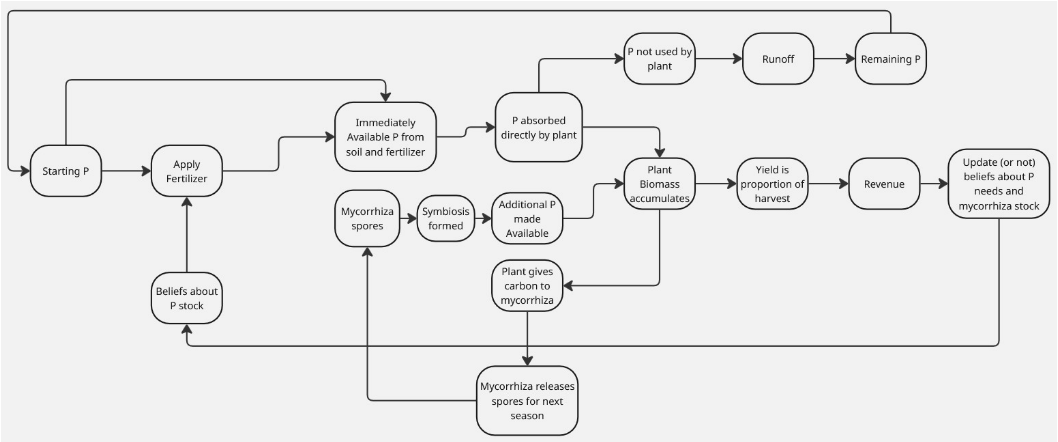

Figure 1: Conceptual Diagram of the Model

of fertilization for one, fixed unit of land. The producer can also choose to add mycorrhizal spores to the soil with a commercial inoculum. To maximize profit, the producer must maximize yield with respect to P uptake, subject to cost-minimization. All other production decisions, such as irrigation and crop choice, are considered fixed. Input and output prices are also assumed to be fixed. The producer's objective is described in Equation 1.

$$V(F_0, I_0) = \max_{F_{t+1}, I_{t+1} \in \Gamma(F_t, I_t)} \sum_{t=0}^{\infty} \beta^t \left(pY_t - w_F F_t - w_I I_t - C(Y_t)\right) \tag{1}$$

Here, β is the discount rate, Y_t is the yield at time t, F_t is the amount of P from fertilizer applied at time t, and I_t is the additional mycorrhizal biomass added through commercially-available inocula. The costs of fertilizer and commerical mycorrhizal inocula are represented by w_F and w_I, respectively, and C are any other costs associated with producing a yield of size Y_t.

The crop's yield is a proportion of its biomass, given:

$$Y_t = \rho H_t \tag{2}$$

Each season, the crop's biomass is determined by some base level (proportional to the base yield, per Eq. 2) which reflects the notion that other production decisions are fixed and the outcomes of these decisions are known. There is also nonlinear time trend to account for technological changes and other factors that can cause base yields (and the associated biomass) to change over time (Chen et al., 2022). The plant accumulates biomass based on how much of its optimal amount of P it can absorb, some of which is absorbed via the mycorrhiza. However, there is an upper limit to the biomass benefit of P (Janssen and Guiking, 1990; Tarkalson and Mikkelsen, 2004). Over-applying P will not lead to indefinitely higher yields, in part because the plant will need additional resources to maintain its biomass. This constraint is reflected by the maintenance cost parameter, μ_H (Martignoni et al., 2021). Symbiosis with mycorrhiza gives the plant other benefits that add to the plant biomass.

$$H_t = \alpha + \delta(t) + q_{HP}N_t + \gamma_M M_t - \mu_H(q_{HP}N_t + \gamma_M M_t)^2 \tag{3}$$

Here, α is the base yield, $\delta(t)$ is a time trend, q_{HP} is the conversion from phosphorus to biomass, N_t is the phosphorus observed by the plant (see Eq. 4), $\gamma_M M_t$ are the net benefits to plant biomass (beyond P uptake) from mycorrhizal symbiosis if mycorrhizal biomass is M_t, and μ_H is the maintenance cost to the plant of maintaining its current biomass.

Optimal yields require sufficient Phosphorus (P). The host plant can get this P from three sources: available P in the soil from previous seasons, fertilizer, or mycorrhiza. The total uptake of this nutrient is described in equation 4. Here, P from the soil is denoted P_{t-1}, and P from fertilizer is given by F_t. The additional P made available to the plant via the mycorrhizal association is denoted $f(M_t)(P_{t-1} + F_t)$. In some specifications, $f(M_t)$ is a scalar, such as in Martignoni et al. (2021); Li et al. (2006); Cozzolino et al. (2013). These studies found that plants with mycorrhizal associations absorbed more P than non-inoculated plants, regardless of P fertilization treatments. Other research has suggested, however, that there is a quadratic relationship between soil P and mycorrhizal biomass, such as in Qi et al. (2022); Hoeksema et al. (2010); and Grant et al. (2005).

$$N_t = (P_{t-1} + F_t)\left[1 + f(M_t)\right] \tag{4}$$

The producer has a phosphorus "budget" each season, comprised of phosphorus already in the soil (the legacy P) and phosphorus added through fertilizer. In Eq. 5, P_t is the starting phosphorus, F_t is the amount of phosphorus added through mineral fertilizer, N_t is the P removed from the soil by the crop, and R_t is the amount of P lost to runoff, evaporation, or erosion.

$$P_{t+1} = P_t + F_t - N_t - R_t \tag{5}$$

This model assumes that mycorrhiza are obligate symbionts. That is, their biomass is zero if there is no host. The mycorrhiza produces spores based on the success of its symbiosis; that is, its generation-to-generation reproduction is density dependent (Martignoni et al., 2020). Equation 6 is a discretized logistic population growth model adapted from Neuhauser and Fargione (2004), which in turn is based on a mutualistic adaptation of a classic Lotka-Volterra predator-prey model (Din, 2013; Pastor, 2008; Harper, 1991; Larkin, 1963). In a predator-prey system, the biomass of the prey is negatively affected by the biomass of the predator, whereas the biomass of the predator is positively affected by the biomass of the prey. In the mutualistic adaptation with one obligate symbiont, the biomass of the obligate symbiont is positively affected by the biomass of the other specie, and there is no "independent" reproduction. For the second, non-obligate symbiont, there is a natural growth rate that does not depend on the population density of the first specie in addition to a positive effect of symbiosis.

In Equation 6, the mycorrhizal biomass accumulates between seasons depending on the success of the association with the crop. Although the crop is harvested/terminated at the end of the season, mycorrhiza have evolved to associate with annual plants. The assumption is thus that mycorrhiza will produce spores for the next season based on the success of association in the current period. The benefits that the mycorrhiza receive from symbiosis in this period are captured by γ_H, and the translation of this into growth in the next period is captured by the parameter r_M. In addition, producers will choose a level of inoculation at the time of planting, captured by I_{t+1}.

$$M_{t+1} = M_t \left(1 + r_M \left(1 - \frac{\gamma_H H_t - M_t}{\gamma_H H_t}\right)\right) + I_{t+1} \tag{6}$$

The Bellman Equation for the deterministic model, then, is given in Equation 7. Due to the recursive nature of the problem, the time subscripts are dropped.

$$V(F, I; M, P) = \max_{F,I} \left[pY - w_F F - w_I I - C(Y) + \beta V(M', P')\right] \tag{7}$$

4 Initial parameter estimates

Several parameters will need to be estimated for this model. Table 1 lists each parameter in the deterministic model, defines each, and proposes a data source or academic paper which contains an estimate for the mean and standard error of the parameter.

Parameter	Description	Data Source/Construction
p	Crop price	USDA ARMS data
w_F	Cost of fertilizer	USDA ARMS data
w_I	Cost of mycorrhizal inoculum	Schroeder-Moreno (2021)
$C(Y_t)$	Cost of production	USDA Commodity Costs and Returns
ρ	Harvest index	Kriedemann et al. (2018)
α	Base yield	USDA NASS
$\delta(t)$	Yield time trend	USDA NASS
q_{HP}	Conversion of P to biomass	Martignoni et al. (2021); Janssen and Guiking (1990)
γ_M	Growth response of plant to mycorrhiza	Chaudhary et al. (2016)
μ_H	Maintenance cost of maintaining its current biomass	Martignoni et al. (2021)
κ	Olsen P, or the portion of P immediately available to the plant	
F_t	P added to soil through fertilization	Beegle and Durst (2017); Sawyer (2000); Ruiz-Diaz et al. (2011)
$f(M_t)$	Additional P made available to plant from mycorrhiza	Martignoni et al. (2021); Li et al. (2006); Cozzolino et al. (2013); Qi et al. (2022); Hoeksema et al. (2010); Grant et al. (2005)
R_t	P removed from soil due to runoff	Ruiz-Diaz et al. (2011); Beegle and Durst (2017); Smith et al. (2016)
r_M	Growth rate of existing mycorrhizal biomass	Martignoni et al. (2021), Toth et al. (1991)
γ_H	Growth effect on mycorrhizal biomass of symbiosis with plant	Toth et al. (1991)
M_t	Mycorrhizal biomass	Toth et al. (1991)

Table 1: Parameters and Sources

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