Irreversibility Investment Decisions With Yield and Price Uncertainty

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Presented at Western Agricultural Economics Association 1997 Annual Meeting
July 13-16, 1997
Reno/Sparks, Nevada

July 1997

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Abstract

Dixit and Pindyck's model of a firm's entry and exit decisions under price uncertainty is expanded by adding output uncertainty. While additional uncertainty usually has the effect of further widening the gap between optimal entry and exit thresholds, the analytical results are inconclusive when uncertainty takes the form of a product of two stochastic variables.

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In 1991 Hertzler published an article distilling the literature on Ito control and its implications for dynamic theory of agricultural decisions under risk. The weak assumptions underlying Ito control, implying little sacrifice in realism for a large gain in analytical power, makes Ito control popular in finance and general economics. As noted by Hertzler, Ito control simplifies the stochastic structure of a model and allows for the derivation of optimality conditions. Since Hertzler's article a number of agricultural economics articles, based on Ito control, have emerged. For example, Fousekis and Shortle employ Ito control when considering the effect on investment demand with stochastic depreciation, and Purvis, et al. apply the Dixit-Pindyck model of investment behavior under irreversibility and uncertainty, which is based on Ito control, to uncertainty about investment cost and environmental compliance (Dixit and Pindyck).

Dixit and Pindyck generally examine entry and exit conditions for a firm considering an investment project when only output price is uncertain and follows a geometric Brownian motion. Output is assumed non-stochastic; however, in many instances static expectations regarding output is not realistic. This is especially true in agriculture, for instance with tree crops, where frequent crop losses due to frost, hail, and other weather conditions contribute to the stochastic nature of yield. A natural extension of Ito control, as suggested by Hertzler, is considering correlated prices and yields, where both price and yield each follow some Brownian motion. Prior to Hertzler's article, Stefanou considered the interaction of two stochastic variables, real wage and stock of technical knowledge when investigating technical change, and Dixit and Pindyck developed the optimality conditions for the product of two stochastic variables following Brownian motion. However, the concept of investment behavior under irreversibility and uncertainty considering both price and yield interactions based on Ito control has not been investigated.

This paper presents a methodology for determining optimal entry and exit thresholds for investment when price and yield follow Brownian motion. As an application, the technology adoption decisions facing Georgia peach growers are investigated. Peach growers must consider the cost of adopting a particular technology, given irreversibility and uncertain returns. Two technologies are considered, conventional portable spot irrigation and full-season irrigation.

Model

Uncertainty in the value of a project arises from fluctuations in market price over time, p, and yield, q. Assume both p and q follow a geometric Brownian motion process

(1) $dp = \alpha_p p dt + \sigma_p p dz_p,$

(2) $dq = \alpha_q q dt + \sigma_q q dz_q,$

where dp and dq represent the change in price and output respectively, α is the rate of change or drift rate, σ is the standard deviation, and the subscripts p and q denoted parameters associated with price and quantity, respectively. The increment of a Wiener process is dz, with $E(dz_p^2) = E(dz_q^2) = dt$ and $E(dz_p,dz_q]= \rho dt$, where ρ denotes the correlation coefficient between p and q.

Following Dixit and Pindyck, it is assumed growers are risk neutral and maximize their expected net present value of investment. A further assumption is that revenue per acre, R, the product of price and yield, R = pq, is lognormally distributed, given growers will shut down if revenue is negative. The stochastic process of revenue, R, is determined by the differential of the change in logarithm of R, dr = d*ln*R, following Ito's Lemma

$$(3) \qquad dr = \frac{\partial r}{\partial p} dp + \frac{\partial r}{\partial q} dq + \frac{\partial^2 r}{\partial p \partial q} dpdq + \frac{1}{2}\frac{\partial^2 r}{\partial p^2} dp^2 + \frac{1}{2}\frac{\partial^2 r}{\partial q^2} dq^2.$$

Noting $\partial r/\partial p = 1/p$, $\partial r/\partial q = 1/q$, $\partial^2 r/\partial p^2 = - 1/p^2$, $\partial^2 r/\partial q^2 = - 1/q^2$, and $\partial^2 r/\partial p\partial q = 0$, equation (3) reduces to

$$(4) \qquad dr = \frac{1}{p} dp + \frac{1}{q} dq + \frac{1}{2p^2} dp^2 + \frac{1}{2q^2} dq^2.$$

Substituting (1) and (2) for dp and dq, respectively and noting (dt)(dz) is of order $(dt)^{3/2}$ and in the limit every term with dt raised to a power greater than one will go to zero faster than dt, yields

$$(5) \qquad dr = (\alpha p + \alpha q - ½\sigma_p^2 - ½\sigma_q^2)dt + \sigma_p dz_p + \sigma_q dz_q.$$

Thus, r = *ln*R follows a simple Brownian motion

$$(6) \qquad dr = \alpha_r dt + \sigma_r dz_r,$$

implying dr over an interval T is normally distributed with mean

$$(7) \qquad (\alpha_p + \alpha_q - ½\sigma_p^2 - ½\sigma_q^2)T,$$

and variance

$$(8) \qquad (\sigma_p^2 + \sigma_q^2 + 2\rho\, \sigma_p\sigma_q)T.$$

An increase in the negative correlation between price and output does not influence the mean as indicated in (7), but from (8) reduces the variation in returns, σ^2, by $2\sigma_p\sigma_q$ per unit.

Let $V_0(R)$ denote the expected present value of starting in an idle state with revenue R based on the stochastic process (6), and let $V_1(R)$ denote the expected present value of abandoning for an active firm. A firm switches optimally between idle and active states constrained by these two functions. For an idle firm the payoff from investing is the value of the project minus the cost of the investment. The investment opportunity yields no cash flows until the option is exercised. Therefore, the only return from holding the option is its expected rate of capital appreciation which by the Bellman equation equals the normal return on the investment

(9) $E[dV_0(R)] = \gamma V_0(R)dt,$

where γ denotes the discount rate (Dixit and Pindyck). Note that from Jenson's inequality the expectation of the change in the value of the project, E(dV), will be positive if V is a convex function of R, $\partial^2V/\partial R^2 > 0$. Expanding dV_0 using Ito's Lemma

$$dV_0 = V'(R)dR + \tfrac{1}{2}V''(R)(dR)^2.$$

Incorporating (6), taking the expectation, and noting $E(dz_R) = 0$ yields

$$E(dV_0) = [\alpha_r RV'(R) + \tfrac{1}{2}\sigma_r R^2 V''(R)]dt.$$

Substituting into (9) and dividing by dt

(10) $\tfrac{1}{2}\sigma_r R^2 V''(R) + (\gamma - \delta)RV'(R) - \gamma V_0(R) = 0,$

where $\gamma - \alpha_r = \delta$ and $\delta > 0$ is required for a finite solution. If $\gamma < \alpha_r$ a grower would never adopt because the gain from holding the option would always be greater than the present value of returns from adoption. For an active firm the flow of profit from investment, (R - C)dt, should also be considered, where C represents the investment cost.. Therefore, for an active firm the Bellman equation is

(11) $\frac{1}{2}\sigma_r R^2 V_1''(R) + (\gamma - \delta)RV_1'(R) - \gamma V_1(R) + R - C = 0.$

The solution for (10) and (11) takes the form

(12) $V_0(R) = A_0 R^{\beta_{01}} + B_0 R^{\beta_{02}},$

(13) $V_1(R) = A_1 R^{\beta_{11}} + B_1 R^{\beta_{12}} + \frac{R}{\delta} - \frac{C}{\gamma},$

where A_i and B_i are constants to be determined and β_{i1} and β_{i2} are the positive and negative roots of the fundamental quadratic equation, i = 0, 1 (Dixit and Pindyck).

Equations (12) and (13) can be simplified further by noting when R is high the option to abandon is zero, thus, the coefficient B_0 associated with the negative root in (12) is zero. Similarly, when R is low the option to become active is zero, and thus, A_1 associated with the positive root in (13) is zero.

Equations (12) and (13) must satisfy the following value-matching

(14) $V_0(R_H) = V_1(R_H) - I$ and $V_1(R_L) = V_0(R_L),$

and smooth-pasting conditions

(15) $V_0'(R_H) = V_1'(R_H)$ and $V_1'(R_L) = V_0'(R_L),$

where the optimal strategy for adoption and abandonment will take the form of two per acre revenue thresholds R_L and R_H with $R_L < R_H$. An idle firm will remain idle as long as revenue remains below R_H and will become active only if R reaches R_H. An active firm will remain active until revenue falls to R_L. The optimal strategy for the range between R_L

and R_H is to continue the status quo, whether it be actively operating or waiting.

Conditions (14) require that at the entry (exit) threshold a grower is indifferent between adopting (abandoning) and remaining idle (active). The value of the option to adopt (abandon) is equal to the returns from adoption (abandonment) minus the cost of adoption (abandonment). Equations (15) are the smooth pasting conditions which require that the transition between the functions representing the value of the idle and active firm be smooth. This implies the slope of the functions representing the optimal decision rule meet evenly. If V(R) were not continuous and smooth at the critical point, an improvement in returns could be exercised by switching to an alternative point for adoption or abandonment.

Expanding the equations (12) and (13) using (14) and (15), yields

$$(16)\qquad -A_0 R\underline{H}^{01} + B_1 R\underline{H}^{12} + \frac{R_H}{\delta} - \frac{C}{\gamma} = I,$$

$$(17)\qquad -A_0 R\underline{L}^{01} + B_1 R\underline{L}^{12} + \frac{R_L}{\delta} - \frac{C}{\gamma} = 0,$$

$$(18)\qquad -\beta_{01} A_0 R\underline{H}^{01-1} + \beta_{12} B_1 R\underline{H}^{12-1} + \frac{1}{\delta} = 0,$$

$$(19)\qquad -\beta_{01} A_0 R\underline{L}^{01-1} + \beta_{12} B_1 R\underline{L}^{12-1} + \frac{1}{\delta} = 0,$$

These four equations are highly nonlinear in R_L and R_H and must be solved simultaneously by numerical solution.

Analytical Results

An interesting analytical result can be derived by considering only the adoption decision of an idle firm. In this case the value-matching and smooth-pasting conditions (14) and (15) are

$$V_0(R_H) = R_H - I,$$

and

$$V_0'(R_H) = 1.$$

Given (12) and noting the coefficient B_0 associated with its negative root is zero, the two unknowns, constant A_0 and optimal investment level R^*, maybe derived. The solution for R^* is

$$R^* = \frac{\beta_{01}}{\beta_{01} - 1} I. \tag{20}$$

As demonstrated by Dixit and Pindyck, if the opportunity cost of capital is greater than the drift parameter then $\beta_{01} > 1$, $\beta_{01} /(\beta_{01} - 1) > 1$, and $R^* > I$. This implies the standard net present value rule of investment, suggesting adoption if $R^* \geq I$, is incorrect. Uncertainty and irreversibility require that R^* be greater than I by a factor of $\beta_{01} /(\beta_{01} - 1)$. This factor is called the hurdle rate which is a function of the drift, variance, and discount rate. It represents the level of caution that should be applied to the adoption decision due to the level of investment uncertainty. Similarly, the lower (exit) threshold, R_L, represents the patience that should be applied prior to exercising the abandonment option, due to the possibility that returns will again turn upward.

Dixit and Pindyck further demonstrate an increase in σ_R acts to widen this wedge between entry and exit thresholds. The zone of inaction or hysteresis widens from this increase in uncertainty. As indicated in (8) output uncertainty increases σ_R which will then widen the wedge between R^* and I in (20). Whereas, an increase in the negative correlation between price and output reduces σ_R yielding a reduction in this wedge.

Application

As an application, table 1 lists parameter values employed for considering the investment and technology adoption decisions facing Georgia peach growers. Drift and variance of price along with the correlation coefficient where computed from annual price data obtained from the Georgia Peach Marketing Report over the period from 1978 to 1992.

Davis' estimate of yield variation is employed as a measure of σ_q^2. This estimate is based on a stochastic production function for spot-irrigated Georgia-peach production which distinguishes variability in production due to random external factors, such as weather or pests, from production variation in growers' different input choices. The drift rate α_Q was not calculated by Davis and is assumed to be zero, given little if any average per acre yield increases over the five years of observations. The mean yield for all orchards over all five years is 213 bushels per acre. Based on these drift, variance, and the correlation measurement, the drift and variance for revenue were calculated according to (7) and (8).

A literature review on the effects of irrigation on yield and yield variation, by Davis, indicates on average, across the various studies, yield is increased by 24% and yield variation declines by 30% with the adoption of full-season irrigation (Davis). This implies mean yield increasing from 213 to 264 bushels per acre and a yield variance declines from

0.344 to 0.240.

For peach production the sunk cost of investment includes land, trees, chemicals, machinery, fuel, repair and maintenance, management, labor, overhead, and interest. Three years of operating costs are also incurred before any marketable yield is produced. The present value of investment discounted at 6% to the third year is $3,007.00 for spot-irrigated production and $4,390 for full-season irrigated production (Davis; Harrison et al.). This investment cost includes non-discounted $655.00 for the fixed cost of spot irrigation. The conventional practice in Georgia is to use a portable irrigation system two to three weeks just prior to harvest for enlarging fruit size. The annual variable operating costs after the third year are $899.11 per acre for spot-irrigated peach production and $931.97 for full-season irrigated production (Davis; Harrison et al.).

Results

Table 2 presents the adoption and abandonment thresholds of revenue and yield for spot-irrigated and full-season irrigated production scenarios. As a basis for comparison, consider first the conventional revenue thresholds R_L and R_H for spot-irrigated peach production. These thresholds represent the criteria for adoption and abandonment decisions under the static or myopic approach. If the present value of returns is greater than the costs, R_H, then adopt the technology; if the returns fall below variable costs, R_L, then abandon the technology. With per-bushel peach prices of $11.00 and $15.00 and considering a $4.50 per bushel harvesting and marketing cost, the yield thresholds, Y_L and Y_H, for net prices of $6.50 and $10.50 are also listed in table 2. Note the mean per acre yield of 213 bushels from the survey data is above the Y_H threshold of 166.08 bushels associated with a net price of $6.50, the conventional criteria suggests investment in spot-irrigated peach production is feasible. However, this does not consider the stochastic nature of price and yield, the irreversibility of the investment decision, or the ability to delay the decision. Incorporating this information into the decision for optimal investment in spot-irrigated production results in a large hurdle rate, 274%, that must be breached for investment to be feasible (table 2). The Y_H thresholds for both prices are well above the mean yield per acre, 213 bushels, indicating that any new investment in peaches is infeasible. This implies for the given expectation in yield at either market price, generating the necessary revenue is unlikely. However, the Y_L threshold is well below the mean yield, suggesting growers currently producing peaches with spot irrigation should continue production.

Considering full-season irrigation the yield thresholds drop considerably. At a net price of $10.50, the Y_H threshold is 212.58 bushels per acre. This threshold is now within the feasible range of possible adoption, given a 24% increase in mean production yields 265 bushels. However, at the lower price of $6.50, the required yield of 343.40 bushels per acre is still above the expected yield. The low values for Y_L indicate that a grower already using full-season irrigation would not abandon the investment less yields dropped considerably.

References Cited

Davis, S.B. *Risk, Scattering, and Efficiency in Orchard Farm Production.* Ph.D. Dissertation, Department of Agricultural and Applied Economics, University of Georgia, Athens, 1996.

Dixit, A.K. and R.S. Pindyck. *Investment Under Uncertainty*. Princeton University Press, Princeton, New Jersey, 1994.

Fousekis, P. and J.S. Shortle. "Investment Demand When Economic Depreciation is Stochastic." *American Journal of Agricultural Economics*. 77(1995):990-1000.

Harrison, D.S., A.G. Smajstrla, and F.S. Zazueta. "An Economic Analysis of Irrigation Systems for Production of Citrus in Florida." Proceedings of the Florida State Horticultural Society, 96 (1983):7-11.

Hertzler, G. "Dynamic Decisions Under Risk: Application of Ito Stochastic Control in Agricultural." *American Journal of Agricultural Economics*. 73(1991):1126-1137.

National Agricultural Statistics Service, Crops Branch, *Statistics of Fruits, Tree Nuts, and Horticultural Specialties.* In Agricultural Statistics (1993):168-215.

Purvis, A., W.G. Boggess, C.B. Moss, and J. Holt. "Technology Adoption Decisions Under Irreversibility and Uncertainty: An *Ex Ante* Approach." *American Journal of Agricultural Economics*. 77(1995):531-551.

Stefanou, S. "Technical Change, Uncertainty, and Investment." *American Journal of Agricultural Economics*. 69(1987):158-65.

Table 1. Parameter Values Employed for Georgia-Peach Growers' Investment Decisions

Parameter	Description	Parameter Value	
		Spot-Irrigation[a]	Full-Season Irrigation[b]
α_p	Price Drift Rate	0.011	0.011
α_q	Yield Drift Rate	0	0
σ_p^2	Price Variance	0.012	0.012
σ_q^2	Yield Variance	0.344	0.240
ρ_{pq}	Price and Yield Correlation	-0.124	-0.124
α_r	Revenue Drift Rate	-0.166	-0.115
σ_r^2	Revenue Variance	0.340	0.241
γ	Discount Rate	0.06	0.06
C	Variable Operating Cost	$899.11	$931.97
I	Investment Cost	$3,007.00	$4,390.00

[a] Only employ portable spot irrigation just prior to harvest.

[b] A fixed field irrigation system for season long applications.

Table 2. Adoption and Abandonment Thresholds for Revenue and Yield With Hurdle Rates for Conventional Spot Irrigation and Full-Season Irrigation Production Scenarios

Production Scenario	Revenue Threshold[a]		Price Per Bushel	Yield Threshold[b]	
	R_L	R_H		Y_L	Y_H
Conventional					
Spot Irrigation[c] (213 Bushels)	899.11	1,079.52	6.50	138.33	166.08
			10.50	85.63	102.81
Optimal					
Spot Irrigation[c] (213 Bushels)	543.00	2,957.00	6.50	83.54 (0.60)	454.92 (2.74)
			10.50	51.71 (0.60)	281.62 (2.74)
Full-Season Irrigation[e] (264 Bushels)	344.81	2,232.12	6.50	53.05	343.40
			10.50	32.84	212.58

[a] R_H is the revenue per acre which triggers adoption, R_L is the revenue per acre which triggers abandonment.

[b] Y_L and Y_H are the yield in bushels per acre required to reach R_L and R_H , respectively, for the given price.

[c] Only employ portable spot irrigation just prior to harvest. Expected bushels per acre is in the parentheses.

[d] Hurdle rate in parentheses, where the hurdle rate is the percentage the optimal Y_H is above or Y_L is below the conventional threshold.

[e] A fixed field irrigation system for season long applications. Expected bushels per acre is in the parentheses.